IMPLEMENTING A FEEDBACK HIERARCHY TO SUPPORT MATHEMATICAL PERSEVERANCE IN A DIGITAL SKETCHING APPLICATION

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Abstract

Personalized feedback is integral for effective mathematics learning. An expert teacher looking over the shoulder of a student can provide a personalized hint to a student making a mistake. If the student makes the same mistake again, the teacher may provide the student a different hint based upon their teaching experience. If the hint is too large, the expert teacher will be reducing the cognitive demand of the task and reducing any opportunities for productive struggle, a key ingredient for learning. Conversely if the hint is too small, the student may remain stuck. An automated system to provide personalized feedback within a software application, like an expert teacher, could allow each student to advance at their own pace. To enable this, a hierarchy of feedback that can support all students at critical moments as they make progress with challenging tasks is needed.

This paper presents the design of a feedback hierarchy and the subsequent effects on engagement for students working with a new application called Drawn2Math (D2M). This application was embedded with a feedback hierarchy designed to keep K-8 students engaged with challenging tasks and support their perseverance to learn mathematics conceptually. A key feature of the app is that students draw solutions to mathematics problems, such as visual representations of area models used in fractions tasks. The app can identify if the student's sketch is correct, and whether a mistake they make matches a predefined common misconception. Authors of the assignment define a set of common misconceptions with appropriate feedback. Feedback spans different levels of scaffolding, from low-level conceptual reminders, to medium-level restructuring, to high-level environmental provisioning. The approach utilizes the rich amount of information in student sketches, which illuminates their misconceptions, as well as teacher expertise in authoring the assignments, which captures their classroom experience regarding common mistakes. Each feedback message is correlated to a specific sketch that a student draws, and is designated a hierarchy level (e.g, first hint for a given mistake), the number of times the hint can be triggered, and follow-on hints for the same mistake. A feedback hierarchy table was developed to allow the assignment author to designate their feedback approach.

An exploratory study implemented the feedback hierarchy with 10 fourth-grade students who were learning fractions for the first time. To test the validity of this system, a Wizard of Oz usability experiment was conducted in which students sketched on an iPad and received triggered support from the predefined feedback hierarchy on their screen, but the feedback was administered by a remotely observing researcher acting as the grading algorithm. Our findings showed that participants working on tasks within D2M were able to leverage the personalized feedback to persevere with the task despite working on them without a teacher present, even at moments when they were most challenged and frustrated. All 10 participants showed evidence of this kind of success. All participants made a sketching mistake during their initial attempt at solving, but the feedback helped them stay engaged and continue to make progress. These findings suggest that feedback hierarchies embedded within this education technology can be carefully designed to help students stay engaged with challenging mathematics and learn conceptually, even when working independently or even remotely.

Keywords: mathematics education, application, feedback hierarchy, perseverance, sketching.

1 INTRODUCTION

One of the potential advantages of educational software is that it ideally will allow each student to progress at their own pace and receive personalized feedback based upon their individual needs. However, for a software package to have the ability to provide personalized feedback it needs to be able to identify what type of mistakes students are making and infer what misconceptions are tied to these mistakes. It is through this kind of personalized feedback that a software package could keep students engaged with challenging mathematics tasks and help them persevere in their problem-solving. Such perseverance is imperative in mathematics education and implies an encounter with a setback that spawned productive new efforts that pay dividends toward learning [1]. Consequently, recent mathematics education research ([1], [2], [3]) have explicitly attended to nurturing perseverance in the classroom, namely by providing students consistent "opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" [4]. It is a key goal of this paper to describe how a mathematics software package could support such student perseverance in a mathematics context.

Many common mathematics education software packages are limited in their ability to infer why students are making various mistakes, and thus ultimately their potential in providing personalized feedback is limited. Most mathematics education software packages are based upon multiple choice or numerical input answers (e.g., Aleks, Teachley, Amplify Fractions). However, these types of inputs provide limited information regarding the reasons behind student mistakes. A multiple choice response may be due to a guess or to elimination of incorrect options rather than true ability to solve a problem. Accordingly, when a student makes a mistake on a multiple choice question, monitoring software would have a limited ability to infer the misconception behind that mistake. For problems with numerical input, a software package may note if the answer is too high or too low, and may even compare a mistake to known numerical mistakes. But ultimately a single numerical input by a student contains limited information about the thought process of the students or why they made a certain mistake. For these reasons, personalized feedback is limited in current mathematics educational software.

This paper presents the implementation of a new application called Drawn2Math (D2M). This application was designed for K-8 students, and its initial modules teach fractions suitable for 4th and 5th graders. A key feature of the app is that students draw solutions to mathematics problems, such as drawing rectangles as visual representations of area models used in fractions tasks. When students draw models of fraction concepts and operations, instead of simply choosing a multiple choice selection or making a numerical entry, they have the opportunity to grapple with various ways of interpreting fractions [5]. This includes understanding fractions as the result of dividing two numbers, as the ratio of two quantities, as operators, as measures, and as parts of wholes or parts of sets. The act of freehand sketching visual representations has been shown to help students develop conceptual understandings, problem-solving, logical reasoning, and mathematical discovery [6]. Moreover, an incorrectly drawn sketch can give insight into the thought process behind the student's mistake which could be used by a teacher to support productive struggle. Several cognitive and neurological studies of mathematical thinking have shown the importance of engaging with and creating visual representations for mathematics learning ([7], [8]) This combination of learning through engagement with visual representations and freehand sketching underpins an effective practice for mathematics learning, especially in the context of fractions. There are an infinite number of sketches that a student can draw in response to a question, and an expert teacher that sees a student's sketch may often be able to infer the conceptual errors the student is making. Our goal is to embed the D2M app with the ability of an expert teacher to provide personalized feedback based upon student sketches.

Personalized feedback can be framed as levels of scaffolding, ranging from developing low-level conceptual thinking scaffolds to medium-level explaining, reviewing, and restructuring scaffolds, to high-level environmental provisioning [9]. Such scaffolding can also be gentle reminders for students to attend to the connections between the big ideas at play in a mathematical situation. Scaffolding is often most effective when it is personal to the actual engagement of the student. Personalized feedback has been shown to support mathematics achievement and help students rely less on memorizing procedures and more on their own thinking to persevere with a challenging mathematics task ([10], [11]). However, consistently providing attentive support for all students at specific moments of struggle is logistically difficult for teachers in face-to-face educational settings. Thus, our goal is to equip the D2M app with such feedback to be administered automatically via certain cues or triggers.

Additionally, this paper presents a method whereby an expert teacher authors assignments and incorporates a hierarchy of feedback into the D2M grading algorithm. The hierarchy specifies which feedback message or image to provide the student based upon the history of sketches the student submits. A trial with 10 students in 4th grade working independently on a test version of the D2M app is described. The test version of the app used a person to simulate the computer algorithm, but still was able to demonstrate student engagement with the assignment and perseverance in overcoming challenges.

2 METHODOLOGY

2.1 Authoring Assignments with Feedback Hierarchy

The author of the D2M assignments defines the possible feedback messages and their triggers at the time of authoring the assignment. Some of the hints are triggered based upon relatively general student sketches, and some are triggered based upon anticipated common student mistakes. Fig. 1 shows an example assignment.

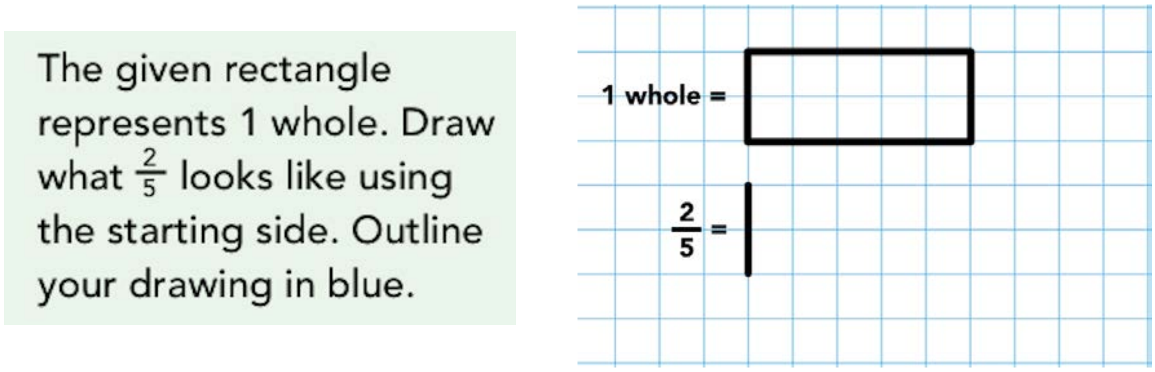
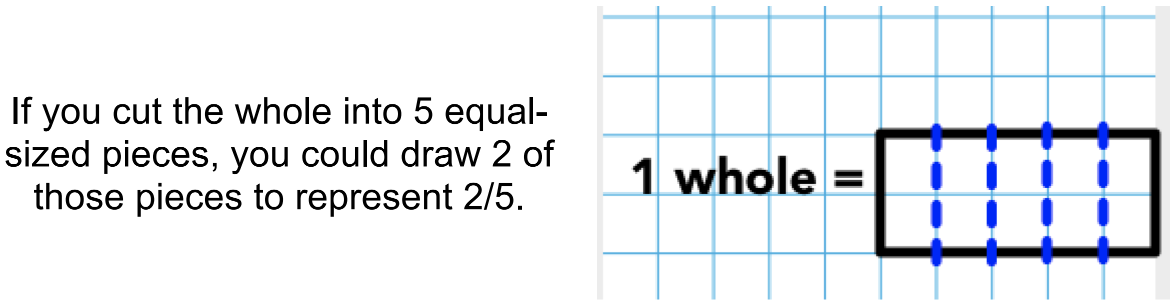


Figure 1. Example Sketching Assignment

For this assignment, the author defines feedback options in a variety of categories. First, the *hierarchy* defines which feedback trigger is evaluated first. Then, the *number of times a hint can be repeated* is established; this can vary from 1 to infinity. Next, the *hint trigger* is specified according to the details of the specific task. There are a range of options by which to specify a hint trigger such as if very little sketching was done, the solution is close to correct, a sketch is missing a pre-drawn component, a scribble was detected, the sketch matches an anticipated common error, a region of the sketch is correct indicating a part of the sketch is graded as correct but not the complete sketch, and a region of the sketch is incorrect indicating that a specific part of the sketch is incorrect. Then, the *feedback text* is established which is provided as the feedback message to the student after submitting a solution. Next, a *feedback image* is specified; this can occur for specific kinds of feedback, such as a tutorial image based on the hint trigger. Last, the *action options* are defined, which can include encouragement for the student to retry the assignment, advice to move on to an easier assignment, or an offer to peek at the correct solution.

An example feedback for the assignment shown in Fig. 1 is shown in Fig. 2. This feedback is provided after a set number of failed attempts and is considered a high-level environmental provision [10].

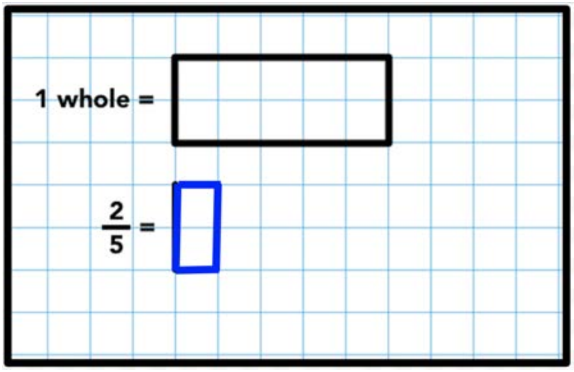


Feedback Message *Feedback Image*

Figure 2. Example Feedback Message and Image Shown After a Number of Failed Attempts

The sketch grading algorithm compares each student's sketch to the solution or solutions if multiple valid solutions are possible. The student’s sketch is also compared to expected common errors specified by the assignment author. The grading algorithm allows for some variation in student drawings and does not require precise lines. Fig. 3 shows an example of an anticipated common error for the assignment that was provided in Fig. 1. This feedback is considered a low-level conceptual thinking scaffold [9].

It looks like you drew ⅕ instead of ⅖. Can you try again?



Feedback Message *Anticipated Common Error*

Figure 3. Feedback Message for the Case of an Anticipated Student Error

The author interface for creating the feedback hierarchy is shown in Fig. 4. The hierarchy is based upon row number. Accordingly, the feedback algorithm evaluates the hint trigger row by row. This first row for which the trigger is valid is used to create the feedback message. Once a feedback hint is provided to the student, the counter for that hint is decremented. If a hint counter reaches zero, it is no longer used, unless all of the hints are reset. In this fashion, a wide range of hint scenarios can be triggered. For example, a certain hint can be set at a high level in the hierarchy for a specific number of times. Once these hints are used, the next hints will move on to lower-level hierarchies (i.e., higher scaffolding). The original hint could be repeated at an even lower-level hierarchy if the author wants that hint condition to be triggered again. This interface was designed to be easy to use for the educator, while also allowing for a wide range of flexibility. Pull-down menus are used to designate author choices wherever possible to simplify assignment creation and avoid typing errors. Once assignment authoring is completed, automated checking is performed to ensure all components are present and consistent in the hierarchy table.

Non-Unit Assignment 1 Hint Hierarchy					
# of Times Repeated	**Hint Trigger**	**Error Image**	**Feedback Text**	**Feedback Image**	**Action Options**
Infinite	Sketch is Not Outlined in Color		Outline Solution with final		Retry
Infinite	Very little sketching done		Try drawing more of		Retry
Infinite	Scribble Detected		Try drawing more carefully		Retry
Infinite	Close to Correct Solution		Close! Make sure you draw		Retry
Infinite	Sketch Matches Common Mistal	*mistake file*	It looks like you drew 1/5		Retry
1	Any Incorrect Sketch		The size of your 2/5 incorre		Retry
1	Any Incorrect Sketch		Remember, drawing 2/5		Retry
2	Any Incorrect Sketch		If you cut the whole into 5	*hint image file*	Retry

Figure 4. Author Interface for Specifying the Hint Hierarchy

2.2 D2M Student Interface

The student sees the interface shown in Fig. 5. The problem description is shown on the left and the drawing area encompasses the grid area. In many assignments, as is the case in Fig. 5, the starting line of the student's solution is specified. Figure 5 also shows a button for both a Draft Pen and a Final Pen. The student is instructed to start drawing using their Draft Pen and can use the Draft Pen for any marks they want. The Draft Pen is thin and gray. Once a student is comfortable with their solution, they create their solution with the Final Pen, often tracing over some of their draft lines. The Final Pen is thick and blue. The use of the Draft Pen allows for greater insight into the student thought process. The assignment author can specify any hint trigger to be based off of the Draft Pen or Final Pen.

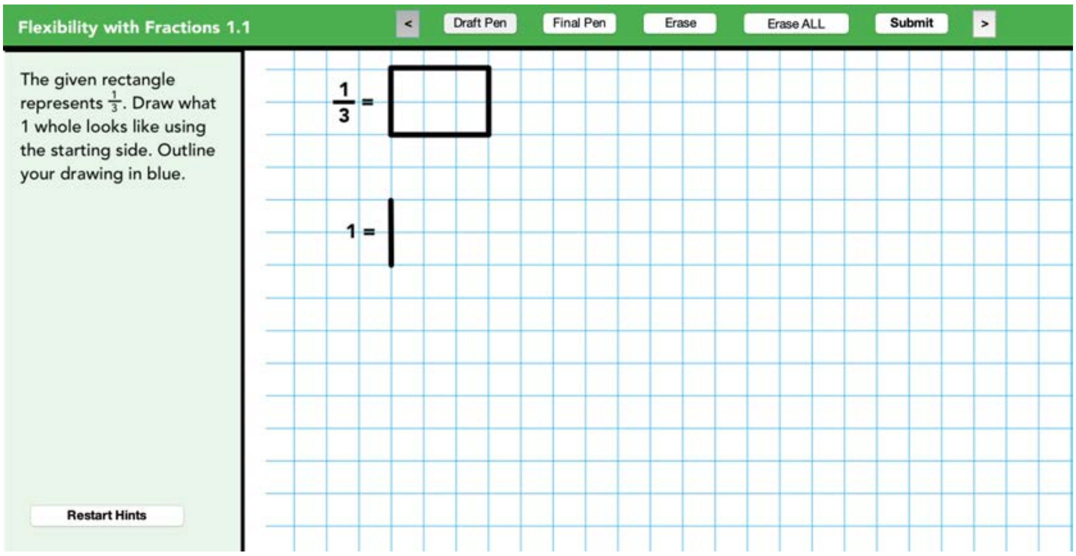


Figure 5. Student Interface for D2M

2.3 Implementing Wizard-of-Oz Trial

At the time of study, the D2M software was still in the development stage, therefore we engaged students in a simulation of the application to study the effectiveness of the overall approach. We used a Wizard of Oz approach (see the 2.3.2 Data Collection section below) to simulate many salient features of the app. These features undergird the development philosophy of D2M as an application that could perform like an expert teacher looking over the shoulder of each student that will provide just the right amount of guidance to support the conceptual learning of mathematics.

2.3.1 Participants

The participants for this study were 10 fourth-grade students from the Mid-Atlantic region of the United States. Each participant was from the same fourth grade class with the same teacher and was learning about fractions for the first time. All participants were purposely chosen to have demonstrated, via pretest, that they had not yet learned or mastered certain fraction concepts. In this study, approximately 6 hours of student work were evaluated.

2.3.2 Data Collection

To collect data, each participant engaged with a simulation of the D2M application. During the simulation, participants thought aloud as they problem-solved with up to 18 sketching-based fraction tasks on a touchscreen device and received personalized mathematical feedback when necessary. This feedback varied and was administered strictly based on the feedback hierarchy. Participants' activity on the touchscreen device was video recorded, the student's voice was audio recorded, and field notes were taken. To simulate the D2M application, we used a Wizard of Oz approach ([12]). The Wizard of Oz approach is a usability trial in which a participant interacts with an unfinished system or product while a researcher ("the Wizard") simulates the behavior of the system. In our case, participants used an iPad sketching program, Pixelboard, through which a remote researcher (the first author) could see in real-time what they are drawing. Thus, each participant worked on their sketching task individually and were monitored by the researcher remotely. The researcher's job was to act as the D2M grading algorithm – to administer predefined and personalized mathematical feedback when participants submitted a certain incorrect sketch, or to mark a submission correct and advance the participant to a new task. The Wizard's feedback was only via windows of text and/or images that appeared on the participant's iPad. The Wizard did have a line of sight to the participant so they could observe body language. Participants were informed that they were allowed to stop working at any time.

2.3.3 Data Analysis

We analyzed the data to discern the frequency and rate at which participants were persevering as they engaged with the simulated D2M application. We used the Three-Phase Perseverance Framework (3PP) [1] as a qualitative analytical tool. The 3PP has been used in research to operationalize perseverance in problem-solving ([1], [2], [3]). It was designed to reflect perspectives of concept [13], problem-solving actions [14], self-regulation [15], and making and recognizing mathematical progress [16]. The three-phases of the 3PP are (1) the Entrance Phase, which considers if the task at hand necessitates perseverance, (2) the Initial Attempt Phase, which considers if a participant decided to initiate and sustain their effort toward a task solution and if those efforts were mathematically productive, and (3) the Additional Attempt Phase(s), which considers if a participant decided to re-initiate and re-sustain their new effort toward a task solution after a substantial setback(s) and if those new efforts were mathematically productive. A participant could theoretically experience multiple Additional Attempt Phases if they encountered multiple substantial setbacks while working.

In this study, we were interested in participants' perseverance after they submitted an incorrect sketch and subsequently received personalized mathematical feedback. Therefore, our analysis focused on participants' Additional Attempt Phases. This implies that participant data for tasks solved during an initial attempt, and thus participants never submitting an incorrect sketch, were not considered in this paper. We defined a substantial setback as moments when a participant submitted a mathematically incorrect answer while working on a D2M sketching-based task and subsequently received personalized feedback. We defined mathematical productivity as submitting a correct answer or as demonstrating progress toward better understanding a mathematical idea compared to previous evidence of understanding. Thus, our analyses considered the frequencies of participants' substantial setbacks and coinciding receival of personalized mathematical feedback (denoted as *A*), the frequencies of participants' re-initiated and re-sustained new efforts after substantial setbacks (denoted as *B*), and the frequencies of participants' mathematical productivity as a result of re-initiated and re-sustained new efforts (denoted as *C*).

We report about two constructs in our results: the Reengagement Rate (*RR*) and the Perseverance Success Rate (*PSR*). Both of these constructs stemmed from the foci of our analyses. The Reengagement Rate captured participants' willingness to reengage with the task and not give up after submitting a mathematically incorrect answer for a D2M sketching-based task. This describes the resilience of a participant after a substantial setback and receival of subsequent personalized mathematical feedback. Thus, the Reengagement Rate was calculated as the quotient of the frequency of participants' re-initiated and re-sustained new efforts after substantial setbacks (B) and the frequency of participants' substantial setbacks (A), or $RR = B/A$. The Perseverance Success Rate captured participants' mathematical productivity as a result of their willingness to reengage with the task and not give up after submitting a mathematically incorrect answer for a D2M sketching-based task. This describes participants' productive struggle after a substantial setback and receival of subsequent personalized mathematical feedback, that is, that participants' resiliency paid dividends of mathematics learning. Thus, the Perseverance Success Rate was calculated as the quotient of frequency of participants' mathematical productivity as a result of re-initiated and re-sustained new efforts (C) and the frequency of participants' substantial setbacks (A), or $PSR = C/A$. In sum, the Reengagement Rate and the Perseverance Success Rate both capture evidence of resiliency in participants' engagement, but the Perseverance Success Rate alone captures the mathematical productivity of such resilience.

3 RESULTS

In this section, we first describe participants' perseverance working on D2M sketching-based tasks, after they submitted an incorrect sketch and subsequently received personalized mathematical feedback. Recall, we only considered participants' experiences with tasks on which they made mistakes – participants' experiences with tasks on which they made no incorrect submissions were not considered in our perseverance analysis. Additionally, recall that participants were working on these tasks alone during the D2M simulation, without a teacher present. We report participants' collective Reengagement Rate and Perseverance Success Rate as findings and include a descriptive example.

3.1 Participants' Perseverance on Sketching-based Tasks

In general, our findings showed that participants working on challenging D2M sketching-based tasks were often able to leverage the personalized mathematical feedback to reengage and persevere with the task, even after making substantial mistakes (see Table 1 for more details of these findings). While working on up to 18 sketching-based tasks, all 10 participants encountered substantial setbacks by submitting at least one incorrect answer. The frequencies of incorrect submissions varied by participant, with the minimum number of incorrect submissions being two and the maximum number of incorrect submissions being 19. Across all participants, the total number of incorrect submissions was 65. Recall that personalized mathematical feedback accompanied each incorrect submission. Thus, participants received personalized mathematical feedback 65 times after each incorrect submission. This feedback varied, depending on the mathematical mistake, and was administered in strict accordance to the feedback hierarchy.

Table 1. Perseverance on sketching-based tasks for which the initial submission was incorrect

	Incorrect Submissions + Personalized Feedback (A)	New Efforts (B)	Productive New Efforts (C)	Reengagement Rate (RR)	Perseverance Success Rate (PSR)
Participant 1	19	17	14	89%	74%
Participant 2	2	2	2	100%	100%
Participant 3	2	2	2	100%	100%
Participant 4	11	11	9	100%	82%
Participant 5	7	7	7	100%	100%
Participant 6	7	5	5	71%	71%
Participant 7	9	9	8	100%	89%
Participant 8	2	2	2	100%	100%
Participant 9	3	3	3	100%	100%
Participant 10	3	3	3	100%	100%
Totals	**65**	**61**	**55**	**94%**	**85%**

Legend: Reengagement Rate (RR) = B/A; Perseverance Success Rate (PSR) = C/A

After receiving personalized mathematical feedback, participants made a new effort by re-initiating and re-sustaining their effort toward a task solution 94% of the time. This is noted by the Reengagement Rate in Table 1. This suggests that participants were motivated by the personalized mathematical feedback to stay engaged with the mathematics, despite a substantial setback.

Related, after receival of personalized mathematical feedback, participants were mathematically productive in their new effort toward a task solution 85% of the time, as noted by the Perseverance Success Rate in Table 1. This suggests that not only did the personalized mathematical feedback help participants to stay engaged with the task despite a substantial setback, but such feedback supported them to productively struggle to make continued mathematical progress.

For instance, consider Participant 4’s (P4) experience with a task prompting them to draw an area model of 1/6 given a rectangular area model representing 1/3. The given model of 1/3 had an area of 6 square units (2 units by 3 units). Reading the instructions silently, P4 initiated and sustained their effort toward a task solution by quickly sketching a rectangle with an area of 2 square units (2 units by 1 unit) as their answer. It seemed they were sketching one-third of the given rectangle. They submitted this answer, and it was graded as incorrect, marking the first substantial setback (A = 1). P4 reacted silently with their body language expressing disappointment. This initial incorrect answer again initiated a piece of lower-level scaffolded feedback designed to gently remind them to refocus their attention on the given information. The feedback said, “The size of your 1/6 is incorrect” (see Fig. 6, left). P4’s mistake may have stemmed from understanding that 1/6 is a smaller quantity than 1/3, but not completely understanding the exact relationship between the size of sixths compared to thirds. P4 responded resiliently to this setback and re-engaged to make a new effort toward a task solution (B = 1). They re-initiated a new effort by deciding to explore some new ideas using the Draft Pen. They re-sustained a new effort by sketching some circular area models depicting 2/6 and 1/3 (see Fig. 6, center). They said, “I know these are the same,” referring to the equivalence of 2/6 and 1/3. After studying their scratch work, P4 drew a rectangle with an area of 2 square units (2 units by 1 unit) as their answer, which was the same answer as their previous incorrect submission. Despite P4’s new efforts to explore these ideas using a different kind of visual representation, this new effort was determined to be not mathematically productive (C = 0). They submitted this answer and it was again graded as incorrect, marking their second substantial setback (A = 2). P4 responded, “Yeah, I know but what else can I do?” to the notification of their incorrect answer, suggesting they were substantially unsure how to proceed with this task. At this point they received their second piece of personalized feedback, which said “The size of your 1/6 is too small” (see Fig. 6, right). This piece of medium-level scaffolded feedback was designed to provoke the student to think about a more exact relationship between the size of sixths compared to thirds.

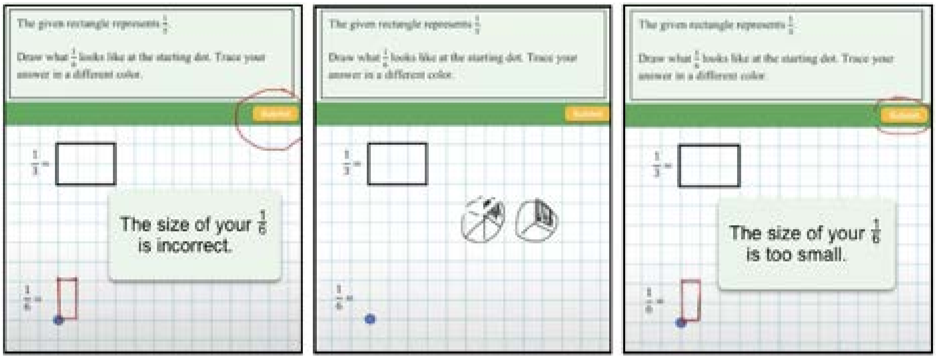


Figure 6. P4's Experience on a Fractions Task

P4 again responded resiliently to this feedback and re-engaged to make another new effort toward a task solution (*B* = 2). P4 re-initiated this new effort by again deciding to explore some new ideas using the Draft Pen. They re-sustained their effort by sketching a copy of the given rectangle that represented 1/3 and partitioning it into 6 equal pieces (see Fig. 7, left). They narrated their work by saying, "I'm going to cut this up," referring to their partitioning work with the given model of 1/3. After pondering their scratch work, P4 sketched a rectangle with an area of 4 square units (2 units by 2 units) as their answer, which was incorrect. They said, "I know it's like half, this is close" as they submitted their answer. Despite the incorrect answer, these new efforts were determined to be mathematically productive (*C* = 1) because P4 was using their scratch work to think about how to use partitioned pieces of the given 1/3 to build their model of 1/6. Additionally, P4 mentioned in their think-aloud that they knew that 1/6 was one half the size of 1/3, which was correct, yet for some reason they sketched a rectangle that was non-representative of that relationship. Thus, their submitted answer was graded as incorrect, marking their third substantial setback (*A* = 3). P4 reacted, "There is no other way," seemingly convinced that this task was not possible to solve. At this point they received their third piece of personalized feedback, which said "Try using a horizontal line to help you split up the given rectangle into smaller pieces" (see Fig. 7, center). This piece of higher-level scaffolded feedback was designed to suggest a practical problem-solving strategy that may have eluded the student thus far.

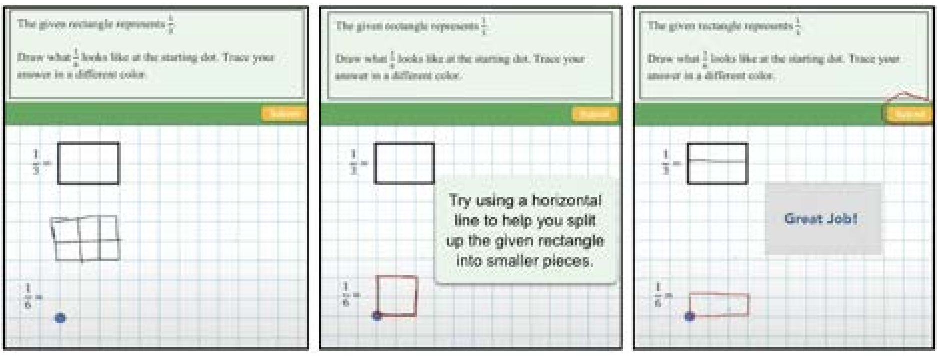


Figure 7. P4's Experience on a Fractions Task (continued)

P4 immediately and resiliently responded to this feedback by saying, "Ohh, I didn't know I could do that! I didn't know how to draw three boxes." P4's reaction provided some insight into their previous incorrect submission. It seemed that through P4's previous scratch work in which they partitioned the area of model of 1/3 into 6 equal pieces, they came to understand that 1/3 was equivalent to 2/6, and half of 2/6 would represent 1/6, or 3 of the square-unit boxes on the digital page. However, a method of sketching 1/6 as an area of 3 square units did not occur to P4 in these moments, so they sketched 1/6 as an area of 4 square units instead, which they believed to be "close." After their reaction, P4 re-engaged to make another new effort toward a task solution (*B* = 3). They re-initiated this new effort by again deciding to use the Draft Pen and re-sustained their new effort by partitioning the given model of 1/3 into 2 equal pieces using a horizontal line. P4 then sketched their new answer as a rectangle with an area of 3 square units (1 unit by 3 units). They submitted this answer, and it was graded as correct (see Fig. 7, right), thus this new effort was determined to be mathematically productive (C = 2). On this task, P4 encountered three substantial setbacks (A = 3), made three new efforts in response to those setbacks (B = 3), and two of those new efforts were mathematically productive (C = 2). Thus, on this task their Reengagement Rate was 100% and their Perseverance Success Rate was approximately 67%.

Many participants encountered several substantial setbacks across their work with a challenging sketching-based task(s); it follows that they received several pieces of personalized feedback that helped them resist the urge to give up and manage their struggle and frustration over time. Further, the 85% Perseverance Success Rate suggests that this continued engagement paid dividends for these participants in terms of their mathematics learning.

4 CONCLUSIONS

This paper presented the design of a feedback hierarchy and the subsequent effects on engagement and perseverance for students working in a simulation of the Drawn2Math application. The feedback hierarchy was designed to keep K-8 students engaged with challenging tasks and support their perseverance to learn mathematics conceptually. Our findings showed that participants working on these challenging tasks were often able to leverage the personalized mathematical feedback to reengage and persevere with the task, even after substantial setbacks and experiencing ample frustration. All 10 participants showed evidence of this kind of success. Participants were reengaging and persevering with these sketching-based tasks despite working on them alone, remotely, without a teacher present. These findings suggest that implementing feedback hierarchies into mathematics education technology can help students stay engaged with challenging mathematics and learn conceptually, even when working remotely. Future work should include conducting a similar study using a functional application that uses an algorithm to implement the personalized feedback in the feedback hierarchy. This continued research will provide important contributions to advancing theory of how personalized feedback within a software application impacts perseverance and conceptual learning.

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