Statistics of individual eigenchannels of diffusive random medium

A. Yamilov[1]*, N. Bender[2], H. Yilmaz[2], H. Cao[2],
[1]*Missouri University of Science and Technology, Missouri, USA*
[2]*Yale University, Connecticut, USA*
* *yamilov@mst.edu*

Abstract: We measure correlations between individual transmission eigenchannels in a unique on-chip photonic platform that allows both selective coupling of light into a single eigenchannel and direct probe of its spatial structure inside the random medium.
OCIS codes: (030.1670) Coherent optical effects; (290.4210) Multiple scattering; (290.1990) Diffusion

1. Eigenchannels of scattering media

Transmission eigenchannels (TEs) are building blocks of wave propagation in scattering media and mesoscopic physics, they "quantize" transport of classical and quantum waves through a system. Such intrinsic eigenchannels and the corresponding eigenvalues can be found by a singular value decomposition of the transmission matrix relating the input and output waves, $\hat{\boldsymbol{t}} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Lambda}}\hat{\boldsymbol{V}}^{+} = \sum \boldsymbol{u}_n \boldsymbol{\lambda}_n \boldsymbol{v}_n^{+}$, where $\boldsymbol{v}_n$ and $\boldsymbol{u}_n$ are orthonormal incoming and outgoing singular vectors, the singular values $\boldsymbol{\lambda}_n$ are the square root of the transmission eigenvalues $\boldsymbol{\tau}_n$. Eigenchannels have been widely used to interpret such hallmark mesoscopic effects as conductance fluctuations and sub-Poissonian shot noise. Because it is impossible to prescribe incident wavefunction in the electronic systems, direct experimental demonstration of TEs is not feasible there. In contrast, analogy between coherent electron transport and photon transport and recent technological advances in spatial light modulators (SLM) have offered an exciting opportunity to study experimentally TEs with light, leading to total transmission enhancement, focusing and imaging applications in opaque media. In this work, we consider a planar waveguide filled with scattering medium [1] we directly probe the eigenchannels inside the 2D system from the third dimension in order to study their spatial structures and statistical properties as well the cross-correlations between them.

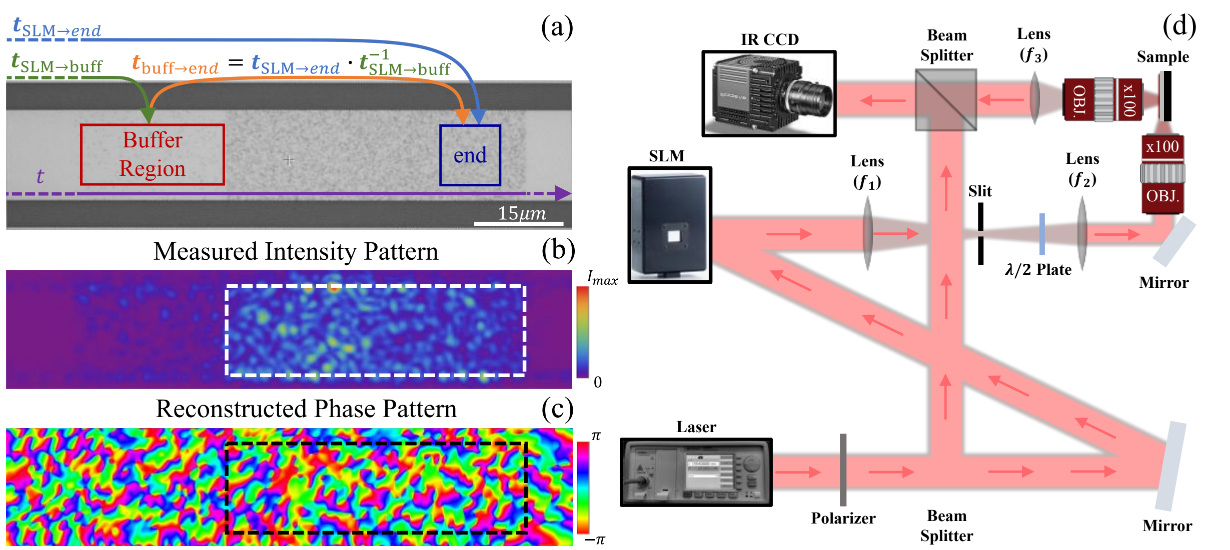

Figure 1. (a) Scanning electron microscope image of the disordered waveguide. Superimposed are schematic depictions of different transmission matrices. (b,c) depict measured intensity distribution of an open eigenchannel and its reconstructed phase. (d) Schematic depiction of the experimental setup.

2. Excitation of individual transmission eigenchannels

In our prior work [2] on controlling wave propagation on-chip with wavefront shaping was based on the feedback mechanism, which allowed us to maximize or minimize the overall transmission through the system by enhancing or suppressing contributions of open channels. This method, however, is not suitable for accessing the individual eigenchannels of the system. We overcame this major experimental obstacle by applying the interferometric measurement technique on-chip, Fig. 1. By measuring the complex transmission matrices from SLM to a weakly scattering region in front of the disordered medium $\boldsymbol{t_{SLM\to buff}}$ and at the end of the sample $\boldsymbol{t_{SLM\to end}}$, we were able to reconstruct the field transmission matrix $\boldsymbol{t_{buff\to end}} = \boldsymbol{t_{SLM\to end}} \cdot \boldsymbol{t_{SLM\to buff}^{-1}}$ of the disordered waveguide. Singular value decomposition of the matrix gave us information about the input wavefronts for individual TEs. After injecting light into a specific channel, we probed its intensity inside the disordered waveguide by collecting the light scattered out of plane by the air holes with an objective lens and projecting onto a camera. We recorded the 2D intensity distribution at different positions (y, z) inside the disordered waveguide for each eigenchannel $\boldsymbol{\alpha}$ and computed the normalized cross-section averaged intensity profiles $\boldsymbol{I_\alpha(z)}$.

3. Correlations between transmission eigenchannels

Although the wavefunctions of TEs are orthogonal to each other in any given system, their intensity fluctuations are correlated. To compare with experiment, we developed numerical model (see Refs. [1-4]) and computed intensity of TEs under experimental relevant conditions. We specifically focused on intensity correlations between eigenchannels as defined by $\langle \tilde{C}_{\alpha\beta}(z,z)\rangle_z \equiv \left\langle \overline{I_\alpha(z) I_\beta(z)} - \overline{I_\alpha(z)} \cdot \overline{I_\beta(z)} \right\rangle_z$, where $\overline{\ldots}$ denotes statistical average and $\langle \ldots \rangle_z$ is average over the longitudinal coordinate. The results of experiment and the simulation reveal that different TEs are correlated and the degree of correlation depends on eigenchannel, Fig. 2(a-c). The diagonal elements correspond to the fluctuation of intensity of individual eigenchannels, c.f. Fig. 2(d). We find that open eigenchannels have smaller fluctuations and are less correlated with other eigenchannels, that is extremely promising from the point of view of practical applications. Finally, we compared contributions of the diagonal and off-diagonal terms in $\langle \tilde{C}_{\alpha\beta}(z,z)\rangle_z$. Remarkably, the overall contribution from the off-diagonal terms is larger than from the diagonal ones.

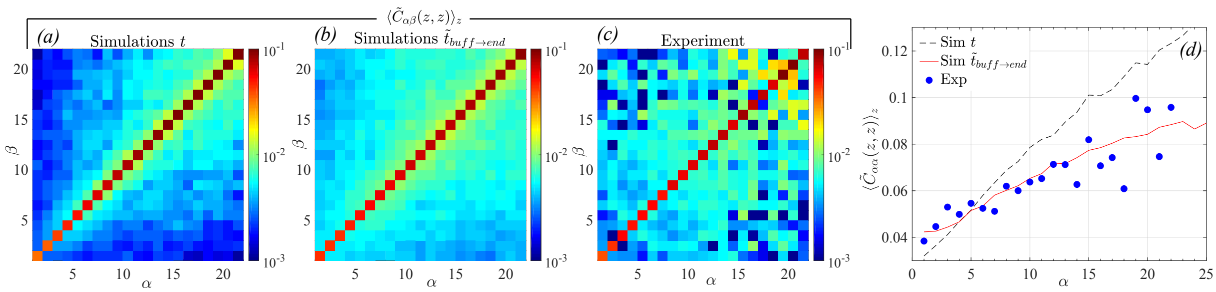

Figure 2. Correlations between different eigenchannels $\langle \tilde{C}_{\alpha\beta}(z,z)\rangle_z$ computed based on the transmission matrix t and on $t_{buff\to end}$ (a,b). Panel (c) shows the experimentally measured correlations of TEs. The diagonal elements from (a-c) are compared in (d).

4. Conclusions

Compared to electronic systems, robustness of coherence effects for photons at room temperature makes optical systems ideal for the in-depth fundamental studies of coherent wave transport. Our work sheds light on the long-standing questions regarding the nature and statistical properties of individual transmission eigenchannels and the role they play in wave transport is complex media. The above results together with our recent discoveries of the transverse localization of transmission eigenchannels [3] and enhanced memory effect of high-transmission eigenchannels in a diffusive system of open slab geometry provide the first glimpse of a wealth of undiscovered physics in wave transport in scattering media. The results obtained in our study of TEs in complex photonic media are also applicable to the propagation of other wave, e.g. microwaves and acoustic waves, informing their applications in wireless (microwave) and underwater (acoustic) communications.

5. References

[1] A. Yamilov, R. Sarma, B. Redding, B. Payne, H. Noh, and H. Cao, "*Position-dependent diffusion of light in disordered waveguides*," Phys. Rev. Lett. **112**, 023904 (2014)

[2] R. Sarma, A. Yamilov, S. Petrenko, Y. Bromberg, H. Cao, "*Control of energy density inside disordered medium by coupling to open or closed channels*," Phys. Rev. Lett. **117**, 086803 (2016)

[3] H. Yılmaz, C. W. Hsu, A. Yamilov, H. Cao, "*Transverse localization of transmission eigenchannels*," Nat. Phot. **13**, 352 (2019)

[4] H. Yılmaz, C. W. Hsu, A. Goetschy, S. Bittner, S. Rotter, A. Yamilov, H. Cao, "*Angular memory effect of transmission eigenchannels*," Phys. Rev. Lett. **123**, 203901 (2019).