Spatial Demography
https://doi.org/10.1007/s40980-021-00075-w

Comparative Spatial Segregation Analytics

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Accepted: 1 January 2021

Abstract
Comparative segregation analysis holds the potential to provide rich insights into urban socio-spatial dynamics. However, comparisons of the levels of segregation between two, or more, cities at the same point in time can be complicated by different spatial contexts as well as ethnic, racial, and class distributions. The extent to which differences in segregation between two cities is due to differences in spatial structure or to differences in composition remains an open question. This paper develops a framework to disentangle the contributions of spatial structure and composition in carrying out comparative segregation analysis. The approach uses spatially explicit counterfactuals embedded in a Shapley decomposition. We illustrate this framework in a case study of the 50 largest metropolitan statistical areas in the U.S.

> All of the segregation indexes have in common the assumption that segregation can be measured without regard to the spatial patterns of white and nonwhite residence in a city (Duncan and Duncan 1955).

1 Introduction

Comparison of the levels of segregation across U.S. cities is a popular pursuit in both academia[1] and in the popular press.[2] Most often, these comparisons follow a similar strategy involving the calculation of an index of segregation for a collection of cities at one point in time, followed by a ranking of the values for the index. The resulting rankings invariably garner widespread attention. Yet, from a methodological point of view, they also raise a number of questions. The ordinal nature of

[1] http://www.censusscope.org/us/print_rank_dissimilarity_white_black.html.
[2] https://www.washingtonpost.com/graphics/2018/national/segregation-us-cities/?noredirect=on&utm_term=.b8e434f29177.

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Published online: 24 January 2021

these summaries is often emphasized. For example, finding that in 2000 Chicago ranked 6th[1] while Newark ranked 7th conveys a different impression than knowing the former had a dissimilarity index of 83.6 relative to the latter's index of 83.4. The question of whether these differences are significant often goes unasked, and therefore, unanswered. This is curiously distinct from much quantitative social science research where questions of inference are central to the investigation. One of the main reasons for the descriptive orientation of much of the segregation literature is the limited amount of work developing comparative approaches.

Dozens of segregation indices have been proposed and applied in the literature, and regardless of which is applied, an index reflects a variety of underlying urban attributes. These might include the overall demographic mix in the city, the density of its urban development (particularly if a spatial segregation index is employed), the size and configuration of administrative aggregation units, or the total size (in either population or geographic terms) of the city. As such, when examining how racial segregation in Newark compares to that in Chicago, indices computed for each city are difficult to understand relative to one another because it is unclear how the underlying urban attributes combine into a single measure and whether (and which) attributes are stronger or weaker drivers of the observed difference. More simply, based on the numbers alone, we can see that Newark may be less segregated than Chicago, but we cannot discern whether this is because Chicago is simply more cosmopolitan, with more people of different backgrounds sharing less space, or whether it is because Chicago's urban structure isolates minority residents into high-density, low-cost apartments which are located exclusively in one part of the city.

The ambiguity concerning the underlying drivers of measured segregation levels is a major impediment for urban policymakers seeking to use housing and transportation investments in service of greater social justice. If, in the example above, it were clear that segregation in Chicago is driven by single-use zoning and poorly-served transit, then increasing service frequency and implementing an inclusionary zoning policy would be natural responses. If it were simply the case that the city had a small minority population that prefers to settle in a close-knit community, the policy implications are far less clear.

Furthermore, federal housing policies have long sought to incentivize housing voucher holders to relocate into integrated neighborhoods (Joseph et al. 2007; Pendall and Carruthers 2003; Kline 2007; DeLuca et al. 2013; Ellen et al. 2016), but these objectives can sometimes conflict with other goals, such as ensuring that carless voucher holders remain near transit (Blumenberg and Pierce 2014; Pendall et al. 2015). This case would indicate a tension between land use (transit) and demography (race or income) with competing objectives and no clear winner for social justice. As such, providing quantitative evidence of this situation would provide justification for the housing authority to adjust its location incentives to best serve its constituents–but such a condition requires the unpacking of a given segregation index to understand the underlying drivers.

Segregation indices are known to be sensitive to the number, sizes, shapes, and arrangement of the enumeration units used as well as the spatial extent of the community under examination (Massey 1978; Jakubs 1981; Wong 2003; Lee et al. 2008; Clark and Östh 2018). Moreover, commonly used segregation measures, such as the dissimilarity index, can be sensitive to the city-wide composition of the minority group, as all other things equal, a small minority population is more likely to be unevenly distributed within a city than a larger minority population (Allen et al. 2015). Further complicating matters is the interaction effect where small enumeration units can exacerbate the impact of minority composition on segregation indices.

If a pair of cities were identical in their enumeration units *and* city-wide minority composition, then any difference in the segregation measures between the two cities could be attributed to differences in the spatial distribution of the minority groups within each of the cities. Unfortunately, matters are not so simple in applied comparative segregation research as cities differ in the structure of their enumeration units, minority composition, and their spatial distribution of the minority population over their enumeration units. As such, researchers are currently unable to ascertain how much of the difference in the two segregation indices is due to variations in spatial structure[3] or to differences in city-wide population composition between the two cities. By spatial structure, we mean the structure of the city's enumeration units and the spatial distribution of the minority population across those units. For these reasons, we argue that by decomposing segregation indices into their underlying components, namely those driven by population structure and spatial structure, scholars and policymakers would be armed with a new and highly informative set of urban analytics that can lead to better policymaking and reduced inequality. That is what we propose here.

In this paper we consider comparative segregation analysis from a spatially explicit perspective to make several contributions. We propose a framework for disentangling the roles of varying spatial structure and composition when carrying out comparative segregation analysis. Our framework is based on a decomposition of differences in segregation between two cities, or the same city at two points over time, due to spatial and attribute differences between the two cities. The approach relies on novel counterfactual distributions for the comparison cities together with a Shapley value decomposition defined on these counterfactuals. We also provide empirical insights on the magnitude of these differences across 50 metropolitan areas in the US over the period 2000-2010.

The remainder of the paper is organized as follows. We first revisit the literature on comparative segregation and examine the complications that spatial structure and effects pose for such analyses. In Sect. 3, we present our framework for comparative segregation analysis that is designed to address some of these issues. We then provide an empirical illustration of our framework in Sect. 4. The paper concludes with a summary of key points and suggestions for future areas of research.

[3] A referee pointed out that a more general term might be "aggregation unit" rather than spatial structure. We agree that our framework can be generalized, but we see the concept of the aggregation unit as only one component of a city's spatial structure.

2 Comparing Measures of Segregation

Much of the segregation literature focuses on developing improvements to statistical measurement techniques (Massey 1978; Wong 1993; Massey et al. 1996; Wong 1999; Reardon and Firebaugh 2002; Wong 2003, 2004; Dawkins 2004; Reardon and O'Sullivan 2004; Wong 2005; Reardon et al. 2008; Chodrow 2017), while a parallel body applies these metrics to the study of gender, ethnic, racial, occupational, educational, income, and other forms of segregation (Mare and Bruch 2011). For decades, these two strands have fed off one another, with empirical studies revealing undesirable properties of common segregation indices, and statistical work proposing alternative techniques or corrections to account for the identified shortcomings. Despite a constant stream of incremental improvements, and dozens of segregation indices proposed and applied throughout the literature, there remains considerable room for improvement in methods designed for segregation analysis, particularly from the perspective of comparative frameworks. Put differently, while there has been vast improvement in the theoretical and computational measurement of segregation, the past century has seen limited innovation that is able to overcome the "problems of inter-urban comparative work that arise because of the nature of available census data sets" (Johnston 1981, p.246). Herein we review these problems and the ways in which various scholarship has tried to address them.

2.1 Comparisons Over Space

In the canon of comparative segregation studies, the most common methodological technique is for researchers to choose and defend the use of a particular segregation index, calculate index values for a set of cities or regions, and rank and compare the resulting values describing the ordinal structure across cities. Thereafter, researchers sometimes examine how these ordinal rankings differ for alternative indices. For decades, scholars have deployed these descriptive methods successfully to compare residential segregation in a wide range of contexts, cultures, and time periods. Both canonical and recent work has examined segregation by race and class in American cities thoroughly (Clark 1986; Ihlanfeldt and Scafidi 2002, 2004; Bringegar and Leonard 2008; Hwang 2015; Wang et al. 2018). But scholarship is by no means limited to American cities or social constructs. Elsewhere, researchers have compared segregation measurements between global cities (Harsman and Quigley 1992; Marcińczak et al. 2015; Musterd et al. 2017), between countries (Goering 1993; Johnston et al. 2007), and within cities in countries across the globe (Morgan 1975; Owusu and Agyei-Mensah 2011; Wang and Li 2016). Neither are place-based comparisons limited necessarily to analysis of residential segregation. A large body of work in sociology and labor studies examines occupational segregation by race and gender, and how those patterns compare across countries and/or labor markets (Blackburn et al. 1993), and while these works are conceptually distinct, their formula for comparative analysis is identical.

Comparative studies of this variety are useful because they permit observations such as "in general there is less segregation in Australia and New Zealand than in Canada, the United Kingdom and the United States" (Johnston et al. 2007, p.713), and these patterns can be further analyzed by the differing social contexts of each country, or used to develop a policy agenda. But the simplicity of this analytical technique and the resulting rankings masks a crucial uncertainty because inter-urban comparisons are never truly "apples-to-apples". It is impossible to compare segregation in City A versus City B while accounting appropriately for the idiosyncratic differences between them in composition, size, scale, and configuration.

Critiques in comparative segregation research often arise over concerns about data quality and measurement approaches. Chief among the criticisms is that segregation measurements are sensitive to (at least) two critical features of urban areas beyond control of the analyst. First, since indices operate on population ratios, they are notably sensitive to the relative size of different population groups in each city. Small shares of minority populations can inflate widely used measures like the index of dissimilarity (Cortese et al. 1976; Clark 1986; Massey 1978; Reardon and O'Sullivan 2004). Second, while residential segregation is a multiscalar phenomenon[4] whose smallest scale manifests at the housing-unit level, the census data used to calculate segregation measurements necessarily relies on aggregations to larger polygons to protect confidentiality. As a result, "all measures of spatial and aspatial segregation that rely on population counts aggregated within subareas are sensitive to the definitions of the boundaries of these spatial subareas" (Reardon and O'Sullivan 2004)[p.124]. That is, segregation indices are significantly affected by the size and shape of the census tracts (or other spatial units) that serve as the basis of such measures (Jakubs 1981; Massey 1978). To overcome issues related to census enumeration units, Reardon and O'Sullivan (2004) interpolate census blocks to a regular grid so that spatial units approximate a continuous surface, and several others have adopted this technique (Lee et al. 2008; Reardon et al. 2006) in the literature. While this method skirts issues of census boundary configuration, kernel-based interpolation of this variety relies on what may be restrictive assumptions about population density, and explicitly ignores important physical features like impassable terrain or uneven development. As a result, the population surfaces are often inaccurate, raising questions about the validity of segregation measures generated by these techniques.

Apart from technical issues inherent in the properties of particular indices and the applicability of available data, methods for comparative analysis still leave much to be desired since, as Clark (1986) points out, the conceptual distinction between indices can lead to significantly different interpretations in applied settings. The

[4] Multiscalar in this context refers to the fact that groups of people can be separated by residential housing patterns. Micro-level segregation may manifest when small pockets of self-similar neighborhoods are interspersed with one another, whereas macro-level segregation may manifest when residents of two different groups inhabit opposite sides of a large region. For further discussion the reader is directed toward Reardon et al. (2008).

Table 1 Generic structure of a data set of a given city. $n^{a,t}_{1,y}$ is the population of group y in tract 1 of city a in time period t

Tract	Group x	Group y	Total
1	$n^{a,t}_{1,x}$	$n^{a,t}_{1,y}$	$n^{a,t}_{1,.}$
2	$n^{a,t}_{2,x}$	$n^{a,t}_{2,y}$	$n^{a,t}_{2,.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
I	$n^{a,t}_{I,x}$	$n^{a,t}_{I,y}$	$n^{a,t}_{I,.}$
Total	$n^{a,t}_{.,x}$	$n^{a,t}_{.,y}$	$n^{a,t}_{.,.}$

difference in segregation between City A and City B may look trivial when measured with the index of isolation, but appear significantly larger when measured with the Gini index. Current techniques leave no recourse for this problem other than argumentation regarding which index is the superior, trustworthy measure.

In their classic study, Massey et al. (1996) describe five conceptual dimensions of segregation they term evenness, exposure, concentration, centralization, and clustering, and while there is debate over whether these represent the "true" dimensions of residential segregation, there is nonetheless agreement that multiple dimensions are worthy of consideration. Thus, dozens of segregation indices persist in the literature, thanks in part to their desirable sensitivity to various different dimensions. In applied comparative research, however, differential sensitivity can by definition lead to ambiguous results. In problematic cases, segregation indices disagree by wide margins, as discussed by Clark (1986, p. 97) who shows that "Baltimore (Table 1) was almost twice as segregated as San Jose on the dissimilarity index in 1970, but the exposure index suggested that while Baltimore was substantially segregated, San Jose was not". Explaining the gap between these measures for the two cities presents an interesting avenue for further study, but also an impasse for statistical comparative work. To our knowledge, no existing quantitative techniques are capable of analyzing whether the segregation measures for each city are significantly different in either semantic or statistical terms.

2.2 Comparisons Over Time

A natural extension of comparative segregation analysis is the inclusion of time as an important dimension. Incorporating temporality into the study of urban segregation typically assumes one of three flavors; A large body of work examines the experience of individual households, and whether minority members are able to escape segregated neighborhoods in successive generations (Bischoff and Reardon 2013). This work grows from the life course tradition in sociology to address questions pertaining to the long-term experience of neighborhood and community realized by members of minority groups (McAvay 2018). While this literature sheds considerable light onto the prevalence and persistence of intergenerational segregation, it typically takes segregation measures as given, and focuses the analysis on migration patterns that expose families to higher or lower levels of urban segregation. The emphasis here is less on the measurement of segregation and more on the residential mobility patterns that bring individuals into contact with segregation, and thus is less useful for comparative work.

Temporal comparative analysis provides a unique window into the dynamic structure of segregation and the ways in which urban areas are evolving. Comparisons over time help us understand the paths that cities follow, and whether they trend toward integration or separation, though these analyses too suffer a variety of drawbacks. Among the chief criticisms of temporal comparisons is that they necessarily rely on decennial census data, which captures “just one part of the picture, applying only to the population present and captured at both time points” (Bailey 2012, p.709). Decennial data are severely limited for segregation analysis because they fail to capture the volatility in metropolitan housing markets that occurs over a 10 year period. Apart from issues of data concurrency, temporal comparisons suffer other shortcomings as well. As with place-based comparisons discussed above, temporal comparisons rely on census data, which are retabulated each year according to changes in population density. This means that in theory, the segregation levels measured in a particular city could change over time even if population ratios and spatial allocation remained constant, but tabulation blocks were redrawn between the two decades (Jakubs 1981; Massey 1978; Reardon and O’Sullivan 2004; Logan et al. 2014).

2.3 Comparisons Over Space-Time

A final mode of comparison in the field of segregation analytics is that between places over time. As the most data intensive (since it requires multiple observations for both space and time), this is naturally the smallest of the three reviewed fields. Extending temporal comparisons, researchers making space-time comparisons between segregation measures tend to plot the linear trends for each city, compare cross sectional measures between cities, and compare the trend lines between cities to facilitate statements such as “From 2000 to 2010... economic segregation increased in 72 CZs [and] larger metro areas tend to be more segregated than less populous metros” (Acs et al. 2017).

Much like its cousins, space-time comparisons in segregation research also span the globe (van Ham and Tammaru 2016) and in a variety of spatial and aspatial social science contexts (Clark 1986; Blackburn et al. 1993; Johnston et al. 2004; Lichter et al. 2007; Fowler 2016). But as a methodological amalgamation of spatial and temporal comparisons, space-time comparisons indeed suffer the combined flaws of each. This literature, too, makes clear that segregation measurement strategies are fraught with difficulty since, “at a minimum we would expect satisfactory measures to provide consistent comparisons across place and over time” (Blackburn et al. 1993, p.340) but this is not the case. Even with modern, spatially explicit segregation indices, space-time comparisons are particularly problematic because each urban system has multiple variables changing in concert. Each city experiences changes in its population structure and urban development patterns (and thus, census enumeration units) and there are no methods that permit researchers to decompose the measured differences to understand *which* variable is a larger contributor to the results. Nor are there guidelines that help researchers analyze whether the magnitude in either segregation slopes or point-estimates are meaningful.

2.4 Beyond Ordinal Rankings

While it dominates much of the literature, ordinal comparisons are not the only strategy employed by researchers to investigate patterns of urban segregation. Another common strategy is to calculate segregation indices to serve as dependent variables in regression models. For example researchers have examined whether density (Pendall and Carruthers 2003), land use regulation (Lens and Monkkonen 2016), or population diversity (Johnston et al. 2004) explain differing levels of segregation in American cities. Recently, Garcia-López and Moreno-Monroy (2018) find that spatial structure, in the form of mono/polycentricity affects measured income segregation in Brazilian cities. These last two studies are especially poignant because they begin to highlight the importance of both demographic structure and spatial structure and their effects on the resulting measurements of segregation in an urban area. The literature makes clear that the geometric size and configuration of the tabulation units on which segregation measures rely affect the resulting indices significantly (Massey 1978; Jakubs 1981; Wong 2004; Krupka 2007; Lee et al. 2008; Clark and Östh 2018)

Regression approaches that hold constant certain aspects of spatial structure, like development intensity or polycentricity attempt to rectify this situation, but since such approaches also fail to account for other aspects of spatial structure like the total size of a city or the shape and configuration of its infrastructure networks, housing unit makeup, or neighborhood configuration, it is impossible to disentangle the effects of spatial structure from aggregate segregation measures. Thus, rather than control for the effects of spatial structure, we instead *leverage* it to assess how much of the difference between two measures of segregation is attributable to physical layout as opposed to demography.

Our review of the existing work on comparative segregation analytics elucidates two clear gaps in the research. First, when comparing two places, researchers lack a framework for understanding the means through which the difference arises. Existing methods fail to provide information about whether the difference between two segregation measures arises from differences in the physical layout of a city, or what we call its spatial structure reflected in the sizes, shapes, and arrangement of enumeration units, and how the minority population is distributed over these units, or because of the demographic makeup of its population (its composition). In the sections to follow, we address this question in detail by developing a novel method for decomposing differences in segregation indices into their spatial and social components.

3 A Framework for Comparative Segregation Analysis

Our analytical framework uses the following structure. Consider Table 1 which reports data for a particular city at one point in time. The rows correspond to the enumeration units (census blocks or tracts), while the second and third columns are

Table 2 Cross-sectional Decomposition of Segregation Differences.

		Spatial Structure	
		City 1	City 2
Attribute	City 1	G_A	G_B
Distribution	City 2	G_C	G_D

G_A and G_D are the observed segregation indies for cities 1 and 2, respectively. G_B is the segregation index obtained for the counterfactual where the population composition of city 1 is imposed on the spatial structure of city 2. G_C is the segregation index obtained for the counterfactual where the population composition of city 2 is imposed on the spatial structure of city 1

associated with each ethnic/racial group. We assume that $n_{i,j}^{a,t}$ is the population of unit $i \in \{1, ..., I\}$ of group $j \in \{x, y\}$ in city a and period t. We, usually, consider group x as being the group of interest (also called the *minority* group). In addition, the marginal and total sums are given by $\sum_j n_{i,j}^{a,t} = n_{i,.}^{a,t}$, $\sum_i n_{i,j}^{a,t} = n_{.,j}^{a,t}$, $\sum_i \sum_j n_{i,j}^{a,t} = n_{.,.}^{a,t}$ which are, respectively, the total population of unit i, total city population of group j and total city population. We also define $\tilde{s}_{i,j}^{a,t} = \frac{n_{i,j}^{a,t}}{n_{i,.}^{a,t}}$ as the share of tract i's population belonging to group j (also called *unit composition*) and $\hat{s}_{i,j}^{a,t} = \frac{n_{i,j}^{a,t}}{n_{.,j}^{a,t}}$ as the share of the city's population in group j that resides in tract i.

We adopt the perspective of Allen et al. (2015) and view segregation as an assignment process that distributes values to the internal cells of Table 1 subject to the row and column constraints. In comparing different cities across space, or the same city over time, it is important to note that not only does the distribution of the values over the internal cells of the table matter but also the marginal row and column distributions. Small overall proportions of minority groups can result in the minority group being unevenly distributed by chance, relative to groups with larger shares of the city's population.

3.1 Decomposition

We formulate a general structure that supports the comparative analysis of segregation across different contexts. Depending on the nature of the context, (spatial, temporal, or spatio-temporal) different formulations arise; the same general structure, however, can be used to identify the key dimensions of each comparison. Two dimensions are relevant for comparative segregation analysis: the composition of the population and the spatial distribution of that population.

3.1.1 Cross-Sectional Segregation Decomposition

Table 2 provides an illustration of these dimensions for a cross-sectional comparison case involving two cities at one point in time. Here interest centers on comparison of the segregation indices measured for City 1 versus City 2, corresponding to the two segregation indices associated with cases A and D in the table. To illustrate our approach, we first develop a generalized spatial Gini index. Following the logic of Reardon and O'Sullivan (2004), "we can derive spatial generalizations of all Reardon and Firebaugh (2002)'s aspatial multigroup segregation measures (D, G, H, C, P, R)". Following the notation of Reardon and O'Sullivan (2004), we calculate local environments (otherwise known as "egohoods") of our study area using the following equation

$$\tilde{\pi}_{pm} = \frac{\int\limits_{q \in R} \tau_{qm} \phi(p,q) dq}{\int\limits_{q \in R} \tau_{q} \phi(p,q) dq}$$

which describes the proportion of population group m at location p, with τ_p and τ_{pm} as the total population count and population count of group m, respectively. Here, p is the local environment, measured from the centroid of each census tract, and $\phi(p,q)$ is a triangular function of Euclidean distance between p and q up to a horizon of 2000m (or roughly a 20 minute walk). Using these new inputs to the classic Gini formulation, we achieve a generalized spatial Gini index[5]

Thus, the observed difference $\Delta G_{A,D} = G_A - G_D$ may be due to differences in the way the respective populations are distributed in space (i.e. the configuration of enumeration units and the spatial allocation of population across those units) as well as differences in the population composition between the two cities. To parse the observed differences across these dimensions, we adopt a Shapley decomposition approach (Shorrocks 2013). In formal terms, we define a function:

$$\Delta G = G(S_1, A_1) - G(S_2, A_2) \tag{1}$$

and then define the Shapley contributions of the spatial S and attribute A components, given respectively as C_S and C_A, as:

$$F(S, A) = C_S + C_A = \Delta G \tag{2}$$

with:

$$C_S = \frac{1}{2}\left[G(S_1, A_1) - G(S_2, A_1) + G(S_1, A_2) - G(S_2, A_2)\right] \tag{3}$$

and:

$$C_A = \frac{1}{2}\left[G(S_1, A_1) - G(S_1, A_2) + G(S_2, A_1) - G(S_2, A_2)\right]. \tag{4}$$

Focusing on the spatial component, C_S, in (3), two estimates are obtained. The first holds the attribute distribution constant, and set to that of City 1, while the spatial

[5] We note that all code data, figures, and tables necessary to recreate the full analysis are open-source and available upon request. The empirical analysis in this paper uses the **segregation** package (Cortes et al. 2019) from the Python Spatial Analysis Library PySAL.

Table 3 Temporal Decomposition of Segregation Differences

		Spatial Structure	
		Period 1	Period 2
Attribute	Period 1	G_A	G_B
Distribution	Period 2	G_C	G_D

structure varies between the two cities. In the second estimate, spatial structure varies but the attribute distribution is constant and taken from City 2. The final spatial component is taken as the average of these two estimates. Note that in each of these estimates, a counterfactual is produced and used against the observed spatial Gini index for a particular City. In the first sub-estimate the counterfactual is $G(S_2, A_1)$ which calculates the spatial Gini for a realization where the population composition for City 1 is imposed on the spatial structure of City 2. The difference between the spatial Gini from this counterfactual and that from the observed spatial Gini $G(S_1, A_1)$ is attributed to changing the spatial distribution since it is the only component that varies between the two cases. In the second estimate, we obtain the counterfactual from imposing the attribute distribution (i.e. the relative share of each population group) of City 2 on the spatial distribution of City 1.

In other words, each city is compared against a hypothetical "counterfactual region" in which the total population of each metro region is unchanged, but we switch the relative shares of minority and majority populations using the composition distribution from the opposing city. This provides a counterfactual dataset that describes how each city would look if the spatial population distribution in each city remains constant, but the relative shares of minority and majority residents are taken from the opposing region. Below we discuss how these counterfactuals are constructed. Here we focus on the interpretation of the decomposition. To estimate the Shapley contribution of the attribute distribution, a similar approach is taken in (4), save that we now obtain two estimates by holding spatial structure fixed to that in one of the cities, while allowing the attribute distribution to vary.

3.1.2 Temporal Segregation Decomposition

A comparative analysis of the same city at two points in time can be viewed in a similar fashion, as shown in in Table 3. Here the difference in segregation indices for this city over time is $\Delta G_{A,D} = G_A - G_D$, and now the question is how much of the *change* in the city's segregation is due to changes in spatial structure versus changes in its population composition over the two periods. For example, if a region's segregation index increased between two decennial censuses, the decomposition technique we present allows an investigation into whether this increase is due to a reconfiguration of census tracts (due to densification in certain parts of the region) or due to a change in migration and relative population shares.

We use equations (3) and (4) to estimate the Shapley contributions of changes in spatial structure and changes in the city's attribute distribution, only here the interpretation changes as the subscript for the arguments to the Gini functions refer to either time period 1 or time period 2.

3.2 Counterfactual Distributions

To generate the counterfactual distributions that are used in the Shapley decomposition, we first estimate the tract-level composition of a particular group in each city. Using the notation from Table 1 we use the fact that $\tilde{s}_{i,j}^{1,t}$ is the unit composition of group j in tract i of City 1 in the period t.

Next, we form the cumulative distribution functions (CDF) for these values taken over all the tracts in City 1: $F^{(1)}(\tilde{s}_{i,j}^{1,t})$, and City 2: $F^{(2)}(\tilde{s}_{i,j}^{2,t})$. To create a counterfactual distribution that imposes the tract-level composition distribution of City 2 on the spatial structure of City 1 we take $p_{i,j}^{1,t} = F^{(1)}(\tilde{s}_{i,j}^{1,t})$ and then generate $n_{i,j}^{1,t}|_{attr=2} = F^{(2)^{-1}}(p_{i,j}^{1,t})n_{i,.}^{1,t}$, where $attr = 2$ means that this population is calculated given the composition distribution of City 2. This process is done for all tracts of a given group in City 1 and its complementary group population is given by the difference $n_{i,.}^{1,t} - n_{i,j}^{1,t}|_{attr=2}$. The populations for City 2 are generated analogously.

The intuition behind the counterfactuals is as follows. In Table 2, the counterfactual for case B involves imposition of the unit composition CDF from City 1 on the spatial structure of City 2. This respects the spatial distribution of unit composition rankings in City 2, only the level of the unit composition is taken from the value corresponding to the same percentile but from City 1. In other words, the *location* of tracts with high minority composition follows the distribution from City 2, but the *value* of the minority share is obtained from City 1. For the second counterfactual, the situation is reversed; here the values of the minority shares are obtained from the quantile function for City 2 using the observed values of the CDF for each unit's share in City 1.

Each counterfactual can then be compared against a different observed case. When comparing the spatial Gini from case B to case A, we are asking how segregation would differ if the composition of City 1 was to be imposed on the spatial structure of City 2 rather than its own geography. Comparing case D to case B asks the question how segregation in City 2 would differ if its unit composition distribution was replaced with that of City 1.

4 Illustration

4.1 Los Angeles Versus New York: Cross-Sectional Comparison in 2010

To Illustrate our cross-sectional comparison approach, we first use 2010 census data from Logan et al. (2014) for the metropolitan statistical areas (MSAs) of Los Angeles and New York, and focus on segregation measures for the non-Hispanic Black population using tract level data. Figure 1 illustrates how the non-Hispanic Black population is distributed differently between these two MSAs. The maps in (a) and

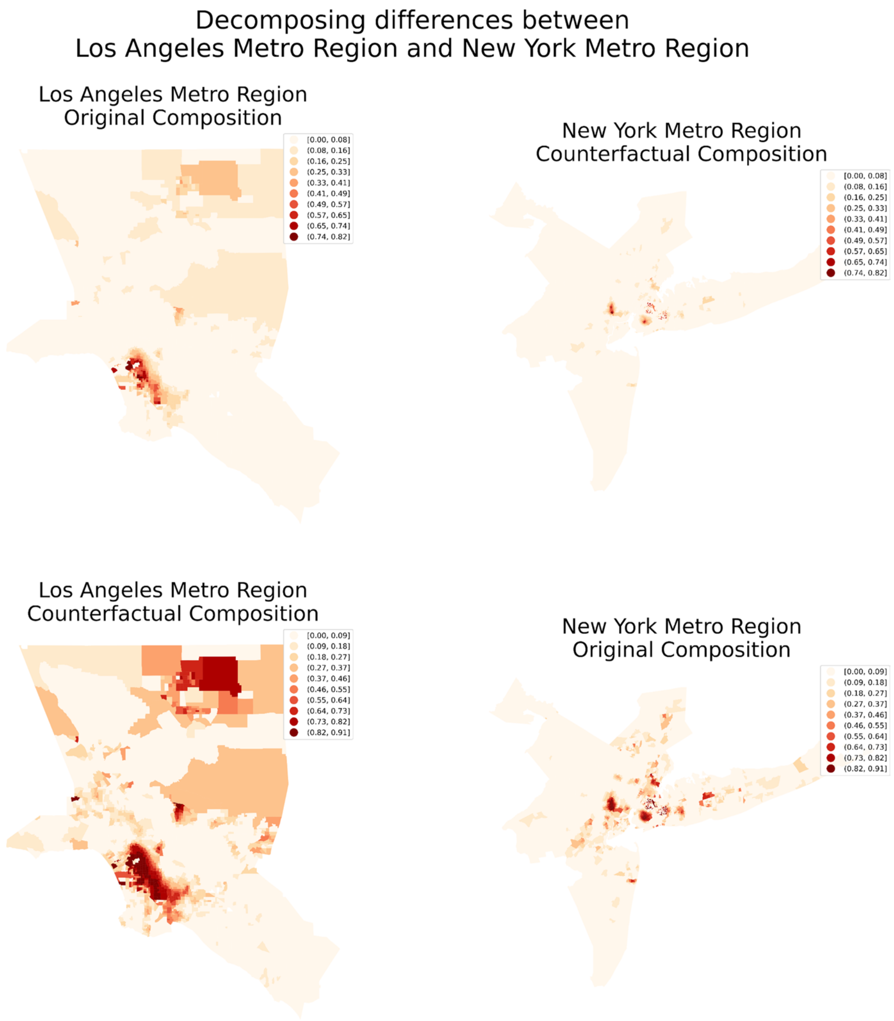

Fig. 1 Los Angeles and New York original and counterfactual non-Hispanic Black population distribution

(d) show the observed spatial distribution of this group in their respective metropolitan areas in 2010.

In Los Angeles, shown on the left hand side of the figure, there is a clear pattern of spatial concentration and unevenness in the racial makeup, in that the non-Hispanic Black population appears heavily concentrated into a single area of the city. New York, by contrast, shown on the right side of the figure, reveals a pattern markedly distinct from Los Angeles, with multiple locations having large shares of non-Hispanic Black population, mostly concentrated in Kings County, a portion of Queens and, with less intensity, in the Bronx. In the parlance of regional science, we would argue the structure of segregation for non-Hispanic

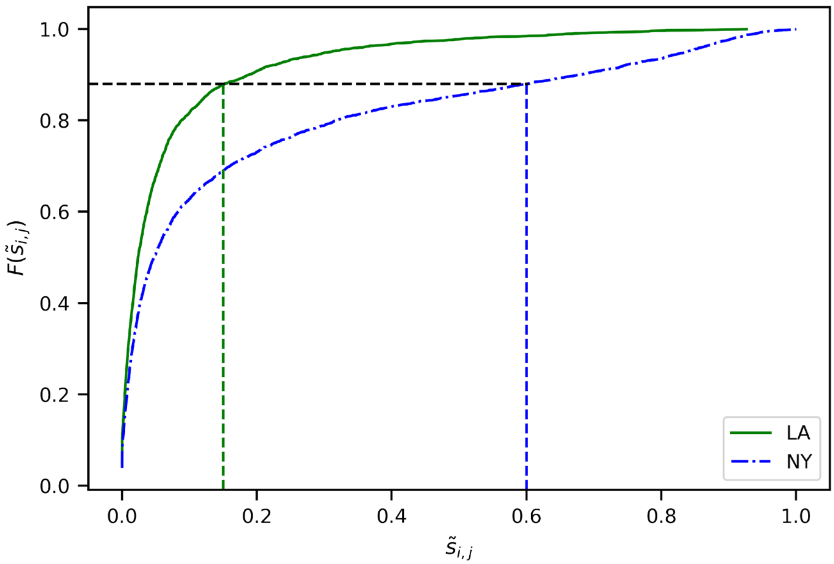

Fig. 2 Cumulative Distribution Functions of non-Hispanic Black population unit-composition: New York and Los Angeles 2010. The dashed lines trace out the 85th percentiles for LA and NY

Blacks in Los Angeles appears essentially monocentric whereas the structure in NYC is clearly polycentric, a curious reversal of their economic forms. According to their spatial Gini indices, the segregation estimate in Los Angeles was 0.671 and for New York was 0.733.

Given all of these differences, both in terms of spatial context and population compositions, comparing segregation between the two cities poses a considerable challenge. The difference between spatial Gini values for the two cities (-0.062) could be caused by a variety of these factors, and we employ the Shapley decomposition approach described earlier to disentangle them. The first thing to consider for the Shapley decomposition is the CDF for the tract-level population composition of each city shown in Fig. 2. The higher minority share in New York City (16.8% non-Hispanic Black) relative to Los Angeles (7.3%) is reflected in the these unit composition distributions, as New York's curve has a lower prevalence of low minority tracts.

To build the counterfactual distributions underlying the Shapely decomposition of Table 2, we utilize the two unit composition cumulative distributions functions in Fig. 2 to generate the two maps for each MSA in Fig. 1. For each tract in the "counterfactual maps" in Fig. 1, we obtain the observed tract composition value from the opposite city's cumulative distribution function. Figure 2 illustrates an example for a specific case of a tract in New York that was 60% Black. This represents the 85th percentile for New York's unit composition distribution. We then look up the corresponding 85th percentile from the unit composition distribution for Los Angeles (15% Black) and replace the original observed share in the New York tract with this percentile. The counterfactual

Table 4 Cross-Sectional Decomposition of Segregation Differences: Gini Coefficients, Los Angeles and NY 2010

		Spatial Structure	
		Los Angeles	New York
Attribute	Los Angeles	0.671	0.642
Distribution	New York	0.757	0.733

Table 5 Temporal Decomposition of Segregation Differences in Los Angeles: LA in 2010 and LA in 2000

		Spatial Structure	
		2010	2000
Attribute	2010	0.671	0.673
Distribution	2000	0.682	0.685

Black population in the New York tract is then generated by applying this percentile against the total population of the tract. We repeat this process for all tracts in New York to generate the counterfactual distribution.

The counterfactual distribution for Los Angeles is generated by going in the reverse direction. Continuing with Fig. 2, a specific tract with a unit composition of 15% Black represents the observed 85th percentile for Los Angeles. This is then replaced with the 85th percentile from the New York unit composition distribution (60%) to generate a synthetic composition for the Los Angeles tract. Again, we repeat this process for all tracts in Los Angeles.

For each of the two counterfactual maps we then calculate the spatial Gini coefficient and add these with the observed spatial Gini coefficients in Table 4. The first thing to notice in this decomposition, is that differences within rows are considerably smaller than differences within columns. From Eqs. 3 and 4 we estimate that the attribute component C_A plays a much more important role in the difference between spatial Gini indices in the two cities, since $C_S = 0.027$ and $C_A = -0.089$. In general terms, this result implies that the difference in segregation between Los Angeles and NYC (as measured by the spatial Gini Index) is attributable primarily to the fact that the cities have different shares of residents that identify as Black, white, and other races. If instead column differences were greater than row differences, it would imply that the way Black, white, and mixed-race neighborhoods are distributed through space is the greater contributor to measured differences between the cities.

In addition to the relative magnitudes of these components, it is also interesting to explore their directional effects. When holding the attribute distribution constant, a shift to the spatial structure of New York in place of Los Angeles results in a lowering of the segregation index. Focusing on the attribute distribution component, swapping in New York's population composition for that of Los Angeles results in an increase in the segregation index, regardless of the spatial structure.

4.2 Los Angeles versus Los Angeles: Temporal comparison between 2010 and 2000

For temporal comparative segregation analysis, we examine the evolution of Los Angeles between 2000 and 2010 for the same non-Hispanic Black population. Table 5 reveals that the difference in Gini was -0.014 where, once again, the attribute component played an important role as $C_S = -0.003$ and $C_A = -0.012$.

This is to be expected, as the amount of change in the spatial structure of a city over a 10-year period is likely to be dwarfed by demographic change. That being said, the difference in spatial structure between 2000 and 2010, while small, worked to reduce segregation (i.e., case B vs. A, or case D vs. C).

4.3 Multiple Metropolitan Regions Across US: Cross-Sectional Comparison in 2010

Given this decomposition illustration using the context of LA and NY, one might be interested in how this behaves for the rest of the country. We extend our method to the 50 most populated MSAs in the United States and decompose the spatial Gini index for each of the 1225 pairwise comparisons. The values of the spatial Gini indices for each of the 50 MSAs are shown in Table 6. In this table, Detroit is the most segregated MSA with a Gini of 0.873 whereas San Jose has the lowest value of 0.286.

Figure 3 displays the distribution of each of the Shapley components along with the point difference in segregation. In this figure, the attribute component is clearly more influential than the spatial component overall as it "dominates" the variability of the point difference. The spatial components typically have values close to zero, whereas the attribute values have considerable variance.

We can look in more detail, however, at each of the comparisons by analyzing the values themselves for the pairwise comparisons between MSAs.[6] We note that by analyzing each component in isolation, each metric's sign is meaningless, as it depends exclusively on the order in which the comparison is being decomposed. For this reason, we calculate pairwise results for all 50 metropolitan areas which are available online and upon request.

To examine these instances, we select from our set comparisons where $|C_S| > |C_A|$ and sort the data according to the difference given by $|C_S| - |C_A|$ to check whether the comparison produced a spatial component more relevant than the attribute component. We identify 67 cases where $|C_S| > |C_A|$. As such, our results reveal that in the vast majority of cases, the attribute difference accounts for a larger share of the difference in segregation. That is, typically, the Shapley attribute component "dominates" the spatial one since in most cases the attribute component is larger that the spatial component in absolute value.

6 Since we have 1225 point estimates, supplementary materials with all comparisons is available upon request.

Table 6 Ranked Gini values for all 50 MSAs

Metro	Gini	Rank	Metro	Gini	Rank
Detroit-Warren-Dearborn, MI Metro Area	0.873	1	Denver-Aurora-Lakewood, CO Metro Area	0.672	26
Cleveland-Elyria, OH Metro Area	0.846	2	Los Angeles-Long Beach-Anaheim, CA Metro Area	0.671	27
St. Louis, MO-IL Metro Area	0.846	3	Richmond, VA Metro Area	0.671	28
Chicago-Naperville-Elgin, IL-IN-WI Metro Area	0.834	4	New Orleans-Metairie, LA Metro Area	0.655	29
Milwaukee-Waukesha-West Allis, WI Metro Area	0.827	5	Orlando-Kissimmee-Sanford, FL Metro Area	0.624	30
Birmingham-Hoover, AL Metro Area	0.800	6	Houston-The Woodlands-Sugar Land, TX Metro Area	0.621	31
Buffalo-Cheektowaga-Niagara Falls, NY Metro Area	0.790	7	Charlotte-Concord-Gastonia, NC-SC Metro Area	0.620	32
Miami-Fort Lauderdale-West Palm Beach, FL Metro Area	0.769	8	Oklahoma City, OK Metro Area	0.617	33
Cincinnati, OH-KY-IN Metro Area	0.761	9	Dallas-Fort Worth-Arlington, TX Metro Area	0.609	34
Kansas City, MO-KS Metro Area	0.753	10	San Antonio-New Braunfels, TX Metro Area	0.595	35
Indianapolis-Carmel-Anderson, IN Metro Area	0.748	11	Minneapolis-St. Paul-Bloomington, MN-WI Metro Area	0.587	36
Louisville/Jefferson County, KY-IN Metro Area	0.740	12	Portland-Vancouver-Hillsboro, OR-WA Metro Area	0.586	37
Baltimore-Columbia-Towson, MD Metro Area	0.735	13	San Francisco-Oakland-Hayward, CA Metro Area	0.574	38
New York-Newark-Jersey City, NY-NJ-PA Metro Area	0.733	14	Virginia Beach-Norfolk-Newport News, VA-NC Metro Area	0.569	39
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD Metro Area	0.730	15	Seattle-Tacoma-Bellevue, WA Metro Area	0.541	40
Memphis, TN-MS-AR Metro Area	0.724	16	Raleigh, NC Metro Area	0.541	41
Columbus, OH Metro Area	0.724	17	Austin-Round Rock, TX Metro Area	0.521	42
Pittsburgh, PA Metro Area	0.724	18	Sacramento–Roseville–Arden-Arcade, CA Metro Area	0.505	43
Washington-Arlington-Alexandria, DC-VA-MD-WV Metro Area	0.702	19	San Diego-Carlsbad, CA Metro Area	0.496	44
Atlanta-Sandy Springs-Roswell, GA Metro Area	0.701	20	Providence-Warwick, RI-MA Metro Area	0.492	45
Jacksonville, FL Metro Area	0.694	21	Salt Lake City, UT Metro Area	0.467	46
Tampa-St. Petersburg-Clearwater, FL Metro Area	0.689	22	Riverside-San Bernardino-Ontario, CA Metro Area	0.447	47
Hartford-West Hartford-East Hartford, CT Metro Area	0.688	23	Phoenix-Mesa-Scottsdale, AZ Metro Area	0.429	48
Boston-Cambridge-Newton, MA-NH Metro Area	0.678	24	Las Vegas-Henderson-Paradise, NV Metro Area	0.329	49

Table 6 (continued)

Metro	Gini	Rank	Metro	Gini	Rank
Nashville-Davidson–Murfreesboro–Franklin, TN Metro Area	0.675	25	San Jose-Sunnyvale-Santa Clara, CA Metro Area	0.286	50

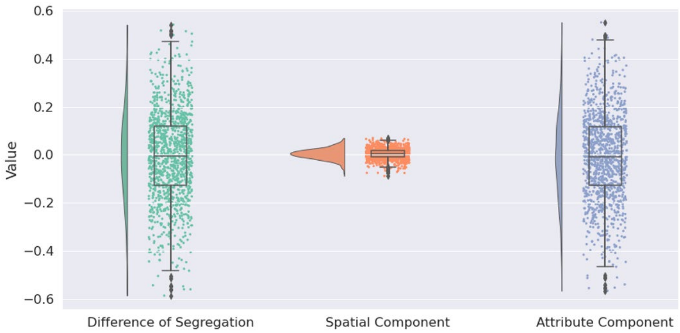

Fig. 3 Shapley Components Raincloud Plots. The density plot, the boxplot and all the values in a scatter plot for each segregation component

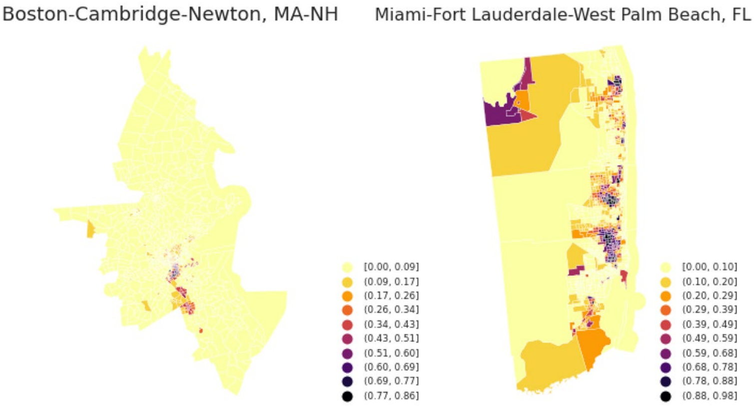

Fig. 4 Non-Hispanic Black population census tract composition in 2010 for the highest spatial component share in absolute values ($C_S = -0.039$ and $C_A = -0.004$) in all pairwise comparisons: Boston versus Miami

The highest value of $|C_S| - |C_A|$ is between Boston-Cambridge-Newton, MA-NH, and Miami-Fort Lauderdale-West Palm Beach, FL, as illustrated in Fig. 4. A cursory view of the maps reveals that the metros have remarkably different segregation patterns, with much of the Boston region's Black population concentrated into a single area, whereas the Miami metro region concentrates in at least three distinct loci, as well as spread through other areas. In this case, therefore, it is the spatial component which is responsible for contributing a greater difference in the measured spatial Gini index between the two cities because it is the configuration of their high minority census tracts that differs more than their population makeup.[7] Figure 5 shows the

[7] Results for all 50 pairwise comparisons are available upon request.

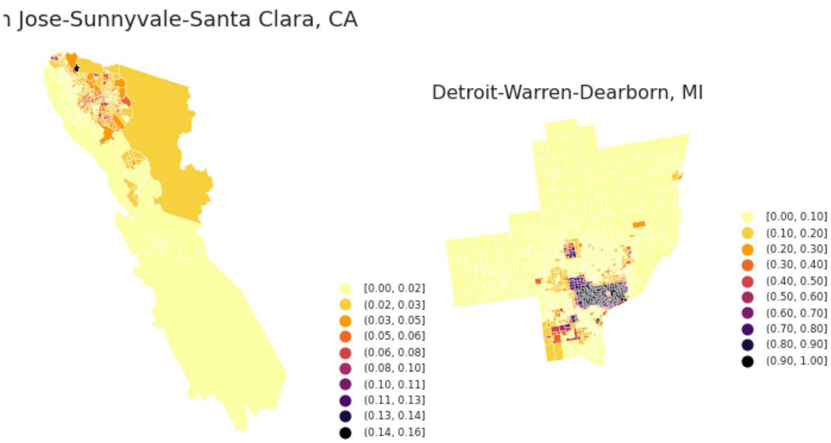

Fig. 5 Non-Hispanic Black population census tract composition in 2010 for the highest attribute component share in absolute values ($C_S = 0.02$ and $C_A = 0.57$) in all pairwise comparisons: San Jose versu Detroit

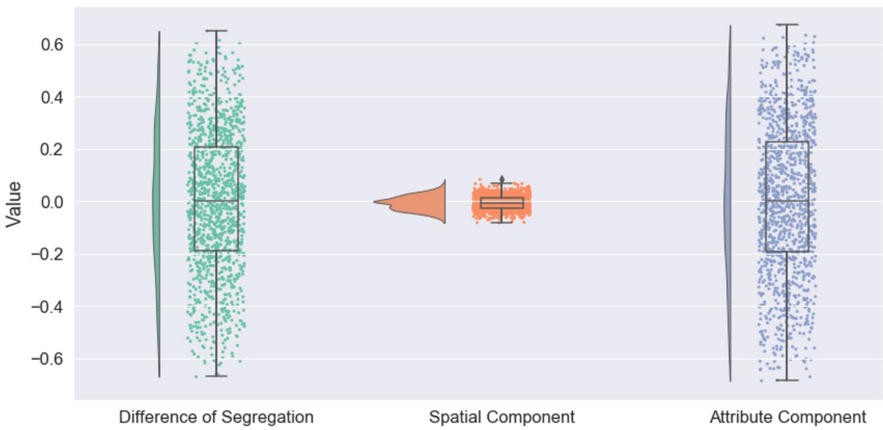

Fig. 6 Isolation Index: Shapely Components for all 1225 pairwise components

pair of cities with the largest attribute component. Here we see that San Jose has a much smaller minority component relative to that found in Detroit.

4.4 Decomposition Under Different Dimensions of Segregation

The Gini index used in the previous section is a measure that assesses the degree of **evenness** of a considered group in a given society. However, different dimensions of segregation can be assessed through different indexes and, according to Massey and Denton (1988), segregation can be considered to have five dimensions: evenness, isolation, clustering, concentration and centralization. Therefore, to check if

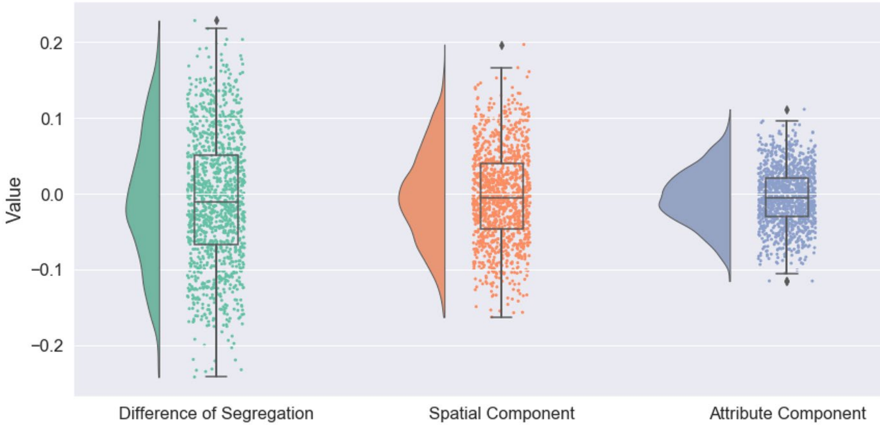

Fig. 7 Clustering Index: Shapely Components for all 1225 pairwise components

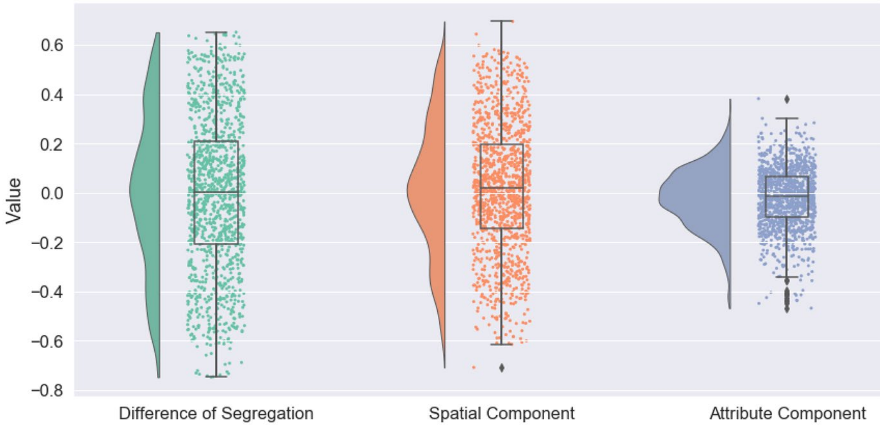

Fig. 8 Concentration Index: Shapely Components for all 1225 pairwise components

the interpretation of the results holds for the application, it is of interest to inspect how some indexes behave for each of these dimensions in the Shapley decomposition introduced in Sect. 3.

We chose to use the Isolation index (*xPx*), the Relative Clustering index (*RCL*), the Relative Concentration index (*RCO*) and the Relative Centralization index (*RCE*) from (Massey and Denton 1988). The results, given by the distributions of each component for every MSAs pairwise comparison in the illustration, are present in Figs. 6, 7, 8 and 9.

From Figure 3 versus Figs. 6, 7, 8 and 9, the differences between the dimensions are clear. The variance in the differences of segregation under the evenness and isolation dimensions is largely driven by the variance in the attribute component, whereas for clustering, concentration and centralization the distributions are more

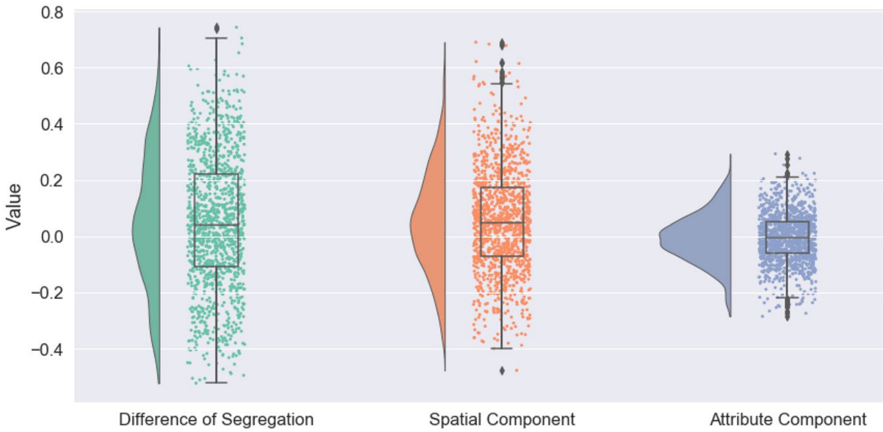

Fig. 9 Centralization Index: Shapely Components for all 1225 pairwise components

mixed, but indicate that the spatial component is more important to these dimensions. The direct reason for this results is that the clustering, concentration and centralization dimensions are, by definition, spatial. That is, the spatial context is always taken into consideration in the construction of the chosen index, while for evenness and isolation this is not necessarily true. In this case, the generalized spatial Gini index that we formulated earlier *does* account for spatial structure, but the Isolation index does not take absolute or relative location into account in its calculation and, therefore, space naturally may not play an important role in the Shapley decomposition.

These results reinforce the inherent difficulty facing researchers working to develop a comparative segregation framework. Because differences in segregation indices measured in different cities can arise from different sources of variation, there remains a clear challenge for analysts seeking to probe further. Our framework provides researchers with insights as to the relative contribution of these sources of variation when carrying out comparative analyses.

5 Conclusion and Discussion

This research advances comparative segregation analysis with a novel approach that combines counterfactual distributions and Shapley decompositions. After reviewing the literature on comparative segregation techniques and highlighting the challenges and opportunities in the field, we formalize our mathematical framework and present an extensive comparative study for the 50 largest Metropolitan Statistical Areas in the U.S. using census tract data.

Our approach is general and can be used to perform comparative segregation analysis for either spatial, temporal, or space-time contexts. For our illustration using the spatial Gini index to compare two different MSAs, the population's racial composition in each region proved to be more relevant than the spatial population distribution thereof in the majority of cases. This characteristic persisted in most of our scenarios when either comparing Los Angeles versus New York, performing temporal evolution comparison of Los Angeles, or when comparing multiple cities in a pairwise fashion. One of the possible reasons behind this is the nature of Gini index. Since this index measures the degree of unevenness, it can be affected by the structure of attribute composition and, therefore, more affected by different shapes of the compositional CDFs. Given that, one of our key findings is that the decomposition results can vary depending on the index used, and that this can reflect different dimensions of segregation such as isolation, clustering, concentration, or centralization.

Our empirical examples show that in major American cities, the difference between measured levels of the spatial Gini index in any two metro regions arises typically due to differences in population composition rather than physical layout. Results from pairwise comparisons among the 50 largest MSAs the U.S. showed that differences between cities measured by segregation indices that capture the dimensions of evenness and isolation are due typically to variance in the city's population structure. For the dimensions of clustering, concentration and centralization, however, the city's spatial configuration usually explains the difference. In future work, we plan to explore how these patterns vary for a broad collection of spatially-explicit segregation indices with differing definitions of space, including multiscalar segregation profiles (Reardon et al. 2006; Fowler 2016). We also plan to examine how our results may differ when extending our Shapley decomposition approach to measures of multigroup segregation.

Our illustration also highlights some particularly interesting comparisons, especially the difference revealed in Boston vs. Miami. Here, the large difference given by the spatial component is noteworthy given the stark contrast between the demographic makeups of the two cities. Whereas metropolitan Boston's population was approximately six percent Black in 2010, metropolitan Miami's population in 2010 was more than three times that share, at roughly nineteen percent. Despite the wide gulf in demographics between the two cities, it is the way each city's population is distributed across the landscape that drives the difference in their spatial Gini indices.

The comparison between San Jose and Detroit (the largest difference in attribute absolute share) is also interesting because while the maps of each metropolitan region do not look similar, they nonetheless share similar features that include small, densely populated tracts in the center of the region surrounded by large sparsely populated tracts on the periphery. At the regional scale, these maps can induce visual distortion because the large tracts dominate the visualization and mask important variation at smaller scales[8], but the underlying similarity in these regions nonetheless appears as a result of the spatial Gini decomposition. This discussion also raises an important issue with respect to the spatial scale of segregation. In this analysis, we have formulated a generalized spatial Gini index,

[8] We note that higher-resolution images are available upon request and can be generated from the open-source code comprising this analysis.

using a threshold distance of two kilometers (or about a 20-minute walk) to define the egohoods underlying the spatial Gini calculation, but it is unclear how consistent our results will remain if the distance threshold parameter is varied.

Much still has to be done for comparative segregation. An important extension to this work is the development of an inferential component to our decompositional framework. As we mentioned earlier, we see the question of segregation measurement as reflecting a reallocation mechanism, and adopting the bootstrap approach of Allen et al. (2015) but applied to our counterfactual distributions seems like a promising direction in this regard. Additionally, other computationally based approaches to inference such as random labeling (Sastre Gutierrez and Rey 2013), and random spatial permutations (Anselin 1995) can be explored to perform comparative inference given a proper specification of a testable null hypothesis mentioned earlier.

Finally, our measure of the spatial structure component is an aggregate one that reflects the combined effects of differences in spatial extent, number, shape, and size of enumeration units between two cities. Expanding the decomposition framework to consider these specific elements of spatial structure is a promising direction. This extension may draw upon analytics from exploratory spatial data analysis (Rey et al. 2015) and spatial ecology (Dale and Fortin 2014) with the goal of unmasking deeper insights about the role of spatial structure in segregation dynamics.

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