## Absence of Evidence for the Ultimate State of Turbulent Rayleigh-Bénard Convection

There are a number of distinct predictions for the asymptotic behavior of heat transport Nu as the Rayleigh number Ra $\to \infty$ in thermal turbulence described by the fundamental model of Rayleigh-Bénard convection [1]. One is $\mathrm{Nu} = \mathcal{O}(\mathrm{Ra}^{1/3})$ [2–5] and another is the so-called "ultimate" scaling $\mathrm{Nu} = \mathcal{O}(\mathrm{Ra}^{1/2})$ [6], possibly modified by logarithmic corrections ranging from $\mathrm{Ra}^{1/2}/(\log \mathrm{Ra})^{3/2}$ [7] to $\mathrm{Ra}^{1/2}/(\log \mathrm{Ra})^{3}$ [8].

He *et al.* [9] reported measurements [10] of Nu for $\mathrm{Ra} \in [3 \times 10^{12}, 10^{15}]$ citing them as evidence of transition to the ultimate state as characterized by the pre-asymptotic multiparameter fit in Ref. [11]. In this Comment, without questioning the veracity of the measurements (they have been questioned [12]) we show that the data do not support the claim.

Figure 1 shows the data with a linear least-squares fit of log Nu to log Ra yielding $\mathrm{Nu} = 0.0502 \times \mathrm{Ra}^{0.336}$. This agrees remarkably with—indeed extends—the $\mathrm{Nu} = 0.0508 \times \mathrm{Ra}^{1/3}$ fit (within about $\pm 5\%$) to experimental data in the overlapping range $\mathrm{Ra} \in [2 \times 10^{11}, 5 \times 10^{13}]$ [13].

He *et al.*'s data, however, suggests more structure than pure power law scaling. The inset of Fig. 1 shows the $\pm 2.9\%$ $(2\sigma)$ deviations from the pure power law fit with a systematic trend that calls for fitting to functional forms capable of capturing the data's convexity. Data and theories without pure scaling can be compared by examining local slopes $d \log \mathrm{Nu}/d \log \mathrm{Ra}$. If data are sufficiently dense then finite difference approximations may be extracted [14] but the data at hand are not, so local slopes can at best be estimated from derivatives of statistically equivalent fits.

For quadratic, cubic, quartic, and quintic polynomial least-squares fits of log Nu to log Ra, residual deviations are, respectively, 1.19%, 1.09%, 1.08%, and 1.07% $(2\sigma)$ variations with no systematic trends [10]. Thus each is an equally valid quantitative description of the data, and Fig. 2 shows local slopes computed from these equivalent fits.

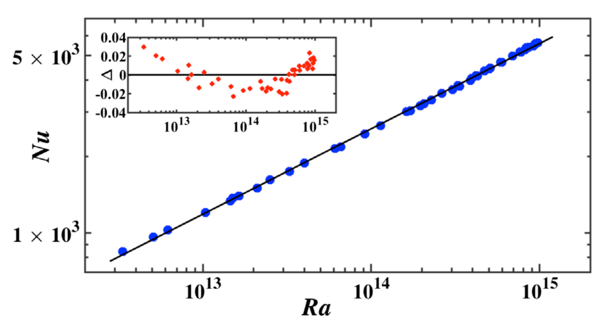


FIG. 1. Nu vs Ra data [10] from Ref. [9] and the power law fit $\mathrm{Nu} = 0.0502 \times \mathrm{Ra}^{0.336}$. Inset: $\Delta \equiv \mathrm{Nu}^{\mathrm{data}}/\mathrm{Nu}^{\mathrm{fit}} - 1$.

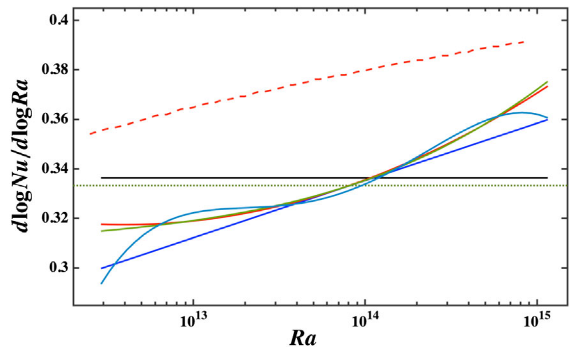


FIG. 2. Solid lines: local slopes from 1st (black), 2nd (blue), 3rd (red), 4th (green), and 5th (light blue) order polynomial fits of log Nu to log Ra. Dashed line: theoretical pre-asymptotic fit from Ref. [11]. Dotted line: $(d \log \mathrm{Nu}/d \log \mathrm{Ra}) = \frac{1}{3}$.

He *et al.* [9] drew a line with $(d \log \mathrm{Nu}/d \log \mathrm{Ra}) = 0.38$ at the high end of their data citing correspondence with a theoretical value from [11] at $\mathrm{Ra} = 10^{14}$, but $0.333 < (d \log \mathrm{Nu}/d \log \mathrm{Ra}) < 0.336$ for all of the equivalent fits at $\mathrm{Ra} = 10^{14}$. They also reported a transition to $\mathrm{Re} \sim \mathrm{Ra}^{1/2}$ Reynolds number scaling (necessary but not sufficient for $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ scaling) for $\mathrm{Ra} > 5 \times 10^{14}$. The scaling fit to those data, however, is $\mathrm{Nu} = 0.0261 \times \mathrm{Ra}^{0.356}$ while local slopes of equivalent fits barely reach $3/8 = 0.375$ (a bound on heat transport dominated by a single horizontal length scale [15]) at $\mathrm{Ra} = 10^{15}$. But the theoretical slope from Ref. [11] is well above 0.39 there. Thus the claim by He *et al.* [9] that their experiment reached the ultimate regime is not justified by their data.

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