Abstract—An opportunistic framework to navigate with differential carrier phase measurements from megaconstellation low Earth orbit (LEO) satellite signals is proposed. A computationally efficient integer ambiguity resolution algorithm is proposed to reduce the size of the integer least-squares (ILS) problem, whose complexity grows exponentially with the number of satellites. The Starlink constellation is used as a specific megaconstellation example to demonstrate the efficacy of the proposed algorithm, showing a 60% reduction in the size of the ILS problem. The joint probability density function of the megaconstellation LEO satellites’ azimuth and elevation angles is derived for efficient and accurate performance characterization of navigation frameworks with LEO satellites, and to facilitate system parameter design to meet desired performance requirements. Experimental results are presented showing an unmanned aerial vehicle (UAV) navigating for 2.28 km exclusively using signals from only two Orbcomm LEO satellites via the proposed framework, achieving an unprecedented position root mean squared error of 14.8 m over a period of 2 minutes.

Index Terms—LEO, megaconstellation, differential carrier phase, navigation, real time kinematic (RTK).

I. INTRODUCTION

The coming decade is slated to witness a space revolution with the launch of tens of thousands of low Earth orbit (LEO) satellites for broadband communication [1]. The promise of utilizing LEO satellites for navigation and timing has been the subject of recent studies [1]–[5]. While some of these studies call for tailoring the broadband protocol to support navigation capabilities [1], [6], other studies propose to exploit existing broadband LEO constellations for navigation in an opportunistic fashion [3]–[5], [7]–[9]. The former studies allow for simpler receiver architectures and navigation algorithms. However, they require significant changes to existing infrastructure, the cost of which private companies such as OneWeb, SpaceX, Boeing, and others, which are planning to aggregate broadly launch tens of thousands of broadband Internet satellites into LEO, may not be willing to pay. Moreover, if the aforementioned companies agree to that additional cost, there will be no guarantees that they would not charge for “extra navigation services.” In this case, exploiting broadband LEO satellite signals opportunistically for navigation becomes the more viable approach. This paper assesses opportunistic navigation with differential carrier phase measurements from broadband LEO satellite signals.

To address the limitations and vulnerabilities of global navigation satellite system (GNSS), opportunistic navigation has received significant attention over the past decade or so [10]–[12]. Opportunistic navigation is a paradigm that relies on exploiting ambient radio signal of opportunity (SOPs) for positioning and timing [13]. Besides LEO satellite signals, other SOPs include AM/FM radio [14]–[16], digital television [17], [18], WiFi [19], [20], and cellular [21]–[27], with the latter showing the promise of a submeter-accurate navigation solution for unmanned aerial vehicles (UAVs) when carrier phase measurements from cellular signals are used [28]–[30].

LEO satellites possess desirable attributes for positioning in GNSS-challenged environments: (i) they are around twenty times closer to Earth compared to GNSS satellites, which reside in medium Earth orbit (MEO), making their received signal power between 24 to 34 dBs higher than GNSS signals; (ii) they will become abundant as tens of thousands of broadband Internet satellites are expected to be deployed into LEO [1]; and (iii) each broadband provider will deploy broadband Internet satellites into unique constellations, transmitting at different frequency bands, making LEO satellite signals diverse in frequency and direction [31]. Moreover, the Keplerian elements parameterizing the orbits of these LEO satellites are made publicly available by the North American Aerospace Defense Command (NORAD) and are updated daily in the two-line element (TLE) files. Using TLEs and orbit determination algorithms (e.g., SGP4), the positions and velocities of these satellites can be known, albeit not precisely. In addition, some of these broadband LEO satellites, such as Orbcomm satellites, are equipped with GPS receivers and broadcast their GPS solution to terrestrial receivers.

This paper considers the problem of navigating exclusively with LEO satellite signals in environments where GNSS signals are unavailable or untrustworthy. To this end, there are several challenges that must be overcome. First, there

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are no publicly available receivers that can produce navigation observables from LEO satellite signals. Second, existing navigation frameworks do not apply in a straightforward fashion to megaconstellation LEO satellites due to the unique error sources associated with megaconstellation LEO satellites. Third, the achievable navigation performance with megaconstellation LEO satellites is not fully characterized. The first two challenges have been partially addressed for Orbcomm satellite signals [8], [32]. This paper makes four contributions that aim to address the aforementioned second and third challenges:

- First, a carrier phase differential (CD)-LEO navigation framework is developed for real broadband LEO satellite signals and an efficient method for resolving carrier phase integer ambiguities in a batch solver is proposed. The performance and complexity of the proposed integer ambiguity resolution method are also characterized.

- Second, the probability density functions (pdfs) of megaconstellation LEO satellites’ azimuth and elevation angles are derived. These pdfs are essential tools to efficiently study the performance of LEO satellite-based navigation.

- Third, the performance of the CD-LEO framework is characterized using the derived pdfs by analyzing (i) the position dilution of precision (PDOP) of megaconstellation LEO satellites, (ii) the measurement residuals due to ephemeris errors, and (iii) the measurement residuals due to integer ambiguity estimation errors as a function of the system design parameters, more precisely, the differential baseline and the batch size. This study allows to design the system parameters to guarantee a desired performance.

- Fourth, novel experimental results are presented showing an unmanned aerial vehicle (UAV) localizing itself with real LEO satellite signals using differential carrier phase measurements to an acceptable level of accuracy.

The high level of precision of carrier phase measurements enables a sub-meter level navigation solution as has been demonstrated in GNSS [33] and cellular SOPs [28]–[30]. However, this precision comes at the cost of added ambiguities that need to be resolved. This paper addresses this challenge for megaconstellation LEO satellites. Consider a receiver onboard a “rover” on Earth making carrier phase measurements to broadband LEO satellites and a “base” station in the vicinity of the rover making carrier phase measurements to the same LEO satellites. One can form the double-difference carrier phase measurements from base and rover measurements and solve for the rover’s position as well as for the resulting integer ambiguities. Without any position priors, the rover cannot perform real-time positioning and must wait until there is enough change in satellite geometry and solve a batch least-squares to estimate its position and the integer ambiguities [34]. To optimally resolve the integer ambiguities, an integer least-squares (ILS) estimator can be employed. However, the complexity of the ILS grows exponentially with the number of ambiguities [35]. With the proposed LEO constellations, hundreds of satellites are expected to be visible at any point in time and almost anywhere on Earth, making the ILS approach infeasible. To address this issue, this paper proposes an integer ambiguity resolution algorithm that approaches the performance of the ILS but with the fraction of its complexity. Once the ambiguities are resolved, the rover can perform real-time positioning.

Aside from integer ambiguities, another major source of error that has to be considered in the CD-LEO framework is the error in the satellite positions obtained from the TLE files. These errors can be on the order of kilometers as the orbit is propagated way beyond the epoch at which the TLE file was generated. Blindly using the satellite positions obtained from the TLE files introduces significant errors in the measurement residuals. Although the double-difference carrier phase measurements will cancel out most of these errors, there will still be significant errors if the base and rover are “too far apart”. These errors are too large to ignore if an accurate navigation solution is desired. This paper characterizes this error and its statistics as a function of the differential baseline, from which the baseline can be designed to guarantee a desirable performance.

The performance of the proposed integer ambiguity resolution algorithm and the magnitude of the CD-LEO measurement residuals due to ephemeris errors heavily depend on the satellite-to-receiver geometry, which is captured by the satellites’ azimuth and elevation angles. Subsequently, it is of paramount importance to characterize the distribution of these angles for LEO megaconstellations. While previous work approximate the angles’ marginal distributions or study the elevation angle distribution for small constellations [36], [37], this paper characterizes the full joint distribution of the azimuth and elevation angles for LEO megaconstellation satellites. This characterization enables several efficient and insightful performance analyses, as well as facilitates performance-driven framework design, i.e., design system parameters to meet desired performance requirements.

The paper is organized as follows. Section II describes the models used, the CD-LEO framework, and the proposed integer ambiguity resolution algorithm. Section III derives the joint pdf of the megaconstellation LEO satellites’ azimuth and elevation angles. Section IV uses these models to characterize the performance of the CD-LEO framework and proposes a methodology to design system parameters to meet a desired performance. Section V presents experimental results demonstrating a UAV navigating with CD-LEO measurements. Concluding remarks are given in Section VI.

II. MODELS AND CD-LEO FRAMEWORK DESCRIPTION

This section describes the models and the CD-LEO framework used in the paper. Note that in the sequel, a satellite will be referred to as a space vehicle (SV).

A. LEO Satellite Position Error

Let \( r_{\text{leo}} \triangleq [x_{\text{leo}}, y_{\text{leo}}, z_{\text{leo}}]^T \) denote the \( l \)-th LEO SV true position vector in the East-North-Up (ENU) frame. If the true LEO SV positions are not known, they may be estimated
utilizing TLE files and orbit determination algorithms (e.g., SGP4), resulting with an estimate \( \hat{\mathbf{r}}_{\text{leo}} \). Denote the estimation error as \( \delta \mathbf{r}_{\text{leo}} \triangleq \mathbf{r}_{\text{leo}} - \hat{\mathbf{r}}_{\text{leo}} \). Due to the large ephemeris errors in TLE files, \( \| \delta \mathbf{r}_{\text{leo}} \|_2 \) can be on the order of a few kilometers, with most of the error being in the along-track coordinate. To illustrate this, the position error of 2 Orbcomm LEO SVs, FM 108 and FM 116, is calculated by differencing (i) the LEO SVs’ position estimate obtained from on-board GPS receivers and broadcasted in the Orbcomm message and (ii) the estimates obtained from TLE files and SGP4 software. The total SV position error magnitude \( \| \delta \mathbf{r}_{\text{leo}} \|_2 \) for each SV and the along-track SV position error magnitude are shown in Fig. 1. Fig. 1 also shows the range residual due to ephemeris errors as observed by a terrestrial LEO receiver, i.e., it is the difference between (i) the true range between the LEO SV and LEO receiver and (ii) the range estimated using the LEO position estimate obtained from TLE files. It can be seen from Fig. 1 that (i) the SV position error can be significant (around 5 km for FM 116), (ii) most of the error is in the along-track direction, and (iii) the range residual is on the order of the SV position error. In order to reduce the effect of ephemeris errors, a navigating vehicle can employ simultaneous tracking and navigation (STAN) framework to estimate the LEO SVs’ states simultaneously with the vehicle’s states [5], [38]. Alternatively, a reference receiver, or base, may be deployed to provide differential corrections, which will significantly reduce the range residuals. This reduction is characterized in Section IV as a function of the SV elevation and azimuth angles. The sequel describes the carrier phase measurement model and the CD-LEO framework.

**B. LEO Carrier Phase Observation Model**

In this paper, availability of carrier phase measurements from LEO SV signals is assumed. For example, the receiver proposed in [8] may be used to obtain carrier phase measurements from Orbcomm LEO SV signals. Note that since LEO satellite orbits are above the ionosphere, their signals will suffer from ionospheric and tropospheric delays. Let \( \delta t_{\text{iono},i}(k) \) and \( \delta t_{\text{trop},i}(k) \) denote the ionospheric and tropospheric delays from the \( i \)-th LEO SV to the \( i \)-th receiver at time-step \( k \), respectively, where \( i \) denotes either the base B or the rover R. An estimate of the ionospheric and tropospheric delays, denoted \( \hat{\delta} t_{\text{iono},i}(k) \) and \( \hat{\delta} t_{\text{trop},i}(k) \), respectively, may be obtained using standard models [34]. After ionospheric and tropospheric delay correction, the carrier phase measurement \( \chi_i(k) \) expressed in meters can be parameterized in terms of the receiver and LEO SV states as

\[
\chi_i(k) = \| \mathbf{r}_i - \mathbf{r}_{\text{leo}}(k) \|_2 + c [\delta t_{\text{leol}}(k) - \delta t_{\text{leol},i}(k)] + \lambda_i N_i(k) + \delta t_{\text{trop},i}(k) + c \delta t_{\text{trop},i}(k) + \nu_i(k),
\]

where \( \mathbf{r}_i \triangleq [x_i, y_i, z_i]^T \) is the \( i \)-th receiver’s position vector in ENU; \( c \) is the speed of light; \( \delta t_{\text{leol}} \) and \( \delta t_{\text{leol},i} \) are the \( i \)-th receiver’s and \( i \)-th LEO SV clock biases, respectively; \( \delta t_{\text{iono},i}(k) \) and \( \delta t_{\text{trop},i}(k) \) are the ionospheric and tropospheric delay errors, respectively; \( \lambda_i \) is the \( i \)-th LEO SV signal’s wavelength; \( N_i(k) \) is the carrier phase ambiguity; and \( \nu_i(k) \) is the measurement noise, which is modeled as a discrete-time zero-mean white Gaussian sequence with variance \( \sigma_i^2(k) \). It is assumed that \( \{ \nu_i(k) \}_{i=1}^L \) are independent and identically distributed, but with different values of \( \sigma_i^2(k) \).

**C. CD-LEO Framework**

The framework consists of a rover and a base receiver in an environment comprising \( L \) visible LEO SVs. The base receiver (B), is assumed to have knowledge of its own position state, e.g., (i) a stationary receiver deployed at a surveyed location or (ii) a high-flying UAV with access to GNSS. The rover (R) does not have knowledge of its position. The base communicates its own position and carrier phase observables with the rover. The LEO SVs’ positions are known through the TLE files and orbit determination software, or by decoding the transmitted ephemerides, if any. Fig. 2 illustrates the base/rover CD-LEO framework.

In what follows, the objective is to estimate the rover’s position using double difference carrier phase measurements. However, such measurements have inherent ambiguities that must be resolved. Recall that \( (L - 1) \) measurements are obtained from \( L \) visible satellites [34], with one unknown ambiguity associated with each double difference measurement. Using only one set of carrier phase measurements with no \textit{a priori} knowledge on the rover position results in an underdetermined system: \( (L + 2) \) unknowns (3 position states and \( (L - 1) \) ambiguities) with only \( (L - 1) \) measurements. Therefore, when \textit{no a priori} information on the position of the rover is
known, a batch weighted nonlinear least-squares (B-WNLS) over a window of \( K \) time-steps is employed to solve for the rover’s position and ambiguities. The rover could either remain stationary or move during the batch window. Subsequently, the rover uses measurements collected at different times in a batch estimator, resulting in an overdetermined system [34]. The total number of measurements will be \( K \times (L - 1) \) in the batch window. If the rover remains stationary, the total number of unknowns will remain \( L + 2 \). Otherwise, the number of unknowns becomes \( 3K + L - 1 \) (3 position states at each time-step and \( L - 1 \) ambiguities). The dimensions of the unknown parameters and the measurement vector set a necessary condition on \( K \) and \( L \) in order to obtain a solution. Once an estimate of the ambiguities is obtained, the rover position can be estimated in real-time using a point-solution weighted nonlinear least-squares (PS-WNLS) estimator. Both the B-WNLS and PS-WNLS estimate the rover’s position from LEO double difference carrier phase measurements, which is described next.

### D. LEO Double Difference Carrier Phase Observation Model

First, define the single difference across receivers adjusted for the base-LEO SV range as

\[
z_i^{(R,B)}(k) = z_i^{(R)}(k) - z_i^{(B)}(k) + ||r_{RB} - \hat{r}_{leo}(k)||_2
\]

\[= ||r_{RB} - \hat{r}_{leo}(k)||_2 + c_\delta t_i^{(R,B)}(k) + \lambda_t N_i^{(R,B)} + \delta t_i^{(R,B)}(k) + \delta \tilde{r}_{leo}(k) + \delta \tilde{r}_{trop}(k) + v_i^{(R,B)}(k),
\]

(2)

where

\[
\delta t_i^{(R,B)}(k) \triangleq \delta t_{RB}(k) - \delta t_{RB}(k),
\]

\[
\lambda_t N_i^{(R,B)} \triangleq \lambda_t N_i^{(R)} - \lambda_t N_i^{(B)},
\]

\[
\delta t_i^{(R,B)}(k) \triangleq \delta t_i^{(R)}(k) - \delta t_i^{(B)}(k),
\]

\[
\delta \tilde{r}_{leo}(k) \triangleq \tilde{r}_{leo}(k) - \tilde{r}_{leo}(k),
\]

\[
\delta \tilde{r}_{trop}(k) \triangleq \tilde{r}_{trop}(k) - \tilde{r}_{trop}(k),
\]

\[
v_i^{(R,B)}(k) \triangleq v_i^{(R)}(k) - v_i^{(B)}(k).
\]

It was observed from real data that \( \delta \tilde{r}_{leo}(k) \) and \( \delta \tilde{r}_{trop}(k) \) are negligible for VHF signals [32]. For higher frequency signals, this difference becomes even less significant as ionospheric delays decrease with the square of the carrier frequency [34]. Subsequently, \( z_i^{(R,B)}(k) \) is approximated as

\[
z_i^{(R,B)}(k) \approx h_i^{(R)}(k) + c_\delta t_i^{(R,B)}(k) + \lambda_t N_i^{(R,B)} + \hat{r}_{leo}(k) + v_i^{(R,B)}(k),
\]

(3)

where

\[
h_i^{(R)}(k) \triangleq ||r_{RB} - \hat{r}_{leo}(k)||_2,
\]

\[
\hat{r}_{leo}(k) \triangleq \hat{r}_{leo}(k) - \hat{r}_{leo}(k),
\]

\[
\hat{r}_{trop}(k) \triangleq \hat{r}_{trop}(k) - \hat{r}_{trop}(k),
\]

\[
\hat{r}_{leo}(k) \triangleq ||r_{RB} - \hat{r}_{leo}(k)||_2 - ||r_{RB} - \hat{r}_{leo}(k)||_2.
\]

In vector form, the measurement equation becomes

\[
z(k) \triangleq h_R(k) + c_\delta t_r^{(R,B)}(k)1_L + \hat{A} + \hat{r}_{leo}^{(R,B)}(k) + \nu(k),
\]

(4)

where \( 1_L \) is an \( L \times 1 \) vector of ones and

\[
z(k) \triangleq \left[ z_1^{(R,B)}(k), \ldots, z_L^{(R,B)}(k) \right]^T
\]

\[
h_R(k) \triangleq \left[ h_1^{(R)}(k), \ldots, h_L^{(R)}(k) \right]^T
\]

\[
A \triangleq \left[ \lambda_t N_1^{(R,B)}, \ldots, \lambda_t N_L^{(R,B)} \right]^T
\]

\[
\hat{r}_{leo}^{(R,B)}(k) \triangleq \left[ \hat{r}_{leo}^{(R)}(k), \ldots, \hat{r}_{leo}^{(B)}(k) \right]^T
\]

\[
\nu(k) \triangleq \left[ \nu_1^{(R,B)}(k), \ldots, \nu_L^{(R,B)}(k) \right]^T.
\]

The covariance matrix of \( \nu(k) \) is given by

\[
R(k) \triangleq \text{diag} \left[ \sigma_1^{(R,B)}(k)^2, \ldots, \sigma_L^{(R,B)}(k)^2 \right],
\]

where

\[
\sigma_i^{(R,B)}(k)^2 \triangleq \sigma_i^{(R)}(k)^2 + \sigma_i^{(B)}(k)^2.
\]

Next, the double difference measurements are obtained. Without loss of generality, the first LEO SV is taken as the reference, yielding the double difference measurements

\[
\bar{z}(k) \triangleq T z(k) = \bar{H}_R(k) + \bar{A} + \bar{r}_{leo}^{(R,B)}(k) + \bar{v}(k),
\]

(5)

where \( \bar{H}_R(k) \triangleq T H_R(k), \bar{A} \triangleq T A, \bar{r}_{leo}^{(R,B)}(k) \triangleq T \hat{r}_{leo}^{(R,B)}(k), \bar{v}(k) \triangleq T \nu(k), \) and \( T \triangleq \left[ -I_{L-1} \ 1 \right]_{(L-1) \times (L-1)} \) is the differencing matrix. Note that the covariance matrix of \( \bar{v}(k) \) is given by \( R(k) = T R(k) T^T \). If \( \lambda_t \) is not equal to \( \lambda_1 \), then \( \bar{A} \) cannot be expressed as \( \lambda N \), where \( N \) is a vector of integers. If \( \lambda_t = \lambda_t \) \( \forall l \), then \( \bar{A} = \lambda N \) and the integer ambiguity resolution algorithm described in Subsection II-G is used to resolve the integers.

### E. B-WNLS Solution

If the rover remains stationary during the batch window, then the parameter to be estimated is given by

\[
x_{\text{stationary}} \triangleq \left[ r_{RB}^{T}(0), \ 1 \right]^T,
\]

otherwise, it is given by

\[
x_{\text{mobile}} \triangleq \left[ r_{RB}^{T}(0), \ldots, r_{RB}^{T}(K - 1), 1 \right]^T.
\]

The parameter \( x_{\text{stationary}} \) or \( x_{\text{mobile}} \) are estimated from the collection of measurements from 0 to \( (K - 1) \) given by

\[
\bar{z}^K \triangleq \left[ \bar{z}^{T}(0), \ldots, \bar{z}^{T}(K - 1) \right]^T,
\]

to yield an estimate \( \hat{x}_{\text{stationary}} \) or \( \hat{x}_{\text{mobile}} \), respectively. Let \( \hat{A} \) denote the estimate of \( A \). For a mobile receiver, the estimation error covariance \( Q_A \) associated with \( \hat{A} \) is given by

\[
Q_A = \left( \sum_{k=0}^{K-1} \Psi_k^T \Omega_k \Psi_k \right)^{-1},
\]

where \( \Psi_k \) is a square-root of \( \Psi_k \triangleq \hat{R}^{-1}(k) \), and

\[
\Omega_k \triangleq \left[ \begin{array}{l} \Psi_k \ \Psi_k \end{array} \right]^T \left[ \begin{array}{l} \Psi_k \ \Psi_k \end{array} \right],
\]

\[
\Psi_k \triangleq \Psi_k^T \left[ \begin{array}{l} H^T(k) \ Y_k \ Y_k \end{array} \right]^{-1} H^T(k) \ Y_k \Psi_k.\]
where $\mathbf{H}(k)$ is the geometry matrix at time-step $k$, which can be parameterized by the SVs’ azimuth and elevation angles $\{\phi_i^L\}_{i=1}^L$ and $\{\theta_i\}_{i=1}^L$, respectively, according to

$$\mathbf{H}(k) = \begin{bmatrix} \cos[\theta_L(k)] \sin[\phi_L(k)] & \cos[\theta_L(k)] \cos[\phi_L(k)] & \sin[\theta_L(k)] \\ \vdots & \vdots & \vdots \\ \cos[\theta_L(k)] \sin[\phi_L(k)] & \cos[\theta_L(k)] \cos[\phi_L(k)] & \sin[\theta_L(k)] \end{bmatrix}. $$

For a stationary receiver, $Q_N$ is given by

$$Q_A = \left[ \sum_{k=0}^{K-1} Y_k - B_K \mathbf{A}_K^{-1} \mathbf{B}_K^T \right]^{-1},$$

$$A_K \triangleq \sum_{k=0}^{K-1} \mathbf{H}^T(k) \mathbf{Y}_k \mathbf{TH}(k), \quad B_K \triangleq \sum_{k=0}^{K-1} \mathbf{Y}_k \mathbf{TH}(k).$$

If $A = \lambda N$, then an estimate of the integers $\tilde{N}$ and an associated estimation error covariance $Q_N$ are obtained according to

$$\tilde{N} = \frac{1}{\lambda} \hat{A}, \quad Q_N = \frac{1}{\lambda^2} Q_A.$$

Note that if all measurement noise variances are equal, i.e., $\sigma_i^2(k) = \sigma^2 \forall i, l, k$, then $Q_A$ and $Q_N$ may be expressed as

$$Q_A = \sigma^2 Q_A, \quad Q_N = \mu^2 Q_A,$$

where $\mu^2 \triangleq \sigma^2/\lambda^2$ and $\bar{Q}_A$ is obtained by setting $R(k) \equiv 2\sigma^2 I_{L \times L}$.

**F. PS-WNLS Solution**

After resolving the ambiguities, a point solution for the rover position can be computed at each time-step. Let $\tilde{N}$ denote the integer estimates of $N$. The double difference measurement vector adjusted for the integer ambiguities is hence expressed as

$$\tilde{z}_f(k) \triangleq \hat{z}(k) - \lambda \tilde{N} = \hat{h}_k + \lambda \tilde{N} + \tilde{z}_{leo}(k) + \bar{v}(k),$$

where $\tilde{N} \triangleq N - \hat{N}$ is the integer ambiguity error. The rover uses $\tilde{z}_f(k)$ to solve for $r_{\tilde{N}_k}(k)$ in a PS-WNLS. For small measurement noise variances, which is the case for high frequency carriers, the positioning performance heavily depends on $\tilde{z}_{leo}(k)$, which is characterized in Section IV.

**G. Reduced-Sized Integer Least Squares Algorithm**

When the proposed LEO constellations are fully deployed, hundreds of LEO satellites will be visible from almost anywhere on Earth. As an example, Fig. 3 shows a heat map of the number of visible Starlink LEO SVs for an elevation mask of 5°. Dozens of satellites will still be visible for even higher elevation masks. For example, 60 Starlink LEO SVs will be visible over Irvine, CA, U.S.A. for a 25° elevation mask. For such number of satellites, it is impractical to solve the ILS, as its complexity grows exponentially with the number of integer ambiguities [35]. This subsection proposes an integer ambiguity resolution algorithm, referred to as reduced-size ILS, which approaches the performance of the Least-squares Ambiguity

**Decoration Adjustment (LAMBDA) method [35], but with a significantly smaller fraction of the LAMBDA method’s complexity. The reduced-size ILS relies on the tradeoff between complexity and performance. That is, for every integer, a test is formulated to determine whether the Integer Rounding (IR) method, which has negligible complexity, is a good estimate of the corresponding integer, or whether the integer must be estimated using an ILS.**

$$\text{Fig. 3. Heat map of the number of visible Starlink LEO satellites at any point on Earth for an elevation mask of 5°.}$$

The test is of the form

$$[Q_N]_{ll} \leq \frac{1}{\mu^2 \eta},$$

where $[Q_N]_{ll}$ is the $l$-th diagonal element of $Q_N$ after decorrelation. The choice of $\eta$ is discussed in Subsection IV-B. An integer estimated by IR is said to be reliable if it satisfies (6). Next, define the set of reliable integers $S_R$ as

$$S_R = \left\{ l \mid [Q_N]_{ll} \leq \frac{1}{\mu^2 \eta} \right\}.$$ (7)

The complimentary set is denoted by $\bar{S}_R$. The next step of the algorithm is reducing the size and performing the ILS on the reduced set $S_R$. To this end, the vector $\tilde{N}$ and its corresponding covariance matrix are rearranged as

$$\hat{N}_P = \begin{bmatrix} \hat{N}_{S_{R_{ith}}}^T \\ \hat{N}_{S_{R_{ith}}}^T \end{bmatrix}, \quad Q_{N_P} = \begin{bmatrix} Q_{S_{R_{ith}}^R} & Q_{S_{R_{ith}}^R S_{R_{ith}}} \\ Q_{S_{R_{ith}}^R S_{R_{ith}}} & Q_{S_{R_{ith}}^R} \end{bmatrix},$$

where each element of $\tilde{N}_{S_{R_{ith}}}$ belongs to the set $\{N_l \mid l \in S_R\}$. Let $\tilde{N}_{S_{R_{ith}}}$ denote the IR solution of $\tilde{N}_{S_{R_{ith}}}$. Subsequently, the original ILS problem may be reduced as

$$\tilde{N}_{eff} = \arg \min_{\tilde{N}_{eff} \in \mathbb{Z}^{L_{eff}}} \| \tilde{N}_{eff} - N_{eff} \|^2_{Q_{S_{R_{ith}}}},$$

where $\tilde{N}_{eff} \in \mathbb{R}^{L_{eff}}$ is the “effective” real-valued estimate of the remaining integers to be resolved using the ILS, and is computed using the minimum mean square error (MMSE) estimate given by

$$\tilde{N}_{eff} = \tilde{N}_{S_{R_{ith}}} + Q_{S_{R_{ith}} S_{R_{ith}}} Q_{S_{R_{ith}}}^{-1} \left( \hat{N}_{S_{R_{ith}}} - \tilde{N}_{S_{R_{ith}}} \right).$$

Let $\tilde{N}$ denote the final integer estimate, which combines the reliable IR estimates and the estimates obtained from the reduced ILS. It is shown in Subsection IV-B that $L_{eff}$ may approach zero at some regimes of $\mu^2$. This implies that the proposed method achieves the LAMBDA method’s performance without any ILS search for many practical realizations of $Q_N$. 
III. DERIVATION OF THE JOINT DISTRIBUTION OF MEGACONSTELLATION LEO SVS’ AZIMUTH AND ELEVATION ANGLES

In this section, the joint pdf of megaconstellation LEO SVs’ azimuth and elevation angles is derived. This pdf offers an efficient way to characterize the performance of the CD-LEO framework as well as to enable performance-driven design of the CD-LEO framework, such as the differential baseline and the B-WNLS batch window. The orbit of a LEO SV is defined by its inclination angle \( i \) and orbital altitude \( h_l \). Define the normalized orbital radius

\[
\alpha_l \triangleq 1 + \frac{h_l}{R_E},
\]

where \( R_E \) is the average radius of the Earth, which is assumed to be spherical. The SV’s position will be uniformly distributed over its orbital plane. The surface over which the LEO SV to be spherical. The SV’s position will be uniformly distributed where

\[
\nu_l \triangleq \frac{\theta_l}{\sin i},
\]

and orbital altitude \( h_l \). Define the normalized orbital radius

\[
\alpha_l \triangleq 1 + \frac{h_l}{R_E},
\]

where \( R_E \) is the average radius of the Earth, which is assumed to be spherical. The SV’s position will be uniformly distributed over its orbital plane. The surface over which the LEO SV can exist is defined as \( B_0(i, R_{h_l}) \), which is a capless sphere of radius \( R_{h_l} \). Let \( \phi_l \) and \( \theta_l \) denote the azimuth and elevation angles, respectively, of the \( l \)-th LEO SV. These angles are specific to a receiver location given by longitude \( \lambda_0 \) and latitude \( \varphi_0 \). Moreover, let \( \gamma(\theta_l) \) denote the angle between the LEO SV and receiver position vectors. Using the law of sines, \( \gamma(\theta_l) \) can be expressed as

\[
\gamma(\theta_l) = \cos^{-1}\left(\frac{1}{\alpha_l} \cos \theta_l \right) - \theta_l.
\]

A. Stationarity of Elevation and Azimuth Angle Distribution

It is important to establish the stationarity of the azimuth and elevation angle distribution. The analysis in Section IV assumes that the elevation angle are stationary and uncorrelated in time. This assumption becomes valid for megaconstellations where the distribution does remain close to stationary. To illustrate this, the Starlink LEO SV constellation is shown in Fig. 5(a). The constellation parameters are obtained from the parameters in Table I. Fig. 5(b) shows an SV orbit with orbital radius \( \alpha R_E \) and phase \( \nu \) between successive SVs.

B. Satellite Longitude and Latitude Distribution Model

Given an SV’s longitude \( \lambda_l \) measured from the ascending node, it can be shown that the SV’s latitude \( \varphi_l \) is given by

\[
\varphi_l = \sin^{-1}\left[\sin \theta_l \cdot \sin \lambda_l\right].
\]

By design, \( \lambda_l \) is uniformly distributed over the \([0, 2\pi]\) interval. Subsequently, using random variable transformation, the pdfs of \( \lambda_l \) and \( \varphi_l \) are given by

\[
f_\lambda(\lambda_l) = \begin{cases} \frac{1}{2\pi} & 0 \leq \lambda_l < 2\pi \\ 0 & \text{elsewhere} \end{cases}
\]

\[
f_\varphi(\varphi_l) = \begin{cases} \pi \sqrt{\sin^2 \theta_l - \sin^2 \varphi_l} & |\varphi_l| < \lambda_l \\ 0 & \text{elsewhere} \end{cases}
\]

with the joint pdf given by

\[
f_{\lambda, \varphi}(\lambda_l, \varphi_l) = f_\lambda(\lambda_l)f_\varphi(\varphi_l).
\]

The histogram obtained from the Starlink constellation and the analytical pdfs for \( i_l = 53^\circ \) are shown in Fig. 6.

C. Satellite Elevation and Azimuth Distribution Model

The joint pdf of \( \phi_l \) and \( \theta_l \), denoted by \( f_{\phi_l, \theta_l}(\phi_l, \theta_l) \), can be obtained from \( f_{\lambda, \varphi}(\lambda_l, \varphi_l) \) through coordinate transformation. To this end, the mapping from the pair \( (\phi_l, \theta_l) \) to \( (\lambda_l, \varphi_l) \) must be established. The result is captured in the following lemma.
Lemma III.1. Given a spherical Earth, an SV orbit characterized by $i_t$ and $\alpha_t$, and a receiver’s longitude $\lambda_0$ and latitude $\varphi_0$, the inverse mapping from $(\phi_t, \theta_t)$ to $(\lambda_t, \varphi_t)$ is given by
\[
y(\phi_t, \theta_t) = \begin{bmatrix} \lambda_t \\ \varphi_t \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left( \frac{a_{02}(\phi_t, \theta_t)}{a_{01}(\phi_t, \theta_t)} \right) \\ \sin^{-1} \left( a_{03}(\phi_t, \theta_t) \right) \end{bmatrix},
\]
where
\begin{align*}
a_{01}(\phi_t, \theta_t) &\equiv \sin \left[ \gamma (\theta_t) \right] f_{01}(\phi_t, \theta_t) + \frac{1}{\alpha_t} \cos \varphi_0 \cos \lambda_0, \\
a_{02}(\phi_t, \theta_t) &\equiv \sin \left[ \gamma (\theta_t) \right] f_{02}(\phi_t, \theta_t) + \frac{1}{\alpha_t} \cos \varphi_0 \sin \lambda_0, \\
a_{03}(\phi_t, \theta_t) &\equiv \sin \left[ \gamma (\theta_t) \right] f_{03}(\phi_t, \theta_t) + \frac{1}{\alpha_t} \sin \varphi_0,
\end{align*}
f_{01}(\phi_t, \theta_t) \equiv \cos \varphi_0 \cos \lambda_0 \tan \theta - \sin \lambda_0 \sin \phi - \sin \varphi_0 \cos \lambda_0 \cos \phi, \\
f_{02}(\phi_t, \theta_t) \equiv \cos \varphi_0 \sin \lambda_0 \tan \theta + \cos \lambda_0 \sin \phi - \sin \varphi_0 \sin \lambda_0 \cos \phi, \\
f_{03}(\phi_t, \theta_t) \equiv \sin \varphi_0 \tan \theta + \cos \varphi_0 \cos \phi.
\]

Proof. For a spherical Earth, the $l$-th satellite position in Earth-centered Earth-fixed (ECEF) may be expressed as
\[
r_{\text{leo}} = \alpha_t R_E \cos \varphi_0 \cos \lambda_0 \cos \varphi + \frac{1}{\alpha_t} \cos \lambda_0 \sin \lambda_0 \sin \varphi_0.
\]
Subsequently, given $r_{\text{leo}}$, the longitude and latitude $\lambda_t$ and $\varphi_t$, respectively, may be obtained according to
\[
\begin{align*}
\lambda_t &= \tan^{-1} \left( \frac{e_{02}^2 r_{\text{leo}}}{e_{01}^2 r_{\text{leo}}^2} \right), \\
\varphi_t &= \sin^{-1} \left( \frac{e_{03}^2 r_{\text{leo}}}{\| r_{\text{leo}} \|_2} \right).
\end{align*}
\]
The SV position in ENU can also be expressed as
\[
r_{\text{enu},l} = d_l \begin{bmatrix} \cos \theta_l \sin \phi_l \cos \theta_l \sin \phi_l \sin \theta_l \sin \phi_l \sin \theta_l \sin \phi_l \sin \theta_l \sin \phi_l \end{bmatrix}^T,
\]
where $d_l$ is the distance between the SV and the receiver. Using the law of sines, $d_l$ may be expressed as
\[
d_l = \alpha_t R_E \frac{\sin \left[ \gamma (\theta_l) \right]}{\cos \theta_l}.
\]

Using coordinate frame transformation, the SV position in ECEF can be obtained from $r_{\text{enu}}$ through
\[
r_{\text{enu}} = R \begin{bmatrix} \varphi_0, \lambda_0 \end{bmatrix} r_{\text{enu},l} + r_{\text{r},l},
\]
where
\[
R = \begin{bmatrix} 0 & \cos \lambda_0 & \sin \lambda_0 \\ - \sin \lambda_0 & \cos \lambda_0 & 0 \\ - \cos \varphi_0 \cos \lambda_0 & - \sin \varphi_0 \sin \lambda_0 & \cos \varphi_0 \cos \lambda_0 \end{bmatrix}.
\]

Equation (17) is readily obtained by combining (18)–(21).

Finally, $f_{\Phi, \Theta}(\phi_t, \theta_t)$ is given by
\[
f_{\Phi, \Theta}(\phi_t, \theta_t) = \begin{cases} \frac{|a_{03}(\phi_t, \theta_t)| \sin \lambda_t}{2 \pi \sqrt[4]{\min \left( \frac{\sqrt{\gamma^2 - \gamma_0^2}(\phi, \theta_t)}{a_{03}(\phi_t, \theta_t)} \right) \sin \lambda_t}} & \text{if } |a_{03}(\phi_t, \theta_t)| \sin \lambda_t \geq \min \left( \frac{\sqrt{\gamma^2 - \gamma_0^2}(\phi, \theta_t)}{a_{03}(\phi_t, \theta_t)} \right), \\
0 & \text{elsewhere}, \end{cases}
\]
where $J_y(\phi_t, \theta_t) \equiv \begin{bmatrix} \frac{\partial \lambda_t}{\partial \phi_t} & \frac{\partial \lambda_t}{\partial \theta_t} \\ \frac{\partial \varphi_t}{\partial \phi_t} & \frac{\partial \varphi_t}{\partial \theta_t} \end{bmatrix}$. The expression of $J_y(\phi_t, \theta_t)$ and its determinant are given in Appendix A.

D. Azimuth and Elevation Joint Distribution for a Set Elevation Mask

Since the visible SVs have non-negative elevation angles, one is interested to know the pdf for $\theta_t \geq 0$. In practice, a positive elevation mask $\theta_{\text{min}}$ is set. The pdf for $\theta_t \geq \theta_{\text{min}}$ is hence given by
\[
f_{\Phi, \Theta}(\phi_t, \theta_t) = \begin{cases} \frac{|a_{03}(\phi_t, \theta_t)| \sin \lambda_t}{C_{\Phi, \Theta} \sqrt{\min \left( \frac{\sqrt{\gamma^2 - \gamma_0^2}(\phi, \theta_t)}{a_{03}(\phi_t, \theta_t)} \right) \sin \lambda_t}} & \text{if } (\phi_t, \theta_t) \in D_{\Phi, \Theta}, \\
0 & \text{otherwise}, \end{cases}
\]
where the domain $D_{\Phi, \Theta}$ is defined as
\[
D_{\Phi, \Theta} = \left\{ (\phi_t, \theta_t) \ | |a_{03}(\phi_t, \theta_t)| \sin \lambda_t \geq \sin \theta_t \right\}.
\]
and the normalization constant $C_{\Phi, \Theta}$ is given by
\[
C_{\Phi, \Theta} = 2 \pi^2 \int_{D_{\Phi, \Theta}} f_{\Phi, \Theta}(\phi_t, \theta_t) d\phi_t d\theta_t.
\]
Note that one can find the average number of visible satellites $\bar{L}$ according to
\[
\bar{L} = L \times \Pr \left[ \theta_t \geq \theta_{\text{min}} \right] = \frac{L C_{\Phi, \Theta} \sin \lambda_t}{2 \pi^2},
\]
where $L$ is the total number of SVs in the constellation.

E. Multi-constellation Azimuth and Elevation Joint Distribution

Recall that the pdf in (23) is constellation-specific, i.e., it is parameterized by one inclination angle $i_t$ and one normalized orbital radius $\alpha_t$. For the case of multi-constellations, as is the case for LEO megaconstellation, the joint pdf for all constellations, each of which defined by is given by
\[
\sum_{j=1}^{J} \frac{L_j}{2 \pi^2} \int_{D_{\Phi, \Theta}} f_{\Phi, \Theta}(\phi_t, \theta_t) d\phi_t d\theta_t.
\]
where $J$ is the total number of constellations, $L_j$ is the number of satellites in the $j$-th constellation, $J f_{\Phi, \Theta}(\phi_t, \theta_t)$ is the pdf of the $j$-th constellation obtained according to (23), and $p_j \equiv \frac{L_j}{\sum_{j=1}^{J} L_j}$ is the probability of a particular SV being part of the $j$-th constellation.

IV. PERFORMANCE CHARACTERIZATION AND PERFORMANCE-DRIVEN CD-LEO FRAMEWORK DESIGN

This section studies the PDOP, shows a methodology to obtain the optimal threshold for the proposed reduced-ILS method, and characterizes the measurement error in the PSWNLs due to satellite position errors.

A. PDOP Characterization

One important measure of the estimability (i.e., degree of observability) of the receiver’s position is the PDOP. Assuming equal measurement noise variances, the PDOP in the CD-LEO framework is given by $\text{PDOP} = \text{trace}[\mathbf{P}]$, where $\mathbf{P}$ is the PDOP matrix given by
\[
\mathbf{P} = 2 \left( \mathbf{H}^T \mathbf{T}^T (\mathbf{T} \mathbf{T}^T)^{-1} \mathbf{H} \right)^{-1}.
\]
Another metric of interest is the horizontal dilution of precision (HDOP), which gives a measure of the estimability of the horizontal components of the position vector. This metric is appropriate to study in the case where the rover is equipped with an altimeter and is using LEO signals mainly to estimate horizontal components of the position vector. This is validated in the HDOP cdf, shown in Fig. 7 for a receiver in Irvine, CA, U.S.A., and for three elevation angle masks: 5°, 25°, and 35°. The HDOP is mostly below 0.6 for elevation angles of 25° or below, and above 2 almost all the time for elevation angles of 35° or below. In fact, the HDOP is mostly below 0.6 for elevation angles of 25°, showing that highly accurate horizontal positioning may be achieved.

B. Reduced-Size ILS Threshold Selection

The optimal threshold $\eta$ for the integer ambiguity resolution algorithm presented in Subsection II-G was characterized numerically for the Starlink constellation with the parameters shown in Table I. The receiver was assumed stationary and was located on the UCI campus. Several realizations of SV elevation and azimuth angles were generated for different values of $\sigma^2$, and the threshold $\eta(\sigma^2)$ was selected as the minimum threshold that maximizes the success rate of the proposed method. The elevation mask was set to 35°.

Fig. 8(a) demonstrates the success rate, i.e., $Pr [N = N]$. [40], for (i) the method proposed in Subsection II-G using the numerically computed threshold, (ii) the IR method, and (iii) the LAMBDA method for $\theta_{\min} = 35^\circ$, $L = 25$, and $K = 7$. It can be observed that the performance of the proposed method approaches that of the LAMBDA method as $\mu^2$ decreases. Fig. 8(b) shows the average size of $N_{\text{eff}}$ denoted by $L_{\text{eff}}$, which is the dimension of the unknown integer vector in the proposed reduced-size ILS algorithm. It can be seen that $L_{\text{eff}}$ is at most 32% of the size of the original ILS problem, reducing the complexity of the ILS search by orders of magnitude.

C. Measurement Errors Due to Ephemeris Errors

Recall that the SV positions are obtained by non-precise ephemerides. The effect of the estimated SV position error onto the CD-LEO measurement is first characterized as a function of the SV elevation angle. Next, the pdf of the elevation angle derived in III is used to obtain the cdf of the measurement error due to ephemeris errors. A first-order Taylor series expansion around $\hat{r}_{i,\text{leo}}$ yields

$$\| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2 \approx \| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2 + h_{i,B}^T \Delta r_b,$$

where $h_{i,B}$ is the unit line-of-sight vector between the $i$-th LEO SV and the $i$-th receiver. A first-order Taylor series expansion around $h_{B,i}$ yields

$$h_{R,i} \approx h_{B,i} + \frac{1}{\| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2} \left( I - h_{B,i} h_{B,i}^T \right) \Delta r_b,$$

where $\Delta r_b$ is the baseline vector between the base and the rover. Subsequently, the residual due to SV position errors can be expressed as

$$\tilde{r}_{i,\text{leo}}^{(R,B)} = \| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2 - \| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2 - \| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2 + \| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2
\Rightarrow \tilde{r}_{i,\text{leo}}^{(R,B)} \approx \frac{\Xi_i \hat{r}_{i,\text{leo}}}{\| r_{r_i} - \hat{r}_{i,\text{leo}} \|^2_2},$$

where $\Xi_i \triangleq \left( I - h_{B,i} h_{B,i}^T \right)$. The residual can be interpreted as the dot product between the baseline projected onto the range-space of $\Xi_i$, denoted $\mathcal{R}(\Xi_i)$, and the SV position error vector also projected onto $\mathcal{R}(\Xi_i)$, as shown in Fig. 9.

It can be seen from Fig. 9 that the magnitude of $\tilde{r}_{i,\text{leo}}^{(R,B)}$ is maximized when the SV’s ground track is collinear with the baseline. In such cases, using (20), the magnitude of $\tilde{r}_{i,\text{leo}}^{(R,B)}$ may be bounded according to

$$| \tilde{r}_{i,\text{leo}}^{(R,B)} | \leq \left( \theta_i^{(B)} \cos \theta \right) \frac{\| \Delta r_b \|^2_2 \| r_{r_i} \|^2_2}{\| r_{r_i} \|^2_2},$$

where

$$g(\theta, \alpha) = \frac{\sin \theta \cos \theta \sqrt{\alpha^2 - \cos^2 \theta}}{\alpha^2 \sin \cos^{-1} \left( \frac{\cos \theta}{\alpha} \right)} - \theta.$$
Subsequently, the cdf of \( \tilde{r}_{\text{leo}(\text{R,B})} \) can be characterized from (26) and the joint distribution of the LEO SVs’ azimuth and elevation angles derived in Section III. To this end, the cdf of \( g(\theta, \alpha) \) is calculated for the Starlink LEO constellation using the parameters in Table I. The receiver was assumed to be on the UCI campus. The cdf was computed for three elevation masks: \( \theta_{\text{min}} = 5^\circ, \theta_{\text{min}} = 25^\circ, \) and \( \theta_{\text{min}} = 35^\circ \). The cdf of \( g(\theta, \alpha) \) is shown in Fig. 10(a), and the expected value of \( g(\theta, \alpha) \), denoted by \( \mathbb{E}[g(\theta, \alpha)] \) is shown in Fig. 10(b) as a function of \( \theta_{\text{min}} \).

Fig. 10. (a) Cdf of \( g(\theta, \alpha) \) for \( \theta_{\text{min}} = 5^\circ, \theta_{\text{min}} = 25^\circ, \) and \( \theta_{\text{min}} = 35^\circ \). (b) Expected value of \( g(\theta, \alpha) \) as a function of \( \theta_{\text{min}} \).

V. EXPERIMENTAL RESULTS

This section presents experimental results of a UAV navigating with signals from Orbcomm LEO SVs via the CD-LEO framework discussed in Section II. First, the experimental setup is discussed. Then, the navigation frameworks implemented in the experiments and their associated results are presented.

A. Experimental Setup

To demonstrate the CD-LEO framework discussed in Section II, the rover was a DJI Matrice 600 UAV equipped with an Ettus E312 USRP, a high-end VHF antenna, and a small consumer-grade GPS antenna to discipline the on-board oscillator. The base was a stationary receiver equipped with an Ettus E312 USRP, a custom-made VHF antenna, and a small consumer-grade GPS antenna to discipline the on-board oscillator. The receivers were tuned to a 137 MHz carrier frequency with 2.4 MHz sampling bandwidth, which covers the 137–138 MHz band allocated to Orbcomm SVs. The LEO carrier phase measurements were produced at a rate of 4.8 kHz and were downsampled to 10 Hz. The the base’s position was surveyed on Google Earth, and the UAV trajectory was taken from its on-board navigation system, which uses GNSS (GPS and GLONASS), an inertial measurement unit (IMU), and other sensors. The hovering horizontal precision of the UAV is reported to be 1.5 meters.

B. CD-LEO Framework Experimental Results

Single difference measurements provide more information on the SV-to-receiver geometry than double difference measurements since the differencing matrix \( T \) is not applied [42]. This comes at the cost of an additional state to be estimated: the common clock bias \( \delta r_{\text{c}}^{\text{(R,B)}}(k) \). To this end, the UAV’s position and velocity states were estimated along with the common clock bias \( \delta r_{\text{c}}^{\text{(R,B)}}(k) \) and the constant ambiguity \( N_{\text{c}}^{\text{(R,B)}} \). Note that \( N_{\text{c}}^{\text{(R,B)}} \) was lumped into \( \delta r_{\text{c}}^{\text{(R,B)}}(k) \). The UAV’s position and velocity were assumed to evolve according to a nearly constant velocity model, and the common clock state was assumed to evolve according to the standard model of double integrator driven by noise as discussed in [43], [44].
UAV. To study the effect of ephemeris errors on the navigation solution, two EKFs were implemented: (i) one that uses the Orbcomm LEO SV positions estimated by the SVs’ on-board GPS receiver and (ii) one that uses the Orbcomm LEO SV positions estimated from TLE files. The estimated trajectories are shown in Fig. 13(b) and Fig. 13(c). The EKF position estimation errors are shown in Fig. 14 along with the $3\sigma$ bounds. Note that since the UAV mainly travels in the North direction, the East direction becomes poorly estimable; hence, the $3\sigma$ bounds in the East direction increase at a higher rate than the $3\sigma$ bound in the North direction, as shown in Fig. 14. The common clock bias estimate and the corresponding $\pm 3\sigma$ bounds are also shown in Fig 14. The 3-D position root mean squared errors (RMSEs) and final errors for both EKFs are shown in Table II.

**C. Non-Differential LEO Framework Experimental Results**

To demonstrate the importance of the CD-LEO framework, a non-differential LEO framework is implemented. To this end, the UAV’s position and velocity are estimated in an EKF using the non-differential measurements in (1). In this case, two clock biases must be estimated capturing the difference between the receiver’s clock bias and each of the Orbcomm LEO SVs’ bias. The same dynamics models and initialization method employed in Subsection V-B were used in the non-differential framework. Similarly to Subsection V-B, two EKFs were implemented: (i) one that uses the Orbcomm LEO SV positions estimated by the SVs’ on-board GPS receiver and (ii) one that uses the Orbcomm LEO SV positions estimated from TLE files. The estimated trajectories are shown in Fig. 13(b) and Fig. 13(c). The EKF position estimation errors are shown in Fig. 15 along with the associated $3\sigma$ bounds. The clock bias estimate associated with FM 108 and the corresponding $\pm 3\sigma$ bounds are also shown in Fig 14. The 3-D position RMSEs and final errors for both EKFs are shown in Table II.

**D. Discussion**

Table II summarizes the experimental results for the CD-LEO and non-differential LEO frameworks. It can be seen from Fig. 1 that the residuals in the non-differential carrier phase measurements are on the order of kilometers, which explains the unacceptably large RMSEs of the non-differential framework. While using the SV positions transmitted by the Orbcomm SVs reduces the RMSEs, the errors remain unacceptably large in the non-differential framework due to other unmodeled errors. Such errors cancel out in the CD-LEO framework, yielding acceptable performance whether SV positions from GPS or TLE are used. The accuracy of these results is unprecedented, considering that (i) only 2 LEO SVs were used, (ii) no other sensors were fused into the navigation, and (iii) these LEO SVs are not intended for navigation and are exploited opportunistically. The double difference residual due to ephemeris errors was calculated, and is shown in Fig. 16. During the experiment, the baseline varied between 20 m and 200 m. According to Subsection II-A, the function $g(\theta, \alpha)$ averages to 1.346 for the Orbcomm constellation, which has an inclination angle of 45° and orbital altitude of 800 km and $\theta_{\text{min}} = 5^\circ$. From the SV position errors in Fig. 1, the expected range of the residuals is from 0.3 to 16 cm. It can be seen from Fig. 16 that the magnitude of the double difference residual is on the order of centimeters and matches the expected values, showing (i) the robustness of the CD-LEO framework against ephemeris errors and (ii) the accuracy of the performance analysis framework discussed in Section IV.

**VI. Conclusion**

This paper proposed a differential framework for opportunistic navigation with carrier phase measurements from megaconstellation LEO satellites. A computationally efficient integer ambiguity resolution algorithm was proposed to reduce the size of the ILS problem, with simulation using the Starlink constellation as a specific megaconstellation example showing a 60% reduction in the size of the ILS problem while maintaining optimality. Moreover, the joint pdf of the megaconstellation LEO satellites’ azimuth and elevation angle was derived. A performance characterization of the proposed CD-LEO framework was conducted using derived joint azimuth and elevation angle pdf, showing the potential of LEO satellite signals for precise navigation. The performance characterization conducted herein also facilitates system parameter design to meet desired performance requirements. Experimental results were presented showing a UAV navigating for 2.28 km exclusively using signals from only two Orbcomm LEO SVs.

**TABLE II**

<table>
<thead>
<tr>
<th>Framework</th>
<th>SV position source</th>
<th>RMSE</th>
<th>Final error</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD-LEO</td>
<td>GPS</td>
<td>14.8 m</td>
<td>3.9 m</td>
</tr>
<tr>
<td>CD-LEO</td>
<td>TLE</td>
<td>15.0 m</td>
<td>4.8 m</td>
</tr>
<tr>
<td>Non-differential</td>
<td>GPS</td>
<td>338.6 m</td>
<td>590.4 m</td>
</tr>
<tr>
<td>Non-differential</td>
<td>TLE</td>
<td>405.4 m</td>
<td>759.5 m</td>
</tr>
</tbody>
</table>
via the proposed framework with an unprecedented position RMSE of 14.8 m over a period of 2 minutes.

Fig. 14. EKF position estimation error and ±3σ bounds for the CD-LEO framework. The estimates and sigma bounds for the case where SV positions are obtained from GPS and the ones for the case where the SV positions are obtained from TLE files are almost identical.

Fig. 15. EKF position estimation error and ±3σ bounds for non-differential LEO framework. The sigma bounds for the case where SV positions are obtained from GPS and the ones for the case where the SV positions are obtained from TLE files are almost identical.

Fig. 16. Double difference residuals due to ephemeris errors for Orbcomm LEO SVs FM 108 and FM 116.

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**APPENDIX A**

**DERIVATION OF THE JACOBIAN**

Define $g_\gamma(\theta_1) \triangleq \partial \sin [\gamma(\theta_1)] / \partial \theta_1 = \left[ \sin \theta_1 / \sqrt{1 - \cos^2 \theta_1} \right] \cos [\gamma(\theta_1)],$

$g_\phi^0(\phi_1, \theta_1) \triangleq \partial f_{01}(\phi_1, \theta_1) / \partial \phi_1 = \sin \lambda_0 \sin \phi_1 - \sin \phi_0 \cos \lambda_0 \cos \phi_1,$

$g_\theta^0(\phi_1, \theta_1) \triangleq \partial f_{02}(\phi_1, \theta_1) / \partial \theta_1 = \cos \phi_0 \cos \lambda_0,$

$g_\phi^0(\phi_1, \theta_1) \triangleq \partial f_{02}(\phi_1, \theta_1) / \partial \phi_1 = -\cos \lambda_0 \sin \phi_1 - \sin \phi_0 \sin \lambda_0 \cos \phi_1,$

$g_\theta^0(\phi_1, \theta_1) \triangleq \partial f_{03}(\phi_1, \theta_1) / \partial \theta_1 = \cos \phi_0 \sin \lambda_0,$

$g_\theta^0(\phi_1, \theta_1) \triangleq \partial f_{03}(\phi_1, \theta_1) / \partial \phi_1 = \sin \phi_0,$

$b_{01}^0(\phi_1, \theta_1) \triangleq \partial a_{01}(\phi_1, \theta_1) / \partial \phi_1 = \sin [\gamma(\theta_1)] g_{01}^0(\phi_1, \theta_1),$

$b_{02}^0(\phi_1, \theta_1) \triangleq \partial a_{02}(\phi_1, \theta_1) / \partial \phi_1 = \sin [\gamma(\theta_1)] g_{02}^0(\phi_1, \theta_1),$

$b_{02}^0(\phi_1, \theta_1) \triangleq \partial a_{03}(\phi_1, \theta_1) / \partial \phi_1 = \sin [\gamma(\theta_1)] g_{03}^0(\phi_1, \theta_1).$

Since by definition $\| r_{\text{vel}} \|_2 = \alpha R_{\text{e}},$ then the following holds

$a_{01}^\alpha(\phi_1, \theta_1) + a_{02}^\alpha(\phi_1, \theta_1) + a_{03}^\alpha(\phi_1, \theta_1) = 1. \tag{27}$

Subsequently, the elements of the Jacobian matrix $J_y(\phi_1, \theta_1)$ are given by

$\partial \lambda_1 \triangleq b_{02}^{\phi_1}(\phi_1, \theta_1) a_{01}(\phi_1, \theta_1) - b_{01}^{\phi_1}(\phi_1, \theta_1) a_{02}(\phi_1, \theta_1),$

$\partial \lambda_1 \triangleq b_{02}^{\phi_1}(\phi_1, \theta_1) a_{01}(\phi_1, \theta_1) - b_{01}^{\phi_1}(\phi_1, \theta_1) a_{02}(\phi_1, \theta_1),$

$\partial \phi_1 \triangleq \sqrt{a_{01}^2(\phi_1, \theta_1) + a_{02}^2(\phi_1, \theta_1)},$

$\partial \phi_1 \triangleq \sqrt{a_{01}^2(\phi_1, \theta_1) + a_{02}^2(\phi_1, \theta_1)},$

and the determinant of $J_y(\theta_1)$ is given by

$|J_y(\theta_1)| = \frac{a_{01} b_{02}^{\phi_1} b_{03}^{\phi_1} b_{02}^{\phi_1} b_{03}^{\phi_1} - a_{02} b_{01}^{\phi_1} b_{03}^{\phi_1} b_{01}^{\phi_1} b_{03}^{\phi_1}}{(a_{01}^2 + a_{02}^2)^2}. \tag{28}$