Evaluating Central Banks' tool kit: Past, present, and future☆

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A R T I C L E   I N F O

A B S T R A C T

We develop a structural DSGE model to systematically study the principal tools of unconventional monetary policy – quantitative easing (QE), forward guidance, and negative interest rate policy (NIRP) – as well as the interactions between them. To generate the same output response, the requisite NIRP and forward guidance interventions are twice as large as a conventional policy shock, which seems implausible in practice. In contrast, QE via an endogenous feedback rule can alleviate the constraints on conventional policy posed by the zero lower bound. Quantitatively, QE1-QE3 can account for two thirds of the observed decline in the "shadow" Federal Funds rate. In spite of its usefulness, QE does not come without cost. A large balance sheet has consequences for different normalization plans, the efficacy of NIRP, and the effective lower bound on the policy rate.

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1. Introduction

In response to the Financial Crisis and ensuing Great Recession of 2007–2009, the Fed and other central banks pushed policy rates to zero (or, in some cases, slightly below zero). With conventional policy unavailable, central banks launched a sequence of experimental unconventional policy interventions. The principal tools of unconventional policy include large scale asset purchases (or quantitative easing, QE), forward guidance (FG), and negative interest rate policy (NIRP). Even ten years subsequent to the Crisis, many advanced economies are still plagued by near-zero policy rates. Further, an emerging consensus is that global forces and demographic trends have lowered the "natural" or "neutral" rate of interest; see Laubach and Williams (2003) and Del Negro et al. (2017). This means that the Great Recession is likely not a one-off event and heretofore unconventional tools will become a more conventional part of central banks' tool kit. The time is therefore ripe for a comprehensive, quantitative review of these tools in a unified modeling framework.

Our paper makes a number of important contributions to the literature. First, rather than introducing them in a piece-meal fashion as in the existing literature, we develop the first quantitative DSGE model that builds in all three different types of unconventional tools together. This allows us to study similarities, differences, and potential interactions between

them. Second, we draw attention to two distinct channels by which NIRP can transmit to the economy. On the one hand, NIRP in our model can be stimulative via a forward guidance type channel, but on the other hand, it erodes the net worth of financial intermediaries, which works in the opposite direction. This countervailing "banking" channel is similar to the theoretical results developed in Ulate (2018). Third, we propose a novel and intuitive endogenous specification for QE that is similar to a conventional Taylor rule. We find that endogenous QE can successfully neutralize the adverse consequences of a binding zero lower bound (ZLB). Despite its usefulness, QE does not come without a cost. A fourth contribution of our paper is to address questions relating to the size of a central bank's balance sheet, including consequences of different normalization plans as well as the interaction between the size of the balance sheet and NIRP. Fifth, we provide a first attempt at endogenizing an effective lower bound (ELB) for policy rates. The ELB depends on the overall size of a central bank's balance sheet. Finally, we introduce a novel timing assumption to model forward guidance, where our approach ameliorates the so-called forward guidance "puzzle" (Del Negro et al., 2015).

Though it shares similarities with canonical medium-scale models (e.g. Christiano et al., 2005, and Smets and Wouters, 2007), our model differs from standard models in some important ways. Firms are required to issue long term bonds to finance part of investment in new physical capital, similarly to Carlstrom et al. (2017). Asset markets are segmented in that households can only indirectly access long term bonds via holding short term debt (i.e. deposits) with financial intermediaries. Financial intermediaries are introduced in a way similar to Gertler and Karadi (2011, 2013). A costly enforcement problem results in an endogenous leverage constraint on intermediaries. This results in excess returns and time-varying interest rate spreads, which allows QE type policies to have real effects. New to the literature, we also formally model the central bank's balance sheet with interest-bearing reserves. Along with a constraint on minimum reserve balances at intermediaries, this allows for the interest rate on reserves to potentially differ from the deposit rate, thereby permitting us to explore the effects of negative interest rate policy.

During normal times, conventional monetary policy in our model involves the central bank setting the interest rate on reserves (what we hereafter refer to as the policy rate), which equals the deposit rate, according to a (Taylor, 1993) type rule.[1,2] QE in our model entails the central bank purchasing (or selling) long maturity private or government bonds, financed via the creation (or destruction) of reserves.[3] QE policies can be undertaken whether conventional policy is constrained by the ZLB or not, though they are more effective when short term rates are fixed. Forward guidance in our model involves cutting the desired short term policy rate implied by the Taylor rule when the actual policy rate is constrained by zero. This signals a commitment to lower the actual short term rate after the ZLB constraint has lifted. We introduce into our model a reduced form parameter capturing the perceived credibility of forward guidance announcements.

Consistent with the experience of many countries that have experimented with NIRP, our model assumes that the policy rate can go negative, while the deposit rate (the economically relevant short term interest rate) cannot. To implement NIRP, the central bank can require intermediaries to hold more reserves than they would like. On the one hand, NIRP affects the economy much like forward guidance – it signals the intent to lower economically relevant short term rates in the future, which in turn can affect long term rates in the present. Because it involves a tangible action rather than mere words, it may be more credible than conventional forward guidance. A countervailing mechanism at work in the model is that NIRP reduces intermediary net worth, which, other factors held constant, works to tighten credit conditions. As we discuss in Section 6, a "tiering" system, such as that recently implemented by the Swiss National Bank, could alleviate this offseting "banking" channel by exempting some fraction of reserves from negative rates.

We begin by comparing exogenous changes in both conventional and unconventional policies. In our model, a 100 basis point expansionary shock to the policy rate results in output rising by about 0.5 percent at peak. To generate a similar output expansion when conventional policy is constrained by the ZLB for two years in expectation, the central bank must expand its balance sheet by about 4 percent relative to steady state output. Generating the same output increase via FG or NIRP at the ZLB requires a cut in the desired policy rate of more than twice the magnitude of a conventional policy shock. With a pre-QE size of the Fed's balance sheet, NIRP is less stimulative than a fully-credible FG, but more than a partially-credible one.

Significant stimulus from NIRP requires very large cuts in policy rates. Such large cuts into negative territory have not been attempted in practice for a variety of political and financial reasons. Forward guidance may be plagued by credibility issues. Quantitative easing, in contrast, is relatively effective in our model. It has also been the most popular of unconventional tools among central banks. Most prominently, the Bank of Japan, the Fed, and the European Central Bank have each expanded their balance sheets via QE operations to roughly 5 trillion dollars, which amounts to anywhere between 25 and 100 percent of their corresponding GDPs.

Having analyzed the effects of exogenous changes in unconventional policy tools, we then focus on endogenous quantitative easing in attempt to speak to the Great Recession experience. In particular, we posit a Taylor-type rule for quantitative

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[1] For a study of Ramsey optimal instrument rules in a model sharing similarities with ours, see Mau (2019).

[2] Several related papers (e.g. Gertler and Karadi, 2011; Gertler and Karadi, 2013 and Carlstrom et al., 2017) do not model reserves at all and instead think of conventional policy as directly setting the deposit rate via a Taylor rule.

[3] Clarida (2012) distinguishes between different types of large scale asset purchases, calling purchases of long term government bonds "quantitative easing" and purchases of privately-issued debt "credit easing." Our model features both government and private sector long maturity bonds. We will use the term "quantitative easing" as it has been used in the financial press to refer to any type of large scale asset purchase (either government or non).

easing when the policy rate is constrained by the ZLB.[4] Endogenous quantitative easing serves as a highly effective substitute for conventional policy. This result holds conditional on different shocks, but more interestingly, also in an unconditional simulation with several adverse demand shocks meant to mimic the experience of the US during the Great Recession. In this simulation exercise, the endogenous QE rule involves expanding the central bank's balance sheet to about 25 percent of GDP, which corresponds to the Fed's balance sheet after QE3. This balance sheet expansion provides stimulus roughly equivalent to a decline in the policy rate of about 2 percent, which is closely in line with the empirical estimates of the shadow rate from Wu and Xia (2016).

Although endogenous QE can serve as an effective antidote to the ZLB, its use does not come without costs. Many central banks around the world have accumulated a large balance sheet over the course of their QE operations, and they are confronting questions related to the cost associated with unwinding now that the ZLB period is over. We quantitatively study different normalization plans, sometimes referred to as quantitative tightening (QT). A smooth balance sheet normalization is preferred to either an immediate normalization after a ZLB period has ended or to carrying a significantly larger balance sheet going forward. Interestingly, the normalization plan after the ZLB is expected to be over can impact how the economy fares during the ZLB. Specifically, if agents expect the central bank to maintain a large balance sheet after the ZLB, the economy could experience a deeper recession even with the central bank accumulating a larger quantity of bonds.

We also study interactions between the size of a central bank's balance sheet and the use of other unconventional tools. Other things being equal, NIRP is less effective the larger a balance sheet a central bank carries. Fixing other parameters but increasing the size of the central bank's steady state balance sheet to 38% of GDP (which matches the Euro area as of the end of 2018), NIRP actually becomes mildly contractionary. Furthermore, there exists a cutoff negative policy rate below which financial intermediaries would voluntarily choose to exit. We refer to this cutoff rate as the effective lower bound (ELB) and find that it can be quite close to zero for an economy in which the central bank carries a large balance sheet.

1.1. Literature

The last decade has witnessed an explosion in work studying unconventional monetary policy. Kuttner (2018) provides an accessible overview. There are both reduced-form empirical papers and quantitative theoretical analyses; our paper is most similar to the latter. To date, the majority of this literature has typically studied unconventional policies in isolation. We are one of the first to propose a theoretical framework in which the three principal types of unconventional policies may be simultaneously studied.

The principal modeling frameworks to study QE are the preferred-habitat model and DSGE models. Hamilton and Wu (2012) and Greenwood and Vayanos (2014) rely on the preferred-habitat model of Vayanos and Vila (2009) to study the empirical effects of the maturity structure of publicly held debt on the term structure.[5] Ray (2019) incorporates a preferred habit framework into a New Keynesian model to study QE. In a similar vein, (Gertler and Karadi, 2011; 2013) and (Carlstrom et al., 2017) build DSGE models with segmented asset markets and financial frictions to quantify the effects of QE on output and other economic aggregates. The structure of financial intermediaries in our model is similar to these papers. We differ in that we model the central bank as financing its operations through the issuance of interest-bearing reserves, which appears to be the operating framework on which the Federal Reserve has settled moving forward. The inclusion of reserves in our model is also central to studying negative interest rate policy. We also focus on endogenous QE policy, which we consider to be more policy-relevant than exogenous QE. Unlike most papers in the literature, (Foerster, 2015) also studies exit strategies from QE programs, and some of our conclusions regarding exit are consistent with his. The difference between our paper and his is the modeling framework. He uses a regime-switching model, whereas we set up a more realistic environment that takes the ZLB into account. In follow-up work to the present paper, (Sims and Wu, 2019; 2020) develop a small-scale version of our modeling framework that focuses solely on QE to study normative issues related to optimal policy. Sims and Wu, forthcoming provide an accessible summary of the results in these papers.

Del Negro et al. (2015) point out that in standard monetary DSGE models, the effects of forward guidance become stronger when the intended rate change happens farther off into the future, which they refer to as the "forward guidance puzzle." McKay et al. (2016) propose a possible resolution of the puzzle relying on heterogeneity and incomplete asset markets.[6] We model forward guidance with a novel timing assumption as a shock to an underlying desired policy rate in the present that, in turn, affects the actual policy rate once a ZLB episode is over. This ameliorates many of the puzzling theoretical results concerning the efficacy of forward guidance. We also introduce a reduced-form parameter to capture the degree of commitment or credibility involved in the implementation of forward guidance.

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[4] Gertler and Karadi (2011) and Ray (2019) both focus on alternative formulations of endogenous QE rules. Ray (2019) writes down a QE rule on the demand shifter for preferred habitat investors that endogenously reacts to inflation; while (Gertler and Karadi, 2011) posit a rule that responds to excess premia. Our rule, like a conventional Taylor rule, instead adjusts the central bank's bond holdings to inflation and output.

[5] Krishnamurthy and Vissing-Jorgensen (2011) use an event study methodology to quantify the effects of the different waves of QE in the United States on a variety of different interest rates. Gagnon et al. (2011) is one of the first papers to empirically study the effects of large scale asset purchases. They find that these purchases were successful in driving down long term interest rates primarily through lower risk and term premia. Bauer and Rudebusch (2014), in contrast, emphasize how large scale asset purchases can affect long term yields by signaling the expected path of short term rates.

[6] Campbell et al. (2012) empirically study the responses of asset prices and interest rates to Federal Reserve communications both before and during the Great Recession.

Several recent papers study NIRP. Abo-Zaid and Garín (2016) argue that in a New Keynesian model with credit frictions and money demand the optimal nominal interest rate may be negative. Wu and Xia (2018) extend the shadow rate term structure model to allow for a negative interest rate and they emphasize a forward guidance aspect to NIRP. Our model is closely related to this mechanism, though their term structure model is mute about the responses of macro aggregates to NIRP. Ulate (2018) and Eggertsson et al. (2019) both construct quantitative DSGE models with a bank-lending channel and argue that negative policy rates erode bank profits, which reduces pass-through from policy rates. A similar channel is at play in our model. Different than these papers, we stress the importance of the size of the central bank's balance sheet for the strength of this channel. Moreover, we discuss an additional forward guidance channel and emphasize that NIRP can be thought of as forward guidance with commitment.

On balance, our conclusions align with a number of recent papers that argue that unconventional policies are an (almost) perfect substitute for conventional monetary policy and that the ZLB (or ELB) on policy rates is ultimately not much of a hindrance to effective stabilization policy. See, for example, Swanson and Williams (2014), Wu and Xia (2016), Wu and Zhang (2017, 2019), Garín et al. (2019), Debortoli et al. (2016), Mouabbi and Sahuc (2017), and Swanson (2018a,b). Different from the literature, we build all three unconventional policy tools in a unified structural model. This facilitates an understanding of underlying mechanisms and allows us to compare and contrast the effectiveness of different types of unconventional policy interventions as well as how these policies interact with one another.

The remainder of the paper is organized as follows. Section 2 lays out the model, and Section 3 describes how our model can accommodate various monetary policy tools. Section 4 compares alternative exogenous monetary policy shocks. Section 5 studies endogenous quantitative easing and recreates the experience of the Great Recession. Section 6 discusses future issues facing central banks related to the size of their balance sheets. Section 7 offers concluding thoughts.

2. Model

In this section, we flesh out the key ingredients of our model. The principal actors in the model are households, labor unions, several types of production firms, financial intermediaries, a fiscal authority, and a central bank.

Although it shares many familiar ingredients with canonical medium-scale DSGE models (i.e. Christiano et al., 2005, and Smets and Wouters, 2007), several features of our model are less common. First, similar to Carlstrom et al., 2017, we assume that production firms must issue perpetual bonds (Woodford, 2001) to finance part of their new investment. These bonds allow for a succinct presentation and can easily be mapped into a long term zero coupon bond for quantitative analysis. Second, we formally model financial intermediaries. As in Gertler and Karadi (2011, 2013), intermediaries finance themselves with net worth and short term debt (what we call deposits) and hold long term bonds issued by firms and the government as well as reserves issued by the central bank. Markets are segmented in the sense that households cannot hold long term bonds directly. Because of a costly enforcement problem, intermediaries face an endogenous leverage constraint that results in excess returns. This feature, in conjunction with our assumption that firms must issue long term bonds to finance investment, generates an "investment wedge" that allows QE type policies to have real economic effects. Third, our framework is also unique in that we model the central bank as financing its operations through the issuance of interest-bearing reserves, which appears to be the operating framework on which the Federal Reserve has settled moving forward. Combined with a constraint on minimum reserve balances at financial intermediaries, this allows us to study negative interest rate policy (NIRP) when the deposit rate is constrained by the zero lower bound.

Below, we only highlight the non-standard ingredients that are directly relevant for incorporating unconventional monetary policy tools; the rest of the model is presented in detail in Appendix A. There are six relevant sectors in the model economy: households are discussed in Appendix A.1, the labor market in Appendix A.3, and the fiscal authority in Appendix A.5; financial intermediaries are discussed in Section 2.2 and the production side of the economy in Section 2.3. A description of monetary policy is in Section 3.

2.1. Long term bonds

A representative wholesale firm (described in Section 2.3) and the fiscal authority (discussed in Appendix A.5) issue long term bonds to finance their activities. We follow Woodford (2001) in modeling these bonds as perpetuities with decaying coupon payments. Let $\kappa \in [0, 1]$ denote the decay parameter for coupon payments.[7] A one unit bond issue in period t for Q_t dollars obligates the issuer to a coupon payment of one dollar in $t + 1$, κ dollars in $t + 2$, κ^2 dollars in $t + 3$, and so on.

Let $CF_{m,t}$ denote the new nominal issuance of these bonds by the representative wholesale firm (which we index by m). The total coupon liability due in period t for the firm based on past issuances is:

$$F_{m,t-1} = CF_{m,t-1} + \kappa CF_{m,t-2} + \kappa^2 CF_{m,t-3} + \cdots \tag{2.1}$$

An attractive feature of these perpetual bonds is that one need not keep track of the entire sequence of past issues. Rather, one need only keep track of the period t and $t - 1$ total coupon liabilities. Iterating (2.1) forward one period, one observes:

$$CF_{m,t} = F_{m,t} - \kappa F_{m,t-1} \tag{2.2}$$

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[7] For simplicity, we assume that κ is the same for both private and government bonds, although this need not be the case.

New bond issuances in period t trade at market price Q_t. Given the decaying coupon structure, bonds issued in period $t - j$ must trade at $\kappa^j Q_t$, for $j \geq 0$. This means that one only needs to keep track of the bond price for the current issue rather than the entire sequence of prices for past issues. In particular, the value of a bond portfolio consisting of all the outstanding private bonds in period t may be written:

$$Q_t F_{m,t} = Q_t CF_{m,t} + \kappa Q_t CF_{m,t-1} + \kappa^2 Q_t CF_{m,t-2} + \cdots \tag{2.3}$$

Nominal government bonds have an identical structure to privately issued bonds. $CB_{G,t}$ denotes new issuances in period t, $B_{G,t}$ denotes the total nominal liability on current and past issuances, and $Q_{B,t}$ is the market price.

2.2. Financial intermediaries

Financial intermediaries are structured similarly to Gertler and Karadi (2011, 2013). Each period there is a fixed mass of intermediaries indexed by i. Intermediaries finance themselves with net worth, $N_{i,t}$, and deposits taken from households, $D_{i,t}$. Each period, a fraction $1 - \sigma$, with $\sigma \in [0, 1]$, stochastically exit and return their net worth to their household owner. They are replaced by an equal number of new intermediaries that begin with real start up funds of X given to them by their household owner.

Intermediaries hold privately issued bonds, $F_{i,t}$; government issued nominal bonds, $B_{i,t}$; and interest-bearing reserves, $RE_{i,t}$, which are held on account with the central bank. The inclusion of reserves is different than (Gertler and Karadi, 2013) and ends up being useful in thinking about negative interest rate policy. The balance sheet condition of a typical intermediary is:

$$Q_t F_{i,t} + Q_{B,t} B_{i,t} + RE_{i,t} = D_{i,t} + N_{i,t} \tag{2.4}$$

A financial intermediary accumulates net worth until stochastically exiting. Net worth for surviving intermediaries evolves according to:

$$N_{i,t} = \left(R_t^F - R_{t-1}^d\right) Q_{t-1} F_{i,t-1} + \left(R_t^B - R_{t-1}^d\right) Q_{B,t-1} B_{i,t-1} + \left(R_{t-1}^{re} - R_{t-1}^d\right) RE_{i,t-1} + R_{t-1}^d N_{i,t-1} \tag{2.5}$$

R_{t-1}^{re} is the (gross) interest rate on reserves, which is set by the central bank and known at $t - 1$. R_{t-1}^d is the deposit rate, which is determined in equilibrium. The first three terms are the excess returns from holding private bonds, government bonds, and reserves relative to the cost of funding via deposits. The last term measures cost savings from financing with net worth as opposed to deposits. R_t^F and R_t^B are the realized holding period returns on private and government issued bonds and satisfy:

$$R_t^F = \frac{1 + \kappa Q_t}{Q_{t-1}} \tag{2.6}$$

$$R_t^B = \frac{1 + \kappa Q_{B,t}}{Q_{B,t-1}} \tag{2.7}$$

The objective of an intermediary is to maximize its expected terminal net worth where discounting is by the stochastic discount factor of the household, $\Lambda_{t,t+1}$. Consider the problem of an intermediary continuing after period t. There is a $1 - \sigma$ probability that it will exit after $t + 1$, a $(1 - \sigma)\sigma$ probability that it will exit after $t + 2$, and so on. Accordingly, its objective is

$$V_{i,t} = \max \ (1 - \sigma) \mathbb{E}_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} n_{i,t+j}, \tag{2.8}$$

where $n_{i,t} = N_{i,t}/P_t$ is real net worth, with P_t the price of final output.

A financial intermediary faces two constraints. One is a costly enforcement problem as in Gertler and Karadi (2011, 2013). A financial intermediary can choose to abscond with some assets at the end of a period rather than continuing as an intermediary. If an intermediary does this, depositors can recover a fraction of the intermediary's assets, with the intermediary retaining the rest. For depositors to be willing to lend to intermediaries, it must not be optimal for the intermediary to divert funds in this way, which we refer to as going into bankruptcy. Accordingly:

$$V_{i,t} \geq \theta_t \left(Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}\right) \tag{2.9}$$

In (2.9), the right hand side of the inequality represents the (real) funds that a financial intermediary can keep should it choose to enter bankruptcy, while the left hand side is the value of continuing as an intermediary.[8] Should it choose to divert, an intermediary can keep a stochastic fraction of its private bonds, θ_t. It can keep the fraction $\theta_t \Delta$ of government bonds, where $0 \leq \Delta \leq 1$. This means that it is easier for the intermediary to divert private bonds than government bonds. We choose to make θ_t stochastic (and exogenous). θ_t may be thought of as a type of credit shock – when θ_t increases, depositors can recover a smaller fraction of an intermediary's assets in the event of bankruptcy, which in turn makes them

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[8] $f_{i,t} = F_{i,t}/P_t$ and $b_{i,t} = B_{i,t}/P_t$ are real private and government bond holdings, respectively.

less willing to lend funds. This will have the effect of driving interest rate spreads up, which is a hallmark of financial crises. We assume that the third type of asset held by intermediaries – reserves – is fully recoverable by depositors in the event of bankruptcy.

The second constraint faced by intermediaries is a reserve requirement. Relative to the recent literature, this is new in our model, though binding reserve requirements are common in older papers. We assume that intermediaries are required to hold a minimum level of reserves that is set by the central bank. The reserve requirement is time-varying and proportional to an intermediary's deposits. Letting $re_{i,t} = RE_{i,t}/P_t$ and $d_{i,t} = D_{i,t}/P_t$ denote real reserve holdings and real deposits, the constraint is:

$$re_{i,t} \geq \varsigma_t d_{i,t} \tag{2.10}$$

For most of the analysis, the reserve requirement will be non-binding. (2.10) is included so as to potentially allow for a negative interest rate on reserves. When this constraint binds for all intermediaries, $\varsigma_t = \frac{re_t}{d_t}$.

All financial intermediaries will behave in the same way with identical optimality conditions. These are

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^F - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta_t \tag{2.11}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_{t+1}^B - R_t^d \right) = \frac{\lambda_t}{1 + \lambda_t} \theta_t \Delta \tag{2.12}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left(R_t^{re} - R_t^d \right) = -\frac{\omega_t}{1 + \lambda_t}, \tag{2.13}$$

where

$$\Omega_t = 1 - \sigma + \sigma \theta_t \phi_t \tag{2.14}$$

$$\phi_t = \frac{1 + \lambda_t}{\theta_t} \mathbb{E}_t [\Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1}] R_t^d - \frac{\omega_t re_t}{n_t \theta_t}. \tag{2.15}$$

(2.11) - (2.13) are the key equilibrium conditions in the model. $\lambda_t \geq 0$ is the multiplier on the costly enforcement constraint, (2.9), while ω_t is the multiplier on the reserve requirement, (2.10). If neither constraint binds, then to first order expected returns on all three types of assets must equal the cost of funds (i.e. the deposit rate). If the costly enforcement constraint binds, then there will be excess returns of long term private and public bonds over the deposit rate. $\Delta < 1$ means that excess returns on government bonds will be lower than excess returns on private bonds. In principle, we could allow Δ to be time-varying and exogenous; this would disassociate the term premium from the corporate spread. In the interest of tractability, we leave this extension to future work. The interest rate on reserves can never exceed the deposit rate, but it could be less than the deposit rate when the reserve requirement is binding. (2.14) is an auxiliary variable introduced to simplify the analysis. See derivations in Appendix A.2.

The value of an intermediary satisfies

$$V_{i,t} = \theta_t \phi_t n_{i,t}. \tag{2.16}$$

When the constraint in (2.9) binds,

$$\phi_t = \frac{Q_t f_{i,t} + \Delta Q_{B,t} b_{i,t}}{n_{i,t}}, \tag{2.17}$$

which is an endogenous leverage ratio, whose equilibrium condition is in (2.15). The constraint makes the financial intermediary less levered than it would find optimal. This endogenous leverage constraint is ultimately what can give rise to excess returns.

We show from (2.15) that:

$$\theta_t \phi_t \geq 1 + \lambda_t - \frac{\omega_t re_t}{n_t} \tag{2.18}$$

If neither (2.9) nor (2.10) bind, then $\lambda_{t+j} = \omega_{t+j} = 0$ for all j, which implies that $\theta_t \phi_t = 1$.[9] Intuitively, this means that net worth is as valuable to a household as to an intermediary. In this case, returns on all assets are equal. Hence, whether an intermediary invests in F_t, B_t, D_t or RE_t is irrelevant.

The two different constraints have competing effects on the value of an intermediary. Other things being equal, when the costly enforcement constraint, (2.9), binds, then $\lambda_t > 0$ and $\theta_t \phi_t$ is larger. In this case, there exist excess returns on holding long term assets (private and government bonds). Hence, net worth is more valuable inside an intermediary as opposed to a household (who cannot hold these assets and hence cannot take advantage of these excess returns). In contrast, when the reserve requirement, (2.10), binds, $\theta_t \phi_t$ gets smaller. A binding reserve requirement pushes the interest rate on reserves below the deposit rate, which means that intermediaries suffer a loss for each dollar of reserves they hold.

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[9] In this circumstance, $\Omega_t = 1$.

2.3. Production

There are four different types of production firms in our model. A competitive final good producer aggregates retail output into a final good available for consumption and investment. A continuum of retail firms repackage wholesale output for resale to the final good firm. Retail firms behave as monopolistic competitors and are subject to price stickiness. A representative investment good firm purchases final output and transforms it into new physical capital subject to a convex adjustment cost. A representative wholesale firm produces output using its own capital, accumulated via purchases of new capital from the investment good firm, and labor hired from labor unions. The key departures from a standard framework involve the wholesale firm, and so we focus on that in the text. The rest of the production side is presented in detail in Appendix A.4.

The representative wholesale firm produces output according to a Cobb-Douglas technology:

$$Y_{m,t} = A_t (u_t K_t)^\alpha L_{d,t}^{1-\alpha} \tag{2.19}$$

$Y_{m,t}$ is flow output and $L_{d,t}$ is labor input. $0 < \alpha < 1$ is the exponent on capital services in the production function. A_t is an exogenous productivity variable that obeys an exogenous stochastic process. u_t is capital utilization. K_t is the stock of physical capital, which the firm owns. The cost of utilization is faster depreciation, where $\delta(u_t)$ maps utilization into depreciation. Physical capital accumulates according to a standard law of motion:

$$K_{t+1} = \widehat{I_t} + (1 - \delta(u_t))K_t \tag{2.20}$$

Similar to Carlstrom et al. (2017), we assume that the wholesale firm must issue perpetual bonds to finance the purchase of new physical capital, $\widehat{I_t}$. Different than them, we only require the firm to finance a constant fraction, $\psi \in [0, 1]$, of investment, not the entirety. This gives rise to a "loan in advance constraint" of the form:

$$\psi P_t^k \widehat{I_t} \leq Q_t CF_{m,t} = Q_t (F_{m,t} - \kappa F_{m,t-1}), \tag{2.21}$$

where P_t^k is the price at which the wholesale firm purchases new physical capital.

The wholesale firm hires labor in a competitive spot market at nominal wage W_t. Its nominal dividend is

$$DIV_{m,t} = P_{m,t} A_t (u_t K_t)^\alpha L_{d,t}^{1-\alpha} - W_t L_{d,t} - P_t^k \widehat{I_t} - F_{m,t-1} + Q_t (F_{m,t} - \kappa F_{m,t-1}) \tag{2.22}$$

The firm maximizes the present discounted value of real dividends, where discounting is by the stochastic discount factor of households. The first order conditions are

$$w_t = (1 - \alpha) p_{m,t} A_t (u_t K_t)^\alpha L_{d,t}^{-\alpha} \tag{2.23}$$

$$p_t^k M_{1,t} \delta'(u_t) = \alpha p_{m,t} (u_t K_t)^{\alpha-1} L_{d,t}^{1-\alpha} \tag{2.24}$$

$$p_t^k M_{1,t} = \mathbb{E}_t \Lambda_{t,t+1} \left[\alpha p_{m,t+1} A_{t+1} K_{t+1}^{\alpha-1} u_{t+1}^\alpha L_{d,t+1}^{1-\alpha} + (1 - \delta(u_{t+1})) p_{t+1}^k M_{1,t+1} \right] \tag{2.25}$$

$$Q_t M_{2,t} = \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} [1 + \kappa Q_{t+1} M_{2,t+1}] \tag{2.26}$$

$$\frac{M_{1,t} - 1}{M_{2,t} - 1} = \psi \tag{2.27}$$

$w_t = W_t/P_t$ is the real wage, $p_{m,t} = P_{m,t}/P_t$ is the relative price of wholesale output, and $p_t^k = P_t^k/P_t$ is the relative price of new capital. (2.23) is the standard static first order condition for labor demand. $M_{1,t}$ is one plus the product of ψ with the multiplier on the constraint that firms must issue bonds to finance investment, (2.21), while $M_{2,t}$ is simply one plus the multiplier on the constraint. (2.24) is the first order condition for utilization. (2.25) and (2.26) are optimality conditions for capital and bonds, respectively. If the constraint did not bind, then $M_{1,t} = M_{2,t} = 1$ and (2.25)-(2.26) would reduce to standard asset pricing conditions. $M_{1,t}$ serves as an endogenous "investment wedge" and $M_{2,t}$ can be thought of as a "financial wedge." These wedges distort the standard asset pricing decisions and fluctuations in these wedges are the mechanism through which QE type policies transmit to the real economy. See details in Appendix A.4.2.

3. (Un)conventional monetary policy tools

In this section, we formally discuss how our model can accommodate both conventional as well as unconventional monetary policy. To date, the literature has tended to study unconventional policy tools in isolation. We are the first to build a model in which all three types of policies can simultaneously be studied.

Before discussing unconventional policies, we must first specify what is meant by conventional policy. We define conventional policy as the adjustment of a short term interest rate, R_t^{TR}, and characterize it via an endogenous feedback rule similar to Taylor (1993):

$$\ln R_t^{TR} = (1 - \rho_r) \ln R^{TR} + \rho_r \ln R_{t-1}^{TR} + (1 - \rho_r)[\phi_\pi (\ln \Pi_t - \ln \Pi) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_r \varepsilon_{r,t}, \tag{3.1}$$

where R^{TR} and Π are steady state values of the policy rate and the inflation target, $0 < \rho_r < 1$, and ϕ_π and ϕ_y are non-negative parameters. We restrict attention to parameter values giving rise to a determinate equilibrium, i.e. we assume $\phi_\pi > 1$. The policy rate adjusts in response to deviations of inflation from target and output growth from trend (which in our model is zero).[10]

During "normal" times, we assume that (i) the central bank sets the interest rate on reserves equal to the underlying policy rate, and (ii) the reserve requirement, (2.10), is non-binding, so that

$$R_t^d = R_t^{re} = R_t^{TR}. \tag{3.2}$$

Unconventional policies of the sort described and analyzed below have by-and-large been attempted during an environment in which short term rates were constrained from below by zero. To implement a conventional ZLB constraint, we suppose that the reserve requirement is non-binding, so that the deposit rate and interest rate on reserves are equal, and that the net interest rate on reserves must be non-negative:

$$R_t^{re} = R_t^d = \max\left\{1, R_t^{TR}\right\} \tag{3.3}$$

3.1. Quantitative easing

Perhaps the most prominent unconventional policy tool implement by central banks throughout the world is quantitative easing (QE). It was first introduced by the Bank of Japan in the early 2000s. In the aftermath of the Great Recession and subsequent ZLB period, the United States, Euro Area, and United Kingdom (among others) all adopted this policy tool. In the United States, by the end of QE3 the Federal Reserve had expanded its balance sheet to 4.5 trillion dollars (or approximately 25 percent of GDP). By the end of 2018, the ECB's balance sheet had grown to 4.7 trillion Euros, which was about 38 percent of GDP in the Euro Area, and the Bank of Japan held 552 trillion in Yen, an amount in excess of 100 percent of Japanese GDP.

As in Gertler and Karadi (2011, 2013) and Carlstrom et al. (2017), we interpret quantitative easing as central bank purchases of private or government issued bonds. These purchases are financed via the creation of interest-bearing reserves held by financial intermediaries. This is a realistic description of how QE policies played out in practice in the United States subsequent to the crisis. This also distinguishes our model from both Carlstrom et al., 2017 and Gertler and Karadi, 2011; 2013.[11]

In our model, the central bank's balance sheet is given by:

$$Q_t F_{cb,t} + Q_{B,t} B_{cb,t} = RE_t \tag{3.4}$$

The central bank can hold either private, $F_{cb,t}$, or government, $B_{cb,t}$, bonds. These assets are financed via the creation of interest-bearing reserves. Any operating surplus is returned to the fiscal authority via a lump sum transfer. Market-clearing requires that bonds issued by the wholesale firm and debt issued by the government must be held by either financial intermediaries or the central bank.

QE policies can have real effects to the extent to which financial intermediaries are constrained via the costly enforcement problem, (2.9). When intermediaries are constrained, bond purchases financed via the creation of reserves ease this constraint. In this case, central bank bond purchases do not simply crowd out intermediary bond purchases, in such a way that the total demand for bonds increases. This results in higher bond prices, which in turn eases the loan-in-advance constraint facing the wholesale firm. This results in higher investment and aggregate demand. If the constraint on intermediaries is non-binding, or if the wholesale firm does not need to finance investment via debt (i.e. $\psi = 0$ in (2.21)), then bond purchases have no economic effects. Provided $\Delta < 1$ and both of these constraints bind, central bank purchases of private bonds will have a stronger effect on excess returns compared to government bond purchases, and will hence be more stimulative.

We consider both exogenous and endogenous QE policies. For exogenous policy, we simply assume that central bank bond holdings follow exogenous AR(1) processes:

$$f_{cb,t} = (1 - \rho_f)f_{cb} + \rho_f f_{cb,t-1} + s_f \varepsilon_{f,t} \tag{3.5}$$

$$b_{cb,t} = (1 - \rho_b)b_{cb} + \rho_b b_{cb,t-1} + s_b \varepsilon_{b,t} \tag{3.6}$$

where f_{cb} and b_{cb} denote steady state (real) central bank private and government bond holdings, ρ_f and ρ_b are parameters constrained to lie between zero and one, and shocks are drawn from standard normal distributions (with s_f and s_b denoting standard deviations of the shocks). Endogenous QE policies are considered in Section 5. Bond purchases may be undertaken either in "normal" times, when the policy rate is set according to (3.1), or during periods in which the policy rate is constrained by zero as in (3.3).

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[10] In the model there is no analytical expression for "potential" output and with the myriad nominal and financial frictions it is not even theoretically clear what concept of potential ought to be used. Responding to output growth may be both theoretically desirable (e.g. Walsh, 2003 and Orphanides and Williams, 2006) and empirically more in-line with observed central bank reaction functions (e.g. Coibion and Gorodnichenko, 2011).

[11] Carlstrom et al. (2017) do not model the central bank's balance sheet at all, and Gertler and Karadi (2011, 2013) assume that the central bank finances purchases by issuing short term debt directly to households (though Gertler and Karadi, 2013 discuss the similarity of such a setup to one with interest-bearing reserves).

3.2. Forward guidance

Forward guidance entails promises from a central bank about the future path of its policy rate (Campbell et al., 2012 and Del Negro et al., 2015). Some degree of forward guidance has been used as a communications tool by central banks around the world for some time, though it has gained more attention recently due to the ZLB constraint on short term policy rates. Language used in central bank communications has varied widely across both space and time. For example, the Federal Reserve started by communicating its expected duration of extremely low policy rates, with announcements to the effect of "we expect conditions to warrant exceptionally low levels of the federal funds rate for/through... " Initially the forecast of the duration of low rates was ambiguous, with phrases such as "for some time" or for "an extended period." Later the Fed turned to more specific guidance regarding the duration of low policy rates. As the duration of the ZLB period wore on, the Fed also resorted to more explicitly contingent language, for example promising low rates until a target unemployment rate of 6.5 percent was achieved. Mario Draghi of the ECB used words to influence policy with his famous "whatever it takes" remark in 2012.

We model forward guidance as a shock to the underlying desired policy rate in (3.1) at a time when the ZLB constraint, (3.3), is binding. Taking the expected duration of the ZLB, H, as given, a negative shock to the desired policy rate portends a lower interest rate on reserves (and hence a lower deposit rate, assuming the two are equal) *after* the ZLB is over. As a reduced form way to capture the credibility of forward guidance announcements, we scale the shock to the desired policy rate in a forward guidance experiment by $\gamma \in [0, 1]$. $\gamma = 1$ entails perfect credibility, whereas $\gamma = 0$ means that forward guidance announcements are not believed by agents in the model. This parameter can be motivated by the empirical findings of Campbell et al. (2019). To the extent to which forward guidance announcements are credible, such shocks can impact long term yields in the present via the logic of the expectations hypothesis.

Our modeling of forward guidance is different from the extant literature, which typically captures forward guidance as an anticipated shock to a policy rule after a ZLB period is expected to be over. The difference in timing is important – a one unit shock to the desired policy rate at t has a much smaller effect on the economy than an announced shock of the same size to the policy rate in $t + H$. This is because of decay due to interest smoothing (i.e. $0 < \rho_r < 1$) – the change in the expected path of future deposit rates is smaller in our experiment compared to a more standard forward guidance shock. In a somewhat mechanical way, therefore, our new timing assumption leaves our model immune from the so-called "forward guidance puzzle," which refers to the fact that forward guidance becomes more powerful as it targets policy rates in the more distant future (McKay et al., 2016).[12] Another way to interpret our new timing assumption is that the size of the shock gets smaller with the horizon of the announcement, which is consistent with recent empirical work from Bundick and Smith (2016).

3.3. Negative interest rate policy

Although it has not been attempted by the Federal Reserve, a number of central banks around the world have experimented with negative interest rate policy (NIRP) in the last several years – the Euro Area and Japan being prominent examples. While recent experience suggests that interbank interest rates can go negative, deposit rates in countries experimenting with NIRP have not. Accordingly, we implement NIRP in our model by assuming that the ZLB constraint applies only to the deposit rate and not to the interest rate on reserves.[13] The inclusion of interest-bearing reserves in our model therefore allows us to incorporate NIRP into a model that can also be used to study other unconventional policy tools.

Formally, we assume that the central bank sets the interest rate on reserves equal to the desired underlying policy rate, R_t^{TR}. For the purposes of the presentation here, there is no constraint on how negative the policy rate can go. We defer a discussion of the ELB until Section 6.2. We only assume that the deposit rate cannot go negative via a modification of (3.3):

$$R_t^d = \max\{1, R_t^{re}\}, \quad R_t^{re} = R_t^{TR} \tag{3.7}$$

We model NIRP as a negative shock to the underlying desired policy rate, met by an equal reduction in the interest rate on reserves, at a time where the deposit rate is constrained by zero. Implementation of such a policy requires that the reserve requirement, (2.10), which is non-binding in all of the policy experiments discussed above, binds. Intuitively, absent a reserve requirement, intermediaries are willing to hold an indeterminate amount of reserves so long as $R_t^{re} = R_t^d$. They would not wish to hold any reserves if $R_t^{re} < R_t^d$. To get intermediaries to hold reserves with a negative gap between the

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[12] To be perfectly transparent, in our model a shock which lowers the deposit rate by a given amount H periods into the future has a *smaller* stimulative impact than a shock which lowers the deposit rate by the same amount $H + 1$ periods into the future. While it is therefore afflicted by the "forward guidance puzzle" so-defined, we should note that the puzzle is *less severe* in our model than in a version of our model with unconstrained financial intermediaries.

[13] The conventional logic for the ZLB is that deposits and cash are substitutes, and cash pays a zero nominal return, so rates on deposits cannot go below zero or else households will hoard cash. Our model is cashless, so it is not immediately clear why such logic would carry over. We could instead allow the household to hold a non-interest bearing asset called cash, and could generate a non-zero demand for it by assuming that households receive utility from holding money balances. As long as the utility from money balances is additively separable from utility from consumption and leisure, this would not affect the equilibrium dynamics of the model.

rate on reserves and the rate on deposits, we simply assume that the central bank requires them to do so, which makes (2.10) binding.

NIRP affects the economy through two distinct channels in our model. First, NIRP has an effect similar to forward guidance – to the extent to which ρ_r (the interest rate smoothing parameter) is large relative to H (the duration of the ZLB), lowering the interest rate on reserves in the present signals lower deposit rates in the future after the ZLB has ended. Since implementation of NIRP involves an observable action (lowering the interest rate on reserves), it may well be more credible than pure forward guidance. The second channel through which NIRP operates works in the opposite direction. $R_t^{re} < R_t^d$ works like a tax on intermediaries in the evolution of net worth, (2.5). This lowers net worth and exacerbates the costly enforcement constraint facing intermediaries; other things being equal, this results in higher excess returns. This channel is similar to what is emphasized both theoretically and empirically in Ulate (2018) and Eggertsson et al. (2019). If forward guidance is completely credible (i.e. $\gamma = 1$), NIRP will therefore be less stimulative than pure forward guidance. If the central bank is not perfectly credible ($\gamma < 1$), then this may not be the case. How important this offsetting effect of NIRP coming through the evolution of net worth is depends on how many reserves intermediaries hold. This in turn has implications for the size of the central bank's balance sheet.

3.4. Discussion

Relative to the existing literature, one of the important contributions of our paper is to simultaneously incorporate the three principal tools of unconventional monetary policy together into one coherent, quantitatively realistic modeling framework. Doing so in a tractable manner necessarily involves some tradeoffs and taking some shortcuts, and our model abstracts from some other features and mechanisms that have been highlighted in the literature. We briefly draw attention to some of these issues in this subsection.

Our modeling of the channel through which QE can impact the real economy is based on constrained financial intermediaries and follows Gertler and Karadi (2011). Of the three unconventional tools, our modeling of QE is the most detailed, and indeed for the quantitative exercises that follow, we focus the bulk of our attention on QE. Nevertheless, our model still abstracts from additional channels by which QE could work. One is the portfolio balance channel as emphasized by Vayanos and Vila (2009), which has some empirical support as documented for example by Hamilton and Wu (2012); Cahill et al. (2013); and Greenwood and Vayanos (2014). Another channel from which our model abstracts is the signaling channel emphasized by Bauer and Rudebusch (2014). Furthermore, our model also abstracts from any potential costs associated with QE, especially those associated with purchasing private debt as discussed in Kurtzman and Zeke (2019). These costs become stronger when the central bank's balance sheet grows larger, which bears some resemblance to some of the issues we discuss with regard to the size of central bank balance sheets in Section 6.

Our model eliminates the forward guidance puzzle in an admittedly reduced-form way. First, as discussed above, we introduce a timing convention which necessarily makes the effectiveness of forward guidance weaker the further out the time horizon. Second, we allow for imperfect credibility as captured by the parameter γ. Alternative and more involved approaches might include formally modeling deviations from full information rational expectations. Such approaches are beyond the scope of our paper. While reduced-form, our approach is consistent with the empirical results in Campbell et al. (2019), who find that central banks have limited ability to impact expectations of the policy rate at horizons over which the forward guidance puzzle is likely to occur.

To break the equivalence between the deposit and policy rates, we incorporate a reserve requirement. Without such a requirement, arbitrage would imply that the deposit and policy rates always be equal, which in our setup would mean that the ZLB applies to the policy rate as well and NIRP is not a viable tool. We think of this reserve requirement as reflecting more than just statutory reserve requirements; indeed, in periods and locations in which negative interest rates have been deployed, banking systems have been awash with reserves. Rather, we think of the requirement as a reduced-form shortcut to capture other costs that might result in banks being willing to hold reserves at negative rates. A more involved approach might include explicitly modeling interbank lending markets, such as in Gertler and Kiyotaki (2010). In their model, banks are heterogeneous with respect to investment opportunities. With negative reserve rates, banks with poor investment opportunities will be willing to lend in interbank markets at negative rates so long as those rates are higher than the negative rate on reserves. These negative rates in turn can be passed onto lending rates from banks with more profitable investment opportunities. Such a channel would amplify the pass-through relative to our framework. Nevertheless, even with a more involved setup such as this, the counterveiling banking channel emphasized in our model, and supported by Ulate (2018) and Eggertsson et al. (2019), would still be present.

4. Comparing alternative monetary policy

In this section, we compare and contrast *exogenous* changes in alternative unconventional policy tools to an exogenous shock to conventional policy. We wish to assess the efficacy of unconventional policy interventions in affecting output and other macroeconomic aggregates.

Table 1
Calibrated parameters.

Parameter	Value or Target	Description
κ	$1 - 40^{-1}$	Coupon decay parameter / Bond duration
ψ	0.81	Fraction of investment from debt
σ	0.95	Intermediary survival probability
θ	$400(R^F - R^d) = 3$	Recoverability parameter / steady state spread
X	Leverage $= 4$	Transfer to new intermediaries / steady state leverage
Δ	1/3	Government bond recoverability
b_{cb}	$\frac{b_{cb}Q_B}{4Y} = 0.06$	Steady state central bank Treasury holdings
f_{cb}	0	Steady state central private bond holdings
ρ_θ	0.98	AR credit
ρ_b	0.8	AR central bank Treasury
ρ_f	0.8	AR central bank private bonds
$\bar{b}_G$	$\frac{B_G Q_B}{4Y} = 0.41$	Steady state government debt
G	$\frac{G}{Y} = 0.2$	Steady state government spending

Note: this table lists the values of calibrated parameters or the target used in the calibration.

4.1. Calibration

The model is solved via a linear approximation about the non-stochastic steady state.[14] When considering the effects of the ZLB, we solve a piecewise linear version of the model using the approach suggested by Guerrieri and Iacoviello (2015). Many of the parameters in the model take on relatively standard values drawn from the literature. These parameters are listed and discussed in Appendix B. In the text, we focus only on the less standard parameters. These are given in Table 1.

The relatively non-standard parameters in our model relate to financial intermediaries, private and government bonds, the fraction of investment the wholesale firm must finance by issuing bonds, and the size of the central bank's balance sheet. We follow Carlstrom et al. (2017) in setting κ to imply a ten year duration for both private and government bonds. We choose σ, the survival probability for intermediaries, to be 0.95, which is in-line with Gertler and Karadi (2011, 2013). We target a steady state excess return of private bonds over the deposit rate, $R_t^F - R_t^d$, of 300 basis points and that of government bonds over the deposit rate, $R_t^B - R_t^d$, of 100 basis points (both at an annualized frequency). These are chosen to roughly match observed spreads of Baa yields over the Federal Funds rate and the ten year Treasury yield over the Federal Funds rate.[15] These targets imply a steady state value of $\theta = 0.58$ and a value of the parameter $\Delta = 1/3$. The value of X (the transfer of startup funds to new financial intermediaries) is chosen to be consistent with a steady state leverage ratio (ratio of assets to aggregate net worth) of four, which is similar to Gertler and Karadi (2011, 2013). The fraction of investment the wholesale firm must finance by issuing debt is $\psi = 0.81$. This is chosen to match the observed ratio of the value of outstanding private debt to annualized nominal GDP immediately prior to the Crisis of 1.68.[16] The steady state values of government spending and government debt are chosen to match a government spending share of output of 20 percent and a steady state debt-to-GDP ratio of 41 percent, the latter of which is the observed ratio of federal government liabilities to annualized nominal GDP in the fourth quarter of 2007. The steady state (real) value of central bank private bond holdings, f_{cb}, is set to zero because the Fed only held Treasury debt at the end of 2007. The steady state central bank holding of government bonds, b_{cb}, is chosen to match the fraction of the Fed's assets over annualized GDP immediately prior to the Great Recession of 6 percent.

Although it can accommodate many more, our model features a parsimonious selection of exogenous shocks – aside from policy interventions, these include shocks to productivity, credit, and government spending. Turning off all exogenous monetary policy shocks (both conventional and unconventional), the standard deviations of these three remaining shocks are then chosen to loosely match a small set of selected moments from the data. More details are provided in Appendix B.
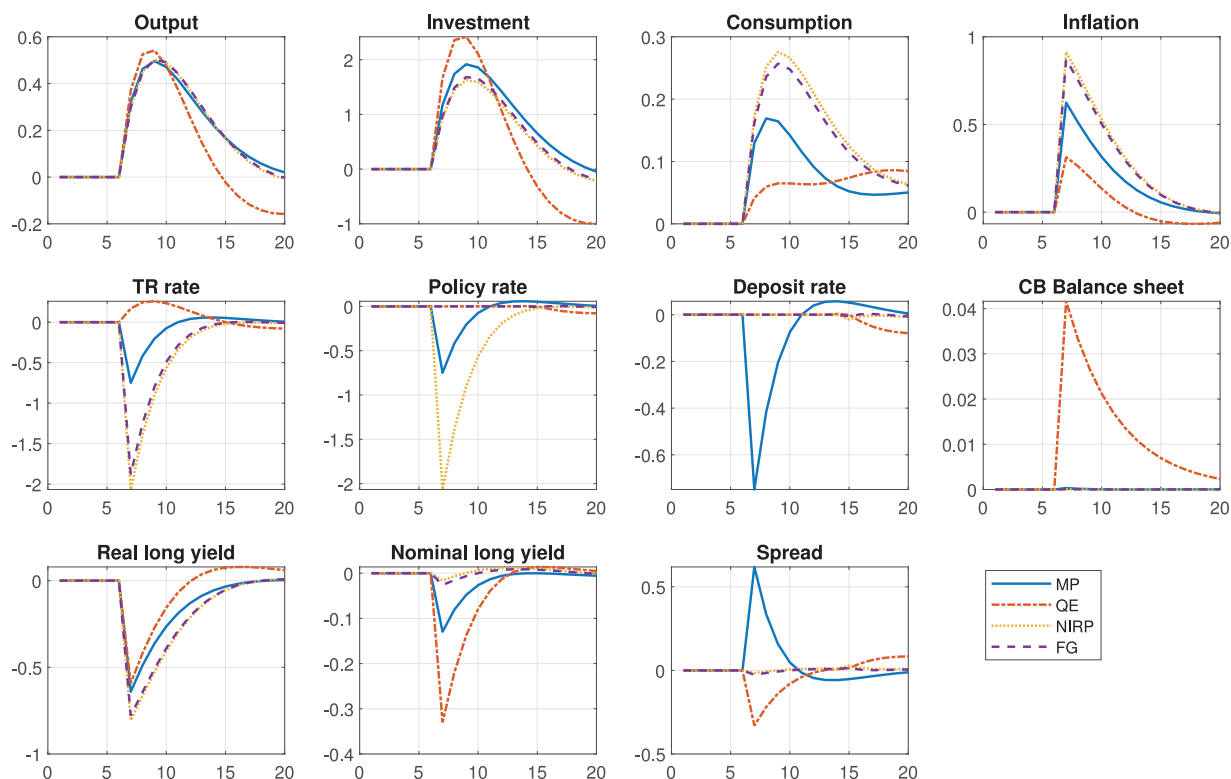
4.2. Quantitative analysis

In this section, we seek to quantify how much an unconventional policy tool must be moved so as to generate similar aggregate responses to a conventional policy shock. The comparison is depicted in Fig. 1, which plots impulse responses of different macroeconomic aggregates to various policy interventions.

The impulse responses are constructed by considering two simulations. In both simulations there are a sequence of credit shocks in periods 1–6. These are used to make the ZLB bind. In the second simulation, there is an additional shock that hits the economy in period 7. The plotted impulse responses are the differences between these two simula-

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[14] In this steady state neither the ZLB nor the reserve requirements bind, so that the steady state reserve and deposit rates are equal.

[15] Note that excess returns and yields to maturity (defined and discussed below) are not necessarily the same outside of steady state, but are identical in the steady state. Over the period 1960–2008, the average Baa-FFR spread is 272 basis points and the average 10 Year-FFR spread is 86 basis points. Over a shorter sample, from 1984 to 2008, the average spreads are 352 and 142 basis points, respectively. Targeting spreads of 300 and 100 basis points, respectively, therefore represents a middle ground.

[16] Outstanding private debt is measured by total credit to the private non-financial sector for the United States from FRED.

Fig. 1. Exogenous monetary policy. *Notes:* Blue solid lines: a −1% shock to the annualized policy rate. Red dash-dotted lines: a QE shock, where the central bank increases its balance sheet by about 4 percent of steady state GDP by purchasing private debt, where $\rho_f = \rho_r$. Yellow dotted lines: NIRP with a shock of −2.4% to the annualized policy rate. Purple dashed lines: forward guidance with a shock of −2.2% to the desired policy interest rate. All these shocks hit the economy in period 7. For all but the blue solid lines, we generate a binding ZLB with a sequence of credit shocks of 1.5 standard deviations in each of the periods 1 through 6. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

tions. Output, investment, and consumption are expressed as percentage deviations from the steady state; inflation and interest rates are in annualized percentage points; the central bank's balance sheet is expressed relative to steady state output.

The solid blue lines depict responses to a conventional policy shock to the Taylor rule, (3.1). The size of the shock is −1 percent at an annualized percentage rate, which results in about a 0.75 percentage point decline in the policy rate on impact (the difference being accounted for by the endogenous reaction of the policy rate to inflation and output growth). The responses of aggregate variables to this shock are familiar. Output increases by about 0.5 percent on impact and follows a hump-shaped pattern. The responses of investment and consumption are similar, though investment responds about four times as much as output and consumption about one third as much as output. Inflation rises by about 0.6 percentage points on impact and remains above trend for about three years.

Responses to unconventional policy interventions are depicted via colored dash lines (red for QE, purple for FG, and orange for NIRP). For these experiments, we assume that a sequence of credit shocks drives the economy to the ZLB; from the period the policy shock hits (period 7), the ZLB is expected to bind for nine quarters, or just over two years. This is in-line with the expected duration of the ZLB as estimated by Bauer and Rudebusch (2016) and Wu and Xia (2016).[17] The quantitative easing experiment involves purchasing private bonds (a purchase of government bonds has a qualitatively similar, albeit smaller, effect on macroeconomic aggregates, as we discuss further below). The magnitudes of the unconventional policy interventions are chosen to roughly match the impact change in output to the conventional policy shock. The autoregressive coefficient in the quantitative easing experiment, ρ_f, is set equal to the Taylor rule smoothing parameter, ρ_r, to facilitate comparison.

The unconventional shocks all have qualitatively similar effects on output compared to the conventional policy shock, although the output response to QE is less persistent than to the other interventions. The investment dynamics mirror the output response for all of the unconventional shocks. The consumption dynamics are similar for FG and NIRP compared to conventional policy, while the consumption response to a QE shock is quite muted in comparison. This is because QE has

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[17] In the case of the quantitative easing and forward guidance experiments, the policy rate (interest rate on reserves) and the deposit rate are both constrained by zero. For the NIRP experiment, as discussed above, only the deposit rate is constrained.

no effect on the deposit rate during the period of the ZLB, which is what is relevant in the household's Euler equation. All the shocks are inflationary, with the QE shock being the least so, while FG and NIRP are more inflationary compared to the conventional policy shock. To generate a similar output increase as a conventional policy shock, the QE experiment requires the central bank to increase the size of its balance sheet by about 4 percent relative to annualized steady state output.

Translating into the equations of our model, we can relate the shock sizes in (3.1) and (3.5) as follows:

$$s_f \varepsilon_{f,t} = \Psi_f s_r \varepsilon_{r,t}, \tag{4.1}$$

where $\Psi_f = -7$ implies the shock size on $f_{cb,t}$ is seven times as large as the shock to the policy rate, and in the opposite direction. In other words, an easing policy could be implemented by lowering the policy rate or by increasing the size of the central bank's balance sheet. We will elaborate on this conversion rate later.

The NIRP and FG interventions are shocks to the Taylor rule of -2.4 and -2.2 percent, respectively, resulting in roughly a 2 percentage point decline in the Taylor rule rate on impact. This is more than twice as large as the change in the policy rate for the conventional policy shock. The main difference between FG and NIRP is that the policy rate does not react for the duration of the ZLB in the case of FG but falls immediately in the NIRP experiment. Forward guidance and NIRP shocks must be larger than conventional policy shocks to generate a similar change in output because these only affect the deposit rate in the future rather than in the present. The longer the expected duration of the ZLB, the larger must be the shock to FG or NIRP to generate a given output movement. FG is slightly more expansionary than a NIRP intervention of the same amount, which is why the NIRP shock is slightly larger than the FG shock. This is because a decline in the policy rate relative to the deposit rate reduces the profitability of intermediaries, as discussed in Section 3.3. This in turn lowers net worth and exacerbates the constraint giving rise to equilibrium spreads. Because the model is solved about a steady state in which the size of the central bank's balance sheet is small, intermediaries do not hold large quantities of reserves and consequently the differences between FG and NIRP are small. This need not necessarily be the case, as we discuss further below.

The different policy shocks have comparable effects on output because they have similar effects on long term interest rates. To be precise, the (gross) yield to maturity, RL_t^F, on the private investment bond is implicitly defined via:[18]

$$Q_t = \frac{1}{RL_t^F} + \frac{\kappa}{\left(RL_t^F\right)^2} + \frac{\kappa^2}{\left(RL_t^F\right)^3} + \cdots$$

Consequently,

$$RL_t^F = \frac{1}{Q_t} + \kappa \tag{4.2}$$

We further define the overall spread as the difference in the net long term private yield and the net deposit rate:

$$\ln RL_t^F - \ln R_t^d \tag{4.3}$$

We define the credit spread as the difference in net yields on private and government bonds:

$$\ln RL_t^F - \ln RL_t^B \tag{4.4}$$

We define the real long term yield as:

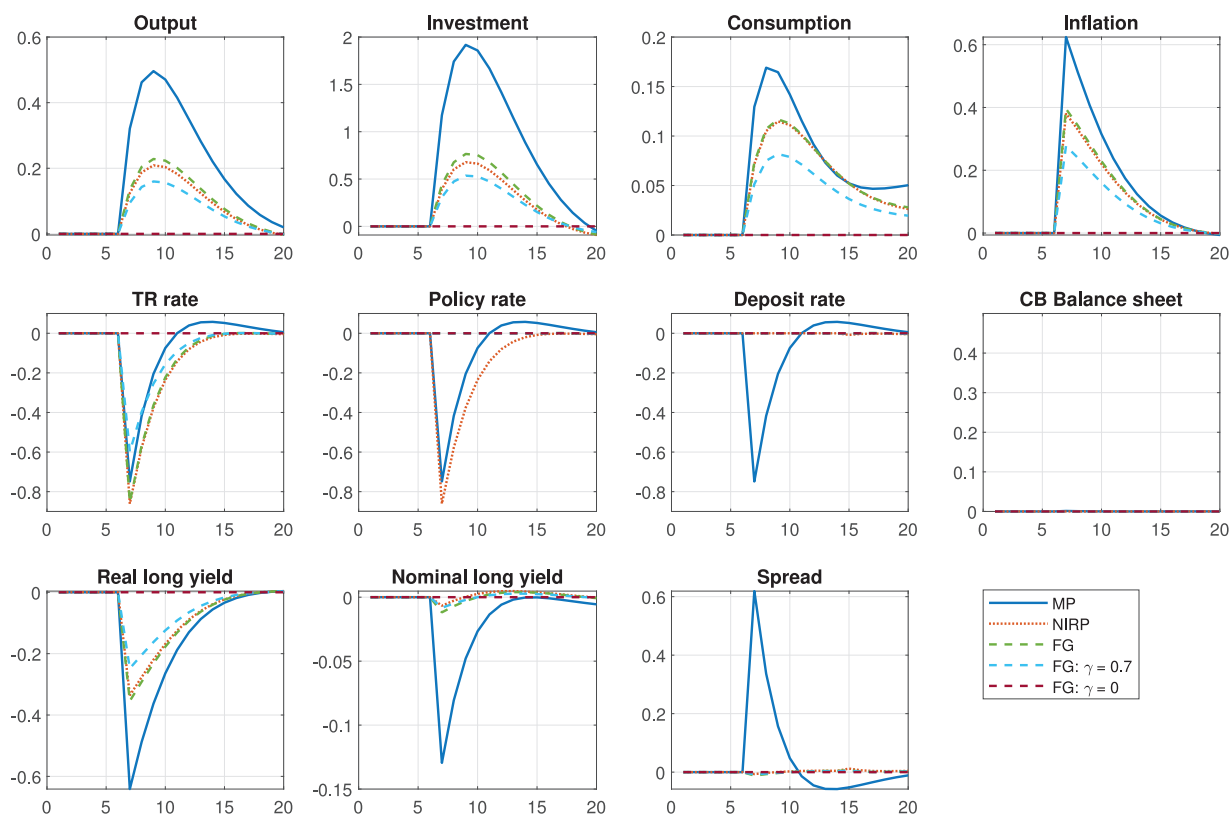$$r_t^L = \ln RL_t^F - \mathbb{E}_t \ln \Pi_{t+1} \tag{4.5}$$

More precisely, what we call the real long term (private) yield equals the spread (which consists of both the term spread and the credit spread) plus the real deposit rate.[19] This turns out to be the relevant metric for assessing monetary policy transmission in the model. One observes that the real long term yield declines by a similar amount both on impact and dynamically in response to both the conventional and unconventional policy interventions. This, in turn, results in very similar output dynamics to each of the policy interventions. Long term yields being the main driving force in the economy is consistent with Wu and Zhang (2017, 2019), who use the shadow rate of Wu and Xia (2016) as a summary statistic for the effects of unconventional policies.

Even though conventional and unconventional policy shocks have similar effects on the real long term yield, they have quite different effects on the overall spread between the long term private yield and the deposit rate. Conventional policy shocks indirectly affect long term rates by influencing short term rates. Since long term yields react less than short term rates, the overall spread increases. In contrast, unconventional shocks affect longer term rates without impacting the deposit rate in the short run. This results in the overall spread declining rather than rising.

Fig. 2 compares and contrasts forward guidance and NIRP across several different specifications. Blue lines depict responses to the conventional policy shock; these are identical to the solid blue lines in Fig. 1 and are included to facilitate comparison. For the unconventional shocks, we generate the ZLB in the same manner as in Fig. 1. Different than

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[18] The expression for the yield is similar for the government bond.

[19] In particular, the real net deposit rate is $\ln R_t^d - \mathbb{E}_t \ln \Pi_{t+1}$. Summing this and (4.3) gives (4.5).
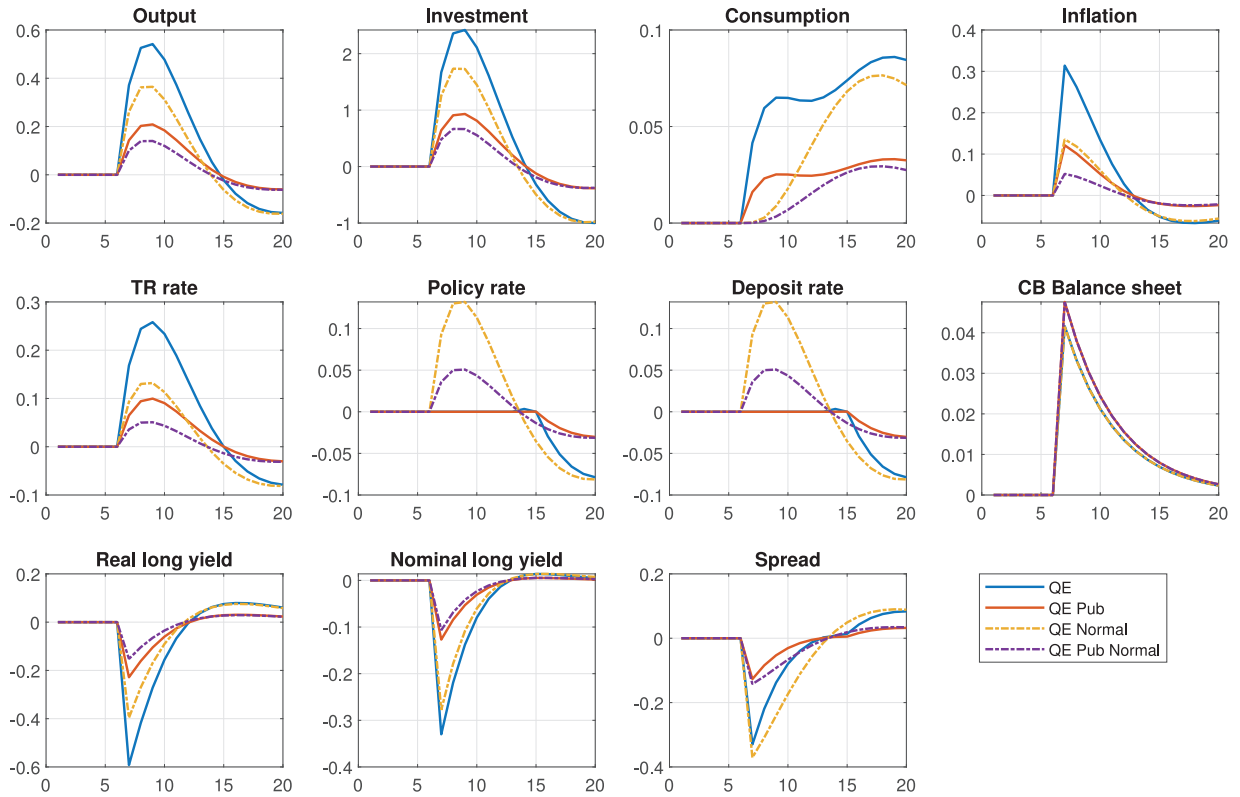
Fig. 2. Exogenous NIRP vs. FG. *Notes:* Blue solid lines: responses to the conventional monetary policy shock. Red dotted lines: response to a NIRP shock. Green dashed lines: fully credible forward guidance; light blue dashed lines: partially credible forward guidance with $\gamma = 0.7$; dark red dashed lines: non-credible forward guidance with $\gamma = 0$. All shocks are −1% annualized and hit in period 7. For all but blue solid lines, we generate a binding ZLB with a sequence of credit shocks of 1.5 standard deviations in each of the periods 1 - 6. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 1, all shocks are the same magnitude of −1 percent annualized (i.e. the shock magnitudes are not adjusted so as to produce similar output responses). Dotted lines are responses to a NIRP shock. Dashed lines depict responses to forward guidance with green being fully credible ($\gamma = 1$), light blue partially credible ($\gamma = 0.7$), and dark red non-credible ($\gamma = 0$).

There are a couple of noteworthy results. First, although a fully credible forward guidance policy is more stimulative than NIRP, for a central bank with limited credibility, NIRP is likely a more effective policy. In this sense, one can interpret NIRP in our model as commitment device a central bank can employ to carry out forward guidance. Pure forward guidance involves nothing more than a verbal promise of future low rates, which could be interpreted by the private sector as cheap talk, especially when the central bank has a low γ. NIRP, in contrast, involves an observable action in the present. Second, conditioning on equal sized shocks, the stimulative effect of a NIRP intervention is about half as large as a conventional policy shock. Put differently, to achieve a given stimulus on output the policy rate must be cut twice as much when the deposit rate is constrained by zero in comparison to normal times. In practice, this likely limits the usefulness of NIRP as an unconventional policy tool. A number of political and social constraints likely limit how negative interbank lending rates can go. In practice, no central bank has pushed policy rates deeply negative (e.g. the ECB's lowest policy rate was only -40 basis points). Later, in Section 6, we will use our model to shed some light on an effective lower bound for policy rates. We also return to NIRP and discuss its efficacy in relation to the total size of the central bank's balance sheet.

Fig. 3 assesses different types of quantitative easing experiments. The blue solid lines are the benchmark responses, which correspond to a purchase of private debt when the policy and deposit rates are constrained by the ZLB. These responses are identical to the red dash-dotted responses in Fig. 1. The red solid lines are responses to a purchase of public debt (of the same magnitude as the benchmark) when short term rates are constrained. As in Gertler and Karadi (2013), private debt purchases are more stimulative than buying government debt. This result is essentially built into the model via our calibration of Δ, which governs the steady state credit spread. The dashed lines depict impulse responses to QE interventions during periods in which short term rates are unconstrained (i.e. "normal" times). QE is more stimulative when short term policy rates are constrained than when not, and private bond purchases remain more stimulative than government bond purchases.

Fig. 3. Exogenous QE. *Notes:* Blue solid lines: purchasing private debt at the ZLB; red solid lines: purchasing public debt at the ZLB. Yellow dashed lines: purchasing private debt during normal times; purple dashed lines: purchasing public debt during normal times. The QE shocks hit the economy in period 7, and are the same across the four specifications. Where relevant, we generate a binding ZLB with a sequence of credit shocks of 1.5 standard deviations in each of the periods 1 - 6. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In summary, various unconventional policy interventions can in principle have similar macroeconomic effects as a conventional cut in the short term policy rate. The requisite forward guidance and NIRP interventions are quite large, however, and the efficacy of FG depends crucially on a central bank's credibility. Further, implementing very large negative policy rates seems implausible in practice. For these reasons, we focus on QE as an endogenous policy tool to ameliorate the adverse consequences of a binding ZLB in the next section.

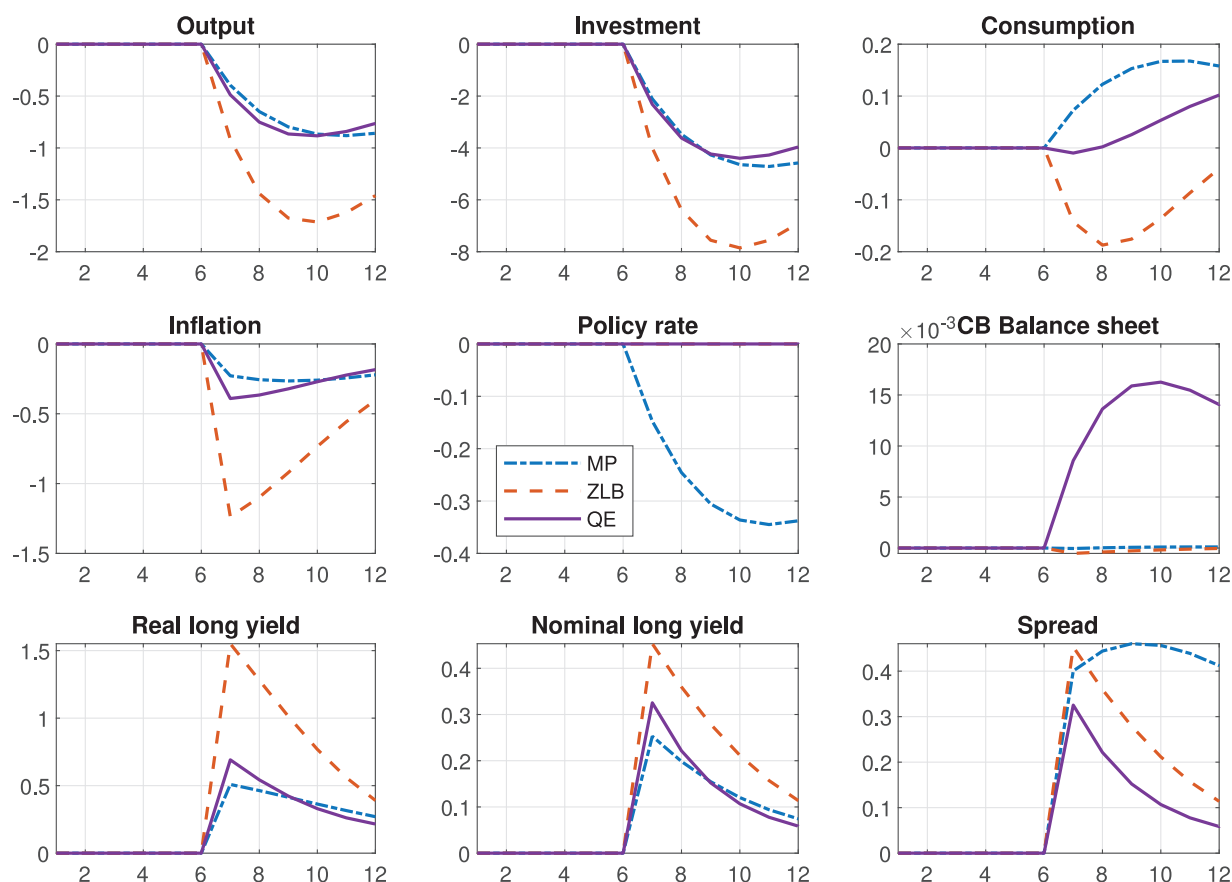## 5. Endogenous quantitative easing and the great recession

The previous section established that exogenous changes in different unconventional policy tools can have similar economic effects as exogenous changes in short term policy rates. In practice, rather than exogenous shocks, effective monetary policy design entails the endogenous adjustment of policy to changing economic conditions. In this section, we study endogenous quantitative easing in response to other shocks. We do so both conditioning on each shock one at a time as well as in a simulation with a sequence of adverse credit shocks meant to mimic the experience of the US in the Great Recession. We wish to address the question of whether QE can serve as a close substitute for conventional policy when the ZLB binds. If so, how much QE is required?

So as to facilitate a comparison with conventional policy, we propose that the central bank's private bond holdings obey a Taylor type reaction function:[20]

$$f_{cb,t} = (1 - \rho_f)f_{cb} + \rho_f f_{cb,t-1} + (1 - \rho_f)\Psi_f[\phi_\pi (\ln \Pi_t - \ln \Pi) + \phi_y (\ln Y_t - \ln Y_{t-1})] + s_f \varepsilon_{f,t} \tag{5.1}$$

The values of ϕ_π and ϕ_y are the same as in (3.1), and to further facilitate comparison we assume that $\rho_f = \rho_r$. The parameter Ψ_f measures the intensity of the reaction of bond-holdings, relative to the reaction of the desired policy rate, to deviations of inflation and output growth from target. For the exercises which follow, we assume that endogenous QE is only operative when the policy rate is constrained by the ZLB. That is, outside of the ZLB, $\Psi_f = 0$, and (5.1) collapses to (3.5). At the ZLB, we allow $\Psi_f \neq 0$. For the quantitative results which follow, we assume a value of $\Psi_f = -7$, which is the

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[20] We could similarly specify an endogenous reaction function for central bank holdings of government debt. All of the results which follow would be similar, though the value of Ψ_b required to mimic conventional policy would be larger.

Fig. 4. Credit shock and endogenous QE. *Notes:* This figure plots impulse responses to a one standard deviation credit shock. The shock hits in period 7. Blue dash-dotted lines: conventional monetary policy without the ZLB. Red dashed lines: the ZLB with no unconventional monetary policy. Purple solid lines: the ZLB with endogenous QE. We create the ZLB with a sequence of credit shocks of 1.5 standard deviations from periods 1 - 6.
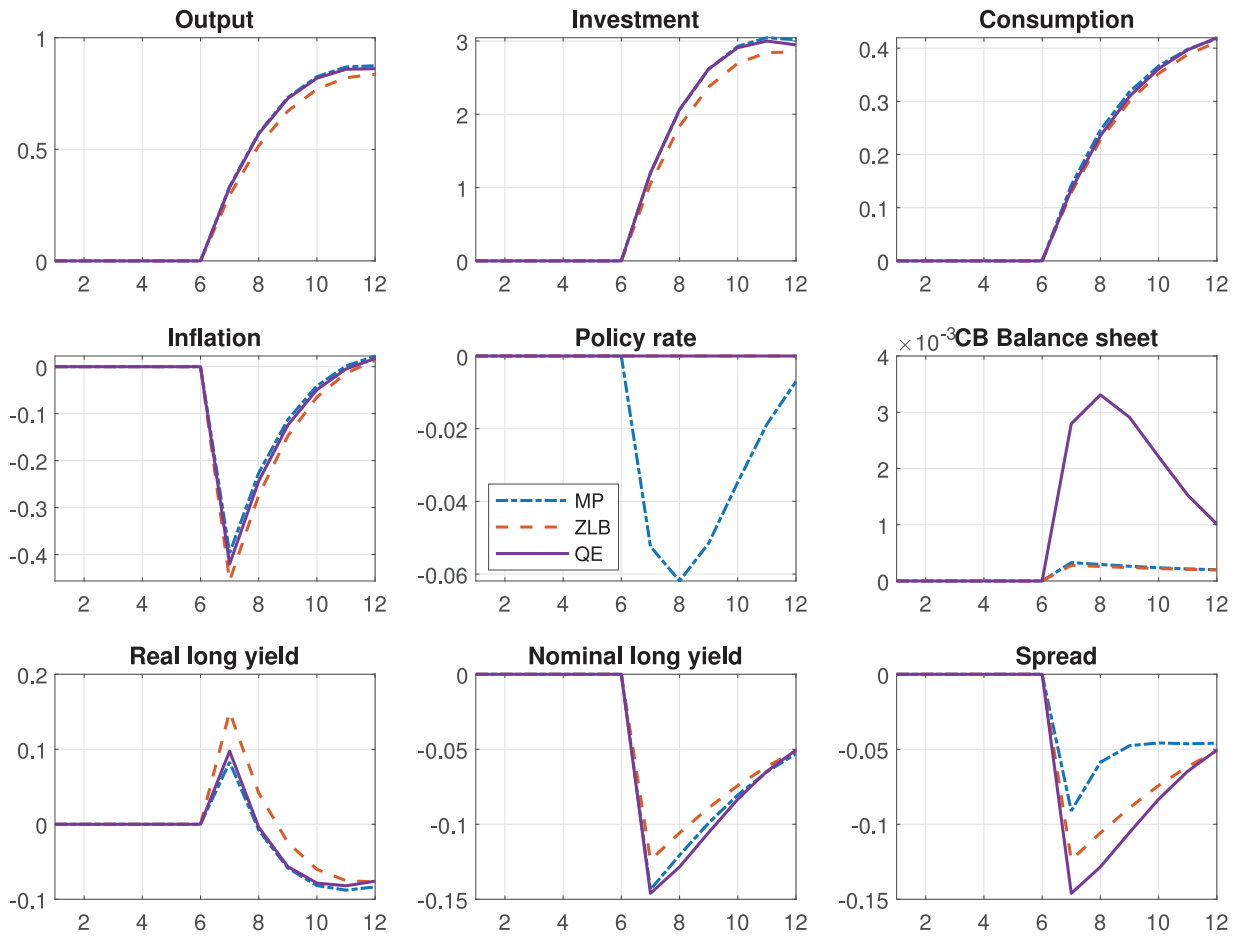
same conversion factor we used above in (4.1) when studying exogenous QE. As we shall show, this conversion rate results in endogenous QE policy when the policy rate is constrained by zero closely mimicking conventional policy.

5.1. Endogenous monetary policy responses to economic shocks

In this subsection, we study the conditional responses to different exogenous shocks when the policy rate is constrained by the ZLB. We find that endogenous QE effectively ameliorates the effects of the ZLB in response to each shock.

Credit shock. Consider first a shock to θ_t. In the model, this measures the fraction of assets with which an intermediary may abscond in the event of bankruptcy; equivalently, $1 - \theta_t$ is the fraction of assets recoverable by creditors. An increase in θ_t therefore makes it more difficult for households to recover funds in the event of failure, which, other things being equal, makes them less willing to lend to intermediaries. We therefore refer to this shock as a credit shock – an increase in θ_t results in creditors pulling funds out of intermediaries, which in equilibrium results in a decline in the supply of credit and an increase in interest rate spreads. This represents a simple way for the model to capture the "run" dynamics on the shadow banking system during the recent crisis as described, for example, in Gorton and Metrick (2012).

Fig. 4 plots impulse responses to a one standard deviation credit shock. The shock occurs in period seven. The procedure to generate the ZLB is exactly the same as in the exogenous policy experiments in Section 4. With conventional monetary policy (blue dash-dotted lines) unconstrained by the ZLB, a one standard deviation credit shock lowers output by about 1 percent at the peak. The shock results in a sharp increase in both the real and nominal long term yields. There is a consequent sharp decline in investment and inflation. The conventional policy response involves lowering short term rates, which partially offsets the decline in aggregate demand. The dashed red lines depict how the economy would react to the same shock if monetary policy were constrained by the ZLB and there is no unconventional policy response. With an inability to lower the policy rate, output falls substantially more in response to the shock – output declines by about 1.7 percent at peak. The shock is substantially more deflationary at the ZLB and the real and nominal long yields rise by significantly more in comparison to when policy is unconstrained.
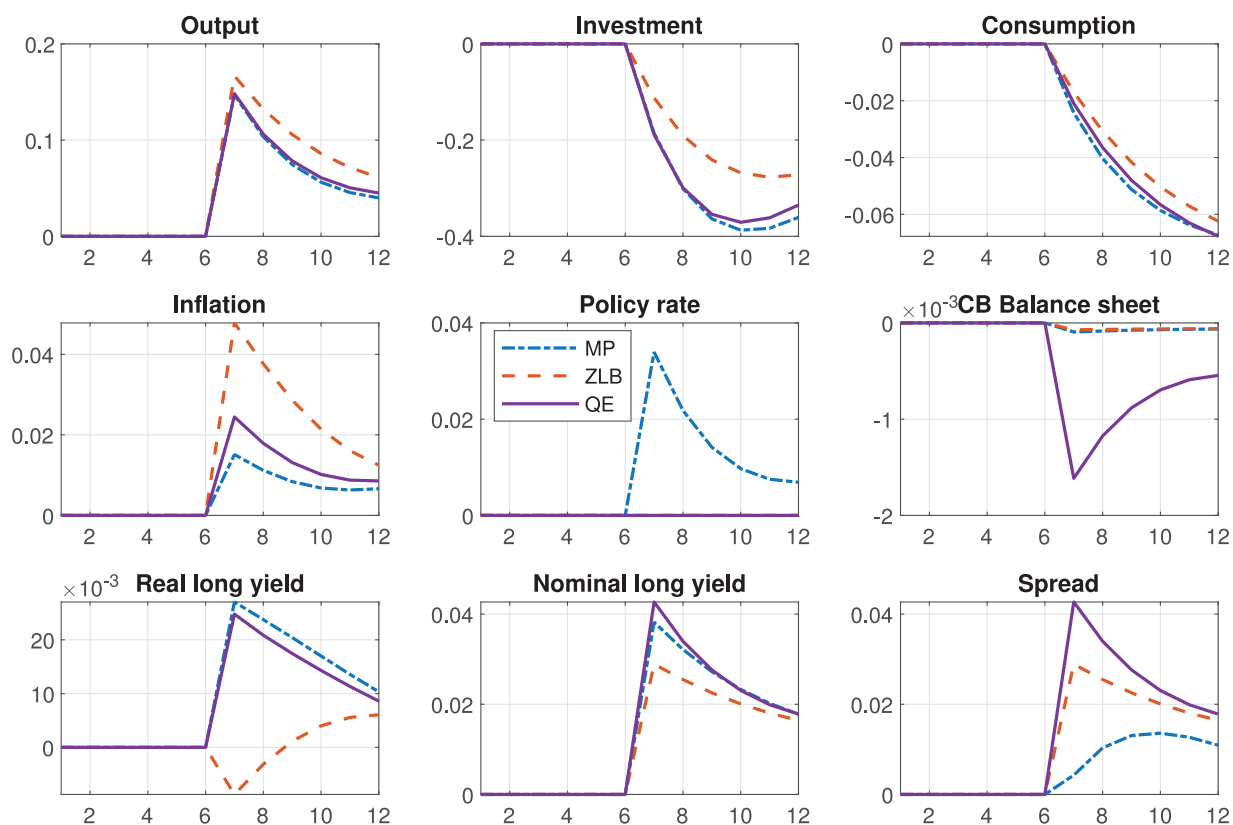
Fig. 5. Technology shock. *Notes:* This figure plots impulse responses to a one standard deviation technology shock. The shock hits in period 7. Blue dash-dotted lines: conventional monetary policy without the ZLB. Red dashed lines: the ZLB with no unconventional monetary policy. Purple solid lines: the ZLB with endogenous QE. We create the ZLB with a sequence of credit shocks of 1.5 standard deviations from periods 1 - 6.

The solid purple lines in Fig. 4 plot responses when the policy rate is constrained by the ZLB but the central bank implements endogenous QE via (5.1). Endogenous QE is able to closely replicate the responses of output and investment to the shock under conventional policy. To replicate the responses under conventional policy (which entails an eventual 35 basis point cut in the policy rate), it is necessary for the central bank to increase the size of its balance sheet by about 1.5 percent of steady state output at the maximum.

The underlying reason why endogenous QE is effective in response to the credit shock is similar to what is discussed above for exogenous policy shocks. Endogenous QE prevents long term interest rates from rising as much as they would when policy is constrained by the ZLB, thereby lowering the slope of the yield curve. This encourages investment and results in similar output dynamics in comparison to conventional policy. Consumption dynamics are different because it depends on the short term interest rate (evident in the upper right panel of the figure), but consumption contributes little to output fluctuations.

Technology shock. Consider next the impulse responses to a one standard deviation technology shock. Responses under different policies scenarios are depicted in Fig. 5.

Under conventional policy, output, consumption, and investment all jump up on impact and continue to grow for a number of periods. Inflation and the policy rate both fall. When the policy rate is constrained and there is no unconventional policy response, output and other aggregates still rise on impact, but by less in comparison to normal times. The central bank would like to lower the policy rate to accommodate the increase in aggregate supply, but the inability to do so results in long term interest rates being higher than they otherwise would be, which dampens demand. When the central bank can endogenously adjust its balance sheet by purchasing bonds, in contrast, the responses of output, investment, and consumption virtually lie atop the responses under conventional policy. Note that this is with exactly the same value of Ψ_f as in Fig. 4. The requisite bond purchase to closely mimic the responses under conventional policy is not large – at the peak, the central bank needs to increase the size of its balance sheet by 0.3% of steady state output.

Fig. 6. Government spending shock. *Notes:* This figure plots impulse responses to a one standard deviation government spending shock. The shock hits in period 7. Blue dash-dotted lines: conventional monetary policy without the ZLB. Red dashed lines: the ZLB with no unconventional monetary policy. Purple solid lines: the ZLB with endogenous QE. We create the ZLB with a sequence of credit shocks of 1.5 standard deviations from periods 1 - 6.
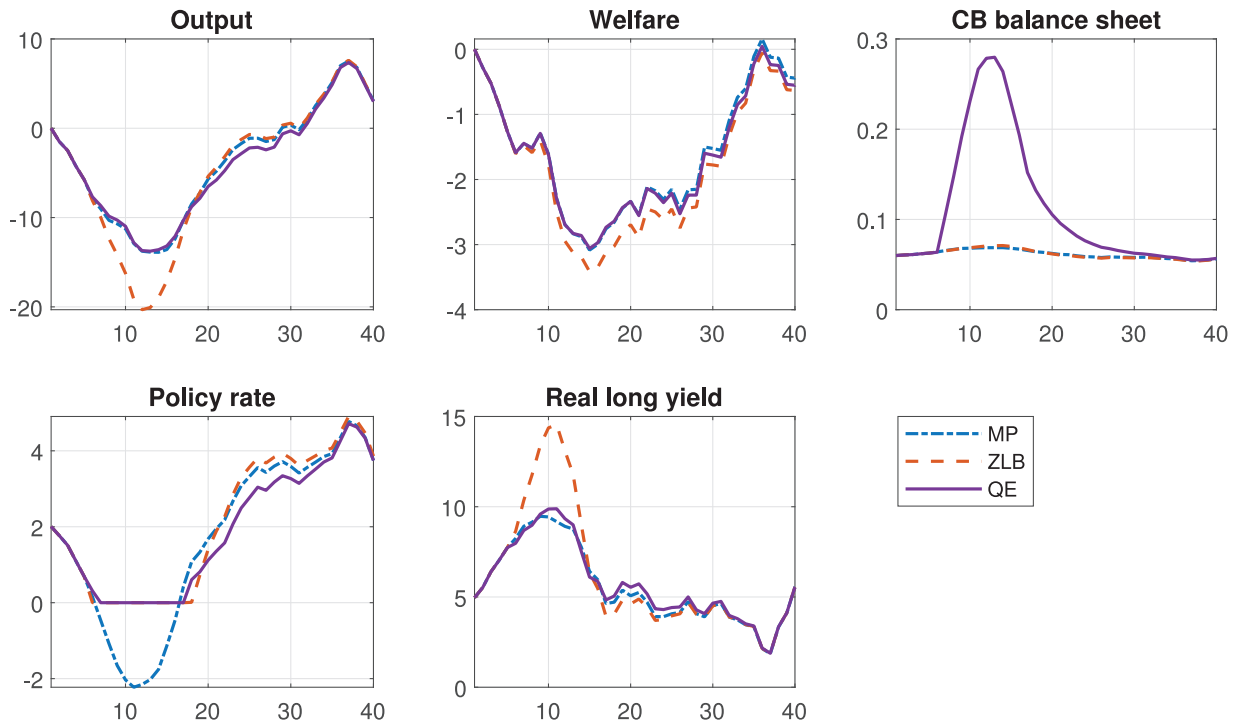
Government Spending Shock. The other exogenous shock in our model is a shock to government spending. The impulse responses under different monetary policy scenarios are shown in Fig. 6.

In normal times, the government spending shock is expansionary for output, but consumption and investment both decline (i.e. the multiplier is less than unity). Inflation and the policy rate both rise, with long term interest rates rising as well. Next, we discuss the case when conventional policy is constrained by the ZLB and unconventional policies are unavailable. Note, to create the zero lower bound environment, we hit the economy with a sequence of credit shocks. The size of these shocks are large enough to keep the economy at the ZLB for the entire duration shown in the plots. Although the government spending shock increases the notional, desired policy rate, we are considering a sufficiently small shock so as to not move the economy away from the ZLB. In the red dashed lines, output expands by more compared to normal times. Consumption and investment both fall by less in comparison to normal times, although for this particular experiment both still decline (i.e. the multiplier remains below unity).[21] This basic pattern is consistent with other papers that argue that the government spending multiplier can be substantially larger at the ZLB – see, e.g., Christiano et al. (2011). With endogenous QE, in contrast, output and other variables react similarly at the ZLB compared to normal times. The underlying mechanism is the same as above conditional on the other shocks – by buying or selling bonds, the central bank can implement similar paths of long term interest rates as when it engages in conventional policy by adjusting short term rates.

5.2. The great recession

In this subsection we conduct a simulation analysis of the model. The economy starts in the non-stochastic steady state. Random technology, government spending, and credit shocks are then drawn for the subsequent 40 periods. To roughly mimic the experience of the Great Recession in the United States, we fix the credit shocks in periods 2 through 10 so as to drive the economy to the ZLB. This sequence of shocks causes the ZLB to bind for roughly four years in expectation. Absent a ZLB constraint, it would result in the desired policy rate falling to two percentage points below zero.

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[21] For longer expected durations of the ZLB, or different parameterizations of the model, the multiplier can exceed unity (i.e. consumption and/or investment rise on impact). However, it is generally the case that our model is less sensitive to the ZLB than an otherwise similar model with frictionless financial markets.

Fig. 7. Simulation. *Notes:* This figure plots simulated paths of different variables from one draw from our simulation. The economy begins in period 1 in steady state. From period 2 on, we draw random shocks to technology, government spending, and credit. We fix the size of the credit shock at 1.5 standard deviations from periods 2 - 10 to create the ZLB environment. Blue dash-dotted lines: normal times with conventional policy; red dashed lines: ZLB; solid purple lines: ZLB with endogenous QE. Output is expressed as percentage deviations from the steady state; welfare is expressed as absolute deviations relative to the steady state; the size of the central bank's balance sheet is expressed relative to current period output; and interest rates are in annualized percentage points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 7 plots simulated values of output (expressed in percentage deviations relative to steady state), welfare (absolute units relative to the steady state), the size of the central bank's balance sheet (relative to annualized output), the policy rate (expressed at an annualized percentage rate), and the real long yield for one particular draw of random shocks.[22,23] The dashed blue lines correspond to simulated values under conventional policy absent a ZLB constraint. The dashed red lines impose the ZLB and feature no unconventional policy reaction. The solid purple lines correspond to simulated paths assuming that endogenous QE becomes operative while the policy rate is constrained by zero. After the ZLB period is over, central bank bond holdings revert to obeying an exogenous AR(1) process.

Absent a ZLB constraint, for this particular draw of shocks output declines by more than 10 percent relative to steady state. It remains persistently low and does not return to steady state until about period 30, which is when the adverse sequence of initial large credit shocks that are employed to make the ZLB bind have mostly abated. Taking the ZLB constraint into account and assuming no unconventional policy response, in contrast, results in a much sharper output contraction. In particular, output falls a little more than 20 percent below steady state at peak. Put slightly differently, the ZLB causes output to decline by roughly twice as much in comparison to a version of the model ignoring the ZLB. The mechanism behind this amplification is the behavior of the real long yield (bottom right panel of the figure). The combination of deflation plus a fixed short term rate results in a very large spike in the real long yield.

It is apparent from Fig. 7 that endogenous QE largely mitigates the effects of the binding ZLB. In fact, for the first 18 or so periods of the simulation, it is virtually impossible to distinguish between the simulated paths of output and the real long yield under endogenous QE relative to their simulated paths ignoring the ZLB. There is some difference evident after about period 20 of the simulation, where output under the ZLB with endogenous QE lies slightly below the simulated path

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[22] Welfare is defined as the present discounted value of utility flows of the representative agent in the model, i.e. $\mathbb{W}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{t+j} - bC_{t+j-1} \right) - \frac{\chi L_{t+j}^{1+\eta}}{1+\eta} \right\}$. In Fig. 7, welfare does not jump down immediately because agents do not anticipate the shocks we use to drive the economy into the recession.

[23] Note that the model is solved via a first-order perturbation, which is necessary to implement the occasionally binding algorithm of Guerrieri and Iacoviello (2015). Because the steady state of our model is distorted relative to an efficient allocation (because of price and wage markups and constrained intermediaries), alternative policy regimes are relevant for welfare even to first-order.

of output under conventional policy. This is driven by the unwinding of the balance sheet subsequent to the ZLB period and is discussed further in Section 6.

The simulated path of welfare is very similar to that of output. To put this into interpretable units, we can translate the welfare losses to consumption equivalent units.[24] Even without factoring in the ZLB and instead assuming conventional monetary policy, the sequence of adverse credit shocks fed into the model would generate, at peak, a welfare decline equivalent to permanently reducing consumption by 1.32 percent. Taking the ZLB into account without any additional policy interventions produces a welfare decline equivalent to a permanent 1.50 percent decline in consumption. Put differently, the existence of the ZLB without QE results in a 0.18 percentage point decline in welfare. In contrast, implementing endogenous QE brings the welfare loss down 1.31 percent of consumption. While this might seem small in an absolute sense, it represents a very large welfare gain relative to much of the literature on optimal policy (see, e.g., Schmitt-Grohe and Uribe, 2007). Implementing QE undoes all of this welfare loss from the ZLB and is essentially equivalent to conducting conventional policy via the Taylor rule.[25]

The sequence of shocks used to generate Fig. 7 pushes the desired short term policy rate more than 2 percentage points below zero. This is roughly in-line with the estimated decline in the shadow rate series of Wu and Xia (2016). To generate the same response of output, a central bank following our endogenous QE rule purchases bonds in an amount that pushes the size of its balance sheet to about 25 percent of GDP. This is roughly the experience of the US during the Great Recession, where between QE1 and QE3 the Fed increased the size of its balance sheet to roughly 25 percent of GDP.

Fig. 7 focuses on one particular draw of shocks over a 40 quarter time horizon. In Fig. 8 we graphically depict results from repeating this exercise 1000 different times. The figure shows scatter plots of moments of output when the short rate is subject to the ZLB (with and without endogenous QE) on the vertical axes against moments of output under conventional policy on the horizontal axes. The moments on which we focus are the mean and standard deviation of output, expressed in percentage deviations from steady state. Red circles correspond to moments with a binding ZLB but no endogenous QE, while purple stars are moments at the ZLB with endogenous QE. For ease of interpretation, we include 45 ° lines in each plot.

Focus first on the upper panel of Fig. 8, which is based on the first 15 periods of each of the 1000 simulations. The ZLB binds during these periods for all of the 1000 simulations. The red circles lie far below the 45 ° line in the case of the mean, and far above the 45 ° line in the case of the standard deviation. In other words, when the economy is at the ZLB and unconventional policies are unavailable, output is on average significantly lower and more volatile relative to a situation in which there is no ZLB. The purple stars, in contrast, align tightly with the 45 ° line for both the mean and standard deviation scatter plots. This suggests that endogenous QE can serve as a highly effective substitute for conventional policy during periods in which the ZLB binds.
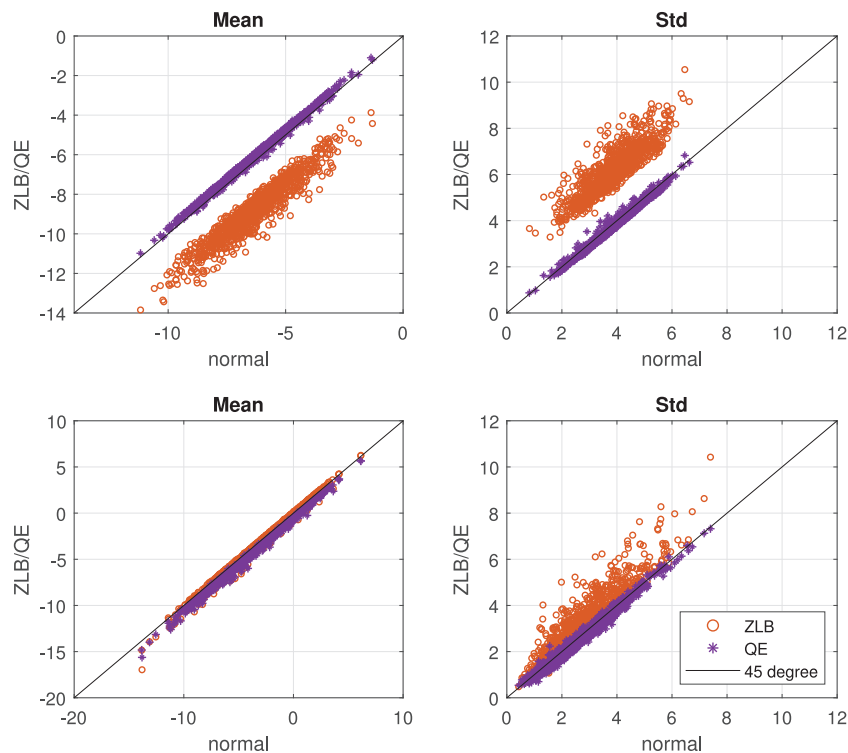
Next, focus on the bottom panel of Fig. 8, which computes moments based on periods 16 to 40 of each simulation. The ZLB begins to lift in period 16. As a result, there is little observable difference between mean output with and without endogenous QE. Relative to the upper panel of the figure, the purple stars on average lie slightly below the 45 ° line in the case of mean output. This suggests that in the simulations where endogenous QE is undertaken during a ZLB period, the economy produces slightly less on average after the ZLB has ended. This is also evident in Fig. 7 for one draw. In terms of the volatility of output, the purple stars still align closely with the 45 ° line in the lower panel. There is, on average, slightly more volatility in periods 16–40 in the economy without endogenous QE, though it is markedly less than in periods 1–15 as shown in the upper panel. What is driving this is that, as noted above, while the ZLB begins to lift in period 16, in many of the simulations it lasts longer. In some other draws, the economy temporarily reverts to the ZLB subsequent to the liftoff.

Table 2 summarizes these same moments by reporting averages across the 1000 simulations. The table corroborates the visual impressions from Fig. 8. Even without taking the ZLB into account, output would be substantially below the steady state given the large sequence of credit shocks fed into the model. The ZLB with no unconventional policy exacerbates this. Focusing on the first fifteen periods of the simulation, which correspond to the first rows of each block of the table, the level of output would be on average about 2.5 percentage points lower and about 60 percent more volatile taking the ZLB into account versus not. The reason for the lower mean and enhanced volatility is the behavior of the real long yield, which is on average both higher and more volatile when taking the ZLB into account. In contrast, with endoge-

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[24] We do so as follows. First, note that we can split welfare up into two additive recursive components (see footnote [22]): $\mathbb{W}_t = \mathbb{W}_t^c + \mathbb{W}_t^l$, where $\mathbb{W}_t^c = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{t+j} - b C_{t+j-1} \right) \right\}$ and similarly for $\mathbb{W}_t^l$. Then we measure welfare in each period after the sequence of shocks as shown in Fig. 7, focusing on the welfare decline in period 18 when the economy exists the ZLB. We then solve for the fraction of steady state consumption, $\iota \in [0, 1]$, that would have to be taken away from the representative agent to generate an equivalent welfare decline in steady state. We measure three different ι for the three different policy cases conditioning on the same sequence of credit shocks – conventional monetary policy, the ZLB with no unconventional policy, and the ZLB with the endogenous QE rule.

[25] In fact, in our simulations the welfare loss is actually ever so slightly *smaller* when the policy rate is constrained by the ZLB and endogenous QE is undertaken. This result should not be taken to mean that our model suggests conventional policy ought to be abandoned in favor of QE all the time. For one, the parameters of our Taylor rule are not optimally chosen to minimize welfare losses. With an optimal Taylor rule it is likely that the welfare loss of 1.32 percent would be significantly smaller. Second, this result is conditional on the realization of credit shocks; as emphasized in Sims and Wu (2019), in this kind of model QE is particularly well-suited to address credit market shocks, but not necessarily other kinds of shocks (e.g. productivity or other types of demand shocks) for which adjustment of the short-term policy rate is more appropriate.

Fig. 8. Simulation. *Notes:* We generate 1000 different simulations of 40 periods each. For each simulation, from period 2 on we draw random shocks to technology, government spending and credit. We fix the size of the credit shock at 1.5 standard deviations from periods 2 - 10 to create the ZLB environment in each simulation. We compute the mean and standard deviation over periods 1 - 15 (top row) or over periods 16 - 40 (bottom row) for output for each of the 1000 different draws. We compare between ZLB (red circles) and endogenous QE (purple stars) on the vertical axes against normal times on the horizontal axes. Each of the 1000 circles or stars represent moments from each of the 1000 draws. Solid black line: 45 ◦ line. Units: percentage deviations from the steady state. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

nous QE, during the ZLB period the mean and volatility of output are roughly equal to what they would be absent a ZLB constraint.

The second rows in each panel of Table 2 show moments calculated from periods 16 to 40 of each simulation. These moments are calculated mainly during periods after the ZLB has ended. Output would still be well below steady state in these periods even if there were no ZLB constraint. Interestingly, the version of the model taking the ZLB into account with no endogenous policy comes closer to matching the mean of output than does the version of the model with endogenous QE. This suggests a tradeoff – endogenous QE results in significantly better performance during a ZLB period at the expense

Table 2
Simulated neans and volatilities.

	Mean			Std		
	MP	ZLB	QE	MP	ZLB	QE
Output	−6.36	−8.85	−6.12	3.84	6.15	3.73
	−3.48	−3.67	−4.09	2.70	3.21	2.57
Real long yield	7.65	9.39	7.68	1.33	3.15	1.48
	5.81	5.77	6.05	0.86	0.98	0.86

Notes: We generate 1000 different simulations of 40 periods each. For each simulation, from period 2 on we draw random shocks to technology, government spending, and credit. We fix the size of the credit shock at 1.5 standard deviations from periods 2 - 10 to create the ZLB environment in each simulation. We compute the mean and standard deviation over periods 1 - 15 (first row) or over periods 16 - 40 (second row) for each draw. The table reports the average across 1000 draws. MP stands for conventional monetary policy with no ZLB constraint; ZLB refers to the zero lower bound; and QE denotes the ZLB on short term rates but with endogenous QE. Units are percentage deviations from the steady state for output and annualized percentage points for the real long yield.

of slightly worse outcomes once the ZLB has ended (at least in terms of mean output relative to steady state).[26] As alluded to above, this is driven by unwinding of the balance sheet once the ZLB has ended. This unwinding exerts a mild contractionary effect on the path of output, and raises questions about the optimal speed of unwinding. We turn to such questions next.

6. Issues going forward: QT, NIRP, and ELB

Combatting a sequence of negative demand shocks and minimizing the adverse consequences of the ZLB potentially requires a large expansion in the size of a central bank's balance sheet. This is true both in our model as well as in practice for the myriad countries that engaged in aggressive bond purchasing programs in the wake of the Great Recession. In this section, we discuss three broad issues related to the post-QE size of central bank balance sheets going forward. First, an imminent issue facing policymakers around the globe is how to unwind their balance sheets while minimizing potential disruptions to the economy. We discuss the unwinding process, also known as quantitative tightening (QT), in Section 6.1. Second, there exists an interaction between the size of a central bank's balance sheet and the efficacy of NIRP. This is discussed in Section 6.2. Third, the size of the balance sheet constrains how negative policy rates can go. We make a first attempt at endogenizing the effective lower bound (ELB) in our model in Section 6.3.

6.1. Quantitative tightening

In the previous section, we showed that endogenous QE can serve as an effective substitute for conventional monetary policy during ZLB periods. At the same time, we showed that the economy tends to perform somewhat worse *after* the ZLB period has ended when endogenous QE is undertaken as an antidote to the ZLB. What drives this is the unwinding of the central bank's balance sheet as it transitions away from bond purchases.
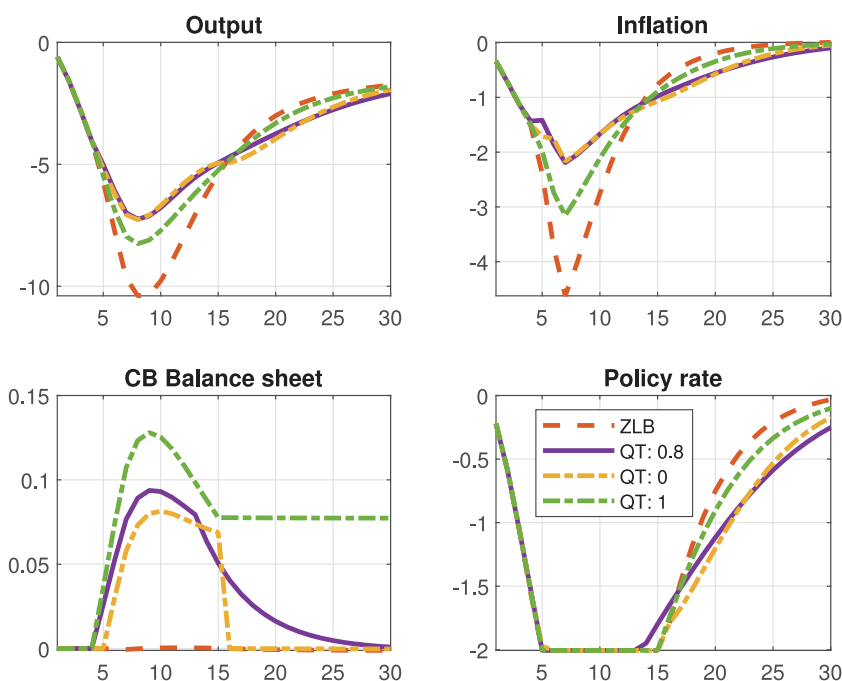
We wish to study the consequences of different paces of balance sheet normalization. We design our QT experiment as an extension to the impulse responses depicted in Fig. 4. Instead of showing the responses of variables to the incremental one standard deviation θ shock, we plot the responses based on all the shocks, including those that we use to create the ZLB environment. In this way, we are able to generate a realistic depiction of the post-Great Recession experience. This is shown in Fig. 9.

Red dashed and purple solid lines are the same as those in Fig. 4, except that the sequence of credit shocks used to generate the ZLB are no longer differenced out. The red dashed lines plot simulated paths of variables under the ZLB constraint with no endogenous QE. The purple solid lines show paths of variables under the ZLB constraint with endogenous QE, followed by QT, with bond-holdings reverting to the AR(1) process as specified in (3.5) with the same autoregressive parameter of $\rho_f = 0.8$. The dash-dotted lines also implement QT after QE, but with different speeds of unwinding (i.e. different values of ρ_f). The pace of unwinding is especially relevant and has appeared prominently in many recent policy discussions.

The case of immediate unwinding is shown in orange. This was not considered a viable possibility in policy circles. Relative to our baseline case of $\rho_f = 0.8$, the orange lines with $\rho_f = 0$ are similar during the ZLB period but show a slightly lower trajectory of output once the ZLB has ended. Another extreme is for the central bank to carry forward the large balance sheet accumulated during QE with no unwinding. This possibility has been raised in policy discussions, and is captured by the green dash-dotted lines with $\rho_f = 1$. Interestingly, no normalization results in a much more severe recession and worse deflation during the ZLB period, although a slightly higher trajectory of output once the ZLB has abated. Because of the worse economic conditions during the ZLB period, no balance sheet normalization requires a larger expansion for the central bank's balance sheet.

Both of these results are somewhat surprising and cut against the grain of conventional wisdom in policy circles, where it was thought that immediate unwinding would be detrimental but that carrying forward a large balance sheet would be inconsequential. In contrast, in our model immediate unwinding is fairly similar to our baseline case of smooth QT, whereas no unwinding exacerbates the output decline during the ZLB period. Moreover, our results suggest that the anticipated speed of unwinding after the ZLB ends has an important impact for how the economy fares during the ZLB. Our intuition for these results is as follows. When the central bank credibly promises to unwind the balance sheet after the ZLB lifts, this makes the present a relatively more attractive time to invest relative to the future. In contrast, when there is no credible commitment to reduce the balance sheet in the future, firms can smooth out their investment expenditures across more periods. Hence, aggregate demand in the present reacts more strongly to a given bond purchase when firms expect purchases to be reversed when the ZLB lifts. Consistent with our quantitative results, therefore, QE is a more effective antidote to the ZLB when agents believe QT will be undertaken relatively quickly, with the tradeoff being that rapid QT results in worse economic performance once the ZLB lifts. The bottom line is that, to achieve the desired effectiveness of QE, policy makers need to plan out the entire path ex ante, and credibly communicate that with the public.

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[26] In contrast, as is also evident in the bottom panel of Fig. 8, output remains slightly more volatile in periods 16–40 in the simulation where no endogenous QE is undertaken relative to the simulation with endogenous QE.

Fig. 9. Quantitative tightening. *Notes:* Red dashed lines: ZLB without unconventional monetary policy. Purple solid lines: QT according to (3.5) with $\rho_f =$ 0.8; orange dash-dotted lines: QT according to (3.5) with $\rho_f = 0$; green dash-dotted lines: QT according to (3.5) with $\rho_f = 0.1$. All the variables are relative to the steady state. Output is expressed in percentage points. Inflation and policy rate are expressed in annualized percentage points. The central bank's balance sheet is expressed as a percentage relative to annualized steady-state output. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
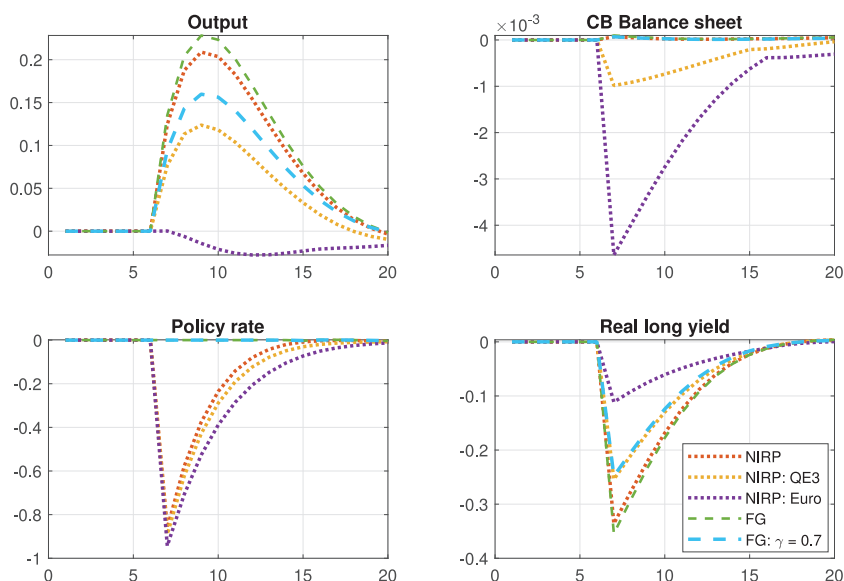
6.2. Interaction between QE and NIRP

Moving forward, it is also important to understand interactions between different types of unconventional policy interventions. The size of a central bank's balance is relevant for the efficacy of NIRP. That is because negative policy rates function as a type of tax on intermediaries; the more reserves intermediaries hold, the more burdensome is the tax. This tends to undo the stimulative signaling feature of NIRP.

Dotted lines in Fig. 10 depict impulse responses to 100 basis point NIRP shocks conditioning on different sizes of the central bank's balance sheet. Red dotted lines are responses when the central bank's balance sheet is about 6 percent of steady state GDP; this is identical to Fig. 2. The orange and purple dotted lines condition on balance sheet sizes calibrated to the size of the Fed's post-QE3 balance sheet and the ECB's balance sheet as of the end of 2018. For comparison, dashed lines plot responses to forward guidance shocks assuming varying degrees of credibility. The green and light blue responses are identical to Fig. 2.

As the central bank's balance sheet becomes large, NIRP becomes less stimulative. For example, conditioning on a balance sheet calibrated to post-QE3 in the US, a NIRP shock becomes half as stimulative as the benchmark in red, and less stimulative than non-credible forward guidance ($\gamma = 0.7$). When the balance sheet is as large as in the Euro area, a cut in the policy rate into negative territory is actually mildly contractionary. This is because the contractionary banking channel outweighs the expansionary forward guidance channel of NIRP. This interaction is especially relevant for countries such as Japan and the Euro area that have accumulated large balance sheets and have already pushed policy rates into negative territory. An interesting implication from our model therefore relates to the timing of unconventional policy interventions. In practice, economies that eventually turned to negative rates all engaged in heavy QE programs prior to doing so. From the perspective of our model, it would have instead been more effective to first push policy rates into negative territory and then to engage in QE programs.

Recently, the Swiss National Bank (SNB) introduced a new form of NIRP. Negative interest will only be charged on the portion of a bank's reserve account balance that exceeds a given threshold. By highlighting two channels through which NIRP can transmit to the economy – a stimulative forward guidance channel and the offsetting banking channel – our model provides a useful framework to think about the potential benefits of the SNB's policy. By exempting a fraction of reserves, this policy design would weaken the offsetting banking channel highlighted in this paper, thereby making NIRP more powerful and the consequences of a large central bank balance sheet less severe.

Fig. 10. Exogenous NIRP and balance sheet. *Notes:* Dotted lines are plots of responses to exogenous NIRP shocks conditioning on different sizes of the central bank's balance sheet. Red dotted lines: NIRP with the steady state central bank's balance sheet of 6% of the GDP (calibrated to pre-QE); orange dotted lines: NIRP with the steady state balance sheet of 25% GDP (calibrated to post QE3); purple dotted lines: NIRP with the steady state balance sheet of 38% of GDP (calibrated to the Euro area to 2018Q4). Green dashed lines: fully credible forward guidance; light blue dashed lines: partially credible forward guidance with $\gamma = 0.7$. We generate a binding ZLB with a sequence of credit shocks of 1.5 standard deviations in each of the periods 1 - 6. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

6.3. Implications of a large balance sheet for the ELB

In practice, no central bank has experimented with moving policy rates into deep negative territory. An important question is how negative rates could conceivably go without adverse consequences.

Though we have not imposed it thus far, in principle there exists a lower bound on the interest rate on reserves in our model. In particular, if $\theta_t \phi_t$ were to fall below one (i.e. net worth is more valuable in a household than in an intermediary), intermediaries might choose to voluntarily exit rather than continue. How low R_t^{re} can go without violating $\theta_t \phi_t \geq 1$ depends on (i) the magnitude of excess returns on private and government bonds and (ii) the total value of reserves issued by the central bank. The lower are excess returns, or the larger is the volume of reserves, the higher will be the ELB.
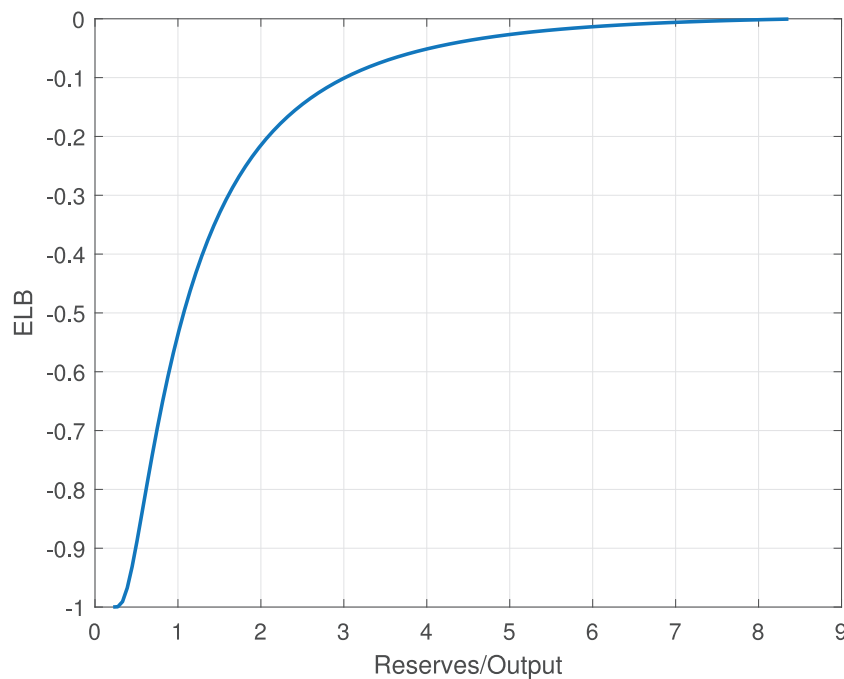
Fig. 11 plots the minimum value of steady state R^{re} consistent with $\theta \phi \geq 1$ as a function of the quantity of reserves (as a fraction of output). When there are no (or few) reserves, there is essentially no limit on how negative R^{re} can go. As there are more reserves in the system, and reserves constitute a higher fraction of intermediary assets, the effective lower bound becomes tighter. The intuition is straightforward. Negative policy rates are essentially a tax on the holders of reserves. The more reserves there are, the more punitive is this tax, and at some point intermediaries might find it undesirable to continue.

Caution is in order when interpreting the quantitative magnitudes in Fig. 11 seriously. Our model abstracts from many real-world features that might further inhibit a central bank's ability to push policy rates into deep negative territory. Nevertheless, we think that the qualitative relationship between the size of the central bank's balance sheet and the ELB is both instructive and intuitive. It is also potentially increasingly relevant for countries such as Japan and the Euro area that have now amassed very large balance sheets. While more work remains to be done, we provide a useful first benchmark to think more seriously about this issue.

7. Conclusion

In this paper, we developed a quantitative DSGE framework to systematically evaluate three different types of unconventional policy tools – quantitative easing (QE), forward guidance (FG), and negative interest rate policy (NIRP) – as well as their interactions. This represents an important step forward in the literature, which has to date tended to focus on each policy tool in isolation. We emphasize two distinct channels by which NIRP can transmit to the economy: a forward guidance channel and a banking channel. We introduce a novel way to model forward guidance, where our approach ameliorates the forward guidance "puzzle."

We find that unconventional policies can all stimulate output as much as a conventional policy shock. Because it has arguably been the most prominent of the unconventional policy tools deployed over the last decade-plus, we focus most of our quantitative analysis on QE. We specify an endogenous feedback rule for QE and show that it can effectively neutralize

Fig. 11. ELB. *Notes:* This figure plots the minimum steady state value of $R^{re} - 1$ (expressed at an annualized rate) as a function of the steady state size of the central bank's balance sheet (relative to output). For this figure, we set $\beta = 1$ so that the deposit rate, R^d, is fixed at its lower bound of one (or zero in net terms).

the adverse consequences of a binding zero lower bound on the short term policy rate. In an application meant to mimic the experience of the US during the Great Recession, we show that our endogenous QE rule, which expands the central bank's balance sheet to 25 percent of GDP (which is roughly in-line with the Fed's balance sheet post-QE3), provides stimulus roughly equivalent to moving the policy rate 2 percentage points below zero. This is in-line with empirical estimates of the shadow rate from Wu and Xia (2016).

Though endogenous QE is an effective substitute for conventional policy when faced with a binding ZLB, its use is not without cost or potential complication. When unwinding the balance sheet, a smooth balance sheet normalization is preferred. On the contrary, carrying a large balance sheet forward would cause a deeper recession and force the central bank to expand their balance sheet even more at the peak. Moreover, a larger balance sheet undermines the efficacy of NIRP or even makes it contractionary. We provide a first attempt at endogenizing an effective lower bound (ELB) for policy rates, which also depends on the overall size of a central bank's balance sheet.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2020.03.018.

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