Photonic Ising Spin-Glass via Chip-Based Degenerate Kerr Oscillators

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Abstract: We demonstrate reconfigurable all-optical coupling between two degenerate optical parametric oscillators in silicon-nitride microresonators. We show in-phase and out-of-phase operation which is achieved at a fast regeneration rate of 400 kHz with a large phase tolerance.

In order to meet the evergrowing demand for solving computation problems that scale exponentially in time and energy, there have been explorations of alternative computing architectures such as quantum computing and coherent computing [1,2]. Recently, there has been considerable interest in the development of a photonic processor to realize a novel form of coherent computing for simulating the Ising model [3-6], which was developed for modeling ferromagnetism and governed by a Hamiltonian that couples discrete variables that represent spin glasses [7]. Solving for the ground state of such a system corresponds to solving a non-deterministic polynomial-time (NP) hard computational problem and can provide an architecture for solving other NP-complete problems through polynomial-time mapping [8]. The physical realization of an Ising machine requires binary degrees of freedom and reconfigurable coupling. Recent efforts have shown that a network of coupled degenerate optical parametric oscillators (OPO's) based on the $\chi^{(2)}$ nonlinearity can be used to realize a coherent Ising machine [3-6]. This is realized through the nonequilibrium phase transition that occurs at the parametric oscillation threshold, which results in two coherent phases states that are offset by π. Controlling the coupling between the OPO's, which is done in these systems via measurement-feedback or time-multiplexing, results in the generation of complex, phase-locked output states that encode the ground state of an Ising model.

Alternatively, a degenerate OPO can be realized though the $\chi^{(3)}$ nonlinearity, where a frequency-degenerate signal/idler pair is generated through four-wave mixing parametric oscillation [9,10]. Previously, such a scheme has been implemented in silicon nitride (SiN, Si_3N_4) microresonators and bi-phase state generation has been achieved enabling quantum random number generation [10]. The SiN platform is ideally suited for scalability to an all-photonic network of coupled OPO's since it is CMOS-process compatible, has low losses in the near-infrared, and allows for phase matching of the degenerate signal through dispersion engineering of the fundamental mode. In addition, the SiN-based OPO system allows for simultaneous oscillation of all OPO's, continuous-wave (cw) operation, and does not rely on long cavity lengths that require stabilization to support the train of femtosecond pulses, offering faster computation speeds with lower power consumption in a compact footprint. Here, we demonstrate an integrated photonic circuit which consists of spatially-multiplexed degenerate OPO's on a single SiN chip. We experimentally show reconfigurability of the coupling phase between the OPO's through thermal control of the coupling waveguide between the two OPO's and show interference measurements between the OPO's indicating in-phase and out-of-phase operation. Our system can operate at a rate of 400 kHz with a convergence time of < 310 ns.

Figure 1(a) shows the microscope image of the 2-OPO system. For routing the pump lasers to the microresonators, we employ an on-chip power splitter using multimode interference (MMI). The two SiN microresonators (OPO_1 and OPO_2) have a free spectral range (FSR) of 500 GHz. To compensate for the slight difference in the FSR's of the two resonators, we use platinum microheaters above each resonator that enables electrical control of the resonances via the thermal-optic effect [11]. We implement reconfigurable unidirectional coupling between the two OPO's by using a coupling waveguide that directs a fraction of the OPO_1 output field to the input of OPO_2 through the bus waveguide. The chip is pumped using two frequency nondegenerate pump lasers that are offset by ±2 FSR's from the degeneracy point. In order to ensure that the OPO builds up from noise for each initiation, we modulate one of the pumps using an acousto-optic modulator with 310-ns pulses at a repetition rate of 400 kHz. Figure 1(b) shows the generated OPO spectra from the two microresonators. For simultaneous oscillation of both OPO's, we use 72 mW of combined pump power in each ring and set the electrical power to the heater in OPO_1 to 13.5 mW such that the resonances of the two

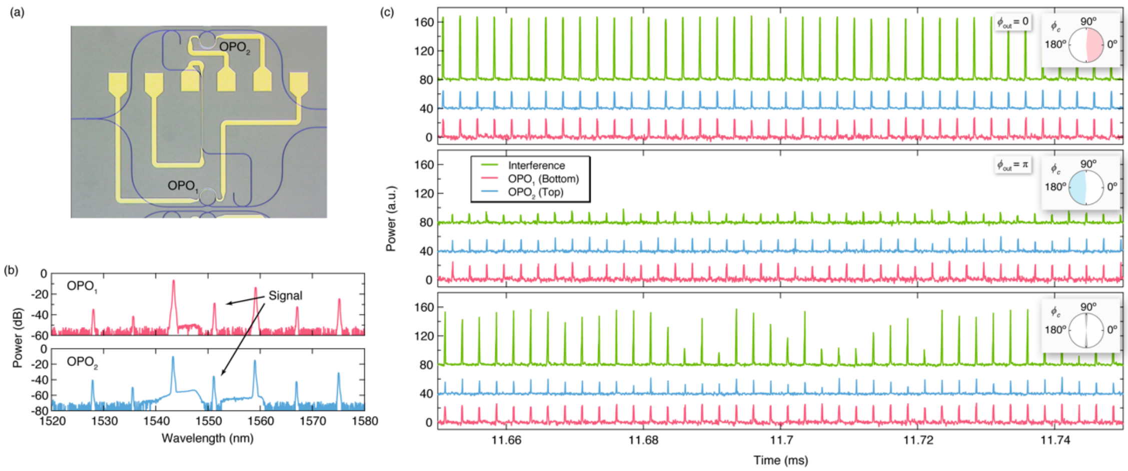

Fig. 1. (a) Microscope image of the device. (b) The measured spectra of the primary OPO (top, OPO_1) and the secondary OPO (bottom, OPO_2). (c) Temporal ensemble of interference measurements of the combined OPO's (green) along with the measurements of OPO_1 (red) and OPO_2 (blue). Inset shows the phase chart of the coupling phase ϕ_c. We indicate the coupled OPO operation regime for in-phase (pink), out-of-phase (blue), and transition regime (grey). (Top) Constructive interference (in-phase, $\phi_{\text{out}} = 0$) is observed for a heater power of 48.3 mW. (Middle) Destructive interference (out-of-phase, $\phi_{\text{out}} = \pi$) is observed for heater power of 50.2 mW. (Bottom) Transition from in-phase to out-of-phase for heater power of 50.2 mW. For clarity, OPO_2 and the interference are offset from OPO_1 in power by 40 and 80, respectively.

microresonators are spectrally overlapped. We readout the coherent phase states of the coupled OPO system by directly measuring the interference between the two OPO's by combining both outputs using a 50/50 beamsplitter. The combined signal is collected using a fiber collimator and sent to a fast photodiode. Before combining the beams, a 50/50 beamsplitter is used in each arm to collect the individual OPO signals.

The generated OPO signal from each microresonator is shown in Fig. 1(b). In our measurements, we manipulate the coupling phase ϕ_c between the OPO's below threshold by controlling the electrical power sent to the integrated heater above the coupling waveguide. Figure 1(c) shows the measured temporal interference signal (green) along with OPO_1 (red) and OPO_2 (blue) for three different heater powers. At 48.3 mW, we observe constrictive interference between the two OPO's corresponding to in-phase operation [Fig. 1(c) top]. For 54.2 mW of heater power, we observe destructive interference, corresponding to out-of-phase operation [Fig. 1(c) middle]. The transition from in-phase to out-of-phase operation occurs at 50.2 mW of heater power, where we observe both constructive and destructive interference [Fig. 1(c) bottom]. Moreover, we observe in-phase operation for heater powers below 48.3 mW and out-of-phase operation for powers above 54.2 mW, indicating that the OPO's above threshold operate in-phase or out-of-phase for a continuous range of coupling phase ϕ_c. The convergence time is well within the pump pulse duration of 310 ns. The system is highly tolerant to the coupling phase between the OPO's, offering flexibility in setting up the system when scaling to larger number of OPO's. To enable reconfigurable coupling between arbitrary OPO's, a fraction of the OPO power will be sent to a matrix of Mach-Zehnder interferometers to perform non-unitary transformations. Our results represent the building blocks towards the realization of a chip-based photonic coherent Ising machine for solving combinatorial optimization problems.

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