Power Management in MIMO Ad Hoc Networks: A Game-Theoretic Approach

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***Abstract*— This paper considers interference characterization and management in wireless ad hoc networks using MIMO techniques. The power allocation in each link is built into a non-cooperative game where a utility function is identified and maximized. Due to poor channel conditions, some links have very low data transmission rates even though their transmit powers are high. Therefore, a mechanism for shutting down links is proposed in order to reduce cochannel interference and improve energy efficiency. The multiuser water-filling and the gradient projection methods are compared with the proposed game theoretic approach in terms of system capacity and energy efficiency. It is shown that using the proposed method with the link shut-down mechanism allows the MIMO ad hoc network to achieve the highest energy efficiency and the highest system capacity.**

***Index Terms*— MIMO, interference, game theory, power control.**

I. Introduction

With the increasing demand for wireless services, the efficient use of spectral resources is of great importance. Multiple Input Multiple Output (MIMO) communication systems hold great promise in using radio spectrum efficiently [1] while power control will improve energy efficiency. In applications like wireless ad hoc networks, battery life is the largest constraint in designing algorithms [2]. Therefore, it is important that power allocation be managed effectively by identifying ways to use less power while maintaining a certain quality of service (QoS).

There has been a considerable amount of research on power management in wireless systems. In [3], [4], power control algorithms were developed for cellular systems. Power control has also been studied with a combination of multiuser detection, beamforming and adaptive modulation [5], [6]. In [7], [8] adaptive algorithms were developed to improve system performance by controlling power allocation and data rate. As the use of MIMO technology in ad hoc networks grows, MIMO interference systems have attracted a great deal of attention. [9], [10] studied the interactions and capacity dependencies of MIMO interference systems and [11], [12] explored methods for power management and interference avoidance in MIMO systems. In recent years there has been a growing interest in applying game theory to study wireless systems. [13], [14] used game theory to investigate power control and rate control for wireless data. A game-theoretic approach to study power allocation in MIMO channels was developed in [15] and a game theory perspective on interference avoidance was provided in [16].

In this paper, we consider a stationary MIMO ad hoc network, where each transceiver pair is hindered by cochannel interference coming from other transceiver pairs operating in the same frequency band. It is known that minimizing interference using power control increases capacity and extends battery life for cellular systems [13]. We investigate optimum signaling for MIMO interference systems with feedback in a realistic ad hoc network environment and study how power control improves energy efficiency by using a game theoretic approach. We use computational electromagnetic simulations [17] to study the effect of interference on a network composed of multiple, cochannel MIMO links. These simulations, given a network topology and environment, calculate the received electromagnetic fields due to all of the multipath rays between every transmitter and every receiver. The simulations are performed using the software system FASANT, which has been used as a tool in system planning and has been validated using urban propagation measurements [18].

This paper is organized as follows. Section II introduces the system model and formulates the optimization problem. Two existing techniques from the literature are also introduced as a basis for comparison. In Section III, a game theoretic approach to power control is proposed where we construct a non-cooperative power control game and show how to design a utility function suitable for MIMO ad hoc networks. Simulation results with all methods are given and discussed in Section IV. Section V provides the conclusion.

II. Problem Formulation

Consider an ad hoc network with a set of links denoted by $\mathcal{L} = \{1, 2, ..., L\}$, where each link undergoes cochannel interference from the other $L-1$ links. Each node uses N_t transmit antennas and N_r receive antennas and the channel between the receive antennas of link l and the transmit antennas of link j is denoted by $\mathbf{H}_{l,j} \in C^{N_r \times N_t}$. For all l of the L links, the transmitted signal vector, $\mathbf{x}_l \in C^{N_t \times 1}$ has covariance matrix $\mathbf{Q}_l = E\{\mathbf{x}_l \mathbf{x}_l^\dagger\}$[1] and the receiver array performs independent single-user detection. The received baseband signal of link l, $\mathbf{y}_l \in C^{N_r \times 1}$, is given by

$$\mathbf{y}_l = \mathbf{H}_{l,l}\mathbf{x}_l + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{x}_j + \mathbf{n}_l \quad (1)$$

[1]In this paper, $E\{\cdot\}$ denotes expectation. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate transpose, $\mathrm{Tr}(\mathbf{A})$ denotes the trace, and $\det(\mathbf{A})$ denotes the determinant if $\mathbf{A}$ is square. $\mathcal{R}_+^n$ denotes the n dimensional nonnegative orthant.

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where $\mathbf{n}_l \in C^{N_r \times 1}$ is the noise vector with independent complex Gaussian entries. We also call $\mathbf{Q}_l$ a power allocation matrix with the transmit power for link l given by $\text{Tr}(\mathbf{Q}_l)$. The instantaneous data rate of link l is obtained as [19]

$$C_l(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \log_2 \det(\mathbf{I} + \mathbf{Q}_l \mathbf{H}_{l,l}^\dagger \mathbf{R}_l^{-1} \mathbf{H}_{l,l}) \quad (2)$$

where $\mathbf{R}_l = \mathbf{I} + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j} \mathbf{Q}_j \mathbf{H}_{l,j}^\dagger$ is the covariance matrix of the interference-plus-noise of link l. The channel matrices $\mathbf{H}_{l,j}$ and $\mathbf{R}_l$ are calculated by our computational electromagnetic simulations. In addition, due to an assumed no-delay channel feedback mechanism, the transmitters instantly know channel conditions.

Each transmitter adjusts its power allocation in an effort to maximize its mutual information. Power adjustment can be done in two ways. In the first technique, for a fixed transmit power of each node, the power is distributed between the multiple transmit antennas to achieve capacity maximization. The second technique allows different power levels for transmitters, i.e., the transmit power for a certain link l, $p_l = \text{Tr}(\mathbf{Q}_l) \leq \bar{p}_l$, can be adjusted. Using this power control technique, the transmitter can follow two courses of action: it can change the total power allotted to the link and it can also allocate this power in different ways between the multiple antennas of the link. For link l, given all other links' power allocation matrices $\mathbf{Q}_j (j \neq l)$, the maximization of link l's data rate with respect to $\mathbf{Q}_l$ can be formulated as the following constrained optimization problem [20].

$$\begin{aligned} \max_{\mathbf{z}_l} \quad & \sum_{i=1}^{N_t} \log_2(1 + z_{l,i}\sigma_{l,i}) \\ s.t. \quad & \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l \\ & z_{l,i} \geq 0, \ 1 \leq i \leq N_t \end{aligned} \quad (3)$$

where $\sigma_{l,i}$ are the eigenvalues of $\mathbf{H}_{l,l}^\dagger \mathbf{R}_l^{-1} \mathbf{H}_{l,l}$ and $\mathbf{z}_l = (z_{l,1}, ..., z_{l,N_t}) \in \mathcal{R}_+^{N_t}$ are the eigenvalues of $\mathbf{Q}_l$ with $z_{l,i}$ representing the power for the i^{th} eigenmode.

For a metric of energy efficiency, we use the ratio of the system capacity over the total power consumption. This metric corresponds to the amount of achievable capacity per unit energy.

$$\lambda = \frac{\sum_{l=1}^{L} C_l}{\sum_{l=1}^{L} p_l} \quad (4)$$

For a single MIMO link l, the optimum signaling problem is to find the optimum $\mathbf{Q}_l$ to maximize $C_l(\mathbf{Q}_1, ..., \mathbf{Q}_L)$. This can be achieved by the well-known independent water-filling method [19]. This approach has also been modified to accommodate fixed interference at the receiver of a link by "whitening the channel matrix" first [9], [21]. In a network with multiple interfering links, the interference correlation seen by each receiver varies with the transmitter correlation matrices of the interfering nodes. A change in the power allocation matrix of one link induces a change in the optimum power allocation matrices of the other co-channel links. Therefore, each link adjusts its power allocation iteratively until the network reaches equilibrium. In [11] a gradient projection (GP) based method was developed in an attempt to maximize the sum transmission rate of all of the links. In this method, the transmit power of each transmitter is set to a fixed value. With the GP method, the transmitters are assumed to cooperate with a centralized control mechanism which has access to the channel state information and covariance matrices of each user.

III. Game Theoretic Approach to Power Control

In this section, we will propose a new technique for interference management in MIMO ad hoc networks using a game theoretic approach.

A. Game Formulation

In the context of game theory, if we assume cooperation between wireless links is not feasible, the problem can be modelled as a non-cooperative game(NCG): $G = [\mathcal{L}, \{\mathbf{A}_l\}, \{u_l(\cdot)\}]$, where $\mathcal{L}$ is the set of links, $\mathbf{A}_l$=$\{\mathbf{z}_l \in \mathcal{R}_+^{N_t} | \sum_{i=1}^{N_t} z_{l,i} \leq \bar{p}_l\}$ is the set of power allocation actions and $u_l(\cdot)$ is the utility function of link l. We further denote the outcome of the game at certain time τ_k by the power allocation vector $\mathbf{q}(\tau_k) = (\mathbf{z}_1(\tau_k), ..., \mathbf{z}_L(\tau_k))$. In order to single out the action of link l, let $\mathbf{q}_{-l}(\tau_k)$ denote the vector consisting of elements of $\mathbf{q}(\tau_k)$ without the l^{th} link. For any link l at time τ_k, the transmitter tries to find $\mathbf{z}_l(\tau_k) \in \mathbf{A}_l$, such that for any other $\mathbf{z}_l'(\tau_k) \in \mathbf{A}_l$, $u_l(\mathbf{z}_l(\tau_k), \mathbf{q}_{-l}(\tau_k)) \geq u_l(\mathbf{z}_l'(\tau_k), \mathbf{q}_{-l}(\tau_k))$. Each transmitter calculates its own link capacity and tries to optimize its utility function.

B. Utility Function Design

Utility is the measure of "satisfaction" or "payoff" that a link obtains from using the channel. For a wireless ad hoc network, we relate the utility function of a particular link l to the achievable data rate of that link, i.e., $g_l = C_l$. This relationship quantifies approximately the demand or willingness of the user to pay for a certain level of service. Each link maximizes its own data rate at the cost of high power consumption, which also causes interference to other users and brings down their data rates. In order to keep a link from selfishly transmitting the highest power available, the system should impose a pricing function. For the sake of simplicity, our pricing function is proportional to link transmit power. As a result, the net utility for the lth link is given by

$$u_l = C_l - \gamma_l p_l \quad (5)$$

where γ_l is a nonnegative scaling factor whose units are bps/Hz/Watt so that the two terms in (5) have the same units. In this paper, we assume there exists an external network controller which assigns a value to γ_l. Different from the central control in the GP method, this external controller does not pass information of channel state and power allocation matrices. Instead, it provides a consensus among all links as to how γ_l is set. The choice of γ_l is further discussed in Section III-E.

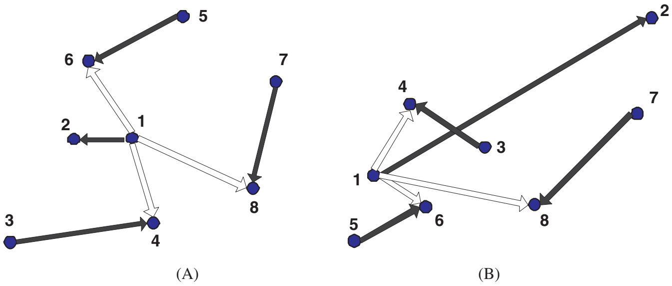

Fig. 1. Illustration of the game theoretic approach with a mechanism for shutting down links (A) Situation in which link 1-2 should reduce transmit power (B) Situation in which link 1-2 should be shut off.

With the new net utility as the objective function, each link tries to optimize (5). This problem can be formulated as:

$$\begin{aligned} \max_{\mathbf{z}_l} \quad & \sum_{i=1}^{N_t} \log_2(1+z_{l,i}\sigma_{l,i}) - \gamma_l \sum_{i=1}^{N_t} z_{l,i} \\ s.t. \quad & \sum_{i=1}^{N_t} z_{l,i} \le \bar{p}_l \\ & z_{l,i} \ge 0,\ 1 \le i \le N_t \end{aligned} \tag{6}$$

C. Link Shut-down Mechanism

To further investigate the net utility function for an ad hoc network, consider the two situations shown in Fig. 1. In both situations, node 1 transmitting to node 2 is the link of interest with the solid arrow showing desired communication links and the hollow arrow showing the propagation of interference. Fig. 1A illustrates the case that if the capacity of a particular link is more than enough to maintain a certain level of QoS, reducing the capacity by decreasing transmit power will mitigate the interference sent to other links. Since each link tries to maximize its net utility which contains transmit power as the pricing term, the transmitter will not be encouraged to transmit at maximum power. In this situation, the pricing factor γ_l enforces the minimum required capacity per unit power. One possible formulation of the objective function in (5) has

$$\gamma_l = \frac{\alpha_l}{p_0} \tag{7}$$

where α_l is a certain capacity value and p_0 is the initial transmit power. This equation describes the pricing factor for the General Game-Theoretic (GGT) technique that will be further described in Section IV.

Another case of interest is shown in Fig. 1B. The achievable data rate for a particular link (node 1 to node 2) is low even when the transmitter sends data at maximum power. If such a low-rate but high power-consumption link exists, it has two major negative effects on the network. First, a low-rate link is not only useless in terms of data transmission but also it may bring down the data rate of other links due to generated interference. Second, the total power consumption of the network is inefficiently increased as other links increase their transmit power in order to counter interference. To avoid these negative effects, it is advisable to shut down links with low data rates. This mechanism can be implemented by setting γ_l to be a large value if a threshold capacity is not exceeded. In this formulation,

$$\gamma_l = \begin{cases} \frac{\alpha_l}{p_0} & C_{l,0} \ge C_l^t \\ \infty & C_{l,0} < C_l^t \end{cases} \tag{8}$$

where $C_{l,0}$ is the initial multiuser water-filling capacity of link l [9], [22] and C_l^t is a capacity threshold assigned to link l by the external network controller. This equation describes the pricing factor for the Game-Theoretic technique with link Shutdown (GTWS) that will be further described in Section III-E.

The decision of whether to shut down a particular link depends on the minimum data rate (C_l^t) that is required by that link. This threshold is adaptively determined by the type of service in which the link is involved as well as the overall channel conditions which relate the QoS level to the threshold. For instance, since streamlined video requires high data rate, a link with video service should set a high threshold for minimum data rate. However, if the link is switched to another type of service which does not require a high data rate, the threshold can be lowered. Also, the overall channel conditions affect the choice of threshold. The receiving nodes in the center of a network usually experience more interference than the nodes at the edge. Therefore, it is beneficial for the threshold be tuned based on the location of the node. The algorithm that will be explained in Section III-D does accommodate adaptively changing the threshold to shut down a link. However, we assume each link is dedicated to a certain type of service at a certain location for a period of time, therefore the threshold is fixed for that time interval. In our simulations we assume a static topology with every link sharing the same QoS requirement, thus a fixed threshold C^t is assigned across the network ($C_l^t = C^t$, for every l).

D. Iterative Power Control with Game Theory

Based on the above utility function, we propose an algorithm in which all links update their power allocation vectors iteratively. We assume that all nodes are powered on simultaneously. Initially each link is given a certain capacity threshold C_l^t to keep the link viable and starts with a fixed transmit power p_0 which is allotted equally to all antennas. For each link its multiuser water-filling capacity $C_{l,0}$ is calculated and a corresponding γ_l is assigned based upon Equation (7) or Equation (8). For any viable link l, the transmitter calculates the optimum $p_l (0 \le p_l \le \bar{p}_l)$ such that the net utility function is maximized. This p_l corresponds to a $\mathbf{z}_l$ which is determined by maximizing (6). The power control process is performed iteratively until the net utility for every link in the network converges.

Algorithm: Power control based on game theoretic approach

1 Initialization:
Set $k = 0$. Each link calculates its multiuser water-filling capacity, then a γ_l is assigned based upon Equation (7)(GGT method) or Equation (8) (GTWS method).

2 Update power allocations:
Let $k = k+1$. For all links $l \in \mathcal{L}$, given power allocation vec-

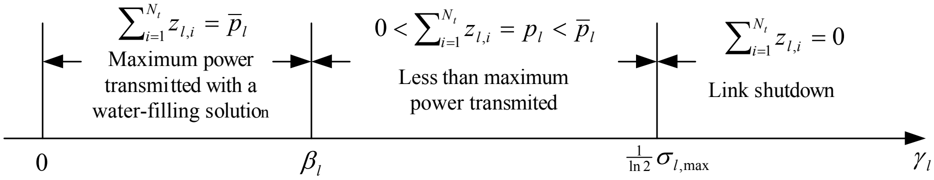

Fig. 2. Power allocation vs γ_l.

tor $\mathbf{q}(\tau_{k-1})$, compute $\mathbf{z}_l(\tau_k) = \arg\max_{\mathbf{z}_l \in \mathbf{A}_l} u_l(\mathbf{z}_l, \mathbf{q}_{-l}(\tau_{k-1}))$. Repeat step 2 until the net utility of every link converges.

E. Implication of Pricing Factor γ_l on Power Allocation

To quantify the implication of γ_l on power allocation, we approach the optimization problem (6) with Lagrange multiplier theory. The Lagrangian function in a standard minimization formulation can be written as

$$L(\mathbf{z}_l, \mathbf{u}) = \sum_{i=1}^{N_t} \left[\gamma_l z_{l,i} - \log_2(1 + z_{l,i}\sigma_{l,i})\right] + \mu_0 \left(\sum_{i=1}^{N_t} z_{l,i} - \bar{p}_l\right) - \sum_{i=1}^{N_t} \mu_i z_{l,i} \quad (9)$$

where $\mathbf{u} = (\mu_0, ..., \mu_{N_t})$ is the Lagrange multiplier vector and $\sigma_{l,1}, ..., \sigma_{l,N_t}$ are constants due to the assumption that all the other links' power allocations ($\mathbf{q}_{-l}$) are given.

According to Karush-Kuhn-Tucker(KKT) necessary conditions [23], let $\mathbf{z}_l$ be a local maximum of (6), then there exists a unique $\mathbf{u}$, such that

$$\frac{\partial L}{\partial z_{l,i}} = \gamma_l - \frac{\sigma_{l,i}}{(1 + \sigma_{l,i} z_{l,i}) \ln 2} + \mu_0 - \mu_i = 0, i = 1, ..., N_t \quad (10)$$

$$\mu_j \geq 0, \quad j = 0, 1, ..., N_t \quad (11)$$

This optimization problem contains all inequality constraints. For any feasible point $\mathbf{z}_l$, if inequality strictly holds for a certain constraint $g_j(\mathbf{z}_l) \leq 0$, the corresponding Lagrange multiplier $\mu_j = 0$ and this constraint is called inactive. Conversely, if $\mu_j > 0$, then that constraint is active because $g_j(\mathbf{z}_l) = 0$.

Theorem 1: Link l will be shut down, if $\gamma_l > \sigma_{l,max}/\ln 2$, where $\sigma_{l,max} = \max_i(\sigma_{l,i})$.

Proof: The KKT condition (10) can be converted to

$$\frac{1}{\gamma_l + \mu_0 - \mu_i} = \left(z_{l,i} + \frac{1}{\sigma_{l,i}}\right) \ln 2 \quad (12)$$

Given $\gamma_l > \sigma_{l,max}/\ln 2$ and from (11), $\mu_0 \geq 0$, so we know

$$\frac{1}{\gamma_l + \mu_0} \leq \frac{1}{\gamma_l} < \frac{\ln 2}{\sigma_{l,i}} \leq \left(z_{l,i} + \frac{1}{\sigma_{l,i}}\right) \ln 2 \quad (13)$$

Comparing (12) and (13), we see that (12) is true only if $\mu_i > 0$, which indicates the constraint $z_{l,i} \geq 0$ is active for any $i = 1, ..., N_t$. Since all $z_{l,i} = 0$, the transmit power of link l is zero and it is shut down. ■

Theorem 1 shows that if γ_l is large enough, link l does not transmit power. In real situations, a transmit node acquires channel information about the link of interest as well as the interfering links, and calculates its data rate and $\sigma_{l,max}$. If the initial multiuser water-filling capacity is lower than the preset threshold, the link is shut down. This mechanism can be implemented by assigning a sufficiently large value to γ_l.

Another strategy for choosing γ_l is to select it to be very small. In the extreme situation when $\gamma_l = 0$, which means no pricing is imposed in the utility function, the power allocation result is obviously the well-known water-filling solution with maximum power transmitted. Generally, when γ_l is smaller than a certain value β_l, link l uses the maximum power available to it and the power allocation is given by $z_{l,i} = \left(\frac{1}{(\gamma_l+\mu_0)\ln 2} - \frac{1}{\sigma_{l,i}}\right)^+$, where μ_0 is chosen to satisfy $\sum_{i=1}^{N_t} z_{l,i} = \bar{p}_l$. β_l is defined here as the cut-off value for power allocation strategies using maximum power versus strategies that use less transmit power. It is nontrivial to give an analytical expression for β_l because of the complex relationship between $\sigma_{l,i}$ and $\bar{p}_l$.

Fig. 2 illustrates different outcomes for power allocation in link l with respect to different values of γ_l. The region of $\beta_l < \gamma_l < \frac{1}{\ln 2}\sigma_{l,max}$ corresponds to a transmit power p_l which is between 0 and $\bar{p}_l$. Mathematically, the constraint on link l's maximum transmit power is inactive, so this link only uses a portion of the power available to it. This situation is desirable for higher energy efficiency, as long as this reduced power consumption allows QoS requirements to be met. In order to avoid unnecessarily transmitting at the highest power which potentially harms network energy efficiency, link l can be assigned a larger γ_l. In Section IV, simulation results will show how γ_l affects the link data rate and power consumption.

In the discussion above, it was assumed that the interference to link l is fixed because all the other links' power allocations do not change ($\sigma_{l,1}, ..., \sigma_{l,N_t}$ are constants). However, during the course of power control, each link adjusts its power allocation from iteration to iteration until power allocation converges for every link ($\sigma_{l,1}, ..., \sigma_{l,N_t}$ fluctuate). For example, suppose at time τ_{k+1} every link except l increases its transmit power to θ times that of at time τ_k, i.e., $\mathbf{q}_{-l}(\tau_{k+1}) = \theta\mathbf{q}_{-l}(\tau_k)$ with $\theta > 1$, and assume $\mathbf{R}_l(\tau_k) >> \mathbf{I}$ element-wise, then it can be shown that $\mathbf{R}_l(\tau_{k+1}) = \mathbf{I} + \sum_{j=1, j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{Q}_j(\tau_{k+1})\mathbf{H}_{l,j}^\dagger \approx \theta\mathbf{R}_l(\tau_k)$ and $\sigma_{l,max}(\tau_{k+1}) \approx \frac{1}{\theta}\sigma_{l,max}(\tau_k) < \sigma_{l,max}(\tau_k)$. Similarly, $\sigma_{l,max}(\tau_{k+1})$ becomes larger if interference reduces. In general, for every link l, $\sigma_{l,max}(\tau_k)$ changes as power control continues across the network. The perturbation of $\sigma_{l,max}(\tau_k)$ causes uncertainty in whether $\gamma_l > \frac{1}{\ln 2}\sigma_{l,max}$ holds throughout the process of power control. Specifically, if γ_l is chosen to be close to $\frac{1}{\ln 2}\sigma_{l,max}(\tau_0)$, γ_l may be less than $\frac{1}{\ln 2}\sigma_{l,max}(\tau_k)$ over the subsequent iterations ($k =$

$1, 2, \cdots$), which may result in that link l resumes transmitting. Therefore, in order to make sure that a low efficiency link is shut down and will not be turned back on, there should be a sufficient margin between γ_l and $\sigma_{l,max}(\tau_0)$, i.e., $\gamma_l \gg \frac{1}{\ln 2}\sigma_{l,max}(\tau_0)$. In Section IV, we set $\gamma_l = \infty$ once it is determined that link l should be shut down.

F. Game Analysis

In the power control game $G = [\mathcal{L}, \{\mathbf{A}_l\}, \{u_l(\cdot)\}]$, the choice of power allocation vector $\mathbf{z}_l$ for link l impacts not only its own link capacity and utility, but also those of other links. If all links reach an equilibrium point as a result of self-optimizing, power allocation of every link does not change since it is in no link's interest to unilaterally change strategy. The concept of Nash equilibrium provides a predictable outcome of a game, although such an equilibrium is not guaranteed to exist. Next we will investigate this 'predictive capability' in the power control game G.

Theorem 2: A Nash equilibrium exists in the NCG: $G = [\mathcal{L}, \{\mathbf{A}_l\}, \{u_l(\cdot)\}]$.

Proof:

In [13] [24], it has been shown that a Nash equilibrium exists, if for any l:

(a) $\mathbf{A}_l$ is a nonempty, convex and compact subset of a finite Euclidean space.

(b) $u_l(\mathbf{q})$ is continuous in $\mathbf{q}$ and quasi-concave in $\mathbf{z}_l$.

It is obvious that for the power control game G, condition (a) is satisfied. The detailed proof is omitted here. For condition (b), (6) indicates that u_l is a continuous function in $\mathbf{q}$. Next we will show that the objective function in (6) is concave in $\mathbf{z}_l$. To do so, we can limit the dimension of $\mathbf{z}_l$ to be 1 because concavity is determined by the behavior of a function on arbitrary lines that intersect its domain [25]. Taking the second-order derivative of u_l with respect to an arbitrary component $z_{l,j}$ yields

$$\frac{\partial^2 u_l}{\partial z_{l,j}^2} = \frac{-\sigma_{l,j}^2}{(1+\sigma_{l,j} z_{l,j})^2} \leq 0 \tag{14}$$

(14) shows that u_l is a concave function in $\mathbf{z}_l$, thus both conditions are satisfied, there exists a Nash equilibrium for the power control game $G = [\mathcal{L}, \{\mathbf{A}_l\}, \{u_l(\cdot)\}]$. ■

We have shown the existence of a Nash equilibrium of the power control game G. In general, it can be beneficial to have only one equilibrium point, because efficient methods exist to calculate this equilibrium point given uniqueness is guaranteed [26]. However, provable uniqueness of Nash equilibrium is a rare property for non-cooperative games. In [3], [26], some sufficient conditions for the uniqueness of Nash equilibrium were presented, where additional assumptions besides the ones in Theorem 2 were made. Note that these conditions are not necessary for the uniqueness of Nash equilibrium. Thus, while the numerical simulations in the next section may not meet the requirements in [3], [26], a unique Nash equilibrium was found in all simulations.

IV. Simulation Results

The ad hoc network is simulated in downtown Philadelphia (shown in Fig. 3) with computational electromagnetics [17].

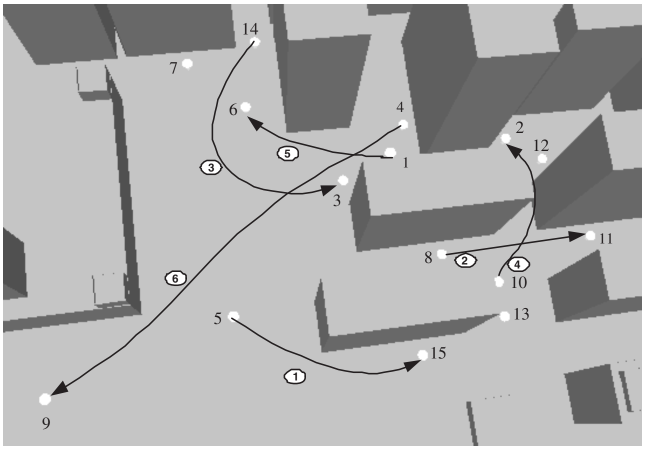

Fig. 3. Illustration of Philadelphia downtown simulation. Node numbers are not circled and link numbers are circled.

The topology is static and contains transmit-receive nodes 5-15 (link 1), 8-11 (link 2), 14-3 (link 3), 10-2 (link 4), 1-6 (link 5) and 4-9 (link 6). All of these links are single-hop and no node relays information. We compare the sum data rate and energy efficiency under different methods, namely, GTWS (γ_l assigned to a link based on Equation (8)), GP approach and multiuser water-filling (MUWF) approach. We also consider a general game-theoretic approach (GGT), meaning a pricing factor γ_l is assigned to a link based on Equation (7), but that link is not necessarily shut down even if its data rate is low. Since game theory is applied to power control, each individual link may not transmit the same amount of power even though the initial transmit power of each link is the same. In order to compare the performance of different methods in a fair way, we fix the total power consumption of the network for GTWS, GP and MUWF methods, which is determined by the GTWS technique, and divide power among links. Specifically, we first set each link to transmit the same amount of power and use GTWS to compute the sum data rate and total power consumption($\sum_{l=1}^{6} p_l$), then divide the total power($\sum_{l=1}^{6} p_l$) equally to all 6 links and compute sum data rate using MUWF and GP method. When applying the GGT method, we let each link start with the same transmit power as it does in GTWS. Since low-efficiency links are not necessarily shut down, the total power consumption in the end may be different from that of the other methods. However, the following comparison will still be fair because perturbed SNR values in the GGT method reflect changes in power consumption. In the simulations, the capacity threshold (C_l^t) is set to 2.4 bps/Hz. For the GGT method (Equation (7)) and the GTWS method (Equation (8)), $\alpha_l = 2$ bps.

Fig. 4 shows the sum data rate of the network for different algorithms. The SNR is calculated on the basis of average transmit power across all links. Comparing the two game theoretic approaches, we can see that for SNR <15dB, the two algorithms have similar performance, while at higher SNR, the GGT method undergoes a setback and the GTWS method is substantially better. This observation can be explained from the data in Table I. Link 1 is an inefficient link in the network, so it is turned off by the hard link shut-down mechanism.

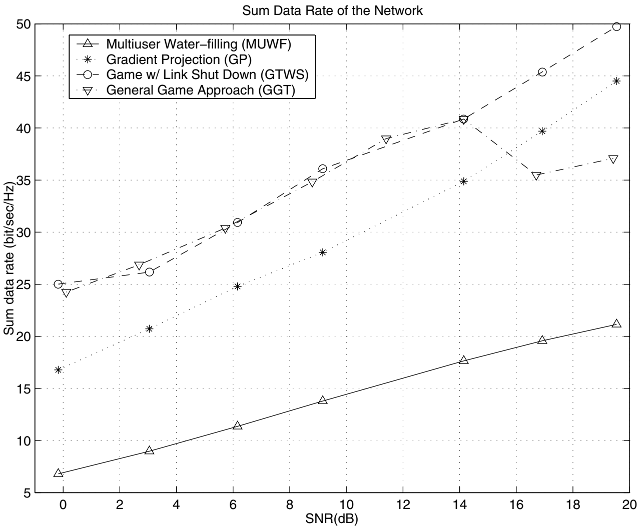

Fig. 4. Sum data rate of different methods.

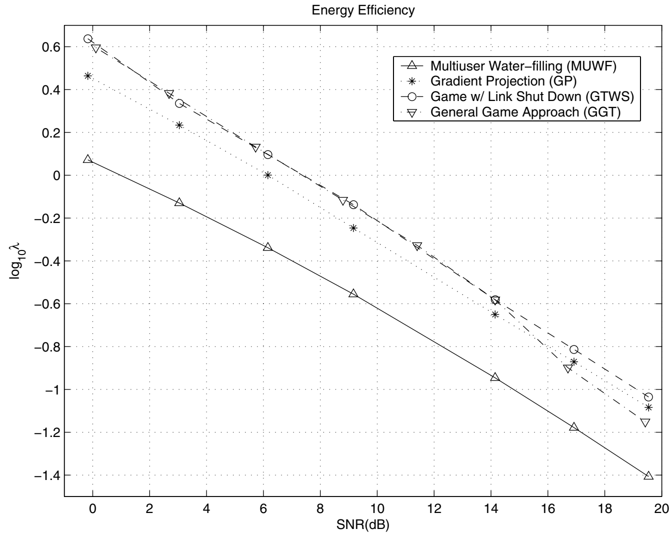

Fig. 5. Energy efficiency of different methods.

TABLE I
LINK CAPACITY AND TRANSMIT POWER

Data	GTWS (14.15dB)	GGT (14.15dB)	GTWS (16.92dB)	GGT (16.92dB)
L1 Tx Power	0	0.03	0	14.84
L2 Tx Power	42.41	42.41	78.73	79.97
L3 Tx Power	22.71	22.71	40.43	39.75
L4 Tx Power	30.45	30.45	65.40	53.76
L5 Tx Power	19.83	19.83	35.88	32.21
L6 Tx Power	40.45	40.45	74.40	74.78
Sum power	155.85	155.88	294.84	295.31
Sum rate (bps/Hz)	40.86	40.86	45.38	35.54

However, in the GGT approach at high SNR, by using the normal pricing factor $\gamma_1 = 2/p_0$ instead of setting $\gamma_1 = \infty$ (as would be done in GTWS), the condition in Theorem 1 is not met. As a result, Link 1 transmits a significant amount of power which causes interference to other users. Therefore the sum data rate is less than that of the GTWS method. On the other hand, at SNR =14.15dB, the normal pricing factor ($\gamma_1 = 2/p_0$) satisfies the condition in Theorem 1, which has the same effect as setting $\gamma_1 = \infty$ (i.e., the link is shut off), thus at this SNR GGT and GTWS methods have nearly identical power allocations. For lower SNR values, the performances of the two algorithms are close but not always exactly the same. This is because the fixed γ_l in GGT may not strictly shut down inefficient links and a small amount of power may leak from these links. This power leakage has little effect on the total power consumption or sum data rate.

Comparing the GTWS, GP and MUWF methods in Fig. 4, we see the GTWS method achieves the highest sum data rate while the MUWF method results in the lowest system capacity[2]. The GP method and MUWF method assume that each transmitter sends a constant amount of power no matter how inefficient that particular link is, which might cause severe interference to other links. For our proposed method, inefficient links are shut down so as to avoid a waste of power. For instance, in the simulation at SNR = 16.92dB, link 1 has such a low capacity that it is unusable. As a result, it is shut off. However, the sum capacity is still higher than the other methods, which indicates that although shutting off inefficient links reduces the number of users in the network, it improves the data rate of existing users.

Among the three approaches without the mechanism of hard link shut-down (i.e., GGT, GP and MUWF), at high SNR ($>$ 16dB)[3], the GP method results in the highest sum data rate. This result is reasonable because its objective function is to maximize the sum data rate while in the GGT and MUWF methods, each link aims to maximize its own data rate or utility. GGT and MUWF are both game-based algorithms [9], [11], in which the selfish utility function introduces conflicts among links and power allocations at the Nash equilibrium are less efficient than other possible power allocations acquired through cooperation. Comparing GGT and MUWF methods, GGT leads to a higher sum data rate and energy efficiency. This result occurs because a power control mechanism has been implicitly built into the utility function (5), and consequently a transmitter is not encouraged to transmit high power in order to obtain high capacity, but to maximize its utility.

Fig. 5 demonstrates the energy efficiency for different methods. A higher value of λ means that a higher data rate is achieved per unit energy. We can see that for every SNR, the GTWS method has the highest λ and thus the energy utilization is the most efficient. Shutting off an inefficient link not only saves the battery life of the node in a particular link, but also reduces interference to other links, allowing other nodes to transmit with less power.

When the GTWS method is used, Fig. 6 shows how sensitive the system capacity and total power consumption are

[2]The convergence point of the GP method depends on the initial condition. However, [11] found out the ergodic mutual information curves are extremely close to each other and one choice of initial condition is not evidently better than another. In this paper, the initial condition is set to equal power allocation.

[3]At low SNR ($<$ 15dB), where a link sends out little power, the GGT solution is equivalent to the GTWS solution. We do not compare this SNR region at this point.

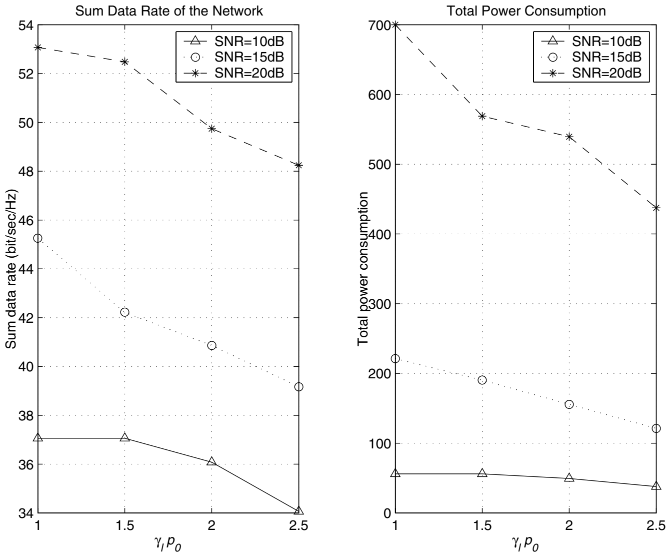

Fig. 6. System capacity and power consumption vs γ_l.

to different values of γ_l. With a fixed SNR or p_0, it can be seen that a higher γ_l tends to result in a lower total transmit power and consequently a lower sum data rate. This is in agreement with the discussion in Section III-E. It should be noted that as long as its data rate is high enough, a link is not encouraged to choose a smaller γ_l, so that unnecessary transmission of power is avoided.

V. Conclusion

In this paper, the topic of power management in MIMO ad hoc networks was addressed. Existing approaches, such as multiuser water-filling and gradient projection, assign a fixed transmit power to each link and each transmitter node allocates power among different antennas in order to optimize the link capacity or sum data rate. If bad channel conditions existed in some communicating links, those methods are not energy efficient.

We proposed a new technique for power management and interference reduction based upon a game theoretic approach. A utility function with an intrinsic property of power control was designed and power allocation in each link was built into a non-cooperative game. To avoid unnecessary power transmission under poor channel conditions, a mechanism of shutting down inefficient links was integrated into the game theoretic approach. Simulation results showed that under the constraint of a fixed total transmit power, if the proposed approach were allowed to shut off inefficient links, the remaining links would still achieve the highest sum data rate and energy efficiency.

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