Experimental Characterization of Resource Allocation Algorithms in MIMO-OFDM Ad Hoc Networks

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Abstract— In this work we have reviewed and field tested several methods for power control in a MIMO-OFDM WLAN. We compared the performance of the Independent Waterfilling, Multi-User Waterfilling, and Gradient Projection techniques and quantified the impact of interference in an indoor environment. The results show the tradeoff between capacity and network overhead. Specifically, the Gradient Projection method results quantify the gains that are possible if global channel and interference state information is shared. However, results also demonstrate that the Gradient Projection technique tends to limit links that have poor channels, which motivates future research to consider methods that include link quality of service specifications.

Index Terms – ad-hoc networking, MIMO systems, OFDM, power control, wireless communications.

I. INTRODUCTION

There is a great potential for wireless communication systems that use Multiple Input Multiple Output (MIMO) technology. Ad hoc and wireless local area networks (WLANs) have both been the focus of recent research [1], [2]. Of particular interest, are resource allocation algorithms that maximize the capacity of MIMO network links in the face of co-channel interference. MIMO communication platforms allow an additional degree of freedom that can be exploited to reduce the interference experienced by network links [3].

Recent work [3], [4], provides interesting insight into methods that can be used to allocate power appropriately in a network such that the system capacity is improved. The purpose of this paper is to quantitatively evaluate the performance of practical these techniques on a MIMO WLAN testbed and extend them to use OFDM signaling.

II. FORMULATION

A. System Model

Consider an ad hoc network with a set of links denoted by $\mathcal{L} = \{1, 2, ..., L\}$, where each link undergoes co-channel interference from the other $L - 1$ links. Each link uses N_t transmit antennas, N_r receive antennas, and N OFDM subcarriers. The matrix channel between the receive antennas of link l and the transmit antennas of link j on subcarrier k

is denoted by $\mathbf{H}_{l,j}^{(k)} \in C^{N_r \times N_t}$. For this paper, we assume $N_t = N_r = 2$.

Under a wideband channel assumption, the channel between the transmit antennas of link j and receiver antennas of link l, $\mathbf{H}_{l,j}$, is given by $\mathbf{H}_{l,j} = diag\{\mathbf{H}_{l,j}^{(k)}\}_{k=0}^{N-1}$.

We can acquire $\mathbf{H}_{l,j}$ through channel training in our MIMO-OFDM ad hoc network testbed (described in Section III-A). For all l of the L links, and k of the N subcarriers, the transmitted signal vector is $\mathbf{x}_l^{(k)} \in C^{N_t \times 1}$. The receiver array is performing independent single-user detection. The received baseband signal of link l on subcarrier k, $\mathbf{y}_l^{(k)} \in C^{N_r \times 1}$, is given by

$$\mathbf{y}_l^{(k)} = \mathbf{H}_{l,l}^{(k)}\mathbf{x}_l^{(k)} + \sum_{j=1,j\neq l}^{L} \mathbf{H}_{l,j}^{(k)}\mathbf{x}_j^{(k)} + \mathbf{n}_l^{(k)} \quad (1)$$

where $\mathbf{n}_l^{(k)} \in C^{N_r \times 1}$ is a noise vector with independent complex Gaussian entries. The transmitted signal $\mathbf{x}_l^{(k)}$ has the covariance matrix $\mathbf{Q}_l^{(k)} = E\{\mathbf{x}_l^{(k)}\mathbf{x}_l^{(k)\dagger}\}^1$. We also call $\mathbf{Q}_l^{(k)}$ a power allocation matrix with the transmit power for link l on subcarrier k given by $\text{Tr}(\mathbf{Q}_l^{(k)})$. By aggregating the power allocation matrices over subcarrier, we can denote the power allocation matrix for link l as $\mathbf{Q}_l = diag\{\mathbf{Q}_l^{(k)}\}_{k=0}^{N-1}$ and express the total link transmission power over all subcarriers as $\text{Tr}(\mathbf{Q}_l)$. The instantaneous data rate of link l is obtained as [5]

$$I_l(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \log_2 \det(\mathbf{I} + \mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}\mathbf{H}_{l,l}) \quad (2)$$

where $\mathbf{R}_l = \mathbf{I} + \sum_{j=1,j\neq l}^{L} \mathbf{H}_{l,j}\mathbf{Q}_j\mathbf{H}_{l,j}^{\dagger}$ is the covariance matrix of the interference-plus-noise of link l. The channel matrices $\mathbf{H}_{l,j}$ and $\mathbf{R}_l$ are determined through field measurements described in Section III-A. In addition, due to an assumed no-delay channel feedback mechanism, the transmitters instantly know channel conditions.

Equation 2 shows the mutual information of link l. The

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[1] In this paper, for a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate transpose, $\text{Tr}(\mathbf{A})$ denotes the trace, and $\det(\mathbf{A})$ denotes the determinant if $\mathbf{A}$ is square.

sum-rate mutual information for the entire wireless network is

$$I(\mathbf{Q}_1, ..., \mathbf{Q}_L) = \sum_{l=1}^{L} \log \det(\mathbf{I} + \mathbf{H}_{l,l}\mathbf{Q}_l\mathbf{H}_{l,l}^{\dagger}\mathbf{R}_l^{-1}). \quad (3)$$

B. Resource Allocation Strategies

There are several strategies considered in this paper to allocate power to antennas (i.e. set $\mathbf{Q}_l$ for link l) in a MIMO-OFDM ad hoc network. These techniques vary in performance and have different computational and network overhead requirements.

1) Independent Waterfilling: For a single MIMO link, l, assuming only local channel knowledge and neglecting the effect of interference (i.e., $\mathbf{R}_l = \mathbf{I}$), the optimum signaling problem is to find the optimum $\mathbf{Q}_l$ to maximize $I_l(\mathbf{Q}_1, ..., \mathbf{Q}_L)$ in Equation 2. This optimization can be achieved by using the "independent water-filling" (IWF) approach [5].

This technique has the smallest computational complexity and incurs little network overhead. However, it does not account for interference effects.

2) Multi-user Waterfilling: If we assume that a transmitter of link l is aware of the interference environment in which the link is operating, the IWF method can be improved (at the cost of higher networking overhead). Specifically, this improvement assumes that the receiver of link l can estimate $\mathbf{R}_l$ and can instantly relay this information back to the transmitter of link l. The IWF approach can be modified to incorporate this interference information by "whitening the channel matrix" first [6], [7]. Specifically, application of a spatial whitening transform to the channel yields

$$\tilde{\mathbf{H}}_{l,l} = \mathbf{R}_l^{-1/2}\mathbf{H}_{l,l}. \quad (4)$$

Substituting $\mathbf{H}_{l,l}$ in the IWF approach with $\tilde{\mathbf{H}}_{l,l}$, we get the multiuser water-filling (MUWF) capacity for link l. In a network with multiple interfering links, the interference correlation seen by each receiver array varies with the transmitter correlation matrices of the interfering nodes.

3) Gradient Projection Method: The final resource allocation method aims to maximize $I(\mathbf{Q}_1, ..., \mathbf{Q}_L)$ from Equation 3, which is the social optimum for the network. To achieve optimum system capacity, the transmitters must cooperate when deciding their power allocation matrices, so that a compromise can be struck between the maximization of an individual links mutual information and the minimization of the interference observed by other users. The gradient projection (GP) method, which is an extension of the unconstrained steepest descent method in convex constrained optimization problems, is a technique that can be used solve this social optimum problem [4].

With the GP method, a centralized controller, which has access to all channel state information and covariance matrices for all users, is necessary. This controller performs the calculation and sends the information to all users so that they can update their power allocation matrices accordingly. The GP technique has the largest networking overhead and has the greatest computational complexity in order to find the optimal allocation of resources.

III. MEASUREMENT CAMPAIGN

A. Testbed

In order to measure and test MIMO ad hoc links, the Drexel University Wireless System Laboratory in collaboration with the Wireless Networking & Communications Group at the University of Texas at Austin has developed a custom software defined multiple antenna mobile ad hoc network. Each node in our experimental platform consists of frequency agile transceivers in the ISM and UNII radio operating bands and a baseband process computer. The baseband chassis provided by National Instruments has two major functional roles. First, the unit runs the analog to digital (A/D) and (D/A) converters required to for two transceivers. The converters operate at 100 MS/s with 14-bit quantization. Second, the baseband unit is a software defined radio (SDR). This allows for the unique ability to tailor the communication scheme for a given experiment. An overview of the testbed can be seen in Figure 1.

The communications scheme used for our experiments to measure the MIMO-OFDM channel is based on the IEEE 802.11a standard. For our measurements, each node employs two antennas. We made channel measurements by transmitting a binary phase shift key test pattern independently over the two transmit antennas. This test pattern was then received and used to construct the channel matrix. The channel matrix could then be used to evaluate power control algorithms detailed in Section II.

The measurements were performed at 2.484 GHz. We used BPSK to generate the analog baseband signal. The bandwidth of the system is 20 MHz, which is separated into $|N| = 64$, 312.5 kHz OFDM subchannels. As given by the the 802.11a standard only 52 for the 64 carriers are used for communication. Two omni-directional antennas with 6 dBi of gain were used at each node with an inter-element spacing of $\lambda/2$.

To analyze the results of the channel measurements, the following scaling factor for link l was found by (5).

$$k_l = \frac{1}{N}\sum_{k=1}^{N}\|\mathbf{H}_{l,l}^{(k)}\|_F^2 \quad (5)$$

Once the k_l was found for all L, the median k_l was selected making the normalization factor, $\sqrt{\frac{k_l}{N_t N_r}}$, which was then applied to all $\mathbf{H}_{l,l}^{(k)}$ and $\mathbf{H}_{l,j}^{(k)}$.

B. Environment

The measurements were taken on the 3rd floor of the Bossone building on the campus of Drexel University. The network topology that we tested had six links and twelve nodes. The nodes were scattered throughout the area and contained a mixture of long-distance and short-distance communications links. The layout of node and link locations is shown in Figure 2. For each transmit location, the channel was measured at every receive location. Therefore, we could analyze the channels of interest ($\mathbf{H}_{l,l}$) and interfering ($\mathbf{H}_{l,j}$) links. The data is available upon request.
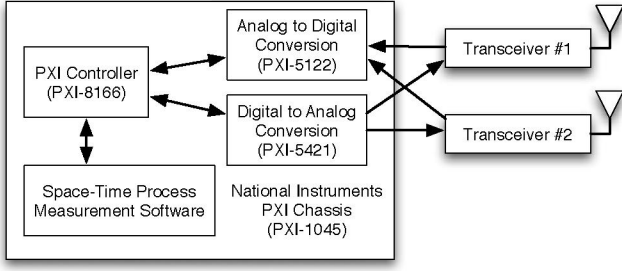
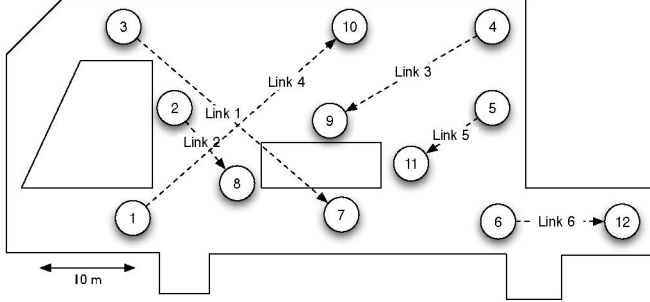Fig. 1. Software Defined Radio Testbed Block Diagram

Fig. 2. Indoor Measurement Testing Environment and Node Locations

Fig. 3. Link Capacity using Independent Waterfilling (IWF) Technique

Fig. 4. Link Capacity using Multi-User Waterfilling (MUWF) Technique

IV. RESULTS

Once the channel measurements were taken, the different power allocation techniques from Section II were evaluated. Figure 3 shows IWF capacity performance of all links versus SNR in an interference limiting environment. Each node makes a decision on power control based upon knowledge of only $\mathbf{H}_{l,l}$, the link's own channel state. Link 6 has the highest capacity, which can be attributed to the fact that the receiver of that link is isolated from the other transmitters. Additionally, the capacity of the other links at higher SNRs is lower due to the the interference created by Link 6. This is due to favorable channel conditions and equal transmission powers. This allocation method has the lowest capacity because each link is determining the best power allocation for itself, without accounting for interference.

Figure 4 shows the capacity of all links versus SNR for the MUWF allocation method. The calculated capacity flattens because as SNR increases for the link of interest, the interference also increases. Link 6 has much greater capacity than the other links because the receiver is affected minimally by the other links. In comparison with the IWF allocation, the links have similar relative performance. Links 4 and 1 have low performance due to the presence of many other transmitting nodes. Table I shows that the MUWF technique outperforms IWF in terms of the sum-rate capacity due to the fact that interference is taken into account.

The results from the GP method can be seen in Fig. 5. Overall, there is higher capacity in comparison to the MUWF. However, to achieve higher rates links 1 and 4 are nearly "shut down" to limit the amount of interference they generate. Both links are long-distance and cause unnecessary interference to
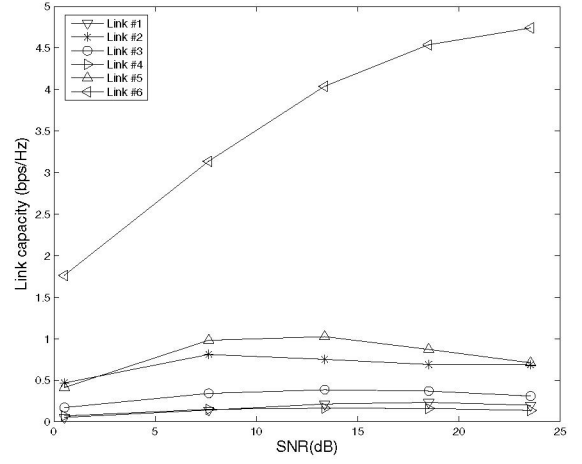
the rest of the network. As is the case, with the other allocation methods, Link 6 has the best performance. Additionally, there is a crossing in the capacity of Links 2 and 5 as SNR increases. This crossing is due to the interference created by Link 2, which limits the capacity of other links that have higher rank channel conditions.

In Fig. 6, we show the normalized power allocation over subcarrier for Links 3 and 5 for the MUWF technique at an SNR of 18.5 dB. Links 3 and 5 were selected due to their proximity to one another, though the allocations are affected by other links in the network. Because this method allocates power to those carriers that have good channel quality and have limited interference, when the power allocation at one subcarrier is high, it is usually low for the same subcarrier on the other link.

Similarly, the normalized power allocation over subcarrier for the GP method at an SNR of 18.5 dB for Links 3 and 5 are compared in Fig. 7. The allocation is different than in Fig. 6 because the GP method allocates power with complete
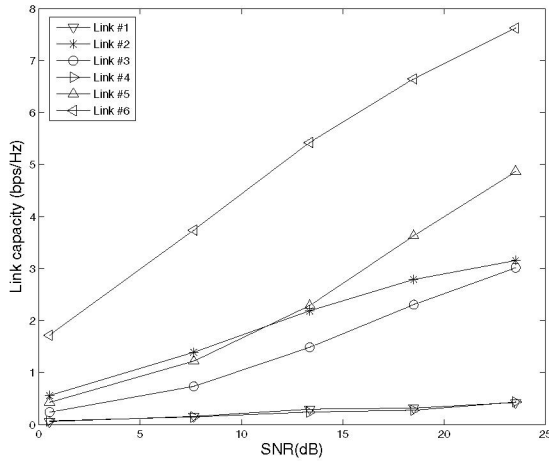
Fig. 5. Link Capacity using Gradient Project (GP) Technique

Fig. 7. Normalized power allocation (with noise power 1) using the Gradient Project (GP) Technique at SNR = 18.5 dB
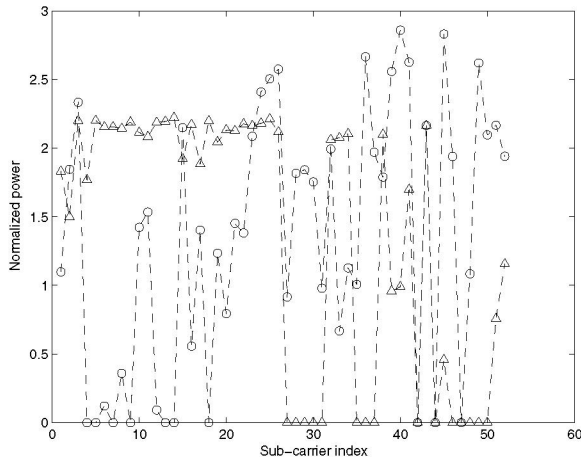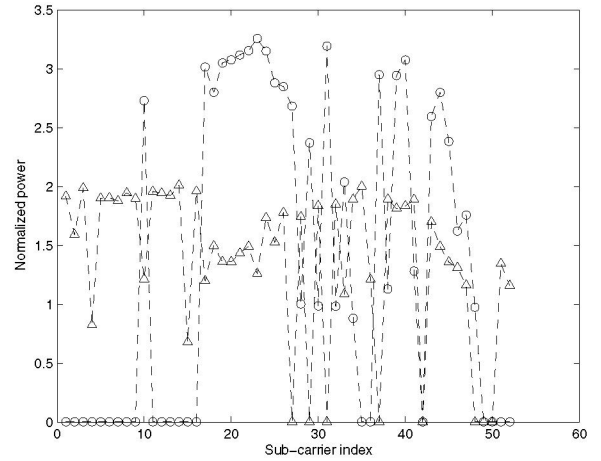

Fig. 6. Normalized power allocation (with noise power 1) using Multi-User Waterfilling (MUWF) Technique at SNR = 18.5 dB

network knowledge. Like MUWF, the power allocation for one subcarrier is not the same for the two links and typically one is much greater than the other.

As a comparison, the sum rate capacity for the network at an SNR of 18.5 dB is presented in Table I. As expected, the GP method provides a higher capacity than the MUWF allocation. Both GP and MUWF also perform better than the IWF method because those techniques account for the channel conditions of other links.

Technique	Sum Rate Capacity (bps/Hz)
IWF	6.9
MUWF	10.5
GP	15.9

TABLE I

COMPARISON OF SUM RATE CAPACITIES

V. CONCLUSION

In this work we have reviewed and field tested several current methods for power control in a WLAN. Specifically, we have measured an interference limited network using a custom built software defined multiple antenna mobile ad hoc network. Comparison of IWF and MUWF techniques quantifies the impact of interference on MIMO ad hoc links in an indoor environment. The results also show the tradeoff between capacity and network overhead. Specifically, the GP method results quantify the gains that are possible if global channel and interference state information is shared. However, the GP technique also shuts down links that have poor channels, which motivates future research to consider methods that include link quality of service specifications.

REFERENCES

[1] "Mobile Networked MIMO (MNM) Web Site," http://www.darpa.mil/ato/solicit/MNM/overview.htm. [Online]. Available: http://www.darpa.mil/ato/solicit/MNM/overview.htm
[2] K. Dandekar and R. H. Jr., "Large-scale capacity characterization of smart antenna equipped mobile ad-hoc network nodes in an urban network," in *Proc. of IEEE Military Communications Conference*, Boston, Mass, USA, 2003.
[3] M. Demirkol and M. Ingram, "Stream control in networks with interfering MIMO links," in *Proc. of 2003 IEEE Wireless Communications and Networking Conference*, 2003, pp. 343–348.
[4] S. Ye and R. Blum, "Optimized signaling for MIMO interference systems with feedback," *IEEE Trans. Signal Proc.*, vol. 51, pp. 2839–2848, Nov. 2003.
[5] T. Cover and J. Thomas, *Elements of information theory*. New York, NY: Wiley, 1991.
[6] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, "Iterative water-filling for gaussian vector multiple access channels," *IEEE Trans. Infor. Theory*, vol. 50, pp. 145–152, Jan. 2004.
[7] M. Demirkol and M. Ingram, "Power-controlled capacity for interfering MIMO links," in *Proceedings of Vehicular Techonology Conference*, vol. 1, Sept. 2001, pp. 187–191.