IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

SPECIAL SECTION ON KEY ENABLING TECHNOLOGIES FOR PROSUMER ENERGY MANAGEMENT

Received September 23, 2020, accepted October 8, 2020, date of publication October 12, 2020, date of current version October 21, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3030195

Contract-Theoretic Demand Response Management in Smart Grid Systems

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The work of Nafis Irtija and Eirini Eleni Tsiropoulou was supported by NSF under Grant CRII-1849739.

ABSTRACT The sheer growth of electricity demand and the rising number of electricity-hungry devices have highlighted and elevated the need of addressing the demand response management problem in residential smart grid systems. In this article, a novel contract-theoretic demand response management (DRM) framework in residential smart grid systems is introduced based on the principles of labor economics. The residential households produce and consume electricity, acting as dynamic prosumers. Initially, the prosumers' personal electricity generation and consumption characteristics are captured by introducing the concept of prosumers' types. Then, the prosumers' and the electricity market's profit is depicted in representative utility functions. Based on the labor economics principles, Contract Theory is adopted to design the interactions among the electricity market, which offers personalized rewards to the prosumers in order to buy electricity at an announced price, and the prosumers, who offer their "effort" by paying for the purchased electricity. The contract-theoretic DRM problem is formulated as a maximization problem of the electricity market's utility, while jointly guaranteeing the optimal satisfaction of the prosumers, under the scenarios of complete and incomplete information from the electricity market's perspective regarding knowing or not the prosumers' types, respectively. The corresponding optimization problems are solved following a convex optimization approach and the optimal contracts, i.e., rewards and efforts, are determined. Detailed numerical results obtained via modeling and simulation, highlight the key operation features and superiority of the proposed framework.

INDEX TERMS Smart grid systems, contract theory, demand response management, labor economics, prosumers, electricity market.

I. INTRODUCTION

Smart grid systems have been introduced as an efficient solution to the global energy crisis given their inherent characteristics of communication, control, and optimization that can conclude to the real-time balance among the power supply and demand [1]. The Demand Response (DR) has become a vital and critical component of the smart grid systems, enabling the consumers to directly interact with the electricity market by dynamically adapting their electricity consumption based on the announced price and the supply availability [2]. Two main types of demand response have been proposed in the literature; (a) direct or price-based DR, where the consumers directly adapt their electricity consumption to the announced price and the utility company has direct control on their consumption [3], and (b) indirect or incentive-based DR, where a dynamic pricing is offered to the consumers as an incentive to voluntarily adapt their electricity consumption [4].

With the evolution of the smart grid systems and towards meeting the electricity demands, the utility companies exploit the renewable energy resources as an alternative solution, due to the lack of availability of fossil fuels [5]. Also, the consumers are encouraged to install small wind turbine power generation systems or rooftop solar photovoltaics to cover partially or completely their electricity needs. Furthermore, the electricity generation surplus from the residential power generation systems is stored in rechargeable batteries (e.g., lithium-ion batteries and liquid electrolyte "flow batteries") or in the electric vehicles (EVs). Both electricity storage alternatives can charge at night when the price is low and the supply is high, while sell back to the smart grid when the electricity demand is high [6]. Thus, the traditional consumers are

The associate editor coordinating the review of this manuscript and approving it for publication was Jiayong Li.

transformed to *prosumers*, becoming a critical component in the smart grid systems' smooth operation.

A. RELATED WORK & MOTIVATION

The problem of Demand Response Management (DRM) has been thoroughly studied in the literature by introducing distributed or centralized solutions and accounting or not for the prosumers' behavioral characteristics and patterns of electricity consumption. In [7], a distributed Stackelberg game-theoretic approach is introduced among one utility company and multiple consumers, where the first one announces the electricity price and the latter ones adjust their electricity consumption. This problem is extended in [8], where a multiple utility companies and multiple consumers smart grid system is considered. The consumers select the utility company wherefrom they will purchase the electricity by following a reinforcement learning approach and, then, a two-stage game-theoretic framework is proposed to determine the optimal electricity price and consumption. In [9], [10], the authors analyze the impact of the communication unreliability among the electricity market (i.e., utility companies) and the prosumers on the DRM performance and the electricity price. Specifically, the authors formulate a joint maximization problem of the DRM performance with respect to the electricity consumption and price, and solve it by leveraging the dual decomposition method.

The profit maximization problem of the utility companies is studied in [11] by formulating it as a finite-horizon continuous-state Markov Decision Process (MDR) problem. The provided solution is also shown that it alleviates the supply-demand imbalance in the smart grid and decreases the prosumers' electricity bills. Towards tackling the high computational complexity of the aforementioned problem, the authors have extended the previous research work in [12]. Specifically, they propose a dual approximate approach that transforms the MDP problem into a linear programming problem that can be solved in a real-time manner. An intelligent residential energy management system is introduced in [13] aiming at the reduction of the prosumers' electricity bills, while guaranteeing the satisfaction of the electricity demand constraints under various examined cases of the household loads and renewable energy resources supply. A smart load estimator based on a neural networks' approach is developed in [14] considering the ambient temperature, the time of the day, the time of use price, and the peak demand constraints imposed by the smart grid operator. Furthermore, an auction-based approach is introduced in [15] among the electric vehicles (EVs) that store electricity and the smart grid electricity aggregator that buys the EVs' electricity surplus. The solution of the auction-based DRM problem determines the optimal amount of electricity sold by each EV, as well as the corresponding price.

Focusing on the prosumer-centric DRM solutions, a privacy preserving DRM framework is discussed in [16], where a reinforcement learning approach is adopted to explore the prosumers' privacy protection behaviors and learn their electricity consumption patterns. In [17], highly resolved electricity consumption models are designed to estimate the residential demand by quantifying the prosumers' electricity use behavior. Thus, the optimal schedule of the prosumers' appliances is determined considering the time-varying electricity prices. Moreover, the prosumers' behavior in terms of consuming electricity has been further studied in [18] towards introducing a pricing-based demand response model for smart homes that consists of various types of household devices. In this research work, the DRM problem is examined under different requirements regarding the user satisfaction levels, which impose different constraints in the corresponding formulated optimization problem. The problem of improving the prosumers' confidentiality, while scheduling the electricity consumption of their personal appliances, is studied in [19], where a mathematical framework is introduced in order to simplify the operation of the advanced metering infrastructure in terms of communication requirements during the DRM process.

More recently, a blockchain-based decentralized DRM framework is introduced in [20] in order to store and process the data generated from the prosumers' smart meters, and exploit the blockchain environment to validate the requests of electricity consumption, dynamic pricing, and electricity transaction executions. Another prosumer-centric DRM solution is proposed in [21] by considering a multi-periodic smart grid DRM problem characterized by shifted demand. The authors consider two major types of players, i.e., prosumers and electricity providers, and they analytically determine the Nash Equilibria towards maintaining the viability of the smart grid infrastructure over the examined time period. Moreover, the authors in [22] have focused their study on the DRM problem related to managing the energy consumption of the lighting and air-conditioner systems, while jointly minimizing the users' discomfort levels. The authors jointly consider the various uncertain environmental factors, as well as the prosumers' uncertain psycho-economic factors and introduce a kernel-based learning approach to determine the optimal price and energy consumption.

Additionally, in [23], a distributed system-wide framework is designed in order to dynamically adapt the system load profile towards minimizing the prosumer's payments, while guaranteeing their comfort and privacy constraints. In [24], a distributed prosumers' utility maximization framework is proposed, where the optimal prices and the demand schedules are determined in order for each prosumer to maximize its net benefit subject to various consumption and power flow constraints. A mixed integer non-linear optimization problem is formulated in [25] towards determining the scheduling of the prosumers' appliances, while considering the electricity price and the penalty associated to peaks of electricity consumption, and at the same time guaranteeing the prosumers' comfort constraints. A non-cooperative game of incomplete information among the prosumers is formulated in [26] and a Bayesian Nash equilibrium solution is derived in order to minimize the peak-to-average ratio within the smart grid system. In [27], a prosumers' classification framework is proposed to categorize the prosumers in two main types based on their electricity consumption behavior, i.e., non-green comfort seeking behavior and green incentive seeking behavior.

B. CONTRIBUTIONS

Despite the efforts made in the previous works, in regards to the demand response management problem, how to incorporate the prosumer's personalized electricity consumption and generation dynamic behavior within the smart grid systems still remains to be an open issue. Moreover, to facilitate the smooth interaction among the electricity market and the prosumers, how to capture their economics-based relationships towards both parties of competing interests concluding to mutually acceptable decisions that jointly maximize their profit is even challenging.

In this work, we strive to tackle these issues. In detail, the design goal is to capture the prosumers' personalized characteristics via modeling their unique personalized types within the smart grid system, depending on their electricity generation capabilities, as well as their electricity consumption needs, and represent their profit in appropriately designed utility functions. Towards dealing with the demand response management, a contract-theoretic approach is introduced based on the principles of labor economics to study the interactions of the electricity market and the prosumers. The proposed contract-theoretic demand response management approach jointly targets at the profit maximization both of the prosumers and the electricity market. The latter novelty is fundamental in the field of DRM problems dealing with prosumers, as the existing research works mainly consider the profit maximization of the electricity market. Contract Theory provides the novelty of treating the prosumers in a personalized manner in terms of their electricity generation and consumption characteristics, and the corresponding price that they are willing to pay in order to be incentivized to participate in the demand response management process. The main contributions of this research work that differentiate it from the rest of the literature, are summarized below.

1) A contract-theoretic demand response management approach is introduced that considers the unique electricity generation and consumption characteristics of the prosumers in order to determine the optimal electricity consumption. Following the principles of labor economics of Contract Theory, the electricity market acts as an ''employer'', offering personalized rewards, i.e., amount of electricity at a corresponding price, to the prosumers. The prosumers act as ''employees'' offering as ''effort'' to the electricity market, the corresponding amount of electricity at the announced price that they are willing to buy. By treating the prosumers in a personalized manner, the electricity market can optimize its profit by exploiting the prosumers' purchasing capacity in an optimal manner, while the prosumers can also jointly optimize their profit by determining the optimal amount of purchased electricity at the announced price.
2) The contract-theoretic DRM problem is studied under complete information, i.e., the electricity market is aware of the prosumers personalized characteristics, and incomplete information. The solution of the contract-theoretic DRM optimization problem concludes to the optimal contracts that consist of the electricity market's offered optimal rewards to incentivize the prosumers to buy electricity at an announced price and the prosumers' optimal amount of purchased electricity to optimize their profit considering their electricity consumption and generation constraints.
3) A series of experiments are performed to evaluate the performance of the overall contract-theoretic DRM framework both in the cases of complete and incomplete information from the electricity market's side, in terms of electricity consumption, electricity market's and prosumers' profit, and social welfare. The results reveal that the contract-theoretic DRM framework achieves only 36.3% reduction of the overall smard grid system's social welfare under the worst case scenario of incomplete information compared to the benchmarking use case of complete information. Also, the impact of considering the prosumers' personalized characteristics in the DRM performance is studied via a thorough comparative evaluation by examining different pricing policies from the electricity market's perspective. Furthermore, a detailed comparative evaluation with alternative DRM approaches demonstrates our proposed framework's superiority and benefits. The results conclude that an average increase of the prosumers' profit by 42% is achieved due to their personalized treatment by the electricity market, while at the same time the latter one increases its profit due to the optimal exploitation of the prosumers' purchasing power.

C. OUTLINE

The rest of the paper is organized as follows. Section II-A explains the system model, while the prosumers' and the electricity market's profits within the demand response management are captured through holistically designed utility functions in Sections II-B and II-C, respectively. The contract-theoretic DRM problem is studied under complete information in Section III, and incomplete information in Section IV. Simulation results are presented in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A. RESIDENTIAL SMART GRID SYSTEM OPERATION & NOTATION

A residential smart grid system is considered consisting of the electricity market, the advanced metering infrastructure, the load controller, the scheduling manager, the renewable energy system, the storage system, and the number of appliances, as presented in Fig. 1. The renewable energy system can consist of rooftop solar photovoltaics or small wind turbine power generation systems. The produced electricity at the residential level is consumed to cover the prosumers' needs, while the surplus is stored at the storage system. The latter consists of rechargeable batteries, such as lithium-ion batteries.

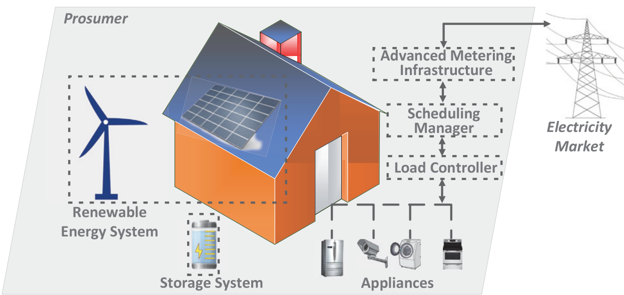

FIGURE 1. **Residential Smart Grid System.**

The residential smart grid system consists of $N = \{1, \ldots, n, \ldots, |N|\}$ prosumers and the time is divided into equal time slots $t, t \in T$, where $T = \{1, \ldots, t, \ldots, |T|\}$. Each prosumer n has a set of appliances $A_n = \{1, 2, \ldots, a_n, \ldots, |A_n|\}$, where part of them have shiftable electricity demands, e.g., washer, dryer, and some of them essential demands, such as refrigerator, alarm system. The operation schedule of each appliance a_n is denoted by $S_{a_n} = \{s^1_{a_n}, s^2_{a_n}, \ldots, s^t_{a_n}, \ldots, s^{|T|}_{a_n}\}$, where $s^t_{a_n} = 1$, if the appliance a_n operates at time slot t, and $s^t_{a_n} = 0$, otherwise. Considering that the appliance a_n consumes $E^{ON}_{a_n}$ when it operates during the time slot t, then, the electricity consumption of the prosumer n is derived as $l^t_n = \sum_{a_n \in A_n} s^t_{a_n} \cdot E^{ON}_{a_n}$. Therefore, the electricity demand vector of all the prosumers in time slot t is given as follows.

$$\mathbf{L} = [l^t_1, l^t_2, \ldots, l^t_n, \ldots, l^t_{|N|}], \quad t \in T \tag{1}$$

It is noted that $l^{min}_n \leq l^t_n \leq l^{Max}_n$, where l^{min}_n represents the prosumer's total essential electricity demands, and l^{Max}_n captures the total electricity demand of the prosumer.

Furthermore, each prosumer can generate electricity through its personal renewable energy resources. The renewable generation output of all the prosumers in time slot t is denoted as follows.

$$\mathbf{G} = [g^t_1, g^t_2, \ldots, g^t_n, \ldots, g^t_{|N|}], \quad t \in T \tag{2}$$

In the following analysis, the prosumer's n overall electricity demand l^t_n and renewable generation output g^t_n are considered to follow uniform distributions, and numerical details are provided in Section V. Given the prosumer's electricity demand and generation, the following two cases are examined.

Case A: If $g^t_n \geq l^t_n$ that means that the prosumer generates more electricity compared to its actual demand during time slot t. Therefore, the prosumer can cover its own electricity needs without entering the electricity market to buy electricity. In the special sub-case that $g^t_n > l^t_n$, then the electricity generation surplus of the prosumer, i.e., $(g^t_n - l^t_n)$, is stored at its residential storage system for future use. Thus, in the next time slot, the available stored electricity is $b^{t+1}_n = b^t_n + (g^t_n - l^t_n)$, where b^t_n denotes the available electricity in the storage system of the prosumer n in time slot t.

Case B: If $g^t_n + b^{t-1}_n < l^t_n$, then the prosumer needs to purchase $l^t_n - g^t_n - b^{t-1}_n$ amount of electricity from the electricity market, as its generated and stored electricity are not sufficient to cover its demand. In this case, the prosumer enters the electricity market demanding $l^t_n - g^t_n - b^{t-1}_n$ amount of electricity and paying a corresponding price.

Let us normalize the electricity bought by each prosumer n from the electricity market as follows.

$$d^t_n = \frac{l^t_n - g^t_n - b^{t-1}_n}{\max\limits_{\forall n \in N^t_{buy}} \{l^t_n - g^t_n - b^{t-1}_n\}} \in (0, 1], \quad n \in N^t_{buy} \tag{3}$$

where $N^t_{buy} = \{n \in N : g^t_n + b^{t-1}_n < l^t_n\} \subseteq N$ is the set of prosumers buying electricity from the electricity market in time slot t.

Given the normalized electricity that is bought from the prosumers that enter the electricity market, we define the type of each prosumer n, $n \in N^t_{buy}$, as follows.

$$\tau^t_n = \frac{d^t_n}{\sum\limits_{n=1}^{|N^t_{buy}|} d^t_n} \in (0, 1], \quad n \in N^t_{buy} \tag{4}$$

The physical meaning of the prosumer's type is that it represents the normalized need of the prosumer to buy electricity from the electricity market compared to the rest of the prosumers that compete for the same resource (i.e., electricity) during each time slot t. It is noted that the prosumers' types dynamically change per time slot considering their electricity demands and generation characteristics and characterize each prosumer in a unique and personalized manner. It is obvious that a prosumer who can support its personal electricity demand without entering the electricity market (Case A) during a time slot t has no type, and it does not compete with the rest of the prosumers for the valuable resource of electricity. In the rest of the analysis, we consider the most heterogeneous and challenging scenario, where each prosumer has its own personal type τ^t_n, thus, $|N^t_{buy}|$ types of prosumers exist during time slot t. Also, for notation convenience in the presentation, we consider that a prosumer of higher type, i.e., $\tau^t_1 < \tau^t_2 < \cdots < \tau^t_n < \cdots < \tau^t_{|N^t_{buy}|}$, has a higher normalized request for buying electricity from the electricity market, i.e., $d^t_1 < d^t_2 < \cdots < d^t_n < \cdots < d^t_{|N^t_{buy}|}$, as it is derived from Eq. 4.

B. PROSUMERS CONTRACT-THEORETIC UTILITY FUNCTION

In the following sections, the principles of labor economics are adopted following the corresponding Contract Theory to drive the examined residential smart grid system to stable and efficient operation points [28]. Specifically, Contract Theory builds labor economics-based relationships among the actors involved in the residential smart grid system, i.e., the electricity market and the prosumers, and incentivizes them to behave in a beneficial manner for the overall system, by providing appropriately designed contracts for both actors [29]. Based on the contract-theoretic model, an employer offers personalized rewards to the employees towards incentivizing them to demonstrate a beneficial behavior for the overall system, while the employees offer their effort to the employer as an exchange. The tuple of {reward, effort} creates a personalized contract among the employer and each employee. If both parties follow the optimal contracts, the overall system concludes to a stable and efficient mode of operation. Contract Theory has already been applied in several fields, such as vehicular networks [30], optimal charging schemes of electric vehicles in smart grid systems [31], federated learning in mobile networks [32], public safety systems [29], and cognitive radio networks [33].

The rationale behind applying the principles of Contract Theory, as well as the novelty of the proposed contract-theoretic demand response management approach, lies in the observation that treating the prosumers in a personalized manner can jointly improve the profit of the electricity market, as well as the prosumers' profit. Specifically, the contract-theoretic DRM approach enables the electricity market to incentivize the prosumers to purchase an optimal personalized amount of electricity at the announced price, while exploiting their personal electricity generation and consumption characteristics.

Within the examined residential smart grid system, the prosumers act as "employees" offering their "effort" to the electricity market. The latter acts as an "employer" offering a personalized reward $r_n^t = \tau_n^t \cdot q_n^t$ to the prosumer. The physical meaning of this formulation is that the prosumer by purchasing q_n^t normalized amount of electricity from the electricity market, it will pay an amount of $p^t \cdot q_n^t$ to the electricity market, where $p^t \in (0, 1]$ is the unitless price of the electricity unit. Thus, the electricity market will gain profit from the prosumers' purchase. On the other hand, the electricity market should incentivize the prosumers to buy electricity at the announced price p^t during time slot t. Thus, it offers a personalized reward r_n^t proportional to their purchased electricity, while accounting for their type τ_n^t within the smart grid system.

Following the above analysis, the personalized contract that is established among the electricity market and each prosumer is the tuple $\{r_n^t(q_n^t), q_n^t\}$. By receiving the reward $r_n^t(q_n^t)$, each prosumer evaluates it in a different manner given the electricity demand (i.e., shiftable or essential) that it covers towards its personalized satisfaction. Thus, we define the prosumer's evaluation function $e_n^t(r_n^t)$ as a strictly increasing, concave function with respect to the prosumer's effort $q_n^t \in (0, d_n^t] \subseteq (0, 1]$, with $e_n^t(r_n^t = 0) = 0$. For demonstration purposes and without loss of generality, in the following analysis we consider $e_n^t(r_n^t) = \sqrt{r_n^t}$. It is noted that the adoption of any other form of the evaluation function that respects the aforementioned properties would not change the theoretical analysis presented below, but the intermediate derivations.

The contract-theoretic utility of each prosumer represents the prosumer's pure personalized perceived satisfaction from the obtained reward (first term of Eq. 5), while considering its personalized cost by investing its effort via buying electricity from the electricity market (second term of Eq. 5). The physical meaning of the first term of Eq. 5 is that each prosumer interprets the received reward $r_n^t(q_n^t)$ in a personalized manner based on its type τ_n^t in order to determine its pure perceived satisfaction. Thus, the greater the type of the prosumers is, which means that the prosumer has greater electricity demand (Eq. 4), the more satisfaction it perceives, as it is enabled to cover more electricity needs. Thus, the prosumer's contract-theoretic utility function is defined as follows.

$$U_n^t(q_n^t) = \tau_n^t e_n^t(q_n^t) - p^t q_n^t \quad (5)$$

It is noted that the prosumer's utility function is unitless to keep the holistic applicability of the proposed framework. In a real-life application of the proposed theoretical framework, the prosumers' effort can be mapped to KWh and the electricity market's reward to monetary units (e.g., discount, coupons).

C. ELECTRICITY MARKET UTILITY

The electricity market has partial or even no available information regarding the prosumers' types. Thus, by offering appropriately designed rewards $r_n^t, \forall n \in N_{buy}^t$ to incentivize the prosumers to buy electricity at the announced price p^t, it aims to implicitly reveal their types and consequently their electricity consumption and generation characteristics. Therefore, the electricity market estimates the prosumers' types with probability Pr_n^t, where $\sum_{n=1}^{|N_{buy}^t|} Pr_n^t = 1$. Thus, the electricity market's utility accounting for the prosumers that buy electricity is defined as follows.

$$U_{EM}^t(\mathbf{q^t}) = \sum_{n=1}^{|N_{buy}^t|} [Pr_n^t(p^t \cdot q_n^t - r_n^t(q_n^t))] \quad (6)$$

where $\mathbf{q^t} = (q_1^t, q_2^t, \ldots, q_n^t, \ldots q_{|N_{buy}^t|}^t)$ is the normalized purchased electricity vector of the $|N_{buy}^t|$ prosumers that buy electricity in time slot t. The physical meaning of Eq. 6 is that the first term denotes the electricity market's revenue, while the second term expresses its cost to provide the rewards to the prosumers. Thus, Eq. 6 captures the electricity market's profit.

Furthermore, the overall social welfare of the examined residential smart grid system is defined as follows.

$$SW^t(\mathbf{q^t}) = U_{EM}^t(\mathbf{q^t}) + \sum_{n=1}^{|N_{buy}^t|} U_n^t(q_n^t) \quad (7)$$

In the following two sections, the contract-theoretic demand response management problem is examined under the cases of complete and incomplete information of the electricity market regarding the prosumers' types. In both cases, our goal is to determine the optimal contracts that will jointly optimize the electricity market's and the prosumers' utilities and conclude to a stable and efficient operation point of the residential smart grid system.

III. PROSUMERS' CONTRACTS UNDER COMPLETE INFORMATION

In this section, the ideal case where the electricity market knows the prosumers types $\tau_n^t, \forall n \in N_{buy}^t$ is examined, mainly for benchmarking purposes. Given the available complete information of the prosumers' types to the electricity market, the latter knows the potential amount of electricity that the prosumers are willing to buy, thus, it can fully exploit the prosumers' purchasing power. The electricity market aims to maximize its utility, while guaranteeing that it will offer personalized rewards to the prosumers, which will satisfy their individual rationality (IR) and finally accept to purchase an amount of electricity that will satisfy their demands. Therefore, the contract-theoretic DRM problem under complete information among the electricity market and each prosumer can be formulated as follows.

$$\max_{\{r_n^t, q_n^t\}_{\forall n \in N_{buy}^t}} [U_{EM}^t(q_n^t) = p^t \cdot q_n^t - r_n^t(q_n^t)] \tag{8a}$$

$$\text{s.t.} \quad U_n^t(q_n^t) = \tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq 0 \quad \text{(IR)} \tag{8b}$$

The electricity market aims to maximize its profit, while guaranteeing the minimum acceptable satisfaction for the prosumers in order to purchase electricity. Thus, the prosumers' IR constraint in Eq. 8b can be reduced to an equality.

Theorem 1: The optimal contract among the electricity market and each prosumer $n, n \in N_{buy}^t$ under the complete information scenario is $\{(\frac{\tau_n^t}{2})^2, \frac{\tau_n^{t\,2}}{2p^t}\}$.

Proof: Based on the reduced IR constraint in Eq. 8b, we have $r_n^t(q_n^t) = (\frac{p^t \cdot q_n^t}{\tau_n^t})^2$. By substituting the $r_n^t(q_n^t) = (\frac{p^t \cdot q_n^t}{\tau_n^t})^2$ in Eq. 8a, differentiating with respect to q_n^t, and equating the outcome to zero, we have $q_n^t = \frac{\tau_n^{t\,2}}{2p^t}$, thus, $r_n^t = (\frac{\tau_n^t}{2})^2$. ∎

The physical meaning of the optimal contract $\{r_n^t, q_n^t\} = \{(\frac{\tau_n^t}{2})^2, \frac{\tau_n^{t\,2}}{2p^t}\}$ is that the prosumers purchase electricity proportionally to their demands and inverse proportionally to the announced price by the electricity market. Also, the electricity market offers rewards aligned with each prosumer's personal needs to purchase electricity towards eventually incentivizing it to perform the purchase at the announced price p^t.

IV. CONTRACT-THEORETIC DEMAND RESPONSE MANAGEMENT UNDER INCOMPLETE INFORMATION

In this section, the general case of incomplete information regarding the electricity market not knowing the prosumers' types, thus, being unaware of the potential amount of electricity that they are interested to buy, is examined. This scenario is the most realistic one in a residential smart grid system, where the electricity market aims at incentivizing the prosumers to buy electricity at an announced price. Our goal is to determine the optimal contract $\{r_n^t, q_n^t\}$ among the electricity market and each prosumer towards maximizing the electricity market's profit, while guaranteeing the optimal satisfaction of the prosumers' electricity demands. Each prosumer should at least receive a positive utility, while purchasing electricity from the electricity market, in order to be incentivized to perform the purchase. This constraint captures the individual rationality (IR) of each prosumer. Furthermore, each prosumer aims at achieving the optimal utility that better captures its own personal electricity consumption and generation characteristics, by receiving a reward from the electricity market to perform its purchase. This constraint captures the incentive compatibility (IC) of each prosumer. The prosumers' IR and IC constraints are formally defined as follows.

Definition 1 (Individual Rationality (IR)): An optimal contract $\{r_n^t, q_n^t\}$ should guarantee a non-negative utility $U_n^t(q_n^t)$ for each prosumer, i.e., $U_n^t(q_n^t) = \tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq 0, \forall n \in N_{buy}^t$.

Definition 2 (Incentive Compatibility (IC)): An optimal contract $\{r_n^t, q_n^t\}$ should be designed in a personalized manner for each prosumer considering its personal electricity consumption and generation characteristics captured via its type τ_n^t, i.e., $\tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq \tau_n^t e_{n'}^t(q_{n'}^t) - p^t q_{n'}^t, \forall n, n' \in N_{buy}^t, n \neq n'$.

Except for the IR and IC constraints that should hold true in order to conclude to the optimal contracts, some additional properties and conditions should be satisfied. The latter ones are described in Propositions 1-3, as follows.

Proposition 1 (Fairness): An optimal contract $\{r_n^t, q_n^t\}$ should provide higher (or equal) reward to the prosumers of higher (or the same) type, i.e., $r_n^t > r_{n'}^t \Leftrightarrow \tau_n^t > \tau_{n'}^t$ ($r_n^t = r_{n'}^t \Leftrightarrow \tau_n^t = \tau_{n'}^t$).

Proof: Both the sufficiency, i.e., $\tau_n^t > \tau_{n'}^t \Rightarrow r_n^t > r_{n'}^t$, and the necessity, i.e., $r_n^t > r_{n'}^t \Rightarrow \tau_n^t > \tau_{n'}^t$ of the fairness condition are shown. Towards proving the sufficiency, we exploit the IC constraint. Thus, $\forall n, n' \in N_{buy}^t, n \neq n'$, we have:

$$\tau_n^t e_n^t(r_n^t) - p^t q_n^t \geq \tau_n^t e_{n'}^t(r_{n'}^t) - p^t q_{n'}^t \tag{9}$$

$$\tau_{n'}^t e_{n'}^t(r_{n'}^t) - p^t q_{n'}^t \geq \tau_{n'}^t e_n^t(r_n^t) - p^t q_n^t \tag{10}$$

We add Eq. 9 and Eq. 10, and we derive the the following expression.

$$\tau_n^t e_n^t(r_n^t) + \tau_{n'}^t e_{n'}^t(r_{n'}^t) \geq \tau_n^t e_{n'}^t(r_{n'}^t) + \tau_{n'}^t e_n^t(r_n^t) \tag{11}$$

Thus, we have $(\tau_n^t - \tau_{n'}^t) \cdot e_n^t(r_n^t) \geq (\tau_n^t - \tau_{n'}^t) \cdot e_{n'}^t(r_{n'}^t)$. Given that $\tau_n^t > \tau_{n'}^t$, we conclude that $e_n^t(r_n^t) > e_{n'}^t(r_{n'}^t)$. The prosumers' evaluation function is strictly increasing with respect to r_n^t and by having the same form for all the prosumers (i.e., $e_n^t = e_{n'}^t = e$), we conclude that $r_n^t > r_{n'}^t$.

Towards showing the necessity of the fairness condition, we know that $r_n^t > r_{n'}^t$, and the evaluation function is strictly increasing, i.e., $e_n^t(r_n^t) - e_{n'}^t(r_{n'}^t) > 0$. By exploiting Eq. 11, we have the following expression: $\tau_n^t[e_n^t(r_n^t) - e_{n'}^t(r_{n'}^t)] \geq \tau_{n'}^t[e_n^t(r_n^t) - e_{n'}^t(r_{n'}^t)]$, thus, $\tau_n^t > \tau_{n'}^t$. Finally, towards examining the special case of $r_n^t = r_{n'}^t \Leftrightarrow \tau_n^t = \tau_{n'}^t$, a similar analysis can be followed. ∎

The physical meaning of Proposition 1 is that a prosumer of higher type, i.e., willing to buy more electricity from the electricity market, should be rewarded with a higher reward for fairness purposes.

Proposition 2 (Monotonicity): A prosumer will receive a higher reward, i.e., $r_1^t < r_2^t < \cdots < r_n^t < \cdots < r_{|N_{buy}^t|}^t$, if it is characterized by higher type, i.e., $\tau_1^t < \tau_2^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}^t$, as it will purchase more electricity, i.e., $q_1^t < q_2^t < \cdots < q_n^t < \cdots < q_{|N_{buy}^t|}^t$.

Proof: The first statement, i.e., $r_1^t < r_2^t < \cdots < r_n^t < \cdots < r_{|N_{buy}^t|}^t \Leftrightarrow \tau_1^t < \tau_2^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}^t$, can be derived from the proof of Proposition 1. Then, we have, $r_1^t < r_2^t < \cdots < r_n^t < \cdots < r_{|N_{buy}^t|}^t \Leftrightarrow \tau_1^t q_1^t < \tau_2^t q_2^t < \cdots < \tau_n^t q_n^t < \cdots < \tau_{|N_{buy}^t|}^t q_{|N_{buy}^t|}^t$ and given that $\tau_1^t < \tau_2^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}^t$, we conclude that $q_1^t < q_2^t < \cdots < q_n^t < \cdots < q_{|N_{buy}^t|}^t$. ∎

The physical meaning of Proposition 2 is that a prosumer of higher type, receives a greater reward in order the electricity market to exploit its full potential (i.e., purchasing power) to purchase a greater amount of electricity.

Proposition 3 (Rationality): A prosumer of higher type, i.e., $\tau_1^t < \tau_2^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}^t$, will receive a greater utility, i.e., $U_1^t < U_2^t < \cdots < U_n^t < \cdots < U_{|N_{buy}^t|}^t$.

Proof: Considering two indicative prosumers $n, n' \in N_{buy}^t$, $n \neq n'$, with $\tau_n^t > \tau_{n'}^t$ and based on the IC constraint, we have $\tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq \tau_n^t e_{n'}^t(q_{n'}^t) - p^t q_{n'}^t \overset{\tau_n^t > \tau_{n'}^t}{\Longleftrightarrow} U_n^t(q_n^t) = \tau_n^t e_n^t(q_n^t) - p^t q_n^t > \tau_{n'}^t e_{n'}^t(q_{n'}^t) - p^t q_{n'}^t = U_{n'}^t(q_{n'}^t)$. Thus, by generalizing this analysis for all the prosumers that purchase electricity during the time slot t, we have $\tau_1^t < \tau_2^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}^t \Leftrightarrow U_1^t < U_2^t < \cdots < U_n^t < \cdots < U_{|N_{buy}^t|}^t$. ∎

The physical meaning of Proposition 3 is that a prosumer of higher type, who will eventually purchase more electricity (Proposition 2), it will experience greater utility, i.e., satisfaction, as it will cover more electricity needs.

The conditions and constraints of individual rationality, incentive compatibility, fairness, monotonicity, and rationality should all hold true in order the prosumers to be incentivized to participate in the demand response management process, while strategically deciding the amount of purchased electricity at the announced price by the electricity market [28]. Thus, the contract-theoretic DRM problem is formulated as a maximization problem of the electricity market's utility (Eq. 12a), while guaranteeing the prosumers' constraints (Eq. 12b - Eq. 12d), as follows.

$$\max_{\{r_n^t, q_n^t\}_{\forall n \in N_{buy}^t}} U_{EM}^t(\mathbf{q^t}) = \sum_{n=1}^{|N_{buy}^t|} [Pr_n^t(p^t \cdot q_n^t - r_n^t(q_n^t))] \quad (12a)$$

$$\text{s.t.} \quad \tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq 0 \quad \text{(IR)} \quad (12b)$$

$$\tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq \tau_n^t e_{n'}^t(q_{n'}^t) - p^t q_{n'}^t, \quad \forall n \neq n' \in N_{buy}^t \text{(IC)} \quad (12c)$$

$$0 \leq r_1^t < r_2^t < \cdots < r_n^t < \cdots < r_{|N_{buy}^t|}^t \quad (12d)$$

In the following analysis, our goal is to reduce the constraints of the non-convex optimization problem (12a) -(12d), in order to solve it in a tractable manner. Initially, we examine the reduction of the IR constraint (12b). By considering the findings of Proposition 2 and given the IC constraint, we have $\tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq \tau_n^t e_{n'}^t(q_{n'}^t) - p^t q_{n'}^t \geq \tau_n^t e_1^t(q_1^t) - p^t q_1^t$. However, we know that $\tau_n^t > \tau_1^t$, thus, we derive that $\tau_n^t e_n^t(q_n^t) - p^t q_n^t \geq \tau_n^t e_1^t(q_1^t) - p^t q_1^t \geq \tau_1^t e_1^t(q_1^t) - p^t q_1^t$. Therefore, we observe that if $\tau_1^t e_1^t(q_1^t) - p^t q_1^t \geq 0$ holds true, then the IR constraint will hold true for every prosumer $n \in N_{buy}^t$. Thus, the IR constraint in Eq. 12b can be reduced to the constraint $\tau_1^t e_1^t(q_1^t) - p^t q_1^t = 0$, considering that the electricity market will try to exploit the maximum benefit from the prosumers' purchasing power.

Then, our goal is to reduce the IC constraints, as presented in Eq. 12c. In the following, we use the terminology of downward and upward IC constraints among the prosumers, as follows: (1) $n, n', n' \in \{1, \ldots, n-1\}$: downward IC constraints, (2) $n, n', n' \in \{n+1, \ldots, |N_{buy}^t|\}$: upward IC constraints, (3) $n, n+1, n \in N_{buy}^t$: local upward IC constraint, (4) $n, n-1, n \in N_{buy}^t$: local downward IC constraint.

Theorem 2: The local downward IC constraint equivalently captures all the downward IC constraints.

Proof: The local downward IC constraints for three prosumers, i.e., $\tau_{n-1}^t < \tau_n^t < \tau_{n+1}^t$, are written as follows.

$$\tau_{n+1}^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t \geq \tau_{n+1}^t e_n^t(r_n^t) - p^t q_n^t \quad (13)$$

$$\tau_n^t e_n^t(r_n^t) - p^t q_n^t \geq \tau_n^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \quad (14)$$

Furthermore, given that $e_{n-1}^t = e_n^t = e_{n+1}^t = e$ and the evaluation function is strictly increasing, we have for $r_n^t > r_{n-1}^t \Leftrightarrow e_n^t(r_n^t) > e_{n-1}^t(r_{n-1}^t) \Leftrightarrow e_n^t(r_n^t) - e_{n-1}^t(r_{n-1}^t) > 0$. Therefore, for $\tau_{n+1}^t > \tau_n^t \Leftrightarrow \tau_{n+1}^t[e_n^t(r_n^t) - e_{n-1}^t(r_{n-1}^t)] > \tau_n^t[e_n^t(r_n^t) - e_{n-1}^t(r_{n-1}^t)] \overset{Eq.\ 14}{\geq} p^t(q_n^t - q_{n-1}^t)$. Thus, by recursively applying the previous outcome, we conclude that $\tau_{n+1}^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t \geq \tau_{n+1}^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \geq \tau_{n+1}^t e_{n-2}^t(r_{n-2}^t) - p^t q_{n-2}^t \geq \cdots \geq \tau_{n+1}^t e_1^t(r_1^t) - p^t q_1^t$. Therefore, we derive the following equivalent constraint,

$$\tau_n^t e_n^t(r_n^t) - p^t q_n^t \geq \tau_n^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \quad (15)$$

which means that the local downward IC constraint equivalently captures all the downward IC constraints. ∎

Theorem 3: The local downward IC constraint equivalently captures all the upward IC constraints.

Proof: Considering again three prosumers, i.e., $\tau_{n-1}^t < \tau_n^t < \tau_{n+1}^t$, we write the IC constraints, as follows.

$$\tau_{n-1}^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \geq \tau_{n-1}^t e_n^t(r_n^t) - p^t q_n^t \tag{16}$$

$$\tau_n^t e_n^t(r_n^t) - p^t q_n^t \geq \tau_n^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t \tag{17}$$

Based on the fairness property, we have $r_n^t > r_{n'}^t \Leftrightarrow \tau_n^t > \tau_{n'}^t$. Therefore, by Eq. 17, we can derive the following expression.

$$\begin{aligned} p^t(q_{n+1}^t - q_n^t) \geq \tau_n^t[e_{n+1}^t(r_{n+1}^t) - e_n^t(r_n^t)] \geq^{\tau_n^t > \tau_{n-1}^t} \\ \tau_{n-1}^t[e_{n+1}^t(r_{n+1}^t) - e_n^t(r_n^t)] \end{aligned} \tag{18}$$

By Eq. 16 and Eq. 18, we have $\tau_{n-1}^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \geq \tau_{n-1}^t e_n^t(r_n^t) - p^t q_n^t \geq \tau_{n-1}^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t$. Therefore, we conclude that $\tau_{n-1}^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \geq \tau_{n-1}^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t$, which means that if the IC constraint is satisfied for the prosumer with type τ_{n-1}^t, then, all the upward IC constraints also hold true. By recursively applying this outcome, we have $\tau_{n-1}^t e_{n-1}^t(r_{n-1}^t) - p^t q_{n-1}^t \geq \tau_{n-1}^t e_{n+1}^t(r_{n+1}^t) - p^t q_{n+1}^t \geq \cdots \geq \tau_{n-1}^t e_{|N_{buy}^t|}^t(r_{|N_{buy}^t|}^t) - p^t q_{|N_{buy}^t|}^t$. Thus, we conclude that the local downward IC constraint can capture all the upward IC constraints. ∎

By considering the above analysis and the outcomes of Theorem 2 and 3, the contract-theoretic DRM problem under incomplete information can be written as follows.

$$\max_{\{r_n^t, q_n^t\}_{\forall n \in N_{buy}^t}} U_{EM}^t(\mathbf{q^t}) = \sum_{n=1}^{|N_{buy}^t|} [Pr_n^t(p^t \cdot q_n^t - r_n^t(q_n^t))] \tag{19a}$$

$$\text{s.t.} \quad \tau_1^t e_1^t(q_1^t) - p^t q_1^t = 0 \tag{19b}$$

$$\tau_n^t e_n^t(q_n^t) - p^t q_n^t = \tau_n^t e_{n-1}^t(q_{n-1}^t) - p^t q_{n-1}^t \tag{19c}$$

$$0 \leq r_1^t < r_2^t < \cdots < r_n^t < \cdots < r_{|N_{buy}^t|}^t \tag{19d}$$

The above optimization problem is a convex one, as both the objective function and the constraints are convex. Thus, it can be easily solved by using standard convex optimization methods, in order to derive the optimal contracts $\{r_n^t, q_n^t\}_{\forall n \in N_{buy}^t}$.

V. NUMERICAL RESULTS

In this section, a detailed numerical evaluation is presented to study the performance and inherent attributes of the proposed contract-theoretic demand response management framework in smart grid systems. Initially, the pure framework's operation under the complete and incomplete information scenarios is presented in Section V-A, while in Section V-B the performance of the proposed framework under different pricing policies, i.e., low, medium, high, is illustrated. Section V-C demonstrates a thorough comparative evaluation between the proposed contract-theoretic DRM framework and a prosumers' type-agnostic DRM approach, as well as the benefits of our framework compared to various prosumers' electricity purchasing strategies in the examined residential smart grid system.

We consider an indicative residential smart grid system consisting of $|N| = 10$ prosumers, who generate g_n^t KWh amount of electricity per time slot t, uniformly distributed in the interval [0, 16] KWh and their electricity consumption is also uniformly distributed in the interval $l_n^t \in [5, 25]$ KWh. The initial storage capacity is $b_n^{t=0} = 0\ KWh, \forall n \in N$, the announced price is $p^t = 0.23$ (unless otherwise stated) and the probability of the prosumers' types Pr_n^t follows a uniform distribution. The proposed framework's evaluation was conducted in an ASUS laptop with AMD Ryzen 5, 2.1GHz Processor and 8Gb available RAM.

A. PURE FRAMEWORK OPERATION EVALUATION

In the following, we present the operational characteristics and the performance of the proposed contract-theoretic DRM framework under the scenarios of complete and incomplete information. Fig. 2a presents the prosumers' types $\tau_1^t < \cdots < \tau_n^t < \cdots < \tau_{|N_{buy}^t|}$ in time slot t, where 10 prosumers needed to purchase electricity from the electricity market, and their corresponding normalized electricity demand d_n^t is presented in Fig. 2b. The prosumers' purchased normalized electricity q_n^t and the prosumers' personalized rewards r_n^t for the complete and incomplete information scenarios, as they have been determined by the optimal contacts solutions of the optimization problems Eq. 8a-8b and Eq. 19a-19d, respectively, are illustrated in Fig. 2c and Fig. 2d, respectively. The results reveal that the prosumer of higher type receives a higher reward (Fig. 2d), following the fairness property (Proposition 1), and purchases more electricity (Fig. 2c), following the monotonicity property (Proposition 2), thus, achieving greater utility (Fig. 2e), based on the rationality property (Proposition 3). Also, it is observed that in the case that the electricity market knows the prosumers' electricity consumption and generation characteristics, i.e., types, under the complete information scenario, it can fully exploit their purchasing power. Thus, the prosumers are incentivized with greater rewards (Fig. 2d) to purchase more electricity (Fig. 2c). Furthermore, given that the electricity market knows the prosumers' types, it offers them the minimum possible rewards based on their purchased electricity in order to marginally satisfy their rationality constraints, thus, $U_n^t = 0, \forall n \in N_{buy}^t$ (Fig. 2e).

Additionally, Fig. 2f illustrates that a prosumer of higher type enjoys greater utility, as well as the contract that is explicitly designed for its type concludes to the best achieved utility. Furthermore, the electricity market's cumulative utility and the overall residential smart grid system's cumulative social welfare are presented in Fig. 2g and Fig. 2h, respectively. The results reveal that better performance is achieved under the complete information scenario, however, the overall system's social welfare is on average reduced by 36.3% under the incomplete information scenario, while this value becomes even smaller for larger populations. The latter

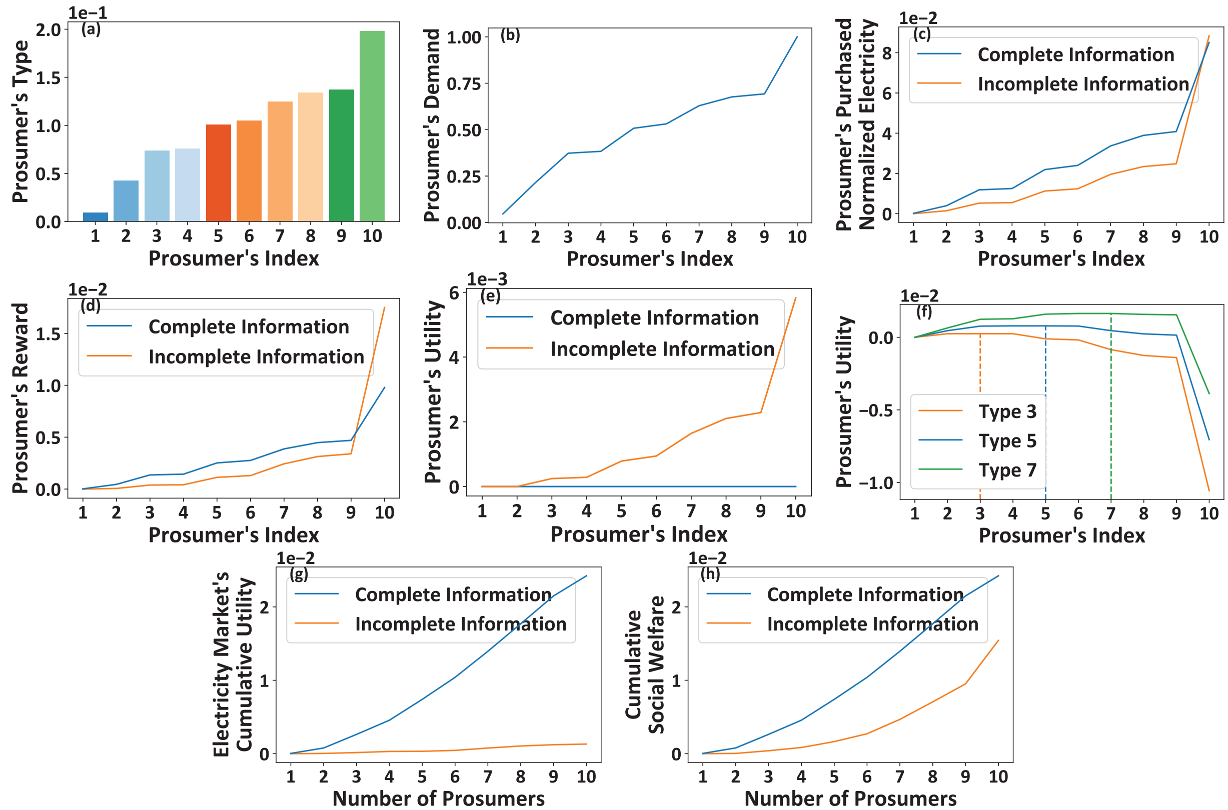

FIGURE 2. Pure operation evaluation of the contract-theoretic demand response management framework under complete and incomplete information.

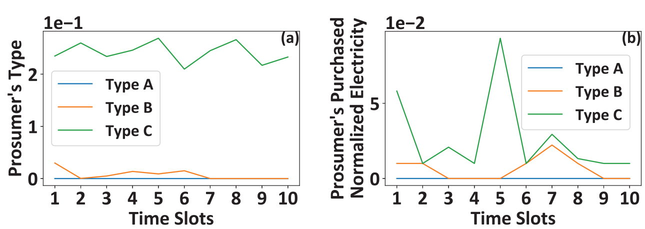

FIGURE 3. Users' types and normalized electricity consumption for three representative types of prosumers.

observation indicates that the proposed framework behaves in an acceptable manner for realistic implementations with complete lack of information regarding the prosumers' characteristics.

Following the previous analysis, we study the prosumers' behavior over a period of $|T| = 10$ time slots. Three indicative types of prosumers are selected: (a) Type A: the prosumer generates a lot of electricity and has a low electricity demand; (b) Type B: the prosumer has both medium electricity generation capability and electricity demand; and (c) Type C: the prosumer generates a small amount of electricity, but its electricity demand is high. Fig. 3a and Fig. 3b present the types τ_n^t of the three examined prosumers, as well as the corresponding normalized purchased electricity q_n^t over the time. The results reveal that the prosumers of Type A can cover their own electricity demands by their generated electricity without entering the electricity market, thus $\tau_n^t = 0$ and $q_n^t = 0$. Also, the prosumers of Type C have a greater type compared to the prosumers of Type B (Fig. 3a), as they have a greater electricity demand d_n^t given their electricity consumption and generation characteristics, as they have been described above, thus, they finally purchase more electricity from the electricity market (Fig. 3b) to cover their needs.

B. PRICING POLICIES & DEMAND RESPONSE MANAGEMENT

In this section, we examine the behavior and performance of the proposed contract-theoretic DRM framework under the incomplete information scenario considering three different pricing policies: (a) Low-cost: the electricity market applies a mild personalized pricing policy, i.e., $p_n^t = \sqrt{\tau_n^t}$ to each prosumer; (b) Medium-cost: a personalized pricing policy is announced, i.e., $p_n^t = (\tau_n^t)^{1/3}$ which is more costly compared to the low-cost policy; and (c) High-cost: a constant high price is announced to all the prosumers, i.e., $p^t = 0.6$. Fig. 4a-4c present the prosumers' utility, the electricity market's utility, and the overall residential smart grid system's social welfare as a function of the prosumer's index for all the three examined pricing policies. The results reveal that the higher-cost the pricing policy is, the less are the prosumers incentivized

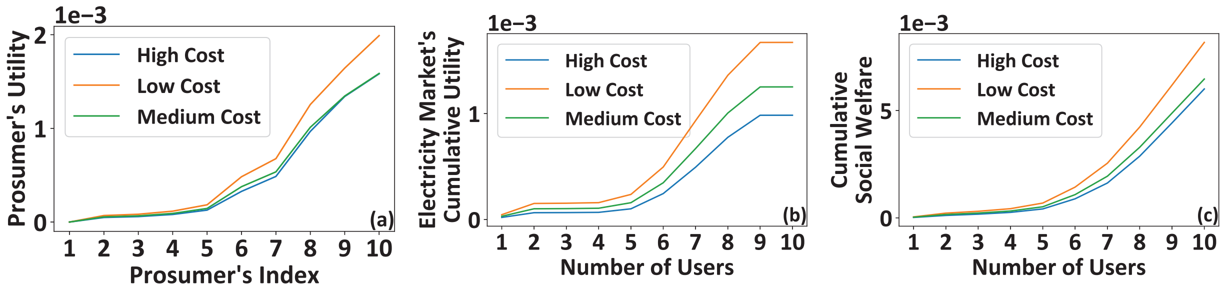

FIGURE 4. Pricing policies and demand response management.

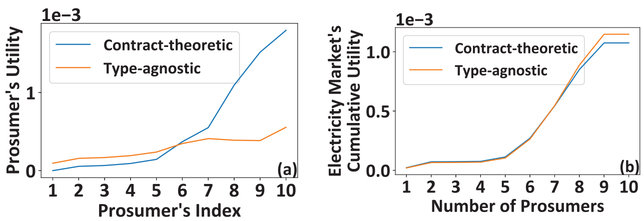

FIGURE 5. Type dependent vs. type-agnostic demand response management.

to buy electricity from the electricity market to cover their electricity demands. The latter phenomenon results in low values of achieved utility by the prosumers (Fig. 4a). Thus, the electricity market makes less profit resulting in decreased values of electricity market's utility, as shown in Fig. 4b. Therefore, for high-cost pricing policies, the social welfare of the overall residential smart grid system remains low (Fig. 4c), given that both the prosumers (Fig. 4a) and the electricity market (Fig. 4b) become less satisfied.

C. COMPARATIVE EVALUATION

In this section, a detailed comparative evaluation is presented to reveal the drawbacks and benefits of the proposed contract-theoretic DRM framework under the incomplete information scenario. Initially, the benefits of treating the prosumers in a personalized manner are presented. Thus, we compare our framework, where the electricity market offers personalized rewards $r_n^t(q_n^t) = \tau_n^t \cdot q_n^t$ to the prosumers based on their types, to a type-agnostic approach that offers a reward proportional to their normalized purchased electricity, i.e., $r_n^t(q_n^t) = \frac{\sum_{n=1}^{|N_{buy}^t|} \tau_n^t}{|N_{buy}^t|} \cdot q_n^t$.

Fig. 5a and Fig. 5b illustrate the prosumers' and the electricity market's utilities, respectively, as a function of the prosumers' index for the two examined comparative scenarios. The results show that the prosumers benefit regarding their achieved utility under our proposed contract-theoretic framework by approximately 42% on average compared to the type-agnostic framework due to the fact that the electricity market offers them personalized rewards to better incentivize them to purchase electricity at the announced price. On the other hand, the electricity market achieves lower utility compared to the type-agnostic framework, as the latter one tends to over-reward the prosumers without considering their personal electricity consumption and generation characteristics.

An additional thorough comparative evaluation is performed considering the different prosumers' decision-making regarding the purchased electricity. Specifically, we compare our proposed contract-theoretic DRM framework to four different scenarios: (1) Minimum effort: the prosumers purchase a minimum amount of normalized electricity, i.e., $q_n^t = \min\{q_n^t\}_{\forall n} \cdot 0.5$; (2) Maximum effort: the prosumers purchase a maximum amount of normalized electricity, i.e., $q_n^t = \max\{q_n^t\}_{\forall n} \cdot 1.5$; (3) Random effort: the prosumers purchase a random amount of normalized electricity, i.e., $q_n^t \in (0, 1]$, and (4) Guided effort: the prosumers purchase electricity following the function $10^{-3} \cdot \exp(0.4 \cdot q_n^t)$.

Fig. 6a - 6c illustrate the prosumers' utility, the electricity market's cumulative utility, and the smart grid system's cumulative social welfare, respectively. The results reveal that the proposed contract-theoretic framework explicitly benefits the prosumers (Fig. 6a) given the personalized treatment and incentivization that they experience, while the electricity market's profit is limited (Fig. 6b). Additionally, by focusing on the system's social welfare, we observe that the proposed framework outperforms concluding to a stable and efficient operation point for the overall examined smart grid system.

Regarding the maximum and minimum electricity purchasing scenarios, we observe that they demonstrate the worst and the second best utility for the prosumers, respectively, showing that the electricity cost becomes dominant in the prosumers' satisfaction. The exact opposite holds true for the electricity market's utility as it collects high and low profit, respectively. Following this discussion, the prosumers' great dissatisfaction in the case of purchasing their maximum needed amount of electricity concludes to the lowest social welfare compared to all other scenarios. Furthermore, the random and guided effort scenarios present an intermediate behavior compared to the other extreme scenarios by adopting the principles of Contract Theory and behavioral economics.

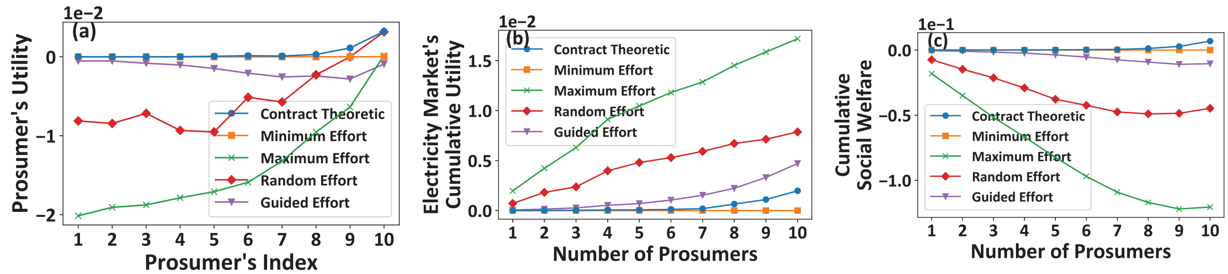

FIGURE 6. Pricing policies and demand response management.

VI. CONCLUSION & FUTURE WORK

In this article, a contract-theoretic demand response management framework is introduced based on the principles of labor economics to support the stable and efficient operation of smart grid systems. The prosumers electricity consumption and generation characteristics are captured to define the prosumers' types. The electricity market's and the prosumers' utilities were designed to represent their profit from participating in the DRM problem. The contract-theoretic DRM problem is formulated as a maximization problem of the electricity market's profit, while jointly guaranteeing the prosumers' profit optimization accounting for their types. The problem is solved under the cases of complete and incomplete information from the electricity market's perspective regarding knowing or not the prosumers' types, respectively, and the optimal contracts among both parties with competing interests are determined. The optimal contracts consist of the amounts of electricity bought by the prosumers and the corresponding rewards offered by the electricity market to incentivize them. Detailed numerical results of the pure proposed framework, as well as comparative ones, are presented to show the drawbacks and benefits of the introduced contract-theoretic DRM framework. Our current and future work focuses on extending the proposed framework to accommodate the prosumers behavioral characteristics. Specifically, our goal is to study the prosumers risk-aware decision-making in purchasing electricity under various price fluctuations.

ACKNOWLEDGMENT

(Nafis Irtija and Fisayo Sangoleye contributed equally to this work.)

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