

Transceiver I/Q Imbalance and Widely-Linear Spatial Processing in Large Antenna Systems

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Abstract—In order to keep the total device costs low, large antenna systems require affordable radio frequency (RF) electronics. Unfortunately, this requirement results in RF impairments and may thus cause performance degradations. In this paper, we show how one of these impairments, namely in-phase/quadrature (I/Q) imbalance, distorts the received signals in an uplink multiuser multiple-input multiple-output (MU-MIMO) system where multiple users are spatially multiplexed into the same time-frequency resource. In addition, we present three receiver (RX) post-processing methods and analyze their performance with different multicarrier scenarios under transceiver I/Q imbalances. The results clearly show that the simple maximum ratio combining (MRC) based RX processing suffers heavily from the presence of multiple spatially multiplexed users, especially in case of I/Q imbalances, and cannot necessarily provide sufficient performance even with the number of RX antennas approaching infinity. In contrast, the linear minimum mean-square error (LMMSE) processing offers more flexible and efficient operation characteristics but is also shown to suffer from performance degradations due to I/Q imbalances. To overcome this problem, we formulate a widely-linear (WL) variant of the MMSE method, called WL-MMSE, which provides good performance also under I/Q imbalances in different multiple access scenarios, and is thus a good candidate for future software defined radios where flexibility is a key concern.

Index Terms—in-phase/quadrature (I/Q) imbalance, large antenna systems, multiuser multiple-input multiple-output (MU-MIMO), widely-linear (WL) processing

I. INTRODUCTION

Large antenna systems, also known as massive multiple-input multiple-output (MIMO), are considered to have an order of magnitude more base station (BS) antennas than active user equipment (UEs) on a given time-frequency resource [1]–[3]. The vast amount of BS antennas demands low-cost and low-power radio frequency (RF) electronics in order to keep the total costs and dissipated power in control. This, in turn, can cause quality degradations in the associated RF circuitry and consequently the overall performance is deteriorated [2].

One of the most severe RF impairments is the so-called in-phase/quadrature (I/Q) imbalance which occurs in direct-conversion transceivers [4]. The roots of I/Q imbalance are twofold. On the one hand, nonideal mixers cause phase imbalance between the I and Q branches. On the other hand, imperfect responses of amplifiers, filters, analog-to-digital and digital-to-analog converters result in gain imbalance between the I and Q branches [4]. The resulting signal distortion is well known to

cause inter-carrier interference in multicarrier systems and thus to degrade the obtainable performance [5]. In multiuser MIMO (MU-MIMO) systems, where multiple UEs are spatially multiplexed into the same time-frequency resource, the influence of I/Q imbalances is even more complex since there the imbalances generate also inter-user interference between the UEs at mirror or image subcarrier pairs [6].

In the literature many methods for I/Q imbalance mitigation are proposed. Stemming from the inter-carrier interference, the so-called augmented or widely-linear (WL) methods where each mirror subcarrier pair is processed jointly have gained lots of attention, see e.g. [7], [8]. However, these studies do not address the influence of spatially multiplexed UEs or possible external interferers and are therefore not directly applicable to MU-MIMO communications or interference-limited systems, such as mobile cellular radio with frequency reuse one.

In this paper, we focus on a very flexible system model where UEs are spatially multiplexed into the same time-frequency resource and thus operate in a challenging radio environment. In addition, we include external interferers to our models in order to model a heterogeneous network framework where users of different radio networks are all operating simultaneously at the same frequencies. We derive a signal model for the received uplink spatial signal vector and show explicitly how I/Q imbalances in the UE transmitters (TXs) and BS receiver (RX) distort the signals. Furthermore, we show with extensive computer simulations how three RX post-processing methods, namely maximum ratio combining (MRC), linear minimum mean-square error (LMMSE) spatial filter and its WL variant called WL-MMSE, handle the challenging data stream separation task with MU-MIMO transmission and under I/Q imbalances. The results clearly show that the simple MRC based spatial RX processing suffers heavily from the presence of multiple spatially multiplexed users, especially in case of I/Q imbalances, and cannot necessarily provide sufficient performance even with the number of RX antennas approaching infinity. The LMMSE based spatial processing, on the other hand, offers more flexible and efficient operation characteristics but is also shown to suffer from performance degradations due to I/Q imbalances. Finally, the WL-LMMSE based spatial processing approach results in clearly the best signal-to-interference-plus-noise ratio (SINR) performance in all considered scenarios, despite of the associated TX and RX I/Q imbalances.

This paper is organized as follows. Section II presents a generic signal and system formulation for MU-MIMO transmission under I/Q imbalances. Then, spatial RX processing methods are introduced in Section III. Section IV evaluates the system performance numerically and finally, we summarize the paper in Section V.

Notation: Vectors and matrices are written with bold characters. The superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ and $(\cdot)^{-1}$ represent transpose, Hermitian (conjugate) transpose, complex conjugate and matrix inverse, respectively. The tilde sign $(\cdot)$ is used for denoting WL

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quantities and the results obtained by WL processing. We write $\text{diag}(x_{11}, x_{22}, \dots, x_{ii}, \dots)$ to denote a diagonal matrix $\mathbf{X}$ with elements x_{ii} on the main diagonal. The statistical expectation is denoted with $\mathbb{E}[\cdot]$.

II. SIGNAL AND SYSTEM FORMULATION

We examine an orthogonal frequency division multiplexing (OFDM) / orthogonal frequency division multiple access (OFDMA) MU-MIMO system scenario where multiple UEs are spatially multiplexed into the same time-frequency resource, as depicted in Fig.1. The subcarriers are indexed with $c \in \{-C/2, \dots, -1, 1, \dots, C/2\}$ where C is the total number of subcarriers. Since OFDM and OFDMA are based on independent subcarrier signals, all analysis is here done for an arbitrary subcarrier c whose mirror subcarrier is denoted by $c' = -c$. The number of UEs at subcarriers c and c' is denoted by U and V , respectively. Furthermore, a single UE at subcarrier c is indexed by u and at c' by v . In OFDM case, the same set of UEs use both subcarriers c and c' . In OFDMA case, in turn, the sets of UEs at subcarriers c and c' are different and the users are thus multiplexed also in the frequency domain. We denote the number of the RX antennas in the BS by N and the number of TX antennas of UE u by M_u . Furthermore, the number of the independent data streams of UE u at subcarrier c is given by $Q_{u,c}$ and the transmitted data vector is equal to $\mathbf{x}_{u,c} \in \mathbb{C}^{Q_{u,c} \times 1}$. Consequently, the total number of transmitted data streams at subcarrier c is equal to $S = \sum_{u=1}^U Q_{u,c}$. The transmitted antenna signal vector of UE u is given by $\mathbf{s}_{u,c} = \mathbf{G}_{u,c} \mathbf{x}_{u,c} \in \mathbb{C}^{M_u \times 1}$ where $\mathbf{G}_{u,c} \in \mathbb{C}^{M_u \times Q_{u,c}}$ denotes the spatial precoder matrix. In order to describe the radio environment flexibly we also include L external interferers to our models. External interferer l has J_l antennas and its transmitted antenna signal vector at subcarrier c is denoted by $\mathbf{s}_{\text{int},l,c} \in \mathbb{C}^{J_l \times 1}$. Throughout the paper, all data vectors refer to frequency domain quantities, i.e., prior to the inverse fast Fourier transform in the UEs and after the fast Fourier transform in the BS.

In uplink transmission, each UE transmits its own precoded data streams towards the BS. The transmitted antenna signal vector of UE u at subcarrier c under TX I/Q imbalance can be modeled as [6], [9]

$$\mathbf{s}_{\text{Tx},u,c} = \mathbf{K}_{\text{Tx}1,u,c} \mathbf{G}_{u,c} \mathbf{x}_{u,c} + \mathbf{K}_{\text{Tx}2,u,c} \mathbf{G}_{u,c}^* \mathbf{x}_{u,c}^*. \quad (1)$$

Here $\mathbf{K}_{\text{Tx}1,u,c} = \text{diag}(K_{\text{Tx}1,1,u,c}, \dots, K_{\text{Tx}1,M_u,u,c}) \in \mathbb{C}^{M_u \times M_u}$ and $\mathbf{K}_{\text{Tx}2,u,c} = \text{diag}(K_{\text{Tx}2,1,u,c}, \dots, K_{\text{Tx}2,M_u,u,c}) \in \mathbb{C}^{M_u \times M_u}$ denote the diagonal TX I/Q imbalance matrices. The matrix entries for TX antenna m of UE u at subcarrier c are given by $K_{\text{Tx}1,m,u,c} = (1 + g_{\text{Tx},m,u,c} e^{j\phi_{\text{Tx},m,u,c}})/2$ and $K_{\text{Tx}2,m,u,c} = (1 - g_{\text{Tx},m,u,c} e^{j\phi_{\text{Tx},m,u,c}})/2$ where the gain and phase imbalance coefficients are equal to $g_{\text{Tx},m,u,c}$ and $\phi_{\text{Tx},m,u,c}$, respectively [5]. Based on (1), TX I/Q imbalance causes cross-talk between the mirror subcarrier signals of an individual UE. This model holds as such for a scenario where subcarriers c and c' are both allocated to UE u . Notice, however, that if subcarrier c' is not allocated to UE u , the resulting transmitted signal at subcarrier c consists only of the first term in (1). However, when subcarrier c' is allocated, through the OFDMA principle, to another UE v which also suffers from TX I/Q imbalance, the corresponding transmitted signal of UE v at subcarrier c is equal to $\mathbf{s}_{\text{Tx},v,c} = \mathbf{K}_{\text{Tx}2,v,c} \mathbf{G}_{v,c}^* \mathbf{x}_{v,c}^*$. For notational convenience we use the latter more general case in our models. The special case where subcarriers c and c' are both allocated for the same set of UEs is obtained from the models by substituting $V = U$ and $v = u$.

The transmitted UE signals propagate then through wireless channels and are eventually received in the BS. The received signal

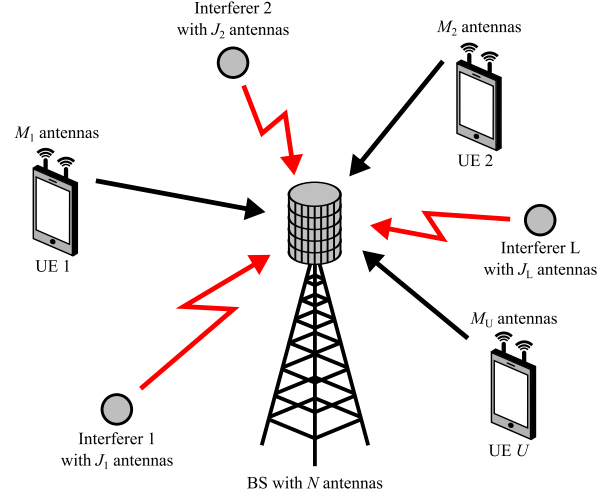


Fig. 1: The considered uplink MU-MIMO scenario with all devices being active at subcarrier c . The BS antenna array is here drawn as a cylindrical array but the signal model is not restricted to any specific array structure.

vector $\mathbf{r}_{\text{TxRx},c} \in \mathbb{C}^{N \times 1}$ under the influence of TX as well as RX I/Q imbalances is then equal to

$$\mathbf{r}_{\text{TxRx},c} = \sum_{u=1}^U \tilde{\Psi}_{u,c} \mathbf{G}_{u,c} \mathbf{x}_{u,c} + \sum_{v=1}^V \tilde{\Omega}_{v,c} \mathbf{G}_{v,c}^* \mathbf{x}_{v,c}^* + \mathbf{K}_{\text{Rx}1,c} \mathbf{z}_c + \mathbf{K}_{\text{Rx}2,c} \mathbf{z}_c^* \quad (2)$$

where we assume perfect time and frequency synchronization between the UEs and the BS. Furthermore, $\mathbf{K}_{\text{Rx}1,c} = \text{diag}(K_{\text{Rx}1,1,c}, \dots, K_{\text{Rx}1,N,c}) \in \mathbb{C}^{N \times N}$ and $\mathbf{K}_{\text{Rx}2,c} = \text{diag}(K_{\text{Rx}2,1,c}, \dots, K_{\text{Rx}2,N,c}) \in \mathbb{C}^{N \times N}$ denote the diagonal RX I/Q imbalance matrices where the entries for RX antenna n are given by $K_{\text{Rx}1,n,c} = (1 + g_{\text{Rx},n,c} e^{-j\phi_{\text{Rx},n,c}})/2$ and $K_{\text{Rx}2,n,c} = (1 - g_{\text{Rx},n,c} e^{j\phi_{\text{Rx},n,c}})/2$. Here the RX gain and phase imbalance coefficients are equal to $g_{\text{Rx},n,c}$ and $\phi_{\text{Rx},n,c}$, respectively [5]. The effective channel matrices, $\tilde{\Psi}_{u,c} \in \mathbb{C}^{N \times M_u}$ and $\tilde{\Omega}_{v,c} \in \mathbb{C}^{N \times M_v}$ including the effects of the wireless channels as well as TX and RX I/Q imbalances are given by

$$\tilde{\Psi}_{u,c} = [\mathbf{K}_{\text{Rx}1,c} \quad \mathbf{K}_{\text{Rx}2,c}] \begin{bmatrix} \mathbf{H}_{u,c} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{u,c}^* \end{bmatrix} \begin{bmatrix} \mathbf{K}_{\text{Tx}1,u,c} \\ \mathbf{K}_{\text{Tx}2,u,c}^* \end{bmatrix}, \quad (3)$$

$$\tilde{\Omega}_{v,c} = [\mathbf{K}_{\text{Rx}1,c} \quad \mathbf{K}_{\text{Rx}2,c}] \begin{bmatrix} \mathbf{H}_{v,c} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{v,c}^* \end{bmatrix} \begin{bmatrix} \mathbf{K}_{\text{Tx}2,v,c} \\ \mathbf{K}_{\text{Tx}1,v,c}^* \end{bmatrix}$$

where $\mathbf{H}_{u,c} \in \mathbb{C}^{N \times M_u}$ and $\mathbf{H}_{v,c} \in \mathbb{C}^{N \times M_v}$ represent the wireless channel matrices of UEs u and v , respectively. Finally, the interference and noise vector $\mathbf{z}_c \in \mathbb{C}^{N \times 1}$ equals

$$\mathbf{z}_c = \sum_{l=1}^L \mathbf{H}_{\text{int},l,c} \mathbf{s}_{\text{int},l,c} + \mathbf{n}_c \quad (4)$$

where $\mathbf{H}_{\text{int},l,c} \in \mathbb{C}^{N \times J_l}$ denotes the wireless channel of external interferer l . Additionally, $\mathbf{n}_c \in \mathbb{C}^{N \times 1}$ denotes additive white Gaussian noise in the RX electronics. As visible in (2), transceiver I/Q imbalances cause inter-user interference between the users in mirror subcarrier pairs. In addition, also the external interference and noise from the mirror subcarrier leak to subcarrier c .

The special case with I/Q imbalance occurring only in the TX (RX) side is obtained from (2) by substituting $\mathbf{K}_{\text{Rx}1,c} = \mathbf{I}$ and $\mathbf{K}_{\text{Rx}2,c} = \mathbf{0} \forall c$ ($\mathbf{K}_{\text{Tx}1,u,c} = \mathbf{I}$ and $\mathbf{K}_{\text{Tx}2,u,c} = \mathbf{0} \forall u, c$).

III. SPATIAL RX PROCESSING

In uplink MU-MIMO the BS exploits its multiple RX antennas for spatial processing. This means that data streams originating

from different UEs can be reliably separated even when they are transmitted over the same time-frequency resource [10]. Furthermore, one of the promising prospects of massive MIMO or large-antenna system in general is that simple RX spatial processing, even classical MRC, can potentially be adopted [11]. In general, the combiner's output signal vector $\mathbf{y}_{\text{TxRx},c} \in \mathbb{C}^{S \times 1}$ under TX and RX I/Q imbalances is

$$\begin{aligned} \mathbf{y}_{\text{TxRx},c} &= \mathbf{W}_c^H \mathbf{r}_{\text{TxRx},c} \\ &= \sum_{u=1}^U \mathbf{W}_c^H \tilde{\Psi}_{u,c} \mathbf{G}_{u,c} \mathbf{x}_{u,c} + \sum_{v=1}^V \mathbf{W}_c^H \tilde{\Omega}_{v,c} \mathbf{G}_{v,c}^* \mathbf{x}_{v,c}' \\ &\quad + \mathbf{W}_c^H \mathbf{K}_{\text{Rx1},c} \mathbf{z}_c + \mathbf{W}_c^H \mathbf{K}_{\text{Rx2},c} \mathbf{z}_c' \end{aligned} \quad (5)$$

where $\mathbf{W}_c \in \mathbb{C}^{S \times N}$ denotes the combiner weight matrix at subcarrier c [6]. In the following subsections we present different methods to select the weight matrix in large antenna systems under I/Q imbalance. The selection methods differ especially in complexity and performance.

A. MRC Approach

Commonly used argumentation with large antenna systems is that the spatial separation should be implemented in a very simple way due to the massive amount of RX antennas, see e.g. [2]. Due to this reason MRC is considered to be one of the most promising solutions for large scale processing, primarily due to its simplicity. In general, the MRC weight matrix of UE u is given simply as $\mathbf{W}_{\text{MRC},u,c} = \mathbf{H}_{u,c} \mathbf{G}_{u,c} \in \mathbb{C}^{N \times Q_u}$ [12]. However, under I/Q imbalances we need to take also the influence of TX I/Q imbalance of UE u and RX I/Q imbalance in the BS into account and thus the weight matrix of UE u becomes equal to

$$\mathbf{W}_{\text{MRC},u,c} = \tilde{\Psi}_{u,c} \mathbf{G}_{u,c}. \quad (6)$$

When stacking the weight matrices of individual UEs at subcarrier c into a total weight matrix we get $\mathbf{W}_{\text{MRC},c} = [\mathbf{W}_{\text{MRC},1,c}, \dots, \mathbf{W}_{\text{MRC},U,c}] \in \mathbb{C}^{N \times S}$ to be adopted in (5). The MRC approach is clearly very simple but it has also its built-in limitations. MRC cannot exploit any information, other than the individual direct channel matrix $\tilde{\Psi}_{u,c}$, and is therefore vulnerable especially in noisy conditions with multiple signal sources. Notice that as the channel state information is in practice anyway obtained from uplink pilot or reference signals, it is indeed the *effective direct spatial channel matrix* that is used to form the MRC spatial filter, as given in (6). We use the MRC method as a benchmark in Section IV with numerical illustrations.

B. LMMSE Processing

In order to perform reliable data stream detection in MU-MIMO, the RX should be able to operate also in conditions with multiple active signal sources. One way to do this is the so-called Wiener or LMMSE processing. It optimizes the weights in such a way that the mean-square error between the spatially filtered received signal and the desired transmitted signal is minimized [13]. This approach implicitly suppresses the influence of any unwanted interference and noise, and therefore enables good separation capabilities also in challenging MU-MIMO schemes. The overall weight matrix for data streams from all UEs is equal to $\mathbf{W}_{\text{LMMSE},c} = [\mathbf{W}_{\text{LMMSE},1,c}, \dots, \mathbf{W}_{\text{LMMSE},U,c}] \in \mathbb{C}^{N \times S}$ and can be directly substituted into (5). Under TX and RX I/Q imbalances the weight matrix $\mathbf{W}_{\text{LMMSE},u,c} \in \mathbb{C}^{N \times Q_u}$ for UE u is of the form

$$\mathbf{W}_{\text{LMMSE},u,c} = \mathbf{R}_{r,c}^{-1} \mathbf{V}_{u,c} \quad (7)$$

where $\mathbf{R}_{r,c} = \mathbb{E}[\mathbf{r}_{\text{TxRx},c} \mathbf{r}_{\text{TxRx},c}^H] \in \mathbb{C}^{N \times N}$ is the covariance matrix of the received signals and $\mathbf{V}_{u,c} \in \mathbb{C}^{N \times Q_u}$ denotes the

cross-correlation matrix between the received signal vector and the transmitted signal of UE u [13]. The column of $\mathbf{V}_{u,c}$ for data stream q is given by

$$\mathbf{v}_{q,u,c} = \mathbb{E}[\mathbf{r}_{\text{TxRx},c} x_{q,u,c}^*] = \sigma_{q,u,c}^2 \tilde{\Psi}_{u,c} \mathbf{G}_{u,c} \mathbf{e}_q \quad (8)$$

where $x_{q,u,c}$ is the q^{th} element of $\mathbf{x}_{u,c}$, $\sigma_{q,u,c}^2$ is its power, and $\mathbf{e}_q$ is a vector whose q^{th} element is one and the rest are zeros. Notice that, again, the RX is deploying, explicitly or implicitly, the knowledge of *effective spatial channels* along with the associated interference and noise covariances.

C. Augmented Signal Model and WL-MMSE Processing

LMMSE processing is an effective tool in MU-MIMO BSs. However, it cannot structurally handle the inter-carrier interference and the corresponding inter-user interference caused by I/Q imbalances in the transceiver electronics. The nature of the signal distortion leads towards WL or augmented signal processing where the received signals at subcarriers c and c' are processed jointly [6]–[8]. In order to model such a method, we define an augmented received signal vector $\tilde{\mathbf{r}}_c = [\mathbf{r}_c^T, \mathbf{r}_{c'}^H]^T \in \mathbb{C}^{2N \times 1}$. Then the output signal of the WL combiner is obtained simply by $\tilde{\mathbf{y}}_c = \tilde{\mathbf{W}}_c^H \tilde{\mathbf{r}}_c \in \mathbb{C}^{S \times 1}$ where $\tilde{\mathbf{W}}_c = [\tilde{\mathbf{W}}_{1,c}, \dots, \tilde{\mathbf{W}}_{U,c}] \in \mathbb{C}^{2N \times S}$ denotes the total WL weight matrix. Here $\tilde{\mathbf{W}}_{u,c} \in \mathbb{C}^{2N \times Q_u}$ represents the WL weight matrix of UE u . When substituting the signal model in (2) into the principle of WL processing we get

$$\begin{aligned} \tilde{\mathbf{y}}_{\text{TxRx},c} &= \tilde{\mathbf{W}}_c^H \tilde{\mathbf{r}}_{\text{TxRx},c} \\ &= \sum_{u=1}^U \tilde{\mathbf{W}}_c^H \tilde{\Xi}_{u,c} \mathbf{G}_{u,c} \mathbf{x}_{u,c} + \sum_{v=1}^V \tilde{\mathbf{W}}_c^H \tilde{\Phi}_{v,c} \mathbf{G}_{v,c}^* \mathbf{x}_{v,c}' \\ &\quad + \tilde{\mathbf{W}}_c^H \tilde{\mathbf{K}}_{\text{RxA},c} \mathbf{z}_c + \tilde{\mathbf{W}}_c^H \tilde{\mathbf{K}}_{\text{RxB},c} \mathbf{z}_c' \end{aligned} \quad (9)$$

where the effective WL channel matrices $\tilde{\Xi}_{u,c} \in \mathbb{C}^{2N \times M_u}$ and $\tilde{\Phi}_{v,c} \in \mathbb{C}^{2N \times M_v}$ are given by

$$\begin{aligned} \tilde{\Xi}_{u,c} &= \begin{bmatrix} \mathbf{K}_{\text{Rx1},c} & \mathbf{K}_{\text{Rx2},c} \\ \mathbf{K}_{\text{Rx2},c}^* & \mathbf{K}_{\text{Rx1},c}' \end{bmatrix} \begin{bmatrix} \mathbf{H}_{u,c} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{u,c}' \end{bmatrix} \begin{bmatrix} \mathbf{K}_{\text{Tx1},u,c} \\ \mathbf{K}_{\text{Tx2},u,c}' \end{bmatrix}, \\ \tilde{\Phi}_{v,c} &= \begin{bmatrix} \mathbf{K}_{\text{Rx1},c} & \mathbf{K}_{\text{Rx2},c} \\ \mathbf{K}_{\text{Rx2},c}^* & \mathbf{K}_{\text{Rx1},c}' \end{bmatrix} \begin{bmatrix} \mathbf{H}_{v,c} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{v,c}' \end{bmatrix} \begin{bmatrix} \mathbf{K}_{\text{Tx2},v,c} \\ \mathbf{K}_{\text{Tx1},v,c}' \end{bmatrix}. \end{aligned} \quad (10)$$

In addition, $\tilde{\mathbf{K}}_{\text{RxA},c} = [\mathbf{K}_{\text{Rx1},c}^T, \mathbf{K}_{\text{Rx2},c}^H]^T \in \mathbb{C}^{2N \times N}$ and $\tilde{\mathbf{K}}_{\text{RxB},c} = [\mathbf{K}_{\text{Rx2},c}^T, \mathbf{K}_{\text{Rx1},c}^H]^T \in \mathbb{C}^{2N \times N}$ denote the augmented RX I/Q imbalance matrices.

Stemming from the above models and processing principles, and the target of MSE minimization, we next formulate the WL-MMSE combiner (originally proposed in [14] for processing non-circular signals) which is able to suppress the co-channel interference as well as the inter-carrier and inter-user interference effectively in the presence of all involved I/Q imbalances. The WL weight matrix for the data streams of UE u is equal to

$$\tilde{\mathbf{W}}_{\text{WL-MMSE},u,c} = \tilde{\mathbf{R}}_{r,c}^{-1} \tilde{\mathbf{V}}_{u,c} \quad (11)$$

where $\tilde{\mathbf{R}}_{r,c} = \mathbb{E}[\tilde{\mathbf{r}}_{\text{TxRx},c} \tilde{\mathbf{r}}_{\text{TxRx},c}^H] \in \mathbb{C}^{2N \times 2N}$ is the covariance matrix of the augmented received signal. In addition, the columns of the cross-correlation matrix $\tilde{\mathbf{V}}_{u,c} \in \mathbb{C}^{2N \times Q_u}$ are of the form

$$\tilde{\mathbf{v}}_{q,u,c} = \mathbb{E}[\tilde{\mathbf{r}}_{\text{TxRx},c} x_{q,u,c}^*] = \sigma_{q,u,c}^2 \tilde{\Xi}_{u,c} \mathbf{G}_{u,c} \mathbf{e}_q. \quad (12)$$

The above WL-MMSE RX utilizes effective spatial channels of all multiplexed UEs at both subcarriers c and c' , together with the associated interference and noise covariances, where also the I/Q imbalance characteristics are implicitly built in. Note that whereas WL-MMSE increases the computational complexity of

TABLE I: Basic simulation parameters

Parameter	Symbol	Simulation scenario	
		1	2
Frequency multiplexing	-	Yes	No
Spatially multiplexed UEs	$U, (V)$	$U = 5,$ $V = 5$	$U = 5$
TX antennas in UEs	$M_u, (M_v)$	1	
UE data streams	$Q_u, (Q_v)$	1	
RX antennas in BS	N	100	
Signal to noise ratio [dB]	SNR	20	
Fluctuation in UE powers [dB]	-	± 3	
Image rejection ratio [dB]	IRR	20	

the combiner block, demanding FFT processing remain the same as with LMMSE.

In the next section we provide an extensive numerical performance analysis for the linear and WL processing methods presented above. In particular, we focus on their operation capabilities in a challenging MU-MIMO environment and under transceiver I/Q imbalances.

IV. NUMERICAL PERFORMANCE ANALYSIS

A. Simulation Setup and Scenarios

In the simulations we consider an uplink MU-MIMO multi-carrier scenario whose parameters are presented in Table I. We analyze two simulation scenarios with both common and different parameters. The common parameters are as follows. Subcarrier c has $U = 5$ single-antenna UEs transmitting simultaneously towards a single BS. We set the number of RX antennas in the BS to $N = 100$ for modeling a large antenna system. Thereby, N is an order of magnitude larger than U which is a commonly used assumption to describe large antenna and massive MIMO systems [1]–[3]. We define the signal-to-noise ratio (SNR) as the ratio between the average received signal power of a single UE and the noise power in the RX electronics and set SNR = 20 dB.

The propagation channels between all UEs and BS RX antennas as well as between subcarriers are independent and Rayleigh distributed. On top of that, we include a uniformly distributed fluctuation from a range of ± 3 dB to $\sigma_{u,c}^2 \forall u, c$ in order to model differences in the uplink power control between the UEs. I/Q imbalance is defined in terms of the image rejection ratio (IRR) given in decibels for a single transceiver branch by $\text{IRR} = 10\log_{10}(|K_1|^2/|K_2|^2)$. We set $\text{IRR} = 20$ dB in all transceiver branches. However, we do include randomness to the I/Q imbalance, both across different antenna branches of a single device as well as across different devices, through the phase and gain imbalance coefficients. At first, we draw $\phi_{\text{Tx},u,m,c}, \forall u, m, c$ and $\phi_{\text{Rx},c,n}, \forall n, c$ independently from $\mathcal{U}(-\alpha, \alpha)$ where α guarantees $\text{IRR} = 20$ dB if the gain imbalance coefficients were equal to one. Then, $g_{\text{Tx},u,m,c}, \forall u, m, c$ and $g_{\text{Rx},n,c}, \forall n, c$ are set in such a way that the resulting $\text{IRR} = 20$ dB with the earlier selected phase imbalance coefficients. The I/Q imbalance parameters at different subcarriers are also assumed to be independent. Finally, in order to clearly illustrate the influence of different scenarios, we set the number of external interferers to $L = 0$.

The scenarios differ in the subcarrier allocation. In scenario 1 we consider an OFDMA system where frequency multiplexing is involved, i.e. subcarriers c and c' have different sets of UEs. In this case, we set the number of UEs at the mirror subcarrier to $V = 5$. In contrast, in scenario 2 we assume an OFDM scheme, i.e., an individual UE u where $u = 1, \dots, U$ operates on subcarriers c and c' . This means that the second term in (2) is replaced with $\sum_{u=1}^U \Omega_{u,c} \mathbf{G}_{u,c'}^* \mathbf{x}_{u,c'}$. Although a single UE operates on both subcarriers c and c' , we assume uncorrelated fading between the mirror subcarriers since, practically, only very closely located subcarriers have significant fading correlation.

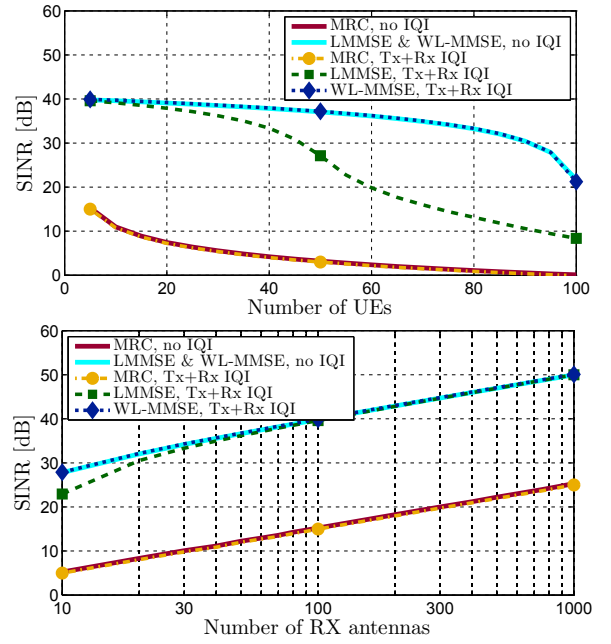


Fig. 2: Scenario 1: frequency multiplexing (OFDMA). The SINR as a function of U (top) and N (bottom) with other parameters as given in Table I. Note the logarithmic x-axis in the lower graph.

All figures discussed in the following illustrate the combiner's output SINR at an arbitrary subcarrier c . The numerator of the SINR is the signal power of a single UE u , i.e. the power of one combiner output element of the first sum in (5) and (9), whereas the denominator includes the total power of the other UEs and noise at subcarrier c as well as the inter-carrier and inter-user interference from the mirror subcarrier due to I/Q imbalance. The SINRs are averaged over all UEs and 1000 independent realizations. For each realization the UE powers, channels and I/Q imbalance coefficients are independently drawn according to the distributions above.

B. Results and Discussion

Scenario 1: Fig. 2 illustrates the average SINR as a function of the number of UEs (top) and the number of RX antennas (bottom) for scenario 1. We notice that the performance of MRC is much worse than that of the MMSE approaches. This is due to the fact that the MRC method is very vulnerable to any interference which, in this case, means the transmissions from multiple spatially multiplexed UEs. The SINR of MRC increases when either the number of UEs decreases or the number of RX antennas increases. We can also see that the SINR of MRC is dominated by the inter-user interference, no matter if the system is under I/Q imbalance or not. In contrast to MRC, the LMMSE combiner provides good results also in a MU-MIMO environment due to its built-in capability for inter-user interference suppression. However, it cannot effectively suppress the inter-carrier interference which is caused by I/Q imbalance. This is especially visible in the upper graph where the increasing number of UEs at subcarriers c and c' increases the inter-carrier-interference and decreases the degrees of freedom of LMMSE. On the contrary to the linear processing methods, the WL-MMSE combiner can suppress the inter-user as well as the inter-carrier interference very efficiently, due to its built-in capability to process signals at c and c' jointly. Consequently, it provides the best performance and, in fact, yields the same SINR as a system under ideal I/Q matching, even when operating under TX+RX I/Q imbalances.

Scenario 2: The results for scenario 2 are presented in Fig. 3. The SINRs of all combiners under perfect I/Q matching, i.e., no I/Q imbalance, are basically identical to their SINRs in scenario 1.

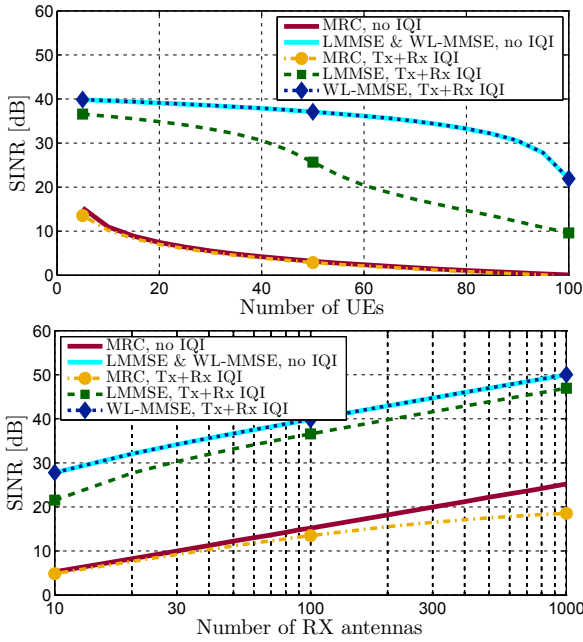


Fig. 3: Scenario 2: no frequency multiplexing (OFDM). The SINR as a function of U (top) and N (bottom) with other parameters as given in Table I. Note the logarithmic x-axis in the lower graph.

Thus we conclude that systems without I/Q imbalances are not sensitive to different subcarrier allocation schemes. However, we do observe big differences under the influence of I/Q imbalances. First of all, the SINR of MRC does not anymore improve with a slope equal to $10\log_{10}(N)$. This is caused by the following fact. The weights of MRC are matched, as shown in (6), to the effective channel $\Psi_{u,c}$ which is dominated by the term $\mathbf{K}_{\text{Rx}1,c}\mathbf{H}_{u,c}K_{\text{Tx}1,u,c}$ as visible in (3). Since the inter-carrier-interference from the same UE, caused already by UE TX I/Q imbalance, propagates through the effective channel $\tilde{\Omega}_{u,c}$ which in this scenario includes a term $\mathbf{K}_{\text{Rx}1,c}\mathbf{H}_{u,c}K_{\text{Tx}2,u,c}$, the only difference comes from different scaling factors $K_{\text{Tx}1,u,c}$ and $K_{\text{Tx}2,u,c}$. This means that the spatial propagation channels of the desired data stream and the inter-carrier interference from the same UE are very similar from the BS perspective and consequently the SINR is restricted to $10\log_{10}(|K_{\text{Tx}1}|^2/|K_{\text{Tx}2}|^2)$. This is exactly the same as the TX IRR, i.e. the SINR of MRC is limited to the TX IRR in scenario 2. In contrast to MRC, LMMSE is again able to provide fairly good SINRs. However, the performance of LMMSE is also deteriorated when compared to scenario 1. Also this is caused by the limited capabilities to suppress the inter-carrier-interference from the same UE, because of the high similarity of the effective spatial channels between the direct linear term and the inter-carrier interference. In this case, however, the weights are given by (7) which is more flexible than the MRC approach. Thus the SINR degradation of LMMSE is much less severe compared to what we observed for MRC. The results also indicate that *MRC as well as LMMSE are sensitive to subcarrier allocation schemes when operating under TX+RX I/Q imbalances*. In contrast, the WL-MMSE processing under I/Q imbalance provides, again, the same performance as a system under perfect I/Q matching. In fact, this is a property which WL-MMSE provides for an arbitrary number of users and RX antennas. Based on the results above, we summarize that WL-MMSE improves the performance considerably when compared to the presented linear methods. It also removes the need for separate I/Q imbalance mitigation and thus simplifies the overall RX structure. As a consequence, WL processing becomes a highly attractive solution for future large antenna systems, with lower-cost RF transceivers, potentially also incorporating software

defined radio technologies where the radio interface must support multiple radio technologies on the one hand and must be flexibly controllable, on the other hand.

V. CONCLUSION

In this paper we have analyzed the influence of TX and RX I/Q imbalances in uplink MU-MIMO transmission with large antenna systems. First, we derived models for both the received antenna signal vector in the BS as well as the corresponding output signal after the BS spatial filter. We also presented three spatial RX processing schemes, namely MRC, LMMSE and WL-MMSE. Using numerical examples, we illustrated that the performance of the MRC method is heavily limited in a MU-MIMO environment where multiple UEs are simultaneously active in the same time-frequency resource. The poor performance was emphasized even more under I/Q imbalances. In fact, it was shown that the SINR of MRC is restricted to the TX IRR in the classical OFDM case without user multiplexing in the frequency domain. The LMMSE method, in turn, can more efficiently suppress the interference from multiple signal sources, and it was shown to operate relatively efficiently and reliably also in challenging MU-MIMO conditions. However, we observed that it cannot effectively suppress the inter-carrier-interference caused by I/Q imbalances which results in severe performance degradation. To overcome this limitation, we formulated the WL-MMSE approach where the mirror-subcarrier signals and all associated spatially multiplexed UEs are processed together. Under TX+RX I/Q imbalances, the WL-MMSE method provides consistently the best performance among the methods under comparison and, in fact, yields equal performance as a system under ideal I/Q matching.

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