DoA Estimation Through Modified Unitary MUSIC Algorithm for CRLH Leaky-Wave Antennas

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***Abstract*—In this paper, we propose a modified unitary multiple signal classification (MUSIC) algorithm for a two-port composite right/left handed (CRLH) leaky-wave antenna (LWA). The algorithm requires only real-valued operations to estimate direction of arrival (DoA) of the received signals, which will simplify future hardware implementation. The CRLH LWA consists of a cascade of metamaterial unit cells, periodically modulated using varactor diodes. By changing the voltages across series and shunt varactors, the antenna is able to uniformly steer its radiation pattern. The proposed modified unitary MUSIC algorithm uses both antennas' input ports to estimate the DoA. Existing MUSIC implementations requires eigenvalue decomposition in the complex-valued signal subspace, which results in highly complex hardware implementation. The computational complexity can be reduced using our proposed modified unitary MUSIC algorithm since it transforms the complex-valued covariance matrix of the received signals to a real-valued matrix using unitary transformations. The performance of the algorithm is experimentally demonstrated by using a CRLH LWA within an anechoic chamber facility and the estimated DoA results are in good agreement with the predicted angles.**

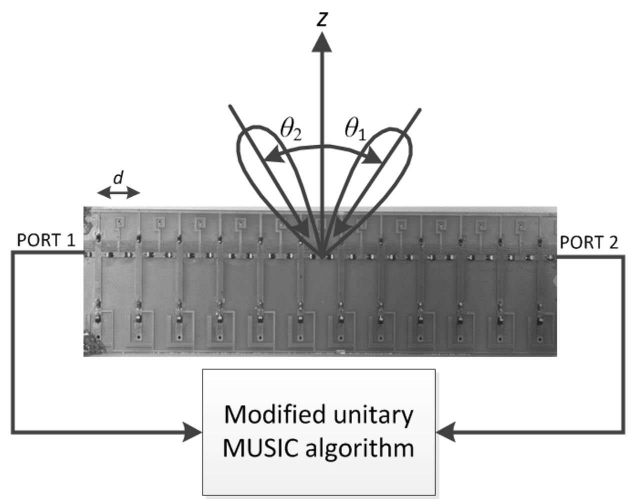

Fig. 1. System model.

I. INTRODUCTION

Direction of arrival algorithms consider the estimation of user direction by analyzing the impinging signals on a receiving antenna array. Due to the large size of conventional antenna arrays, DoA estimation can only be used by the base station [1]. However, when more compact layouts such as leaky-wave antennas are employed, DoA estimation can also be performed in smaller form factor devices. Low manufacturing cost, large beam scanning and absence of extra RF circuitry are additional benefits related to the use of LWA in practical scenarios. In this paper, we formulate and experimentally demonstrate a novel modified unitary MUSIC algorithm, showing how it can be used with a reconfigurable CRLH LWA instead of conventional antenna arrays.

Algorithms, such as MUSIC [2] and the estimation of signal parameters via rotational invariance technique (ESPRIT) [3], are subspace methods which commonly require eigenvalue decomposition of the covariance or correlation matrix [4]. These techniques are usually very hard to implement because of complex-valued operations that are not easily implemented in hardware. In the literature, there are several articles where real-valued subspace methods are presented. For example, a unitary MUSIC algorithm is proposed in [4] and in [5] a unitary ESPRIT algorithm is presented. In our previous work [6], we presented a complex-valued modified MUSIC algorithm for the use with LWAs. We are unaware of other real-valued unitary MUSIC algorithms for DoA estimation using LWA in the literature.

DoA estimation with LWAs does not appear much in the literature. In [7], an energy-based DoA estimation algorithm is introduced and evaluated using CRLH LWA at a fixed frequency. A DoA estimation algorithm for LWAs is presented in [8]. This algorithm is based on calculation of the estimated cross-correlation coefficients between the spectra with added noises and the original spectra. A preliminary evaluation of the MUSIC algorithm using a single CRLH LWA element is introduced in [9]. Other reconfigurable antennas were used for DoA such as the electronically steerable parasitic array radiator (ESPAR) antenna, which is a single port antenna surrounded by parasitic elements. Reactance domain MUSIC algorithms are presented in [10], [11], [12] while, in [13] a reactance domain unitary MUSIC algorithm based on real-valued orthogonal decomposition for ESPAR antennas is introduced.

In this paper, we propose a modified unitary MUSIC

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algorithm using a two-port CRLH leaky-wave antenna, as illustrated in Fig. 1. The conventional MUSIC algorithm creates the correlation matrix of the signal samples received from the elements of an antenna array. When LWAs are used, the conventional MUSIC algorithm is no longer applicable. Therefore, in the case of a N-unit cells LWA, the MUSIC algorithm uses N received signals that are collected from the two input ports of the LWA [6] and a correlation matrix is formed by using these N received signals. This procedure is shown in more detail in our previous work [6].

This paper shows how our proposed real-valued modified unitary MUSIC algorithm transforms the complex-valued correlation matrix of the received signals to a real-valued matrix using unitary transformations. Thus, the computational complexity reduces considerably compared with a complex-valued modified MUSIC algorithm. As results of it, this method can be conveniently integrated into practical hardware of cognitive radio systems. The performance of the modified MUSIC and modified unitary MUSIC algorithms will also be compared in order to evaluate the estimation accuracy in both cases.

This paper is organized as follows: Section II introduces the modified MUSIC algorithm for DoA estimation using LWAs. Section III presents the modified unitary MUSIC algorithm for LWAs. A detailed description of the antenna design is presented in Section IV. Measurement setup and results are described, respectively, in Section V and Section VI. Finally, conclusions are drawn in Section VII.

II. Modified MUSIC algorithm with LWA

Conventionally, the MUSIC algorithm [2] finds the D directions that minimize the projection of the steering vector to the noise subspace and forms a correlation matrix from the received signal that is coming from each element of the antenna array. When LWAs are employed, the spatial diversity can be generated by collecting the same signal M times while the LWA switches between M different radiation patterns [6]. As a result, the correlation matrix is based on these M received signals which are received from the two input ports.

Considering that a LWA has N unit cells, the first step is to measure the transmitted signal u from M different directions. Next, we can define each of these received signals as $y_m(k)$ and a vector $\mathbf{y}(k)$ can be expressed as

$$\mathbf{y}(k) = \mathbf{a}_{\mathrm{mod}} u(k) + \mathbf{z}(k) \tag{1}$$

where $\mathbf{y}(k) = [y_1(k) \cdots y_M(k)]^{\mathrm{T}}$, $\mathbf{z}(k) = [z_1(k) \cdots z_M(k)]^{\mathrm{T}}$ is an additive white gaussian noise vector, and $\mathbf{a}_{\mathrm{mod}} = [a_1 \cdots a_M]^{\mathrm{T}}$ where T is transpose. The elements of the steering vector $\mathbf{a}_{\mathrm{mod}}$ can be expressed as

$$a_{\mathrm{m}} = \sum_{n=1}^{N} I_n \exp[j(n-1)k_0 d(\sin(\theta_m) + \sin(\theta_0))] \tag{2}$$

where k_0 is the free space wave number, $m = 1, ...M$, θ_0 is the angular direction of the received signal, and an exponential function $I_n = I_0 \exp(-\alpha(n-1)d)$, with leakage factor α and structure period d [14]. The LWA's main beam angle θ_m is determined by $\theta_m = \arcsin(\beta/k_0)$ where β is the propagation constant.

After collecting the received signals $y_m(k)$ from the M different directions, we can estimate the correlation matrix, which can be represented as

$$\hat{\mathbf{R}}_{yy} = \frac{1}{N_{\mathrm{s}}} \sum_{k=1}^{N_{\mathrm{s}}} \mathbf{y}(k)\mathbf{y}^{\mathrm{H}}(k) \tag{3}$$

where N_{s} is the number of received symbols for each received signal y_m and H is Hermitian transpose. Next, we can use the singular vector decomposition and form the noise subspace matrix $\mathbf{E}_{\mathrm{n}}$ with eigenvector corresponding to the smallest eigenvalues of $\hat{\mathbf{R}}_{yy}$. Finally, the modified MUSIC spectrum can be expressed as

$$P_{\mathrm{MUSIC}}(\theta) = \frac{1}{\mathbf{a}_{\mathrm{mod}}^{\mathrm{H}}(\theta)\mathbf{E}_{\mathrm{n}}\mathbf{E}_{\mathrm{n}}^{\mathrm{H}}\mathbf{a}_{\mathrm{mod}}(\theta)}. \tag{4}$$

The estimated DoA for the incident signal will be the maximum value of the spectrum $P_{\mathrm{MUSIC}}(\theta)$.

III. Modified unitary MUSIC algorithm with LWA

Commonly, the entries of the correlation matrix in (3) are complex-valued. In addition, the DoA estimation of the incident signal, performed by evaluating (4) and determining the peaks of $P_{\mathrm{MUSIC}}(\theta)$, requires complex-valued operations. Therefore, real-time processing can be difficult to implement. In this section, we show how the correlation matrix can be transformed to a real-valued matrix by using this unitary transformation. This method reduces the original complexity and makes the algorithm more practical for hardware implementation.

First, to determine transformation matrices, the unitary transform $\mathbf{Q}$ can be found as

$$\mathbf{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_{M/2} & j\mathbf{I}_{M/2} \\ \mathbf{J}_{M/2} & -j\mathbf{J}_{M/2} \end{bmatrix} \tag{5}$$

for even M where $\mathbf{I}$ is the identity matrix and $\mathbf{J}$ is a permutation matrix with all its anti-diagonal elements equaling 1 [4], [15], [16]. On the other hand, for odd values of M

$$\mathbf{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_{(M-1)/2} & \mathbf{0}_{(M-1)/2} & j\mathbf{I}_{(M-1)/2} \\ \mathbf{0}_{(M-1)/2} & \sqrt{2} & j\mathbf{0}_{(M-1)/2} \\ \mathbf{J}_{(M-1)/2} & \mathbf{0}_{(M-1)/2} & -j\mathbf{J}_{(M-1)/2} \end{bmatrix} \tag{6}$$

where $\mathbf{0}$ is the Nx1 zero vector. The modified unitary MUSIC algorithm also uses a transform matrix $\mathbf{C}$ composed of zeros and ones, which satisfies $\mathbf{C}^{\mathrm{T}}\mathbf{C} = \mathbf{I}$. The following matrix $\mathbf{C}$

is valid for the case M=12 and is expressed as

$$\mathbf{C} = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}. \quad (7)$$

Now, we can change the $\mathbf{y}(k)$ vector to real-valued by using the matrices described above. It can be expressed as

$$\mathbf{r}(k) = \mathrm{Re}[\mathbf{Q}^{\mathrm{H}}(\mathbf{C}\mathbf{y}(k))]. \quad (8)$$

Then, by using these entries, we can change $\hat{\mathbf{R}}_{\mathrm{yy}}$ to a real-valued symmetric matrix $\hat{\mathbf{R}}_{\mathrm{real}}$ as

$$\hat{\mathbf{R}}_{\mathrm{real}} = \frac{1}{N_{\mathrm{s}}} \sum_{k=1}^{N_{\mathrm{s}}} \mathbf{r}(k)\mathbf{r}^{\mathrm{T}}(k) \quad (9)$$

where $\mathbf{r} = [r_1(k) \cdots r_M(k)]^{\mathrm{T}}$. The correlation matrix $\mathbf{R}_{\mathrm{real}}$ is symmetric because of the tranformation matrix $\mathbf{C}$. Once the correlation matrix is created, we can use a singular value decomposition to compute the noise subspace matrix $\mathbf{E}_{\mathrm{N}}$. If we apply the eigen decomposition for a real-valued correlation matrix $\hat{\mathbf{R}}_{\mathrm{real}}$ in (9), we obtain

$$\hat{\mathbf{R}}_{\mathrm{real}} = \hat{\mathbf{E}}_{\mathrm{s}}\Lambda_{\mathrm{s}}\hat{\mathbf{E}}_{\mathrm{s}}^{\mathrm{T}} + \hat{\mathbf{E}}_{\mathrm{n}}\Lambda_{\mathrm{n}}\hat{\mathbf{E}}_{\mathrm{n}}^{\mathrm{T}} \quad (10)$$

where $\hat{\mathbf{E}}_{\mathrm{s}} = [\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, ..., \hat{\mathbf{e}}_L]$ is the estimated real-valued eigenvectors for the signal subspace, $\Lambda_{\mathrm{s}} = \mathrm{diag}[\lambda_1, \lambda_2, ..., \lambda_L]$ is a diagonal matrix of the largest eigenvalues, and L number of incident sources. Also, $\hat{\mathbf{E}}_{\mathrm{n}} = [\hat{\mathbf{e}}_{L+1}, \hat{\mathbf{e}}_{L+2}, ..., \hat{\mathbf{e}}_M]$ estimate a real-valued noise subspace, $\Lambda_{\mathrm{n}} = \mathrm{diag}[\lambda_{L+1}, \lambda_{L+2}, ..., \lambda_M]$ is a diagonal matrix of M+1-L noise eigenvalues. Once eigenvectors of the noise subspace are estimated, the modified unitary MUSIC spectrum can finally be generated as in (4), except $\mathbf{a}_{\mathrm{mod}}$ is now replaced with $\mathbf{a}_{\mathrm{mod,unitary}}$, which is given by

$$\mathbf{a}_{\mathrm{mod,unitary}} = \mathrm{Re}[\mathbf{Q}^{\mathrm{H}}(\mathbf{C}\mathbf{a}_{\mathrm{mod}})]. \quad (11)$$

We can note that eigen-value decomposition is computed for real-valued matrices instead of complex-valued matrices and finding the peaks of $P_{\mathrm{MUSIC,real}}(\theta)$ requires only real-valued data. Therefore, the complexity calculation and the processing time using the proposed algorithm is reduced considerably compared with a complex-valued modified MUSIC algorithm.

IV. Antenna Design

The proposed unitary MUSIC algorithm has been developed to perform DoA estimation using a two-port reconfigurable LWA. This antenna is made by cascading 12 metamaterial CRLH unit cells, populated by varactor diodes in series and shunt configuration. The DC voltage across the two varactor

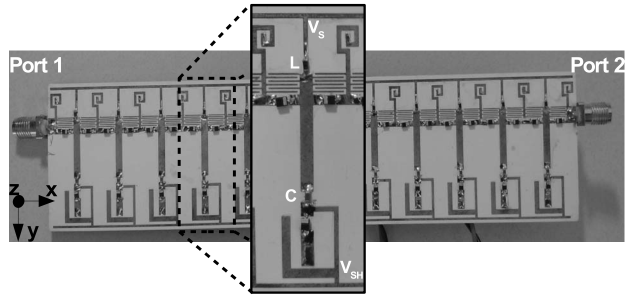

Fig. 2. CRLH unit cell and overall LWA antenna. L = 15.6 cm, H = 3.8 cm.

lines (V_S - V_{SH}) is properly modulated in order to change the propagation constant (β) along the structure and thus steering the radiation beam around the azimuth plane (x-z). The relationship between the radiation angle θ and the propagation constant β can be expressed as

$$\theta = \sin^{-1}\left(\frac{\beta_{S,SH}}{k_0}\right) \quad (12)$$

that is varied through the DC biasing of varactor diodes.

Despite the CRLH-LWA is designed to operate in either right ($\beta > 0$) of left ($\beta < 0$) hand, we have optimized the unit cell in the left hand region in order to achieve the maximum beam coverage by switching between the two antenna ports. That is, when Port 1 is used, the beam can be steered from 0° to +60°, while by switching to Port 2, the beam covers the symmetrical half-plane from 0° to -60°. The spatial resolution between each beam was set to 10° in order to maintain uniform spatial coverage.

Using the full wave electromagnetic simulator HFSS, the antenna has been designed on Rogers 4360 substrate to operate within the 2.4 GHz band. Due to the high dielectric constant (ϵ_r = 6.15) and low loss characteristic ($TanD$ = 0.003), the resulting design was more compact and had enhanced efficiency. The form factor has been further reduced by replacing the usual microstrip $\lambda\backslash 4$ RF-chokes with folded and lumped elements. As shown in Fig. 2, the series voltage V_S is provided by SMD 220 nH inductors, while the shunt voltage V_{SH} passes through folded open stubs. A DC-blocking 0.5 pF capacitor was added to separate the two DC components.

Using an Agilent N5230A Network Analyzer, the measured return loss (S_{11} - S_{22}) exhibit a -10 dB bandwidth of about 70 MHz, covering a frequency range from 2.41 GHz to 2.48 GHz. The insertion loss (S_{12}) between Port 1 and Port 2 resulted from 10 dB to 15 dB for all the configurations, thus maintaining a good isolation between the two ports.

Radiation patterns of all configurations were carried out at 2.46 GHz within the anechoic chamber facility. Thanks to the low loss characteristic of the substrate, the measured gain for all the configurations is 4 to 5 dBi. Nevertheless, for angles above ±50° we experienced a small gain reduction due to a natural CRLH behavior for which the propagation constant

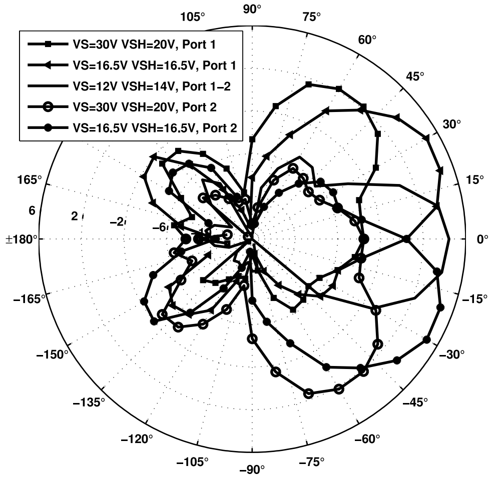

Fig. 3. CRLH-LWA measured radiation patterns, azimuth plane at frequency 2.46 GHz.

starts to move from radiated to propagated regime. In Fig. 3 [6] we show five samples of the radiated beams, the positive angles were generated by feeding Port 1 (and Port 2 closed to a 50 Ω load), while the negative beams were taken through Port 2 (Port 1 closed to a 50 Ω load).

V. Measurement configuration

The performance of the proposed algorithm is evaluated by experimental measurements performed in an anechoic chamber, as shown in Fig. 4. We used the CRLH LWA to transmit a 64 subcarrier signal within the frequency range from 2.452 GHz to 2.472 GHz. Within such limited frequency range, we verified that all the configurations have the same beam angles as measured at 2.46 GHz. The length of a single LWA cell is $d = 1.3$ cm, number of unit cells N = 12, and leakage factor $\alpha = 1$. The LWA transmitted the signal to 12 different directions by properly setting the bias voltages V_S and V_{SH}, thus N = M in our simulations. Four different DoA angles were considered during measurements: -15 °, -25 °, -35 ° and -45 °. The DoA was changed by tilting the CRLH LWA by an angle θ_0, as illustrated in Fig. 4.

First, the LWA Port 1 was fed and then, for each DoA, channel measurements were collected from the following directions: -60 °, -50 °, -40 °, -30 °, -20 °, -10 °. Once the received signals were collected from Port 1, the feed port was switched to Port 2 and the same set of measurements were repeated, collecting signals from positive angles: 0 °, 10 °, 20 °, 30 °, 40 °, 50 °. The wireless channel can be assumed to be reciprocal because of the anechoic chamber environment. The modified unitary MUSIC spectrum was calculated by averaging over the number of subcarriers $N_{\rm sc}$ shown in

$$P_{\rm avg}(\theta) = \frac{1}{N_{\rm sc}} \sum_{n=1}^{N_{\rm SC}} \frac{1}{\mathbf{a}^{\rm H}_{\rm mod,unitary}(\theta)\mathbf{E}_{\rm n}\mathbf{E}_{\rm n}^{\rm H}\mathbf{a}_{\rm mod,unitary}(\theta)}. \quad (13)$$

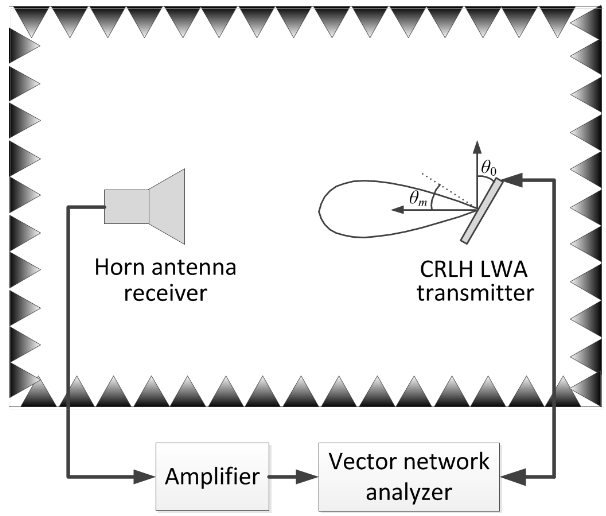

Fig. 4. Anechoic chamber measurement setup.

TABLE I
DoA measurement results

Expected DoA	Unitary MUSIC		MUSIC	
	DoA	Error	DoA	Error
-15°	-19°	3.3 %	-16°	0.8 %
- 25°	-25°	0.0 %	-25°	0.0 %
-35°	-36°	0.8 %	-35°	0.0 %
-45°	-42°	2.5 %	-47°	1.7 %
Average		**1.6 %**		**0.6 %**

VI. Experimental results

The goal of our experimental measurements was to estimate the DoA using the proposed algorithm. The plots, Fig. 5 and Fig. 6, show the power spectrum as function of the DoA angles for both proposed modified unitary MUSIC algorithm and modified MUSIC algorithm [6]. In the figures, the LWA was tilted by -15 °, -25 °, -35 °, and -45 °.

In the figures, we see that both of the algorithms based on the LWA provide DoA values very close to the expected angles. The results are summarized in Table I. It can be seen that the modified MUSIC algorithm has a slightly better accuracy. This difference may be caused by the transformation procedure. The transform matrices transform the correlation matrix to real-valued but do not transform it to centro-symmetric as in the case of the conventional antenna arrays. However, we can see that both of the algorithms are in a very good agreement with the expected DoAs. It demonstrates that the algorithm complexity can be reduced using the unitary approach, while keeping high accuracy in the DoA estimation.

VII. Conclusion

In this work, we have proposed a novel modified unitary MUSIC algorithm for DoA estimation based on a two-port CRLH reconfigurable leaky-wave antenna. The CRLH LWA is able to steer a directional beam by modulating the propagation constant through the DC biasing of varactor diodes. By taking advantage of the LWA's compact dimension and pattern

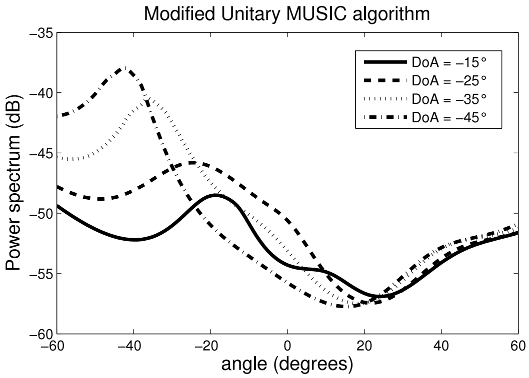

Fig. 5. Modified unitary MUSIC spectrum for all DoAs.

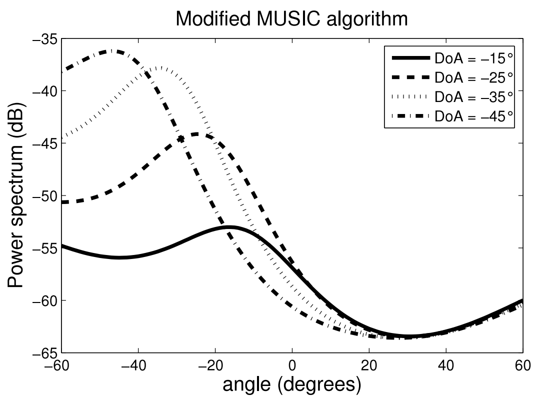

Fig. 6. Modified MUSIC spectrum for all DoAs.

reconfigurability, we developed a modified unitary MUSIC algorithm for direction of arrival estimation. In particular, we have shown how the complex-valued modified MUSIC algorithm can be transformed to the real-valued modified unitary MUSIC algorithm.

Finally, an experimental performance evaluation has been carried out by using a CRLH LWA within an anechoic chamber facility. We have compared the performance of the DoA estimation between the complex-valued modified MUSIC and the real-valued modified unitary MUSIC algorithms. Both algorithm results show good agreement with the predicted DoA angles, demonstrating that the DoA estimation of the impinging wave can be successfully performed by using the two ports of a CRLH LWA. The complex-valued algorithm has a slightly better DoA estimation performance than the proposed real-valued algorithm. However, the proposed unitary algorithm requires much less computational complexity and guarantee good DoA accuracy as well. The proposed algorithm for DoA estimation along with the planar and compact LWA layout can be a valuable solution to enhance the performance of wireless communication in future cognitive radio applications. In our future work, we will test the algorithm in a typical office environment with coherent received signals and the proposed algorithm will be implemented on a field-programmable gate array (FPGA) board. Additionally, we are focusing on the design of LWAs having narrower half power beamwidth (HPBW) and improved beam symmetry in order to further improve the estimated DoA accuracy.

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