# Admittance Control of a Teleoperated Motorized Functional Electrical Stimulation Rehabilitation Cycle ⋆

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**Abstract:** A bilateral teleoperated rehabilitation cycling system is developed for people with movement impairments due to various neurological disorders. A master hand-cycling device is used by the operator to set the desired position and cadence of a lower-body functional electrical stimulation (FES) controlled and motor assisted recumbent cycle. The master device also uses kinematic haptic feedback to reflect the lower-body cycle's dynamic response to the operator. To accommodate for the unknown nonlinear dynamics inherent to physical human machine interaction (pHMI), admittance controllers were developed to indirectly track desired interaction torques for both the haptic feedback device and the lower-body cycle. A robust position and cadence controller, which is only active within the regions of the crank cycle where FES produces sufficient torque values, was used to determine the FES intensity. A Lyapunov analysis is used to prove the robust FES controller yields global exponential tracking to the desired position and cadence set by the master device within FES stimulation regions. Outside of the FES regions, the admittance controllers at the hands and legs work in conjunction to produce desired performance. Both admittance controllers were analyzed for the entire crank cycle, and found to be input/output strictly passive and globally exponentially stable in the absence of human effort, despite the uncertain nonlinear dynamics.

*Keywords:* Functional electrical stimulation (FES), teleoperation, physical human machine interaction (pHMI), rehabilitation robotics.

## 1. INTRODUCTION

Millions of people are affected by neurological conditions (NCs) that result in some form of movement disorder, and require rehabilitation to restore mobility (Kralj and Bajd, 1989). In addition to improving mobility, rehabilitation can lead to improved cardiovascular health and neuroplasticity, increased muscle mass (Bélanger et al., 2000), increased bone density (Mohr et al., 1997), and reduce the occurrence of other negative side effects (Ferrante et al., 2008). A common form of rehabilitation for those with lower-body impairments is the use of functional electric stimulation (FES) of affected muscle groups while engaging in exercise on a recumbent cycle (Ragnarsson, 2008). However, a common side effect of the continuous application of stimulation is rapid muscle fatigue (Kralj and Bajd, 1989). Recent results use switched system control techniques to restrict the application of stimulation to those regions of the cycle rotation which will produce optimal levels of force and torque, thus minimizing the amount of FES effort and delaying the onset of fatigue (Bellman et al., 2017; Cousin et al., 2019; Rouse et al., 2020; Downey et al., 2017) and ultimately extending the length of continuous rehabilitative sessions.

Although rehabilitation has numerous benefits, several factors might limit a person's access to health care and rehabilitation, including lack of facilities in remote rural communities, inability to travel, socioeconomic constraints, and isolation due to compromised immunity (Hjelm, 2005). This has lead to the need for the development of telemedicine systems, including robotic manipulators equipped with haptic force feedback developed to perform telerobotic surgery (Ahmadkhanlou et al., 2009), as well as remotely operated telerobotic rehabilitation devices (Atashzar et al., 2016). Kinesthetic haptic feedback (Puerto et al., 2009) allows the telerobotic operator to experience proportional forces associated with the performance of a rehabilitation participant, creating a simulated physical connection between a clinician and a patient despite their remote locations.

A challenge inherent to physical human machine interaction (pHMI) is the presence of nonlinear dynamics with unknown parameters. In many telerobotic systems, where the operator interacts with a master robotic device, this interaction determines the desired trajectories of a telerobotic system, often a robotic manipulator, where in many cases

⋆ This research is supported in part by NSF Award number 1762829, the Assistant Secretary of Defense for Health Affairs, through the Congressionally Directed Medical Research Program under Award No. W81XWH1910330, and the National Defense Science and Engineering Graduate Fellowship Program. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsoring agency.

full model knowledge of the system is leveraged to produce desired performance (Niemeyer et al., 2016). One approach used to control telerobotic rehabilitation systems is to linearize the model as in (Atashzar et al., 2016). However, this solution does not capture the nonlinear dynamics and uncertainties associated with pHMI, and it is incapable of adjusting to the effects of muscle fatigue and delayed muscle response to stimulation that occurs during FES rehabilitation. Additionally, a representation of the robotic manipulator's interactions with the patient are converted to haptic feedback at the master telerobotic device, often produced using a set of desired (i.e., apparent) inertial, stiffness, and damping parameters. However, when pHMI occurs at both ends of the bilateral teleoperation system (Ferre et al., 2007), unknown parameters exist throughout, creating added difficulties with control and haptic feedback.

This paper develops a bilateral teleoperation system combined with FES rehabilitative cycling techniques (Fig. 1) using the dynamic models developed in (Bellman et al., 2017) and (Downey et al., 2015), which encompass the nonlinear dynamics and uncertainties associated with pHMI and the muscle fatigue associated with FES. The designed control method presented here allows an operator (e.g., a remote physical therapist or the rider) using a separate, mechanically uncoupled device to control the desired performance, including position and cadence of the rider's teleoperated leg cycle, which is powered by closed-loop FES and motor assistance. In addition to providing the desired trajectories for the rider, the telerobotic system also informs the operator of the actual performance of the rider through the use of kinesthetic haptic feedback (Puerto et al., 2009). To produce a more accurate reflection of performance and to facilitate passive interaction, position errors experienced at the rider's cycle are modeled as spring forces, and deviations in desired interaction torques between the rider and the cycle crank set are reflected back to the operator as a damping force proportional to the torque error. In the case presented in this paper, the telerobotic operator is also the rider. The rider using an FES actuated cycle uses a smaller, hand-driven crank set within arm's reach to set the desired cadence of the lower-body leg cycle, producing coordinated motion between the hands and the legs. It is theorized that in addition to restoring the rider's ability to dictate their own cadence, this coordinated motion might serve to further improve neuroplasticity (Ferris et al., 2006) and speed the restoration of mobility (Klarner et al., 2016). To ensure that any asymmetries existing between the rider's legs are accurately captured, it was determined that a split-crank cycle should be used, where the cycle can be independently powered using either the uncoupled left or right leg crank set (Estay et al., 2019; Rouse et al., 2019), where a separate haptic feedback device is dedicated to each side of the cycle. It is expected that as the rider experiences resistance at the hands due to haptic feedback caused by performance errors at the legs, the rider is likely to slow the cadence of the associated hand cycles, thus lowering the intensity of muscle stimulation at the legs.

Admittance control techniques are used to indirectly track desired interaction forces (Cousin et al., 2018) using selected inertial, damping, and stiffness parameters to produce the desired dynamical behavior. The rider/operator experiences apparent (i.e., virtual) model dynamic behavior (Cousin et al., 2018; Keemink et al., 2018), rather than a reflection of the actual uncertain dynamic system, on the hand cycle as well as the teleoperated FES actuated lower-body (i.e., leg) cycle. Using admittance based torque control ensures that motor efforts will be employed to assist or resist as needed to produce desired interaction forces. It was necessary to develop desired interaction functions that would produce bounded admitted trajectories while also guaranteeing positive (i.e., forward) rotation of both cycle crank sets.

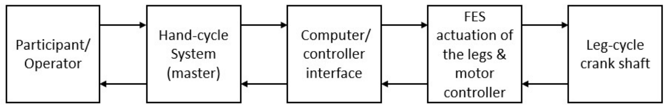


Fig. 1. The bilateral teleoperation system for a rehabilitative FES cycling system.

## 2. DYNAMICS

### A. Decoupled Leg Cycle System

The recumbent cycling system being considered in this paper utilizes a split-crank design, as in (Rouse et al., 2019). For this reason, the dynamics of each leg are independent of one another and so each leg can be considered independently. A detailed analysis of these dynamics are presented in (Rouse et al., 2019) and can be modeled for either leg by

$$\tau_{e_l}(t) + \tau_M(q_l, \dot{q}_l, t) + \tau_{vol_l} = M_l(q_l)\ddot{q}_l + V_l(q_l, \dot{q}_l)\dot{q}_l + G_l(q_l) + P_l(q_l, \dot{q}_l) + b_l\dot{q}_l + d_l(t), \tag{1}$$

where $q_l : \mathbb{R}_{\geq 0} \to \mathcal{Q}_l$ denotes the angular position of the leg cycle crank arm and $\mathcal{Q}_l \subseteq \mathbb{R}$ is the set of all possible measurable leg cycle crank angles. The leg cycle angular velocity (i.e., cadence) is denoted by $\dot{q}_l : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the angular acceleration is denoted by $\ddot{q}_l : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The unknown nonlinear inertial effects, centripetal-Coriolis effects, damping effects, gravitational effects, passive viscoelastic muscle forces, and disturbances are represented by $M_l$, $V_l$, $b_l$, $G_l$, $P_l$, and $d_l$ respectively, where $M_l : \mathcal{Q}_l \to \mathbb{R}$, $V_l : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$, $G_l : \mathcal{Q}_l \to \mathbb{R}$, $P_l : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$ and the subscript $l$ denotes the leg cycle system. The electric motor torque applied to the leg cycle is denoted by $\tau_{e_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The portion of the torque produced about the leg cycle crank axis by the electric motor can be expressed by and is defined as

$$\tau_{e_l} \triangleq B_{e_l} u_{e_l}(t), \tag{2}$$

where $B_{e_l} \in \mathbb{R}_{>0}$ represents the relationship between the electric motor current and the resulting torque, and $u_{e_l}(t)$ denotes the subsequently designed leg cycle motor control input. The independently applied volitional efforts of the rider at the legs are denoted by $\tau_{vol_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The muscle force torques produced about the crank axis by FES in (1) are denoted by $\tau_M : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and are defined as

$$\tau_M \triangleq \sum_{m \in \mathcal{M}} B_m(q_l, \dot{q}_l) u_m(q_l, t), \tag{3}$$

where the subscript $m \in \mathcal{M} = \{G, H, Q\}$ indicates the gluteal ($G$), hamstring ($H$), and quadriceps femoris ($Q$) muscle groups. The term $B_m : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}_{\geq 0}$, $\forall m \in \mathcal{M}$ represents the unknown, nonlinear muscle effectiveness and $u_m : \mathcal{Q}_l \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the FES muscle stimulation intensity (i.e. pulse width). The subsets $\mathcal{Q}_m \subset \mathcal{Q}_l$, $\forall m \in \mathcal{M}$ denote the portion of the crank cycle across which each muscle group is stimulated such that

$$\mathcal{Q}_m \triangleq \{q_l \in \mathcal{Q}_l \mid T_m(q_l) > \varepsilon_m\}, \tag{4}$$

where $\varepsilon_m$ represents a user selected lower bound for each muscle group's torque transfer ratio, $T_m$: $\mathcal{Q}_l \to \mathbb{R}$, such that the designated muscle group's effort only produces positive crank rotation. The area about the crank cycle where at least one muscle group produces a positive crank torque due to FES activation is denoted by $\mathcal{Q}_{FES} \triangleq \bigcup_{m \in \mathcal{M}} \{\mathcal{Q}_m\}$, $\forall m \in \mathcal{M}$.

The stimulation intensity applied to each individual muscle group is defined as (Estay et al., 2019)

$$u_m \triangleq \sigma_m(q_l)\, k_m u_s(t), \tag{5}$$

$\forall m \in \mathcal{M}$, where $k_m \in \mathbb{R}_{>0}$ is a constant selected to ensure participant safety and comfort during stimulation, $u_s(t)$ denotes the subsequently designed FES control input, and $\sigma_m$ represents a switching signal determined using the subsets defined in (4), where $\sigma_m : \mathcal{Q}_l \to \{0, 1\}$ such that

$$\sigma_m \triangleq \begin{cases} 1 & \text{if } q_l \in \mathcal{Q}_m \\ 0 & \text{if } q_l \notin \mathcal{Q}_m \end{cases}. \tag{6}$$

The summation of the motor torque efficiencies, represented by $B_M \triangleq \sum_{m \in \mathcal{M}} B_m \sigma_m k_m$ (Rouse et al., 2018), can be substituted into (3) to produce [1]

$$\tau_M \triangleq B_M u_s. \tag{7}$$

Substituting (2) and (7) into (1) produces the open-loop leg cycle dynamic equation

$$\begin{aligned} B_M u_s + B_{e_l} u_{e_l} + \tau_{vol_l} = M_l \ddot{q}_l + V_l \dot{q}_l + G_l + P_l \\ + b_{c_l} \dot{q}_l + d_l. \end{aligned} \tag{8}$$

*B. Hand Cycle Teleoperation Device*

For the purpose of this paper, the operator driven haptic feedback device will be considered as a hand-cycle, without loss of generality. Although there are no mechanical linkages between the hand cycle and leg cycle systems (i.e. rehabilitation-by-wire), the dynamics of the hand cycle must be considered for the purpose of the control development. Any torque produced by the hand cycle operator is purely volitional, therefore using similar methods as were employed to determine the leg cycle dynamics in (8), the hand cycle dynamics can be modeled as

$$\begin{aligned} B_{e_h} u_{e_h} + \tau_{vol_h} = M_h \ddot{q}_h + V_h \dot{q}_h + G_h \\ + P_h + b_{c_h} \dot{q}_h + d_h, \end{aligned} \tag{9}$$

where $B_{e_h} \in \mathbb{R}_{>0}$ represents the relationship between the hand cycle electric motor current and the resulting torque, and $u_{e_h}(t)$ represents the subsequently designed motor control input to the hand cycle. The operator's volitional torque producing efforts acting about the hand cycle crank axis are denoted by $\tau_{vol_h} \in \mathbb{R}_{\geq 0}$. The angular position of the hand cycle crank arm is denoted by $q_h : \mathbb{R}_{\geq 0} \to \mathcal{Q}_h$, where $\mathcal{Q}_h \subseteq \mathbb{R}$ is the set of measurable hand cycle crank angles. The hand cycle angular velocity is denoted by $\dot{q}_h : \mathbb{R}_{\geq 0} \to \mathbb{R}$, and the angular acceleration is denoted by $\ddot{q}_h : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The unknown, nonlinear inertial effects, centripetal-Coriolis effects, gravitational effects, and passive viscoelastic muscle forces in (9) are represented by $M_h : \mathcal{Q}_h \to \mathbb{R}, V_h : \mathcal{Q}_h \times \mathbb{R} \to \mathbb{R}, G_h : \mathcal{Q}_h \to \mathbb{R}$, and $P_h : \mathcal{Q}_h \times \mathbb{R} \to \mathbb{R}$ respectively, and $d_h$ denotes the unknown disturbances about the hand crank.

[1] For notational brevity, functional dependencies will be eliminated except in the case where they are required for clarity.

*C. System properties*

The leg cycle/rider dynamics in (8) and the hand cycle/operator dynamics in (9) have the following properties and assumptions $\forall i$, $i = \{h, l\}$ (Rouse et al., 2019).

**Property: 1** $\frac{1}{2}\dot{M}_i = V_i$. **Property: 2** $c_{m_i} \leq M_i \leq c_{M_i}$ where $c_{m_i}$, $c_{M_i} \in \mathbb{R}_{>0}$ are known constants. **Property: 3** $|V_i| \leq c_{V_i} |\dot{q}_i| \in \mathbb{R}_{>0}$ where $c_{V_i}$ is a known constant. **Property: 4** $|G_i| \leq c_{G_i} \in \mathbb{R}_{>0}$ where $c_{G_i}$ is a known constant. **Property: 5** $|P_i| \leq c_{P1_i} + c_{P2_i} |\dot{q}_i|$ where $c_{P1_i}$, $c_{P2_i} \in \mathbb{R}_{>0}$ are know constants. **Property: 6** $|b_{c_i}| \leq c_{b_i}$ where $c_{b_i} \in \mathbb{R}_{>0}$ is a known constant. **Property: 7** $|d_i| \leq c_{d_i} \in \mathbb{R}_{>0}$ where $c_{d_i}$ is a known constant. **Property: 8** From (Rouse et al., 2018) it can be shown that that combined muscle efficiency $B_M$ is upper and lower bounded $\forall m \in \mathcal{M}$ such that when $\sum_{m \in \mathcal{M}} \sigma_m > 0$, $B_{\underline{M}} \leq B_M \leq B_{\overline{M}}$ where $B_{\underline{M}}$, $B_{\overline{M}} \in \mathbb{R}_{>0}$.

**Assumption: 1** The position and cadence of the leg and hand cycles are measurable and the electric motor current to torque relationships, $B_{e_i}$, are known constants. **Assumption: 2** The operator's input at the hand cycle is bounded and sufficiently smooth (i.e., $q_h$, $\dot{q}_h$, $\ddot{q}_h \in \mathcal{L}_\infty$), and that the volitional torques and measured interaction torques between the cycles and rider/operator, denoted by $\tau_{int_i} \in \mathbb{R}$, are bounded by known constants such that $|\tau_{vol_i}| \leq c_{vol_i} \in \mathbb{R}_{>0}$ and $|\tau_{int_i}| \leq c_{int_i} \in \mathbb{R}_{>0}$.

## 3. CONTROL DEVELOPMENT

*A. Position and Cadence Control*

The control objective of the FES actuated muscle torques is to track the angular position and velocity (i.e. cadence) of the operator controlled hand cycle system, thus creating a strongly coupled telerobotic system (Ferre et al., 2007). An error signal, denoted by $e : \mathcal{Q}_h \times \mathcal{Q}_l \to \mathbb{R}$, is defined to quantify the difference between the hand and leg cycle crank arm positions as

$$e \triangleq q_h - q_l. \tag{10}$$

An auxiliary error signal, denoted by $r : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, is defined as

$$r \triangleq \dot{e} + \alpha e, \tag{11}$$

where $\alpha \in \mathbb{R}_{>0}$ is a selectable constant.

Premultiplying the time derivative of (11) by $M_l$, and substituting in the second derivative of (10), using (8) and (9), and performing some algebraic manipulation yields

$$\begin{aligned} M_l \dot{r} = \chi_1 - V_l r - e - B_M u_s \\ - B_{e_l} u_{e_l} + M_l M_h^{-1} B_{e_h} u_{e_h}. \end{aligned} \tag{12}$$

The auxiliary term $\chi_1 : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ in (12) can be upper bounded using Properties 2-7 and Assumption 2 as

$$\chi_1 \leq c_1 + c_2 \|z\| + c_3 \|z\|^2, \tag{13}$$

where $z \in \mathbb{R}^2$ is defined as $z \triangleq [e\ r]^T$ and $c_1, c_2, c_3 \in \mathbb{R}_{>0}$ are known constants.

The switched FES control input $u_s$ is designed from the subsequent stability analysis using Property 8, (11), and (12), where

$$\begin{aligned} u_s = \sigma_s \frac{1}{B_{\underline{M}}} (k_1 r + \operatorname{sgn}(r) [k_2 + k_3 \|z\| \\ + k_4 \|z\|^2 + k_5 |u_{e_l}| + k_6 |u_{e_h}|]), \end{aligned} \tag{14}$$

and $k_j \in \mathbb{R}_{>0}$, $j = 1, ..., 6$ are constant control gains, selected as

$$\begin{aligned} k_2 > c_1,\ k_3 > c_2,\ k_4 > c_3, \\ k_5 > B_{e_l}, \qquad k_6 > c_{M_l} c_{m_h}^{-1} B_{e_h}. \end{aligned} \tag{15}$$

The leg stimulation switching signal $\sigma_s : \mathcal{Q}_l \to \{0, 1\}$ in (14) is defined as $\sigma_s \triangleq \begin{cases} 1 & \text{if } q_l \in \mathcal{Q}_{FES} \\ 0 & \text{if } q_l \notin \mathcal{Q}_{FES} \end{cases}$. Substituting (14) into (12) yields the closed-loop error system

$$\begin{aligned} M_l \dot{r} = -\sigma_s \frac{B_M}{B_{\underline{M}}} (k_1 r + \operatorname{sgn}(r) [k_2 + k_3 \|z\| \\ + k_4 \|z\|^2 + k_5 |u_{e_l}| + k_6 |u_{e_h}|]) \\ - B_{e_l} u_{e_l} + M_l M_h^{-1} B_{e_h} u_{e_h} \\ + \chi_1 - V_l r - e. \end{aligned} \tag{16}$$

### *B. Admittance Control at the Legs*

An admittance controller is designed for the electric motor effort to indirectly track the measurable interaction torque, $\tau_{int_l}$, between the leg and the crank set. The desired torque $\tau_{d_l} : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$ is defined as $\tau_{d_l} \triangleq \hat{\tau}_p + \sum_{m \in \mathcal{M}} \sigma_m \tau_m$, where $\hat{\tau}_p : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$ represents the estimated passive torque values produced when the system is operated at the current cadence of the hand cycle and the rider is passive such that $\tau_M, \tau_{vol_l} = 0$. The values of $\hat{\tau}_p$ are determined during an offline, pre-rehabilitation calibration session as in (Cousin et al., 2019). The switching signal $\sigma_m$ has been previously defined in (6), and $\tau_m \in \mathbb{R}_{\geq 0}$, $\forall m \in \mathcal{M}$ represents the selected torque values for each muscle group. The torque error at the legs is defined as

$$e_l \triangleq \tau_{int_l} - \tau_{d_l}. \tag{17}$$

Using admittance control techniques, it is possible to use the calculated error in (17) to determine the admitted position, velocity, and acceleration values, denoted by $q_{\alpha_l}$, $\dot{q}_{\alpha_l}, \ddot{q}_{\alpha_l}$ respectively, that would be produced given the apparent leg cycle system (Cousin et al., 2019; Keemink et al., 2018) such that

$$e_l = M_d \ddot{q}_{\alpha_l} + B_d \dot{q}_{\alpha_l} + K_d q_{\alpha_l}. \tag{18}$$

where $M_d, B_d, K_d$ are the designed inertial, damping coefficient, and spring constant parameters for the admittance filter. To ensure that the admitted trajectory is bounded, given that $\tau_{d_l}$, $e_l \in \mathcal{L}_\infty$, it is sufficient to select $M_d$, $B_d$, and $K_d$ in (17) such that the resulting transfer function between the input, $e_l$, and the output, $q_\alpha$, of (18) is passive (Khalil, 2002). The admittance error signal and an auxiliary error signal are designed as

$$\mu \triangleq q_{\alpha_l} + q_h - q_l, \tag{19}$$

$$\varphi \triangleq \dot{\mu} + \alpha \mu, \tag{20}$$

respectively, where $\alpha$ was previously defined in (11). Premultiplying the time derivative of (20) by $M_l$, substituting in the second derivative of (19), using (8) and (9), and performing some algebraic manipulation yields

$$\begin{aligned} M_l \dot{\varphi} = M_l M_h^{-1} (B_{e_h} u_{e_h} + \tau_{vol_h}) - \tau_l \\ - V_l \varphi - B_{e_l} u_{e_l} - \mu + \chi_2, \end{aligned} \tag{21}$$

where $\tau_l \triangleq \tau_M + \tau_{vol_l}$. The auxiliary term $\chi_2 : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ in (21) can be upper bounded using Properties 2-7 and Assumption 2 as

$$\chi_2 \leq c_4 + c_5 \|\zeta\| + c_6 \|\zeta\|^2, \tag{22}$$

where $\zeta \in \mathbb{R}^6$ is defined as $\zeta \triangleq \left[\ddot{q}_{\alpha_l}\ \dot{q}_{\alpha_l}\ \dot{q}_h\ \psi^T\right]^T$, $\psi \in \mathbb{R}^2$ is defined as $\psi \triangleq [\mu\ \varphi]^T$, and $c_4, c_5, c_6 \in \mathbb{R}_{>0}$ are known constants.

From the subsequent passivity analysis, the leg cycle motor controller is designed as

$$\begin{aligned} u_{e_l} = B_{e_l}^{-1} (k_7 \varphi + \operatorname{sgn}(\varphi) [k_8 + k_9 \|\zeta\| \\ + k_{10} \|\zeta\|^2 + k_{11} |u_{e_h}|]), \end{aligned} \tag{23}$$

where $k_j \in \mathbb{R}_{>0}$, $j = 8, ..., 11$ are constant control gains, selected as

$$\begin{aligned} k_8 > c_4, \qquad k_9 > c_5, \qquad k_{10} > c_6, \\ k_{11} > c_{M_l} c_{m_h}^{-1} B_{e_h}. \end{aligned} \tag{24}$$

Substituting (23) into (21) yields the closed-loop torque tracking admittance error system

$$\begin{aligned} M_l \dot{\varphi} = M_l M_h^{-1} (B_{e_h} u_{e_h} + \tau_{vol_h}) - \tau_l \\ - k_7 \varphi - V_l \varphi - \mu + \chi_2 - \operatorname{sgn}(\varphi) [k_8 \\ + k_9 \|\zeta\| + k_{10} \|\zeta\|^2 + k_{11} |u_{e_h}|]. \end{aligned} \tag{25}$$

### *C. Admittance Control at the Hands*

An admittance controller is designed to track the measurable interaction torque at the hands denoted by $\tau_{int_h}$. The intention is to produce a desired torque at the hands which will inform the operator of any position or torque errors that are occurring within the leg cycle system while using an admitted error system similar to (18).

For this application, to ensure that the desired interaction torque values are positive and bounded within a functional range for the operator, the desired torque $\tau_{d_h}(\dot{q}_h, \tau_{fb}) : \mathbb{R}_{\geq 0} \times \mathbb{R} \to \mathbb{R}$ is defined as $\tau_{d_h} \triangleq \tau_{min_h} + \operatorname{sat}_\beta(\tau_{fb})$, $\beta \triangleq \upsilon \tau_{min_h}$, where the baseline torque value $\tau_{min_h}(\dot{q}_h) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a predetermined function, $\tau_{fb}(\dot{q}_h, e_l, e) : \mathbb{R}_{\geq 0} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ represents the leg cycle feedback, and $\upsilon \in [0, 1]$ is a selectable constant which determines the saturation limit $\beta \in \mathbb{R}$. The saturation function $\operatorname{sat}_\beta(\cdot)$ is included to ensure that the desired torque is bounded in the sense that $0 \leq \tau_{d_h} \leq 2\tau_{min_h}$.

It is possible to produce a variety of kinesthetic haptic feedback scenarios using admittance control, but for the purpose of this application, a spring force related to the position error at the legs as well as a damping force where the damping coefficient is proportional to the torque error at the legs will be modeled such that $\tau_{fb} \triangleq k_d e_l \dot{q}_h + k_s e$, where $k_d, k_s \in \mathbb{R}_{>0}$ are selectable constants. The torque error at the hands is defined as $e_h \triangleq \tau_{int_h} - \tau_{d_h}$, which is implemented into the hand cycle admittance filter

$$e_h = \delta M_d \ddot{q}_{\alpha_h} + \delta B_d \dot{q}_{\alpha_h} + \delta K_d q_{\alpha_h}, \tag{26}$$

to produce the admitted hand cycle trajectories denoted by $q_{\alpha_h}, \dot{q}_{\alpha_h}, \ddot{q}_{\alpha_h}$, where $\delta \in [0,1]$ is selected to produce a passive apparent system (Keemink et al., 2018) proportional to that which was designed for the leg cycle. The hand cycle admittance error signal is defined as

$$\eta \triangleq \dot{q}_{\alpha_h} + \dot{q}_h - \dot{q}_l. \tag{27}$$

Taking the time derivative of (27), premultiplying by $M_h$, and rearranging terms yields

$$\begin{aligned} M_h\dot{\eta} = &\ B_{e_h}u_{e_h} + \tau_{vol_h} - M_hM_l^{-1}\tau_{vol_l} \\ &-M_hM_l^{-1}B_{M_l}u_s - M_hM_l^{-1}B_{e_l}u_{e_l} \\ &-V_h\eta + \chi_3. \end{aligned} \tag{28}$$

The auxiliary term $\chi_3 : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ in (28) can be upper bounded using Properties 2-7 and Assumption 2 as

$$\chi_3 \leq c_7 + c_8\|\xi\| + c_9\|\xi\|^2, \tag{29}$$

where $\xi \in \mathbb{R}^4$ is defined as $\xi \triangleq [\ddot{q}_{\alpha_h}\ \dot{q}_{\alpha_h}\ \dot{q}_l\ \eta]^T$ and $c_7, c_8, c_9 \in \mathbb{R}_{>0}$ are known constants.

From the subsequent passivity analysis, the hand cycle motor controller is designed as

$$u_{e_h} \triangleq -B_{e_h}^{-1}\left(k_{12}\eta + \operatorname{sgn}(\eta)\left[k_{13} + k_{14}\|\xi\| + k_{15}\|\xi\|^2\right]\right), \tag{30}$$

where $k_j \in \mathbb{R}_{>0}$, $j = 12, ..., 15$ are constant control gains, selected as

$$k_{13} > c_7,\ k_{14} > c_8,\ k_{15} > c_9. \tag{31}$$

Substituting (30) into (28) yields the closed-loop hand cycle admittance error system

$$\begin{aligned} M_h\dot{\eta} = &-(k_{12}\eta + \operatorname{sgn}(\eta)[k_{13} + k_{14}\|\xi\| + k_{15}\|\xi\|^2]) \\ &-M_hM_l^{-1}(B_Mu_s + B_{e_l}u_{e_l} + \tau_{vol_l}) \\ &+\tau_{vol_h} - V_h\eta + \chi_3. \end{aligned} \tag{32}$$

## 4. STABILITY ANALYSIS [2]

In Theorem 1, a Lyapunov-like stability analysis is performed for the leg cycle to prove global exponential position and cadence tracking of the hand cycle when the FES controller is active. The admittance motor controller is designed to simultaneously achieve tracking of the desired interaction torques (Cousin et al., 2019) in addition to the trajectory of the hand cycle when $q_l \notin \mathcal{Q}_{FES}$. Therefore, the passivity and input/output stability of both admittance controllers are analyzed, as shown in Theorem 2 for the leg cycle and Theorem 3 for the hand cycle.

### A. Position and Cadence Control

A Lyapunov-based stability analysis is provided for the leg cycle position and cadence controller for the case where $q_l \in \mathcal{Q}_{FES}$ to show exponential tracking within the stimulation regions. Switching times are denoted by $\{t_n^i\}$, $i \in \{s, e\}$, $n \in \{0, 1, 2, ...\}$ where each $t_n^i$ represents the n-th time that the system switches to a stimulation region, denoted by $i = s$, or to an electric motor effort only region, denoted by $i = e$. To facilitate Theorem 1, let $V_1 : \mathbb{R} \to \mathbb{R}_{\geq 0}$ be a radially unbounded, positive definite, Lyapunov function candidate defined as

$$V_1 = \frac{1}{2}M_lr^2 + \frac{1}{2}e^2, \tag{33}$$

[2] PROOFS AVAILABLE ON REQUEST.

such that $\gamma_1\|z\|^2 \leq V_1 \leq \gamma_2\|z\|^2$, where $\gamma_1 \triangleq \min\left(\frac{c_{m_l}}{2}, \frac{1}{2}\right)$ and $\gamma_2 \triangleq \max\left(\frac{c_{M_l}}{2}, \frac{1}{2}\right)$.

*Theorem 1.* For $q_l \in \mathcal{Q}_{FES}$, given the closed-loop error system in (16), global exponential tracking in the sense that

$$\|z(t)\| \leq \sqrt{\frac{\gamma_2}{\gamma_1}}\|z(t_n^s)\|\exp\left[-\frac{\min(k_1, \alpha)}{2\gamma_2}(t - t_n^s)\right], \tag{34}$$

$\forall t \in [t_n^s, t_n^e)$, provided that the gain conditions in (15) are satisfied.

### B. Admittance Control at the Legs

When using admittance control techniques for human machine interaction, it is imperative to show passivity in the sense that the system will not produce energy independent of the inputs and outputs of the system. To facilitate passivity analysis, let $V_2 : \mathbb{R} \to \mathbb{R}_{\geq 0}$ be a radially unbounded, positive definite energy storage function defined as

$$V_2 \triangleq \frac{1}{2}M_l\varphi^2 + \frac{1}{2}\mu^2, \tag{35}$$

such that $\gamma_1\|\psi\|^2 \leq V_2 \leq \gamma_2\|\psi\|^2$.

*Theorem 2.* Given the admittance controller in (18), the closed-loop error system (25), and the energy storage function in (35), the leg cycle is output strictly passive from input $\left|M_lM_h^{-1}\tau_{vol_h}\right| + |\tau_l|$ to output $|\varphi|$ in the sense that

$$\dot{V}_2 \overset{\text{a.e.}}{\leq} |\varphi|\left(\left|M_lM_h^{-1}\tau_{vol_h}\right| + |\tau_l|\right) - \min(k_7, \alpha)\|\psi\|^2, \tag{36}$$

provided that the gain conditions in (24) are satisfied. Furthermore, the closed-loop error system is globally exponentially stable when operated independent of the rider and operator (i.e., $\tau_{vol_h}, \tau_l = 0$) such that $\|\psi(t)\| \leq \sqrt{\frac{\gamma_2}{\gamma_1}}\|\psi(t_o)\|\exp\left[-\frac{\min(k_7,\alpha)}{2\gamma_2}(t - t_0)\right]$.

### C. Admittance Control at the Hands

It must also be shown that the operator controlled hand cycle is passive. To facilitate passivity analysis, let $V_3 : \mathbb{R} \to \mathbb{R}_{\geq 0}$ be a radially unbounded, positive definite energy storage function defined as

$$V_3 \triangleq \frac{1}{2}M_h\eta^2, \tag{37}$$

such that $\gamma_3\|\eta\|^2 \leq V_3 \leq \gamma_4\|\eta\|^2$, where $\gamma_3 \triangleq \frac{1}{2}c_{m_h}$ and $\gamma_4 \triangleq \frac{1}{2}c_{M_h}$.

*Theorem 3.* Given the admittance controller in (26) and the closed-loop error system (32), the hand cycle is output strictly passive from input $|\tau_{vol_h}| + \left|M_hM_l^{-1}(\tau_l + \tau_{e_l})\right|$ to output $|\eta|$ in the sense that

$$\dot{V}_3 \overset{\text{a.e.}}{\leq} |\eta|\left(|\tau_{vol_h}| + \left|M_hM_l^{-1}(\tau_l + \tau_{e_l})\right|\right) - k_{12}\eta^2, \tag{38}$$

provided that the gain conditions in (31) are satisfied. Furthermore, the closed-loop error system is globally exponentially stable when operated independent of external inputs (i.e., $\tau_{vol_h}$, $\tau_{vol_l}$, $\tau_{e_l} = 0$) such that $\|\eta(t)\| \leq \sqrt{\frac{\gamma_4}{\gamma_3}}\|\eta(t_o)\|\exp\left[-\frac{k_{12}}{2\gamma_4}(t - t_0)\right]$.

## 5. CONCLUSION

A teleoperated FES rehabilitation system was introduced with the goal of improving the duration and benefits of rehabilitation for those experiencing lower-body impairments due to NCs. Admittance controllers were designed for the rehabilitation-by–wire hand cycle master device and the teleoperated lower-body cycle, where an inherently stable apparent system was selected based on desired inertial, damping, and stiffness parameters to indirectly track desired interaction torques occurring between the rider/operator and motor controlled robotics. Admitted trajectories were tracked and found to be strictly passive from input to output and globally exponentially stable in the absence of human applied torques. The switched FES control input, designed to track position and cadence, was found to produce global exponential tracking within FES actuated regions. For improved rehabilitation benefit, individually selected desired torques were added for each muscle group when within their stimulation regions, thus ensuring that resistance is applied in response to muscle effort. The admittance controller applies motor effort to assist or resist the rider as needed to produce the desired interaction torque. Similarly, a desired hand cycle torque function is defined to mimic the apparent dynamics at the leg as well as provide haptic feedback to inform the operator of leg cycle performance.

## REFERENCES


Ahmadkhanlou, F., Washington, G.N., and Bechtel, S.E. (2009). Modeling and control of single and two degree of freedom magnetorheological fluid-based haptic systems for telerobotic surgery. *J. Intell. Mater. Syst. Struct.*, 20(10), 1171–1186.

Atashzar, S.F., Shahbazi, M., Tavakoli, M., and Patel, R.V. (2016). A passivity-based approach for stable patient-robot interaction in haptics-enabled rehabilitation systems: modulated time-domain passivity control. *IEEE Trans. Control Syst. Technol.*, 25(3), 991–1006.

Bélanger, M., Stein, R.B., Wheeler, G.D., Gordon, T., and Leduc, B. (2000). Electrical stimulation: can it increase muscle strength and reverse osteopenia in spinal cord injured individuals? *Arch. Phys. Med. Rehabil.*, 81(8), 1090–1098.

Bellman, M.J., Downey, R.J., Parikh, A., and Dixon, W.E. (2017). Automatic control of cycling induced by functional electrical stimulation with electric motor assistance. *IEEE Trans. Autom. Science Eng.*, 14(2), 1225–1234. doi:10.1109/TASE.2016.2527716.

Cousin, C., Duenas, V.H., Rouse, C., and Dixon, W.E. (2018). Admittance trajectory tracking using a challenge-based rehabilitation robot with functional electrical stimulation. In *Proc. Am. Control Conf.*, 3732–3737.

Cousin, C.A., Rouse, C.A., Duenas, V.H., and Dixon, W.E. (2019). Controlling the cadence and admittance of a functional electrical stimulation cycle. *IEEE Trans. Neural Syst. Rehabil. Eng.*, 27(6), 1181–1192.

Downey, R., Merad, M., Gonzalez, E., and Dixon, W.E. (2017). The time-varying nature of electromechanical delay and muscle control effectiveness in response to stimulation-induced fatigue. *IEEE Trans. Neural Syst. Rehabil. Eng.*, 25(9), 1397–1408.

Downey, R.J., Cheng, T.H., Bellman, M.J., and Dixon, W.E. (2015). Closed-loop asynchronous electrical stimulation prolongs functional movements in the lower body. *IEEE Trans. Neural Syst. Rehabil. Eng.*, 23(6), 1117–1127. doi:10.1109/TNSRE.2015.2427658.

Estay, F.I.E., Rouse, C., Cohen, M., Cousin, C., and Dixon, W.E. (2019). Cadence and position tracking for decoupled legs during switched split-crank motorized FES-cycling. In *Proc. Am. Control Conf.*, 854–859.

Ferrante, S., Pedrocchi, A., Ferrigno, G., and Molteni, F. (2008). Cycling induced by functional electrical stimulation improves the muscular strength and the motor control of individuals with post-acute stroke. *Eur. J. Phys. Rehabil. Med.*, 44(2), 159–167.

Ferre, M., Buss, M., Aracil, R., Melchiorri, C., and Balaguer, C. (2007). *Advances in Telerobotics*. Springer.

Ferris, D.P., Huang, H.J., and Kao, P.C. (2006). Moving the arms to activate the legs. *Exercise and Sport Science Reviews*.

Hjelm, N.M. (2005). Benefits and drawbacks of telemedicine. *J. Telemed. Telecare*, 11(2), 60–70.

Keemink, A.Q., van der Kooij, H., and Stienen, A.H. (2018). Admittance control for physical human–robot interaction. *Int. J. Rob. Res.*, 37(11), 1421–1444.

Khalil, H.K. (2002). *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ, 3 edition.

Klarner, T., Bass, T.S., Sun, Y., Kaupp, C., Loadman, P.M., and Zehr, E.P. (2016). Exploiting interlimb arm and leg connections for walking rehabilitation: A training intervention in stroke. *Neural Plast.*

Kralj, A. and Bajd, T. (1989). *Functional electrical stimulation: standing and walking after spinal cord injury*. CRC.

Mohr, T., Pødenphant, J., Biering-Sørensen, F., Galbo, H., Thamsborg, G., and Kjær, M. (1997). Increased bone mineral density after prolonged electrically induced cycle training of paralyzed limbs in spinal cord injured man. *Calcif. Tissue Int.*, 61(1), 22–25. doi:10.1007/s002239900286.

Niemeyer, G., Preusche, C., Stramigioli, S., and Lee, D. (2016). *Springer handbook of robotics*, chapter Telerobotics, 1085–1108. Springer.

Puerto, M.J., Sanchez, E., and Gil, J.J. (2009). Control strategies applied to kinesthetic haptic devices. In *2009 IEEE Workshop on Robotic Intelligence in Informationally Structured Space*, 137–144. IEEE.

Ragnarsson, K. (2008). Functional electrical stimulation after spinal cord injury: current use, therapeutic effects and future directions. volume 46, 255–274.

Rouse, C., Cousin, C., Allen, B.C., and Dixon, W.E. (2019). Split-crank cadence tracking for switched motorized FES-cycling with volitional pedaling. In *Proc. Am. Control Conf.*, 4393–4398.

Rouse, C., Cousin, C., Duenas, V.H., and Dixon, W.E. (2018). Cadence tracking for switched FES cycling combined with voluntary pedaling and motor resistance. In *Proc. Am. Control Conf.*, 4558–4563.

Rouse, C., Downey, R., Gregory, C., Cousin, C., Duenas, V., and Dixon, W.E. (2020). FES cycling in stroke: Novel closed-loop algorithm accommodates differences in functional impairments. *IEEE Trans. Biomed. Eng.*, 67(3), 738–749.