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Robust Power and Cadence Tracking on a Motorized FES Cycle with an Unknown Time-Varying Input Delay

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***Abstract*—Functional electrical stimulation (FES) induced cycling is a common rehabilitative technique applied for those with a movement disorder. An FES cycle system is a nonlinear switched dynamic system that has a potentially destabilizing input delay between stimulation and the resulting muscle force. In this paper, a dual objective control system for a nonlinear, uncertain, switched FES cycle system with an unknown time-varying input delay is developed and a Lyapunov-like dwell-time analysis is performed to yield exponential power tracking to an ultimate bound and global exponential cadence tracking. Preliminary experimental results for a single healthy individual are provided and demonstrate average power and cadence tracking errors of -0.05 ± 0.80 W and -0.05 ± 1.20 RPM, respectively, for a target power of 10 W and a target cadence of 50 RPM.**

***Index Terms*—Functional electrical stimulation (FES), input delay, switched systems, power tracking, rehabilitation robotics, Lyapunov methods.**

I. Introduction

A common rehabilitative therapy for those with lower limb movement disorders is stationary Functional Electrical Stimulation (FES) cycling [1]. There are numerous health benefits associated with FES-induced cycling, such as promoting beneficial changes in the neuromuscular system and facilitating nervous system reorganization [2], improved physiological motor control [3], and many more [1]. However, closed-loop control of FES-cycling is challenging because it deals with uncertain switched nonlinear dynamics [1], uncertain nonlinear muscle activation dynamics [4], fatigue causes the dynamics to be time-varying [5], and FES-induced forces yield a potentially destabilizing time-varying input delay [5].

Over the last two decades, research in closed-loop torque/power tracking investigated increasing the power output (PO) of FES cycling. Prior torque tracking results have utilized discretized average torque tracking [6], tracking when it is efficient kinematically [7], and instantaneous torque tracking [8]–[11]. However, none of these prior results have accounted for the FES-induced input delay, which can potentially lead to instability or reduce efficiency [5].

To compensate for the delayed response of muscle to an FES input, controllers have recently been developed for leg extension exercises [12]–[14], and cadence tracking in FES-cycling in the authors' prior results in [15]–[17]. A unique aspect to coordinated exercises, such as FES cycling, is that both the delay after the application and removal of stimulation should be considered to ensure muscle contractions occur when it is kinematically efficient and to mitigate/eliminate potential undesired antagonistic muscle forces [15]–[17]. The aforementioned results on FES delayed systems have all had control objectives of position and/or cadence tracking. Further health benefits could be achieved by increasing the PO by implementing a power tracking control objective. PO and efficiency increases for FES cycling are desired to cultivate fatigue resistant muscle fibers, which can delay fatigue, and reverse muscle atrophy [18] in addition to other health benefits [11]. However, to date no FES delayed system has included power tracking as a control objective.

Non-FES related input delayed systems have been studied extensively (cf. [19]–[23] etc.), but few of these studies have considered input delays of a switched system [21]–[23]. These non-FES input delayed systems, however, do not provide compensation for critical FES specific factors. For example, the stimulation must be properly timed to yield effective agonist muscle contractions and to reduce antagonistic muscle forces by developing state and delay dependent switching conditions.

In this paper, a dual objective control system is developed for simultaneous power and cadence tracking of a nonlinear, switched, uncertain FES cycle system with an unknown time-varying input delay. To allow for instantaneous power tracking, a running integral is employed, which can more accurately compensate for rider asymmetries [11]. A state and delay dependent trigger condition is developed to schedule the activation and deactivation of the FES for each muscle group so that contractions occur in kinematically efficient regions of the crank. The motor is used to regulate the cadence, similar to clinical practice, but the power is regulated via FES to ensure the participant is contributing a desired amount of effort and to increase the PO. However, FES is not always active, which results in periods where the power objective is not being controlled. The existence

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of uncontrolled regions requires the development of dwell-time conditions to achieve the power tracking objective. Therefore, a Lyapunov-based switched systems analysis is provided, which includes a dwell-time analysis to yield global exponential cadence tracking and exponential torque tracking to an ultimate bound. An initial assessment of the developed controller was provided by a preliminary experiment on a single healthy individual[1]. This experiment demonstrated an average cadence and power tracking error of -0.05 ± 1.20 revolutions per minute (RPM) and -0.05 ± 0.80 Watts (W), respectively, for a desired cadence trajectory of 50 RPM and a target power of 10 W.

II. Dynamics

A. Cycle-Rider Dynamics

Throughout the paper, delayed functions are defined as

$$h_\tau \triangleq \begin{cases} h\left(t-\tau\left(t\right)\right) & t-\tau\left(t\right)\geq t_0 \\ 0 & t-\tau\left(t\right)<t_0 \end{cases},$$

where the time and initial time are denoted by $t, t_0 \in \mathbb{R}_{\geq 0}$, respectively. There exists an unknown time-varying delay [5], known as the electromechanical delay and denoted by $\tau : \mathbb{R}_{\geq 0} \to \mathbb{S}$, between the application (removal) of stimulation and the onset (elimination) of muscle force. The set of all possible delay values is denoted by $\mathbb{S} \subset \mathbb{R}$ [4]. The uncertain, nonlinear motorized cycle rider dynamics are [15], [16][2]

$$\begin{aligned} M\left(q\right)\ddot{q}+V\left(q,\dot{q}\right)\dot{q}+G\left(q\right)+P\left(q,\dot{q}\right)+b_c\dot{q}+d\left(t\right) \\ =\underbrace{\sum_{m\in\mathcal{M}} b_m\left(q,\dot{q},t\right)u_m\left(t-\tau\right)}_{\tau_m\left(q,\dot{q},\tau,t\right)}+\underbrace{b_e u_e\left(t\right)}_{\tau_e(t)}, \end{aligned} \quad (1)$$

where $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$, $\dot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, and $\ddot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote the measurable crank angle, measurable angular velocity (cadence), and unmeasured acceleration, respectively. The set $\mathcal{Q} \subseteq \mathbb{R}$ denotes all possible crank angles. The inertial, gravitational, and centripetal-Coriolis effects are denoted as $M : \mathcal{Q} \to \mathbb{R}_{>0}$, $G : \mathcal{Q} \to \mathbb{R}$, and $V : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, respectively. The rider's passive viscoelastic tissue forces are denoted by $P : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$. The cycle's viscous damping effect is denoted by $b_c \in \mathbb{R}_{>0}$ and the system disturbances are denoted by $d : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The system inputs are from FES-induced muscle contractions and an electric motor, which are denoted by $\tau_m : \mathcal{Q} \times \mathbb{R} \times \mathbb{S} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $\tau_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively.

The uncertain muscle control effectiveness is denoted by $b_m : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$ in (1). The set $\mathcal{M} \triangleq \{RQ,\, RH,\, RG,\, LQ,\, LH,\, LG\}$ contains the right (R) and left (L) gluteal (G), quadriceps femoris (Q), and hamstring (H) muscle groups. The delayed FES input (i.e., pulse width) is represented by $u_{m,\tau} : \mathbb{S} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, $\forall m \in \mathcal{M}$ and defined as

$$u_{m,\tau} \triangleq k_m \sigma_{m,\tau}\left(q_\tau, \dot{q}_\tau\right) u_\tau, \quad (2)$$

where $k_m \in \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$ are selectable constants. The subsequently designed FES control input is represented by $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$. In (2), $\sigma_{m,\tau}\left(q_\tau, \dot{q}_\tau\right)$, $\forall m \in \mathcal{M}$ denotes the delayed switching signals, which indicate the muscle groups that were stimulated at $t-\tau$. To compensate for the delay, state-dependent FES switching signals, denoted by $\sigma_m : \mathcal{Q} \times \mathbb{R} \to \{0,\,1\}$, $\forall m \in \mathcal{M}$, are designed to activate/deactivate each muscle in more efficient regions of the crank, and are defined as

$$\sigma_m\left(q,\dot{q}\right) \triangleq \begin{cases} 1, & q_\alpha \in \mathcal{Q}_m \\ 1, & q_\beta \in \mathcal{Q}_m \\ 0, & \text{otherwise} \end{cases}, \quad (3)$$

where $q_\alpha, q_\beta : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$ represents trigger conditions and are defined as $q_\alpha \triangleq f_1\left(q,\dot{q}\right)$ and $q_\beta \triangleq f_2\left(q,\dot{q}\right)$. The functions $f_1, f_2 : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, use the fact the delay can be bounded such that $\underline{\tau} \leq \tau \leq \bar{\tau}$, where $\underline{\tau}, \bar{\tau} \in \mathbb{R}_{>0}$ are known constants [12], [13]. The trigger conditions determine when to activate/deactivate the stimulation of each muscle group such that FES-induced muscle contractions occur over the entire FES region and simultaneously reduce/eliminate residual torques occuring in antagonistic muscles. The efficient regions of the cycle for each muscle is denoted by $\mathcal{Q}_m \subset \mathcal{Q}$, $\forall m \in \mathcal{M}$ and is defined in [1] as

$$\mathcal{Q}_m \triangleq \left\{q \in \mathcal{Q} \mid T_m\left(q\right) > \varepsilon_m\right\}, \quad (4)$$

$\forall m \in \mathcal{M}$, where $T_m : \mathcal{Q} \to \mathbb{R}$ represents the torque transfer ratio and a selectable lower threshold is denoted by $\varepsilon_m \in (0,\ \max(T_m)]$. By the definition in (4), a muscle contraction in a particular muscle's FES region will efficiently contribute to forward pedaling, that is, positive crank motion. The entire FES region, denoted by $\mathcal{Q}_{FES}$, is defined as $\mathcal{Q}_{FES} \triangleq \underset{m\in\mathcal{M}}{\cup}\{\mathcal{Q}_m\}$. Non-efficient regions of the crank cycle are called kinematic deadzones and are defined as $\mathcal{Q}_{KDZ} \triangleq \mathcal{Q} \setminus \mathcal{Q}_{FES}$.

The motor control effectiveness is a known constant and is denoted by $b_e \in \mathbb{R}_{>0}$ in (1). The electric motor's input (i.e., current), denoted by $u_e : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is defined as

$$u_e\left(t\right) \triangleq k_e u_{cad}\left(t\right), \quad (5)$$

where $u_{cad} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the subsequently designed motor control input and $k_e \in \mathbb{R}_{>0}$ is a selectable constant.

Substituting (2) and (5) into (1) yields[3]

$$B_{m,\tau}u_\tau + B_e u_{cad} = M\ddot{q} + V\dot{q} + G + P + b_c\dot{q} + d, \quad (6)$$

where $B_{m,\tau} \triangleq \sum_{m\in\mathcal{M}} b_m\left(q,\dot{q},t\right) k_m \sigma_{m,\tau}\left(q_\tau,\dot{q}_\tau\right)$ and $B_e \triangleq b_e k_e$.

The switched system in (6) has the following properties [1]. **Property: 1** $c_m \leq M \leq c_M$, where $c_m,\, c_M \in \mathbb{R}_{>0}$ are

[1]Additional experiments are currently stymied due to Covid-19.

[2]For notational brevity, all explicit dependence on time, t, within the terms $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$, and $\tau(t)$ is suppressed.

[3]All functional dependencies are hereafter suppressed, for notational brevity, unless required for clarity of exposition.

known constants. **Property: 2** $|V| \leq c_V |\dot{q}|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant **Property: 3** $|G| \leq c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant. **Property: 4** $|P| \leq c_{P1} + c_{P2}|\dot{q}|$, where c_{P1}, $c_{P2} \in \mathbb{R}_{>0}$ are known constants. **Property: 5** $b_c \dot{q} \leq c_c |\dot{q}|$, where $c_c \in \mathbb{R}_{>0}$ is a known constant. **Property: 6** $|d| \leq c_d$, where $c_d \in \mathbb{R}_{>0}$ is a known constant. **Property: 7** $\frac{1}{2}\dot{M} = V$. **Property: 8** The muscle control effectiveness b_m is lower and upper bounded $\forall m \in \mathcal{M}$, and thus, when $\sum_{m \in \mathcal{M}} \sigma_{m,\tau} > 0$, $c_b \leq B_{m,\tau} \leq c_B$, where $c_b, c_B \in \mathbb{R}_{>0}$ are known constants. **Property: 9** The delay estimate error is bounded such that $\hat{\tau} - \tau \leq \bar{\tilde{\tau}}$, where $\bar{\tilde{\tau}} \in \mathbb{R}_{>0}$ is a known constants and $\hat{\tau} \in \mathbb{R}_{>0}$ is a constant estimate of the delay. [12], [13].

B. Torque Dynamics

The dynamics in (6) are rewritten as

$$\tau_m + \tau_e = \tau_p + \tau_c, \tag{7}$$

where τ_p and τ_c denote the torque by the rider's passive effects and the cycle about the crank axis, respectively. An auxiliary term, denoted by $\tau_{est} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is defined as

$$\tau_{est} \triangleq \tau_p + \tau_c. \tag{8}$$

When no stimulation is applied $\tau_{est} = \tau_e$. **Assumption 1.** The auxiliary term and disturbances are sufficiently smooth such that τ_{est}, $\dot{\tau}_{est}$, $\dot{d} \in \mathcal{L}_\infty$ [11]. **Assumption 2.** As detailed in [11], preliminary testing (see Section V for more information) can be used to generate an estimate of τ_{est}, denoted by $\hat{\tau}_{est} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, that is continuously differentiable such that the estimation error, $\tilde{\tau}_{est} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$\tilde{\tau}_{est} \triangleq \hat{\tau}_{est} - \tau_{est}, \tag{9}$$

can be bounded by a known constant, $c_{est} \in \mathbb{R}_{\geq 0}$, such that $|\tilde{\tau}_{est}| \leq c_{est}$. **Assumption 3.** The system in (6) does not escape to infinity during the time interval $[t_0, t_1^m]$, where t_1^m is the first time instant that muscle forces are present in the system.

The muscle torque, $\hat{\tau}_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$, can be estimated as

$$\hat{\tau}_m \triangleq \hat{\tau}_{est} - \tau_e, \tag{10}$$

where the estimation error of the muscle torque, $\tilde{\tau}_m : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is defined as

$$\tilde{\tau}_m \triangleq \hat{\tau}_m - \tau_m. \tag{11}$$

By substituting (8) into (7) and then using (9)-(11), $\tilde{\tau}_m = \tilde{\tau}_{est}$, and thus $|\tilde{\tau}_m| \leq c_{est}$. Therefore, the torque contribution of the muscles can be separated from that of the motor with the same precision that the passive rider and cycle dynamics can be estimated.

III. Control Development

A. Cadence Error System

One control objective is for the cycle crank to track a desired cadence. The measurable position tracking error is denoted by $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and defined as

$$e_1 \triangleq q_d - q, \tag{12}$$

where the desired trajectory, denoted by $q_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is sufficiently smooth such that $q_d \leq c_{q0}$, $\dot{q}_d \leq c_{q1}$, and $\ddot{q}_d \leq c_{q2}$, where c_{q0}, $c_{q1}\, c_{q2} \in \mathbb{R}_{>0}$ are known constants. The measurable cadence tracking error is defined as $\dot{e}_1 \triangleq \dot{q}_d - \dot{q}$. A measurable auxiliary tracking error, denoted by $e_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is designed to facilitate the subsequent analysis and is defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1, \tag{13}$$

where $\alpha_1 \in \mathbb{R}_{\geq 0}$ is a selectable constant. The composite cadence and position open-loop error system is determined by taking the derivative of (13), multiplying by M, substituting in (6), adding and subtracting e_1, using (12) and (13), and using the fact that $\tau_m \triangleq B_m^\tau u_\tau$ to yield

$$M\dot{e}_2 \;=\; \chi_1 - e_1 - Ve_2 - \tau_m - B_e u_{cad}, \tag{14}$$

where $\chi_1 \triangleq M(\ddot{q}_d + \alpha_1 \dot{e}_1) + V(\dot{q}_d + \alpha_1 e_1) + G + P + b_c \dot{q} + d + e_1$. Based on the subsequent stability analysis and (14), the motor controller is designed as

$$u_{cad} = \frac{1}{B_e}\left(k_1 e_2 + \left(k_2 + k_3 \|y\| + k_4 \|y\|^2\right)\operatorname{sgn}(e_2) - \hat{\tau}_m\right), \tag{15}$$

where k_1, k_2, k_3, $k_4 \in \mathbb{R}_{>0}$ are selectable constants, $y \triangleq \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T$, and $\operatorname{sgn}(\cdot)$ denotes the signum function. The cadence closed-loop error system is obtained by substituting (15) into (14) and using (11) to yield

$$\begin{aligned} M\dot{e}_2 \;=\; & \chi - e_1 - Ve_2 - k_1 e_2 \\ & - \left(k_2 + k_3 \|y\| + k_4 \|y\|^2\right)\operatorname{sgn}(e_2), \end{aligned} \tag{16}$$

where $\chi \triangleq \chi_1 + \tilde{\tau}_m$. Using Properties 1-6 and Assumption 2, χ can be upper bounded by

$$|\chi| \leq c_1 + c_2 \|y\| + c_3 \|y\|^2, \tag{17}$$

where c_1, c_2, $c_3 \in \mathbb{R}_{>0}$ are known constants.

B. Torque Error System

The integral torque tracking objective is denoted by $e_3 : \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and defined as [24]

$$e_3 \triangleq \int_{t_0}^{t} \left(\tau_{m,d}(\theta) - \hat{\tau}_m(\theta)\right) d\theta, \tag{18}$$

where $\tau_{m,d} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denotes the desired torque trajectory that can be bounded by $\tau_{m,d} \leq c_{\tau 0}$ and $\dot{\tau}_{m,d} \leq c_{\tau 1}$. The form of (18) is designed to facilitate the subsequent stability analysis by allowing the torque controller to directly influence the closed-loop torque error system [24]. Applying Leibniz's Rule to (18) yields

$$\dot{e}_3 = \tau_{m,d}(t) - \hat{\tau}_m(t), \tag{19}$$

which represents the instantaneous torque tracking error. To inject a delay-free input term into the closed-loop torque error system, an auxiliary error signal, denoted by $e_4 : \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, is defined as

$$e_4 \triangleq -\int_{t-\hat{\tau}}^{t} u(\theta)\, d\theta. \tag{20}$$

To facilitate the subsequent analysis, an auxiliary torque tracking error, denoted by $r : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, is defined as

$$r \triangleq \alpha_2 e_3 + \alpha_3 e_4, \tag{21}$$

where $\alpha_2, \alpha_3 \in \mathbb{R}_{>0}$ are selectable constants. Taking the derivative of (21), substituting (7)-(10), using the fact that $\tau_m \triangleq B_{m,\tau} u_\tau$, and adding and subtracting $\alpha_2 B_{m,\tau} u_{\hat{\tau}}$ yields the open-loop torque error system

$$\dot{r} = \alpha_2 \chi_2 + \alpha_2 B_{m,\tau} (u_{\hat{\tau}} - u_\tau) - \alpha_3 u + (\alpha_3 - \alpha_2 B_{m,\tau}) u_{\hat{\tau}}, \tag{22}$$

where $\chi_2 \triangleq \tau_{m,d} - \tilde{\tau}_{est}$ and can be upper bounded by $|\chi_2| \leq c_4$, where $c_4 \in \mathbb{R}_{>0}$ is a known constant. Based on the subsequent stability analysis and (22), the motor controller is designed as

$$u = k_s r, \tag{23}$$

where $k_s \in \mathbb{R}_{>0}$ is a selectable constant. Substituting (23) into (22) yields the closed-loop torque error system

$$\begin{aligned} \dot{r} &= \alpha_2 \chi_2 + \alpha_2 k_s B_{m,\tau} (r_{\hat{\tau}} - r_\tau) - \alpha_3 k_s r \\ &\quad + (\alpha_3 - \alpha_2 B_{m,\tau}) k_s r_{\hat{\tau}}. \end{aligned} \tag{24}$$

To facilitate the subsequent stability analysis Lyapunov-Krasovskii functionals, denoted by $Q_1, Q_2 : \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, are defined as

$$Q_1 \triangleq \frac{1}{2}(\alpha_3 - c_b \alpha_2 + \varepsilon_1 \omega_1) k_s \int_{t-\hat{\tau}}^{t} r(\theta)^2 d\theta, \tag{25}$$

$$Q_2 \triangleq \frac{\omega_2 k_s}{\hat{\tau}} \int_{t-\hat{\tau}}^{t} \int_{s}^{t} r(\theta)^2 d\theta ds, \tag{26}$$

where $\varepsilon_1, \omega_1, \omega_2 \in \mathbb{R}_{>0}$ are selectable constants.

IV. Stability Analysis

The torque error system must be analyzed for the case when the muscles are producing forces ($B_{m,\tau} > 0$) and the case when the muscles are not producing forces ($B_{m,\tau} = 0$) along with the analysis of switching between the two cases. For safety and user-comfort, the following initial condition is defined

$$\|z(t_0)\| \leq \gamma_1, \tag{27}$$

where $\gamma_1 \in \mathbb{R}_{\geq 0}$ is a user-defined constant to limit FES intensity. Theorems 1-3 establish the decay rate, growth rate, and boundedness of the torque error system and FES controller. The proof of Theorems 1-3 use a common Lyapunov function candidate to first establish the decay rate for the torque error system when the muscle forces are present and a growth rate otherwise, and to establish an ultimate bound on the integral torque error system. The first two theorems assume that $\|z(\cdot)\| \leq \gamma, \forall \cdot [t_0, t)$, where $\gamma \in \mathbb{R}_{>0}$ is a known constant. Theorem 3 establishes that provided (27) is satisfied, then $\|z(\cdot)\| \leq \gamma, \forall \cdot [t_0, t)$. To facilitate the subsequent analysis, switching times are denoted by $\{t_n^i\}$, $i \in \{m, e\}$, $n \in \{0, 1, 2, ...\}$, which denote the instants in time when $B_{m,\tau}$ becomes nonzero ($i = m$) and the instants when $B_{m,\tau}$ becomes zero ($i = e$).

Theorem 4 establishes exponential stability of the position and cadence error systems and boundedness of the motor control input.

Let $V_1 : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ and $V_2 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_{>0}$ denote positive definite, continuously differentiable Lyapunov function candidates defined as

$$V_1 \triangleq \frac{1}{2} r^2 + \frac{1}{2} \omega_1 e_4^2 + Q_1 + Q_2, \tag{28}$$

$$V_2 \triangleq \frac{1}{2} e_1^2 + \frac{1}{2} M e_2^2. \tag{29}$$

The Lyapunov function candidates can be bounded as

$$\lambda_1 \|z\|^2 \leq V_1 \leq \lambda_2 \|z\|^2, \tag{30}$$

$$\beta_1 \|y\|^2 \leq V_2 \leq \beta_2 \|y\|^2, \tag{31}$$

where $\beta_1, \beta_2, \lambda_1, \lambda_2 \in \mathbb{R}_{>0}$ are known constants and defined as $\beta_1 \triangleq \min\left(\frac{1}{2}, \frac{c_m}{2}\right)$, $\beta_2 \triangleq \max\left(\frac{1}{2}, \frac{c_M}{2}\right)$, $\lambda_1 \triangleq \min\left(\frac{1}{2}, \frac{\omega_1}{2}\right)$, $\lambda_2 \triangleq \max\left(1, \frac{\omega_1}{2}\right)$, y is defined after (15), and the composite error vector $z \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$ is defined as

$$z \triangleq \begin{bmatrix} r & e_4 & \sqrt{Q_1} & \sqrt{Q_2} \end{bmatrix}^T. \tag{32}$$

To facilitate the subsequent analysis, auxiliary constants $\delta_1, \delta_2, \lambda, \lambda_3, \lambda_4, v, v_1, v_2 \in \mathbb{R}_{>0}$ are defined as

$$\begin{aligned} \delta_1 \triangleq \min\Big(&\tfrac{1}{4}\alpha_3 k_s,\ \tfrac{\omega_2}{3 k_s \hat{\tau}^2} - \tfrac{\omega_1 k_s}{\varepsilon_1}, \\ &\tfrac{2\omega_2}{3\hat{\tau}(\alpha_3 - c_b \alpha_2 + \varepsilon_1 \omega_1)},\ \tfrac{1}{3\hat{\tau}} \Big), \end{aligned} \tag{33}$$

$$\delta_2 \triangleq \varepsilon_2 + k_s (\varepsilon_1 \omega_1 + \omega_2), \tag{34}$$

$$\lambda \triangleq \lambda_3 \Delta t_{min}^m - \lambda_4 \Delta t_{max}^e, \tag{35}$$

$$\lambda_3 \triangleq \lambda_2^{-1} \delta_1, \quad \lambda_4 \triangleq \lambda_1^{-1} \delta_2, \tag{36}$$

$$\begin{aligned} v \triangleq\ &\tfrac{v_1}{\lambda_3} \exp(\lambda_4 \Delta t_{max}^e)(1 - \exp(-\lambda_3 \Delta t_{min}^m)) \\ &- \tfrac{v_2}{\lambda_4}(1 - \exp(\lambda_4 \Delta t_{max}^e)), \end{aligned} \tag{37}$$

$$v_1 \triangleq \frac{\bar{\bar{\tau}} \Upsilon^2}{k_s} + \frac{\alpha_2^2 c_4^2}{\alpha_3 k_s}, \tag{38}$$

$$v_2 \triangleq \frac{(\alpha_2 c_4 + k_s c_b \alpha_2 \hat{\tau} \Upsilon)^2}{4 \varepsilon_2}, \tag{39}$$

where $\varepsilon_2 \in \mathbb{R}_{>0}$ is a selectable constant and $\Upsilon \in \mathbb{R}_{>0}$ is a known constant. The minimum allowable dwell-time when muscle forces are present and the maximum allowable dwell-time without muscle forces, denoted by Δt_{min}^m, $\Delta t_{max}^e \in$

$\mathbb{R}_{>0}$, respectively, must be defined in such a way to ensure they can be met. Let $\{t_n^i\}, i \in \{FES, KDZ\}, n \in \{0, 1, 2, ...\}$, denote the known times the crank enters $\mathcal{Q}_{FES}$ (leaves $\mathcal{Q}_{KDZ}$) or $\mathcal{Q}_{KDZ}$ (leaves $\mathcal{Q}_{FES}$), respectively, for the n-th time. Although, the delay is unknown, by the design of the switching condition in (3), it is known that muscle forces are present throughout the entire FES region of the crank cycle. Therefore, the following dwell-time conditions are defined

$$\Delta t_{min}^m \triangleq \min\left(t_n^{KDZ} - t_n^{FES}\right), \forall n, \quad (40)$$

$$\Delta t_{max}^e \triangleq \max\left(t_{n+1}^{FES} - t_n^{KDZ}\right), \forall n. \quad (41)$$

Therefore, after the FES regions are designed, the dwell-time conditions manifest themselves as a minimum and maximum allowable cadence.

Theorem 1. *For $B_{m,\tau} > 0$, the integral torque tracking error in (18) is uniformly ultimately bounded in the sense that*

$$\begin{aligned} \|z(t)\|^2 \leq\ & \tfrac{\lambda_2}{\lambda_1}\|z(t_n^m)\|^2 \exp\left(-\lambda_3\left(t - t_n^m\right)\right) \\ & + \tfrac{v_1}{\lambda_1\lambda_3}\left(1 - \exp\left(-\lambda_3\left(t - t_n^m\right)\right)\right), \end{aligned} \quad (42)$$

$\forall t \in [t_n^m, t_n^e)$, $\forall n$, provided that $\|z(\cdot)\| < \gamma$, $\forall \cdot \in [t_0, t)$, Assumptions 1 and 2 are satisfied, and the following gain conditions are satisfied:

$$\alpha_3 \geq c_B\alpha_2, \quad \omega_2 \geq \frac{3\hat{\tau}^2\omega_1 k_s^2}{\varepsilon_1}, \quad \alpha_3 k_s\hat{\tau} < 1, \quad (43)$$

$$\bar{\bar{\tau}} \leq \tfrac{1}{k_s^2\alpha_2^2 c_B^2}\left(4c_b\alpha_2 - 2\alpha_3 - 4\varepsilon_1\omega_1 - 4\omega_2\right). \quad (44)$$

Proof available upon request.

Theorem 2. *For $B_{m,\tau} = 0$, the integral torque tracking error in (18) can be upper bounded in the sense that*

$$\begin{aligned} \|z(t)\|^2 \leq\ & \tfrac{\lambda_2}{\lambda_1}\|z(t_n^e)\|^2 \exp\left(\lambda_4\left(t - t_n^e\right)\right) \\ & - \tfrac{v_2}{\lambda_1\lambda_4}\left(1 - \exp\left(\lambda_4\left(t - t_n^e\right)\right)\right), \end{aligned} \quad (45)$$

$\forall t \in \left[t_n^e, t_{n+1}^m\right)$, $\forall n$, provided that $\|z(\cdot)\| < \gamma$, $\forall \cdot \in [t_0, t)$, Assumptions 1 and 2 are satisfied, and the gain conditions in (43) and (44) are satisfied.

Proof available upon request.

Theorem 3. *The integral torque tracking error in (18) is ultimately bounded in the sense that*

$$\limsup_{t\to\infty} \|z(t)\| = \sqrt{\gamma^2 - \lambda_1^{-1}c_5}, \quad (46)$$

where $c_5 \in \left(0, \lambda_1\gamma^2\right)$ is a known constant and the integral torque tracking error is bounded for all time in the sense that

$$\|z(t)\| \leq \gamma, \quad (47)$$

$\forall t \in [t_0, \infty)$, provided the initial condition in (27) is met, Assumptions 1-3 are satisfied, and the gain conditions in (43) and (44) are satisfied in addition to the following gain conditions

$$\lambda_3\Delta t_{min}^m > \lambda_4\Delta t_{max}^e, \quad (48)$$

$$\frac{v}{1 - \exp(-\lambda)} + c_5 = \lambda_1\gamma^2, \quad (49)$$

$$\begin{aligned} & \Big[\tfrac{\lambda_2}{\lambda_1}\gamma_1^2 \exp\left(\lambda_4\Delta t_{max}^e\right) \\ & - \tfrac{v_2}{\lambda_1\lambda_4}\left(1 - \exp\left(\lambda_4\Delta t_{max}^e\right)\right)\Big]^{\frac{1}{2}} < \gamma, \end{aligned} \quad (50)$$

Proof available upon request.

Now, Theorem 4 is employed to establish the exponential decay rate of the cadence error system $\forall t$.

Theorem 4. *The composite position and cadence error y is globally exponentially stable in the sense that*

$$\|y(t)\| \leq \sqrt{\frac{\beta_2}{\beta_1}}\|y(t_0)\| \exp\left(-\frac{\min(\alpha_1, k_1)}{2\beta_2}(t - t_0)\right), \quad (51)$$

$\forall t \in [t_0, \infty)$, provided the following gain conditions are met

$$k_2 \geq c_1,\ k_3 \geq c_2,\ k_4 \geq c_3, \quad (52)$$

where c_1, c_2, and c_3 are introduced in (17).

Proof available upon request.

V. Experiment

To validate the controllers in (15) and (23), experiments were performed on an able-bodied participant. The participant provided written informed consent approved by the University of Florida Institutional Review Board. Similar to active therapy, the participant was shown the integral torque tracking error plot in real time (i.e., $e_3(t)$) and was instructed to contribute to the control objective by attempting to minimize the error as stimulation was provided as required. A FES cycle testbed similar to [1] was used.

A preliminary experiment was used to estimate the rider's passive dynamics, $\hat{\tau}_{est}$. To start the experimental protocol, an exponential cadence ramp from 0 to 50 RPM was performed by the motor controller in (15). Upon reaching 50 RPM, the FES controller in (23) was applied and an exponential ramp was applied on the power trajectory from 0 to the desired power (10 W). The experimental protocol lasted for 120 seconds. Although the FES controller in (23) is designed using a torque-based error system, the results will be converted to the measured power, $P : \mathbb{R}_{\geq 0} \to \mathbb{R}$, by using $P \triangleq \hat{\tau}_m\dot{q}$. To determine the range of crank angles at which to stimulate each muscle, the torque transfer ratio's lower threshold values were utilized in addition to (3).

A. Results/Discussion

For clarity, to visualize the results, a 2.4 second (approximately 2 cycles) moving average filter was applied to the measured power, P. Figure 1 depicts the subject's cadence and power tracking performance throughout the experiment. FES was applied over 81% of the duration of the experiment. The controllers demonstrated an average cadence error of -0.05 ± 1.20 RPM and an average power error of -0.05 ± 0.80 W, with a target cadence and power tracking of 50 RPM and 10 W. These preliminary experimental results successfully

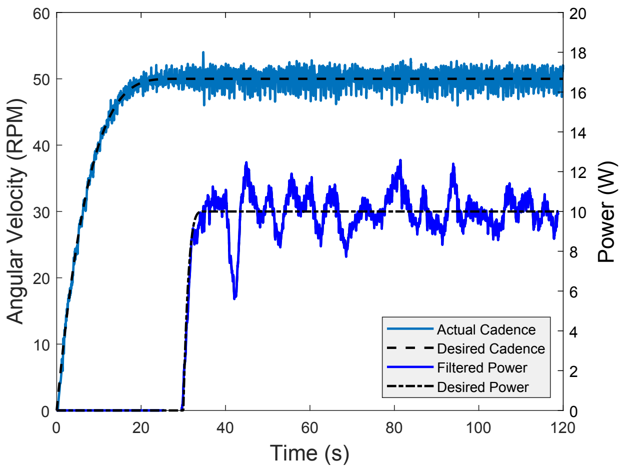

Fig. 1. The desired versus the actual cadence and power.

demonstrate the ability of the controllers in (15) and (23) to achieve 0.1% and 0.5% cadence and power tracking errors despite unknown disturbances, uncertainty in the lower limb dynamics, and an unknown time-varying FES input delay, for a single healthy participant.

VI. Conclusion

A dual objective control system is developed to track cadence and power. A running torque integral is utilized to have real-time torque tracking. The cadence and power are tracked by using the motor and FES, respectively. To compensate for the FES-induced input delay, switching conditions and an auxiliary tracking error (that injects a delay-free FES input into the dynamics) were developed. A Lyapunov-based analysis was performed to guarantee global exponential cadence tracking and exponential power tracking to an ultimate bound. A preliminary experiment was conducted on a single healthy individual to demonstrate the performance of the designed controllers. More thorough experiments will be performed to better quantify the performance of the controller and future efforts will seek additional methods to improve PO during FES cycling.

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