Teleoperated Motorized Functional Electric Stimulation Actuated Rehabilitative Cycling

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***Abstract*—Many people are affected by a wide range of neuromuscular disorders, many of which can be improved through the use of Functional Electrical Stimulation (FES) rehabilitative cycling. Recent improvements in nonlinear, Lyapunov-based FES muscle control with motor assistance in unstimulated regions of the cycle-crank rotation have led to a reduction in muscle fatigue, allowing rehabilitation time to be extended. Studies in rehabilitation have shown that the addition of coordinated movement between the upper limbs and lower limbs can have a positive effect on neural plasticity leading to faster restoration of walking in those who have some neurological disorders. In this paper, to implement coordinated motion during rehabilitation, a strongly coupled bilateral telerobotic system is developed between a hand-cycle system driven by the participant's volitional efforts and a split-crank leg-cycle system driven by the switched application of FES with motor assistance. A variable operator is applied to the leg-cycle's motor input during the FES stimulation regions to provide assistance as required. Lyapunov-based analysis methods are used on the split-crank leg-cycle system to prove global exponential tracking to the desired position determined by the hand-cycle system. Analysis further proves stability of the telerobotic master (i.e. hand-cycle system).**

***Index Terms*—Functional electrical stimulation (FES), teleoperation, human-machine interaction, rehabilitation robotics, switched systems.**

I. INTRODUCTION

Functional electrical stimulation (FES) is a common rehabilitation technique used for people with neuromuscular disorders (NDs) such as stroke or spinal cord injury [1], [2]. FES rehabilitation using a stationary cycle has been shown to have significant physiological and psychological benefits [3]-[8], motivating the desire to extend the duration and efficiency of FES rehabilitative cycling. To this end, recent work in FES rehabilitative cycling has included the use of Lyapunov-based, nonlinear control to adapt for unknown parameters and switching between FES activated muscle effort and motor assistance to ensure that muscle groups are only stimulated in efficient force production regions of the crank rotation [9], thereby delaying muscle fatigue and extending the rehabilitative dosage.

Studies in rehabilitation have shown that by coordinating rhythmic arm and leg movements on a cycle, walking can be improved for stroke patients [10]. This coordination might also serve to improve neural plasticity as neural connections are thought to exist between upper and lower limbs [11]. It has also been shown that motor recovery can be improved with specific, repetitive movement [12] that is enjoyable for the participant and has a positive emotional impact [13]. For those with NDs, rehabilitation is often a long process that should continue after the participant has left the hospital environment, meaning that ideally, rehabilitative therapy is available in the home [14]. However, rehabilitation is often not completed when unsupervised because participants do not find the rehabilitative technology to be motivating enough to encourage them to continue their treatment. This has lead to the creation of several teleoperative and/or game-based rehabilitative systems as in [14]-[16]. To date, these teleoperative rehabilitation systems have informed the participant of desired performance, typically using impedance control, as determined by a remote therapist-operated or computer-based slave system, as in [17]-[19].

Some studies have used mechanically coupled hand-cycles to improve cardiovascular health and overall strength for those with NDs [20], [21]. These mechanically coupled hand-cycles have also been used in recumbent tricycles, serving to also restore mobility and increase quality of life. This paper seeks to further the improvement of FES rehabilitative cycling by adding coordination through the implementation of a robust, bilateral, strongly coupled teleoperation system (Fig. 1), where the rehabilitation participant, or a clinician, pedals an uncoupled hand-cycle master system by their volitional efforts and the FES and motor actuated, switched leg-cycle system acts as the telerobotic slave. It is theorized that using this telerobotic system, rehabilitative participants will feel as though they have an improved ability to choose their desired position and cadence for the entire cycle-rider system; thus, improving their motivation and compliance for home therapies. However, such a telerobotic system introduces several challenges as both the hand-cycle and leg-cycle dynamic systems include unknown nonlinear terms and have the added challenge of requiring the development of switching signals requiring a switched systems analysis.

For the telerobotic system to produce the desired beneficial coordination of the hands and legs, a strong coupling between the systems should be designed so that the slave (leg-cycle system) closely mimics the position and velocity of the master (hand-cycle system) [23]. Therefore, to ensure that each leg is closely coordinated with its paired hand and to

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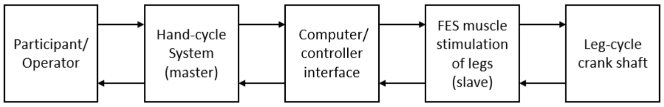

Figure 1. The bilateral teleoperation system for a coordinated upper and lower body rehabilitative FES cycling system. Graphic modified from [22].

address the needs of participants with asymmetric impairments, the leg-cycle system will be a split-crank design, where each leg can independently drive the crank shaft, as in [24]. The resistive control input applied to the hand-cycle system informs the participant of the error between the desired position (set by the hand-crank angle) and the actual position of the associated leg-cycle system. By reflecting feedback from the leg-cycle system to the hand-cycle input, the participant will adjust their cadence, thus allowing for a closed-loop cyber physical human machine interaction.

In [25], a controller was developed for a split-crank leg-cycle system which yielded larger cadence and position tracking errors than typical for a mechanically coupled crank/pedal set. This may be partly because during the split-crank experiments, when the leg is traveling through a return phase of the rotation (i.e. in the hamstring muscle group stimulation region), there is no assistance provided by the drive phase of the opposing leg (i.e. in the quadriceps femoris muscle group stimulation region) [26], thus requiring a significant amount of effort from the hamstring muscle group. This additional effort without any motor assistance quickly leads to FES input saturation causing rapid muscle fatigue and participant discomfort. It is also likely to cause the hand-cycle and leg-cycle systems to become out of phase, leading to a reduction in the possible benefit of coordinated effort. Therefore, this paper also introduces the addition of variable motor effort within the stimulation regions to delay the onset of fatigue, increase participant comfort, and improve position and cadence tracking, thus theoretically extending the rehabilitative dosages.

II. Dynamic Models

A. Lower body leg-cycle system

Using a split-crank cycle as in [25] and [27], the dynamics of a single side of the split-crank cycle and corresponding controlling hand-cycle are modeled independently from the opposing side without loss of generality. The cycle-rider lower body switched dynamics for one side are modeled as [27]

$$\tau_{e_l} = \tau_{c_l}\left(q_l, \dot{q}_l, \ddot{q}_l, t\right) + \tau_{r_l}\left(q_l, \dot{q}_l, \ddot{q}_l, t\right), \tag{1}$$

where $q_l : \mathbb{R}_{\geq 0} \to \mathcal{Q}_l$ denotes the angular position, and $\mathcal{Q}_l \subseteq \mathbb{R}$ is the set of all possible measurable leg-cycle crank angles. The measured angular velocity of the leg-cycle crank arm is denoted by $\dot{q}_l : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the unmeasurable angular acceleration is denoted by $\ddot{q}_l : \mathbb{R}_{\geq 0} \to \mathbb{R}$. In (1), $\tau_{e_l} : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ represents the electric motor torque, $\tau_{c_l} : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ represents the cycle torque, and $\tau_{r_l} : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ represents the rider torque. The unknown nonlinear dynamics for the leg cycle [27] are

$$\tau_{c_l}\left(q_l, \dot{q}_l, \ddot{q}_l, t\right) = J_{c_l}\left(q_l\right)\ddot{q}_l + b_{c_l}\dot{q}_l + d_{c_l}\left(t\right), \tag{2}$$

where the inertial effects, viscous damping effects, and disturbances are denoted by $J_{c_l} : \mathcal{Q}_l \to \mathbb{R}$, $b_{c_l} \in \mathbb{R}_{\geq 0}$, and $d_{c_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. The rider torque in (1) can be divided into its passive elements, $\tau_p : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, the torques produced by muscle forces, $\tau_M : \mathcal{Q}_l \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, and rider disturbances, $d_{r_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$, such that [27]

$$\begin{aligned}\tau_{r_l}\left(q_l, \dot{q}_l, \ddot{q}_l, t\right) = \; & \tau_{p_l}\left(q_l, \dot{q}_l, \ddot{q}_l\right) \\ & -\tau_M\left(q_l, \dot{q}_l, t\right) + d_{r_l}\left(t\right).\end{aligned} \tag{3}$$

The passive rider dynamics in (3) are modeled by [27]

$$\begin{aligned}\tau_{p_l}\left(q_l, \dot{q}_l, \ddot{q}_l\right) = \; & M_{p_l}\left(q_l\right)\ddot{q}_l + V_l\left(q_l, \dot{q}_l\right)\dot{q}_l \\ & + G_l\left(q_l\right) + P_l\left(q_l, \dot{q}_l\right),\end{aligned} \tag{4}$$

where $M_{p_l} : \mathcal{Q}_l \to \mathbb{R}$, $V_l : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$, $G_l : \mathcal{Q}_l \to \mathbb{R}$, and $P_l : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$ represent the unknown, nonlinear inertial effects, centripetal-Coriolis effects, gravitational effects, and passive viscoelastic muscle forces. The muscle torques in (3) are modeled as the summation of all induced muscle forces from individually stimulated muscle groups plus volitional efforts, denoted by $\tau_{vol_l} \in \mathbb{R}_{\geq 0}$, such that [24]

$$\tau_M\left(q_l, \dot{q}_l, t\right) = \sum_{m \in \mathcal{M}} B_{m_l}\left(q_l, \dot{q}_l\right) u_{m_l}\left(q_l, t\right) + \tau_{vol_l}, \tag{5}$$

where the subscript $m \in \mathcal{M} = \{Q, G, H\}$ indicates the quadriceps femoris (Q), gluteal (G), and hamstring (H) muscle groups. The unknown, nonlinear muscle control effectiveness in (5) is denoted by $B_{m_l} : \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}_{\geq 0}, \forall m \in \mathcal{M}$ and the designed FES muscle control input (i.e. pulse width) is denoted by $u_{m_l} : \mathcal{Q}_l \times \mathbb{R}_{\geq 0} \to \mathbb{R}$. The portion of the set $\mathcal{Q}_l$ where each muscle group is stimulated is denoted by $\mathcal{Q}_m \subset \mathcal{Q}_l$ such that [25]

$$\mathcal{Q}_m \triangleq \left\{q_l \in \mathcal{Q}_l \mid T_m\left(q_l\right) > \varepsilon_m\right\}, \tag{6}$$

where $\varepsilon_m \in \left(0, max\left(T_m\right)\right)$ represents a user-defined lower threshold for each muscle group's torque transfer ratio, T_m: $\mathcal{Q}_l \to \mathbb{R}$, such that each muscle group's contribution only acts to produce positive crank rotation. The region about the crank cycle where FES of at least one muscle group produces a positive crank torque is denoted by $\mathcal{Q}_{FES} \triangleq \bigcup_{m \in \mathcal{M}} \left\{\mathcal{Q}_m\right\}, \forall m \in \mathcal{M}$.

The level of stimulation intensity applied to each muscle group is defined as [25]

$$u_m \triangleq \sigma_m\left(q_l\right) k_m u_s\left(t\right), \tag{7}$$

$\forall m \in \mathcal{M}$, where $k_m \in \mathbb{R}_{\geq 0}$ is a selectable constant associated with participant comfort level during stimulation, $u_s\left(t\right)$ represents the subsequently designed FES control input, and σ_m denotes a switching signal determined from (6), where $\sigma_m : \mathcal{Q}_l \to \{0, 1\}$ such that

$$\sigma_m \triangleq \begin{cases} 1 & \text{if } q_l \in \mathcal{Q}_m \\ 0 & \text{if } q_l \notin \mathcal{Q}_m \end{cases}. \tag{8}$$

The electric motor torque produced about the leg-cycle crank axis can be written as [24]

$$\tau_{e_l} \quad \triangleq \quad B_{e_l} u_{e_l}(t), \tag{9}$$

where $B_{e_l} \in \mathbb{R}_{\geq 0}$ represents the unknown, nonlinear relationship between the electric motor current and the resulting torque applied about the crank axis, and $u_{e_l}(t)$ represents the subsequently designed leg-cycle motor control input.

Substituting (2)-(5), (7), and (9) into (1) and rearranging produces

$$\begin{aligned} B_M u_s + B_{e_l} u_{e_l} \quad = \quad & M_l \ddot{q}_l + b_{c_l} \dot{q}_l + d_{c_l} + V_l \dot{q}_l \\ & + G_l + P_l + d_{r_l} - \tau_{vol_l}, \end{aligned} \tag{10}$$

where the combination of the muscle torque efficiencies is represented by $B_M \triangleq \sum_{m \in \mathcal{M}} B_m \sigma_m k_m$ [24] and the system's inertial effects are represented by $M_l \triangleq J_{c_l} + M_{p_l}$ [25].

B. Upper body hand-cycle system

The cycle-rider upper body switched dynamics for one side are modeled by the system

$$\tau_{e_h} \quad \triangleq \quad \tau_{c_h}(q_h, \dot{q}_h, \ddot{q}_h, t) + \tau_{r_h}(q_h, \dot{q}_h, \ddot{q}_h, t) \ , \tag{11}$$

where $q_h : \mathbb{R}_{\geq 0} \to \mathcal{Q}_h$, denotes the angular position of the upper body crank, and $\mathcal{Q}_h \subseteq \mathbb{R}$ is the set of all possible measurable hand-cycle crank angles. The measured angular velocity in the hand-cycle system is denoted by $\dot{q}_h : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the unmeasurable angular acceleration of the crank arm is denoted by $\ddot{q}_h : \mathbb{R}_{\geq 0} \to \mathbb{R}$. Using a similar process as shown for the leg-cycle system and recognizing that the forces applied about the hand-crank axis by the rider, denoted by $\tau_{vol_h} \in \mathbb{R}_{\geq 0}$, are purely volitional, the hand-cycle system can be represented by

$$\begin{aligned} B_{e_h} u_{e_h} + \tau_{vol_h} \quad = \quad & M_h \ddot{q}_h + b_{c_h} \dot{q}_h + d_{c_h} \\ & + V_h \dot{q}_h + G_h + P_h + d_{r_h}, \end{aligned} \tag{12}$$

where $B_{e_h} \in \mathbb{R}_{\geq 0}$ represents the unknown, nonlinear relationship between the electric motor current and the resulting torque applied about the hand-crank axis and $u_{e_h}(t)$ represents the designed hand-cycle motor control input. The unknown, nonlinear inertial effects, centripetal-Coriolis effects, gravitational effects, and passive viscoelastic muscle forces in (12) are represented by $M_h : \mathcal{Q}_h \to \mathbb{R}$, $V_h : \mathcal{Q}_h \times \mathbb{R} \to \mathbb{R}$, $G_h : \mathcal{Q}_h \to \mathbb{R}$, and $P_h : \mathcal{Q}_h \times \mathbb{R} \to \mathbb{R}$ respectively, and d_{c_h} and d_{r_h} denote the unknown cycle and rider disturbances about the hand crank.

C. System properties

The switched leg-cycle system in (10) and the hand-cycle system in (12) have the following properties and assumptions, where $i = \{h, l\}$.

Property 1: $\frac{1}{2}\dot{M}_i = V_i$. **Property 2:** $c_{\underline{M}_i} \leq M_i \leq c_{\overline{M}_i}$ where $c_{\underline{M}_i}$, $c_{\overline{M}_i} \in \mathbb{R}_{\geq 0}$ are known constants. **Property 3:** $|V_i| \leq c_{V_i} |\dot{q}_i| \in \mathbb{R}_{\geq 0}$ where c_{V_i} is a known constant. **Property 4:** $|G_i| \leq c_{G_i} \in \mathbb{R}_{\geq 0}$ where c_{G_i} is a known constant. **Property 5:** $|P_i| \leq c_{P1_i} + c_{P2_i} |\dot{q}_i|$ where $c_{P1_i}, c_{P2_i} \in \mathbb{R}_{\geq 0}$ are know constants. **Property 6:** $|b_{c_i}| \leq c_{b_i}$ where $c_{b_i} \in \mathbb{R}_{\geq 0}$ is a known constant. **Property 7:** $|d_{c_i} + d_{r_i}| \leq c_{d_i} \in \mathbb{R}_{\geq 0}$ where c_{d_i} is a known constant. **Property 8:** $B_{\underline{e}_i} \leq B_{e_i} \leq B_{\overline{e}_i}$ where $B_{\underline{e}_i}$, $B_{\overline{e}_i} \in \mathbb{R}_{\geq 0}$ **Property 9:** The combined muscle efficiency B_M has a lower bound $\forall m$ as in [24] such that when $\sum_{m \in \mathcal{M}} \sigma_m > 0$, $B_{\underline{M}} \leq B_M$ where $B_{\underline{M}} \in \mathbb{R}_{\geq 0}$.

Assumption 1. Due to physical human limitations, q_h is sufficiently smooth (i.e. q_h, $\dot{q}_h$, $\ddot{q}_h \in \mathcal{L}_\infty$) and the volitional torques produced by the participant are upper bounded such that $|\tau_{vol_i}| \leq c_{vol_i} \in \mathbb{R}_{\geq 0}$.

III. Control Development

The control objective is to develop a strongly coupled telerobotic system [23], where the angular position of the hand-cycle is tracked by the FES/motor actuated leg-cycle system. To quantify the objective, the mismatch between the leg and hand crank, denoted by $e \colon \mathbb{R}_{\geq 0} \to \mathbb{R}$, and auxiliary error, denoted by $r : \mathbb{R}_{\geq 0} \to \mathbb{R}$, are defined as

$$e(t) \quad \triangleq \quad q_h(t) - q_l(t), \tag{13}$$

$$r(t) \quad \triangleq \quad \dot{e}(t) + \alpha e(t), \tag{14}$$

where $\alpha \in \mathbb{R}_{\geq 0}$ is a selectable constant.

Taking the time derivative of (14), multiplying by M_l, and using (13) yields

$$M_l \dot{r} \quad = \quad M_l (\ddot{q}_h - \ddot{q}_l + \alpha \dot{e}). \tag{15}$$

Rearranging (10) and (12) and substituting into (15) for $\ddot{q}_l$ and $\ddot{q}_h$ gives

$$\begin{aligned} M_l \dot{r} \quad = \quad & \chi - V_l r - e - B_M u_s \\ & - B_{e_l} u_{e_l} + M_l M_h^{-1} B_{e_h} u_{e_h}, \end{aligned} \tag{16}$$

where the auxiliary term $\chi : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$\begin{aligned} \chi \quad \triangleq \quad & M_l \alpha (r - \alpha e) + V_l (\dot{q}_h + \alpha e) - \tau_{vol_l} \\ & + b_{c_l} (\dot{q}_h - r + \alpha e) + G_l + P_l + e \\ & + d_{c_l} + d_{r_l} + M_h^{-1} (-b_{c_h} \dot{q}_h - d_{c_h} \\ & - V_h \dot{q}_h - G_h - P_h - d_{r_h} + \tau_{vol_h}). \end{aligned}$$

From Properties 2-7 and Assumption 1, χ can be upper bounded as

$$\chi \quad \leq \quad c_1 + c_2 \|z\| + c_3 \|z\|^2, \tag{17}$$

where $z \in \mathbb{R}^2$ is defined as $z \triangleq \begin{bmatrix} e & r \end{bmatrix}^T$ and $c_1, c_2, c_3 \in \mathbb{R}_{\geq 0}$ are known constants.

Based on (14) and (16) and the subsequent stability analysis, the switched FES control input is designed as

$$u_s \quad = \quad \sigma_s \left(k_1 r + \left[k_2 + k_3 \|z\| + k_4 \|z\|^2 \right] \operatorname{sgn}(r) \right), \tag{18}$$

where $k_1, k_2, k_3, k_4 \in \mathbb{R}_{\geq 0}$ are selectable constant control gains and the switching signal, $\sigma_s : \mathcal{Q}_l \to \{0, 1\}$, for leg stimulation is designed as

$$\sigma_s \quad \triangleq \quad \begin{cases} 1 & \text{if } q_l \in \mathcal{Q}_{FES} \\ 0 & \text{if } q_l \notin \mathcal{Q}_{FES} \end{cases} . \tag{19}$$

The result in [25] indicates that position tracking can be difficult on the split-crank bicycle, particularly in the hamstring stimulation region, due to the lack of assistance being provided from the quadriceps femoris muscle group from the opposing leg in a standard joined-crank cycle. This difficulty often leads to saturated stimulation of the hamstring muscle group causing rapid onset of fatigue, poor tracking, and potential participant discomfort. A switching signal for leg-cycle motor effort, $\sigma_e : \mathcal{Q}_l \times \{0,1\} \to [0,1]$, is designed as

$$\sigma_e \triangleq \begin{cases} 1 & \text{if } q_l \notin \mathcal{Q}_{FES} \\ \prod_{m\in\mathcal{M}} (1-\beta_m\sigma_m) & \text{if } q_l \in \mathcal{Q}_{FES} \end{cases}, \quad (20)$$

where $\beta_m \in [0,1]$, $\forall m \in \mathcal{M}$ are constants selected to provide variable motor assistance throughout the FES regions. Based on (14) and (16) and the subsequent stability analysis, the switched leg-cycle motor controller is designed as

$$u_{e_l} = \sigma_e \left(k_5 r + \left[k_6 + k_7 \|z\| + k_8 \|z\|^2 \right] \operatorname{sgn}(r) \right), \quad (21)$$

where $k_5, k_6, k_7, k_8 \in \mathbb{R}_{\geq 0}$ are selectable constant control gains. To ensure resistivity of the hand-cycle motor effort relative to the operator, in the sense that the hand-cycle motor controller should only be applied to resist, or slow, the angular velocity of the hand-cycle rather than act to pull the hand-cycle toward the leg trajectory, a switching signal, $\sigma_h : \mathbb{R} \times \mathbb{R} \to \{0,1\}$, is designed as

$$\sigma_h \triangleq \begin{cases} 1 & \text{if } e\dot{q}_h > 0 \\ 0 & \text{if } e\dot{q}_h \leq 0 \end{cases}, \quad (22)$$

where, if the system is operating using a standard forward rotation ($\dot{q}_h > 0$) and the angular position of the legs is trailing behind the angular position of the hands ($e > 0$), then motor effort will be applied about the hand crank. Likewise, if the system is operating with reversed rotation ($\dot{q}_h < 0$) and the angular position of the legs is larger than the angular position of the hands ($e < 0$), then the motor effort will be applied. The switched hand-cycle error feedback motor controller is designed as

$$u_{e_h} = -k_9 \sigma_h e. \quad (23)$$

Substituting (18), (21), and (23) into (19) produces the closed-loop error system

$$\begin{aligned} M_l \dot{r} = {} & \chi - V_l r - e - B_M \sigma_s k_1 r - B_{e_l} \sigma_e k_5 r \\ & - B_M \sigma_s \left[k_2 + k_3 \|z\| + k_4 \|z\|^2 \right] \operatorname{sgn}(r) \\ & - B_{e_l} \sigma_e \left[k_6 + k_7 \|z\| + k_8 \|z\|^2 \right] \operatorname{sgn}(r) \\ & - M_l M_h^{-1} B_{e_h} k_9 \sigma_h e. \end{aligned} \quad (24)$$

IV. Stability Analysis

A Lyapunov-based stability analysis is provided for two cases; when $q_l \notin \mathcal{Q}_{FES}$ and when $q_l \in \mathcal{Q}_{FES}$. A theorem is presented for each case to evaluate the stability of the switched leg-cycle system. Switching times are denoted by $\{t_n^i\}$, $i \in \{s, e\}$, $n \in \{0, 1, 2, ...\}$ where each t_n^i represents the n-th time that the leg-cycle system switches to the stimulation region (denoted by $i = s$) or the electric motor only region (denoted by $i = e$). For the teleoperation master system (i.e. the hand-cycle) to be considered stable, it must be shown that all system states are bounded [23].

Let $V : \mathbb{R} \to \mathbb{R}_{\geq 0}$ be a positive definite, radially unbounded, common Lyapunov function candidate defined as

$$V = \tfrac{1}{2} M_l r^2 + \tfrac{1}{2} e^2, \quad (25)$$

such that

$$\lambda_1 \|z\|^2 \leq V \leq \lambda_2 \|z\|^2, \quad (26)$$

where $\lambda_1 = min\left\{\frac{1}{2}, \frac{1}{2} c_{\underline{M}_l}\right\}$ and $\lambda_2 = max\left\{\frac{1}{2}, \frac{1}{2} c_{\overline{M}_l}\right\}$. Due to the discontinuous nature of the FES control input and motor controllers, the time derivative of (25) exists almost everywhere (a.e.) within $t \in [t_0, \infty)$ and $\dot{V}(z) \overset{\text{a.e.}}{\in} \dot{\tilde{V}}(z)$, where $\dot{\tilde{V}}$ is the generalized time derivative of (25). Let $z(t)$ for $t \in [t_0, \infty)$ be a Filippov solution to the differential inclusion $\dot{z} \in K[h](z)$, where $h \colon \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $h \triangleq \begin{bmatrix} \dot{e} & \dot{r} \end{bmatrix}^T$ [28]. Solving (14) for $\dot{e}$, using (17), (21), (23), and (24), Property 1, and canceling common terms produces

$$\begin{aligned} \dot{\tilde{V}} \subseteq {} & \chi r - B_M \sigma_s k_1 r^2 - B_{e_l} \sigma_e k_5 r^2 \\ & - M_l M_h^{-1} B_{e_h} k_9 \sigma_h e r - \alpha e^2 \\ & - B_M \sigma_s \|r\| \left(k_2 + k_3 \|z\| + k_4 \|z\|^2 \right) \\ & - B_{e_l} \sigma_e \|r\| \left(k_6 + k_7 \|z\| + k_8 \|z\|^2 \right). \end{aligned} \quad (27)$$

Theorem 1. *For $q_l \notin \mathcal{Q}_{FES}$, where $t \in [t_n^e, t_{n+1}^s)$, the position and cadence error systems are globally exponentially stable in the sense that*

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} \|z(t_n^e)\| \exp\left[-\frac{min(\psi_1, \psi_2)}{2\lambda_2} (t - t_n^e) \right], \quad (28)$$

provided the following gain conditions are met

$$k_6 > \frac{c_1}{B_{\underline{e}_l}}, \; k_7 > \frac{c_2}{B_{\underline{e}_l}}, \; k_8 > \frac{c_3}{B_{\underline{e}_l}}, \quad (29)$$

$$k_5 > k_9 \frac{c_{\overline{M}_l} B_{\overline{e}_h}}{2 c_{\underline{M}_h} B_{\underline{e}_l}}, \quad (30)$$

$$\alpha > k_9 \frac{c_{\overline{M}_l} B_{\overline{e}_h}}{2 c_{\underline{M}_h}}. \quad (31)$$

Proof: When $q_l \notin \mathcal{Q}_{FES}$, $\sigma_s = 0$ and $\sigma_e = 1$. Eliminating u_s, using Properties 2 and 8, and recognizing that $\sigma_h \in \{0,1\}$ $\forall t$, (27) becomes

$$\begin{aligned} \dot{V} \overset{a.e.}{\leq} {} & \|r\| \left(c_1 + c_2 \|z\| + c_3 \|z\|^2 \right) - B_{\underline{e}_l} k_5 \|r\|^2 \\ & - B_{\underline{e}_l} \|r\| \left(k_6 + k_7 \|z\| + k_8 \|z\|^2 \right) \\ & + \tfrac{c_{\overline{M}_l}}{c_{\underline{M}_h}} B_{\overline{e}_h} k_9 \|r\| \|e\| - \alpha \|e\|^2. \end{aligned} \quad (32)$$

Selecting the control gains as in (29) gives

$$\dot{V} \overset{a.e.}{\leq} -B_{\underline{e}_l} k_5 \|r\|^2 - \alpha \|e\|^2 + \tfrac{c_{\overline{M}_l}}{c_{\underline{M}_h}} B_{\overline{e}_h} k_9 \|r\| \|e\|. \quad (33)$$

By Young's Inequality,

$$\dot{V} \overset{a.e.}{\leq} -B_{\underline{e}_l}k_5\|r\|^2-\alpha\|e\|^2 + \frac{k_9 c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}}\left(\|r\|^2+\|e\|^2\right). \tag{34}$$

Selecting the gains k_5 and α as in (30) and (31), respectively, and defining $\psi_1 \triangleq k_5 - k_9\frac{c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}B_{\underline{e}_l}}$ and $\psi_2 \triangleq \alpha - k_9\frac{c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}}$, then

$$\dot{V} \overset{a.e.}{\leq} -\psi_1 r^2 - \psi_2 e^2, \tag{35}$$

is a negative definite function. Using (26) it can be shown that $\dot{V} \overset{a.e.}{\leq} -\frac{min(\psi_1,\psi_2)}{\lambda_2}V$. Solving the differential inequality, using (26), and solving for $\|z(t)\|$ yields (28). From (25) and (35) it can be seen that $e,r\in\mathcal{L}_\infty$, $\forall t\in[t_n^e,t_{n+1}^s)$. Thus, from (18), (21), and (23), it can be seen that $u_s,\ u_{e_l},\ u_{e_h}\in\mathcal{L}_\infty$, $\forall t\in[t_n^e,t_{n+1}^s)$, respectively. ■

Theorem 2. *For $q_l\in\mathcal{Q}_{FES}$, where $t\in[t_n^s,t_{n+1}^e)$, the position and cadence error systems are globally exponentially stable in the sense that*

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|z(t_n^s)\|\exp\left[-\frac{min(\psi_2,\psi_3)}{2\lambda_2}(t-t_n^s)\right], \tag{36}$$

provided the following gain conditions are met,

$$k_2>\frac{c_1}{B_{\underline{M}}},\ k_3>\frac{c_2}{B_{\underline{M}}},\ k_4>\frac{c_3}{B_{\underline{M}}}, \tag{37}$$

$$k_1>k_9\frac{c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}B_{\underline{M}}}. \tag{38}$$

Proof: When $q_l\in\mathcal{Q}_{FES}$, $\sigma_s=1$ and $\sigma_e=\prod_{m\in\mathcal{M}}(1-\beta_m\sigma_m)$. Using Properties 2 and 8, recognizing that $\sigma_h\in\{0,1\}$ $\forall t$, (27) becomes

$$\begin{aligned}\dot{V} \overset{a.e.}{\leq}\ & \|r\|\left(c_1+c_2\|z\|+c_3\|z\|^2\right)-B_{\underline{M}}k_1\|r\|^2\\ & -B_{\underline{M}}\|r\|\left(k_2+k_3\|z\|+k_4\|z\|^2\right)\\ & -B_{\underline{e}_l}\left[\prod_{m\in\mathcal{M}}(1-\beta_m\sigma_m)\right]k_5\|r\|^2\\ & -B_{\underline{e}_l}\left[\prod_{m\in\mathcal{M}}(1-\beta_m\sigma_m)\right]\|r\|(k_6\\ & +k_7\|z\|+k_8\|z\|^2)-\alpha\|e\|^2\\ & +\frac{k_9 c_{\overline{M}_l}B_{\overline{e}_h}}{c_{\underline{M}_h}}\|r\|\|e\|.\end{aligned} \tag{39}$$

Choosing the control gains as in (37) and recognizing that $0\leq\sigma_e\leq 1\,\forall t$ gives

$$\dot{V} \overset{a.e.}{\leq} -B_{\underline{M}}k_1\|r\|^2-\alpha\|e\|^2 + \frac{k_9 c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}}\left(\|r\|^2+\|e\|^2\right). \tag{40}$$

Selecting the gains α and k_1 as in (31) and (38), respectively, and defining $\psi_3\triangleq k_1-k_9\frac{c_{\overline{M}_l}B_{\overline{e}_h}}{2c_{\underline{M}_h}B_{\underline{M}}}$, then

$$\dot{V} \overset{a.e.}{\leq} -\psi_3 r^2-\psi_2 e^2, \tag{41}$$

is a negative definite function. From (41), using similar methods as in Theorem 1, produces (36). From (25) and (41) it can be seen that $e,r\in\mathcal{L}_\infty$, $\forall t\in[t_n^s,t_{n+1}^e)$. Thus, from (18), (21), and (23), it can be seen that $u_s,\ u_{e_l},\ u_{e_h}\in\mathcal{L}_\infty$, $\forall t\in[t_n^s,t_{n+1}^e)$, respectively. ■

Corollary 1. *From Theorems 1 and 2, the equilibrium point $z=0$ of the combined leg and hand-cycle system is globally exponentially stable $\forall t$ in the sense that*

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\|z(t_0)\|\exp\left[-\frac{\zeta}{2\lambda_2}(t-t_0)\right],\ \forall t. \tag{42}$$

Proof: Defining $\zeta\triangleq min(\psi_1,\psi_2,\psi_3)$, from (26), (35) and (41), and solving the resulting differential inequality, it can be shown that

$$\dot{V}\overset{a.e.}{\leq}-\zeta\|z(t)\|^2\overset{a.e.}{\leq}-\frac{\zeta}{\lambda_2}V,$$

$$V(t) \leq V(t_0)\exp\left[-\frac{\zeta}{\lambda_2}(t-t_0)\right]. \tag{43}$$

Using (26) and solving the differential inequality in (43) for $\|z(t)\|$, as in [29], yields (42). Recalling that $z\triangleq\begin{bmatrix}e & r\end{bmatrix}^T$, $|e|,|r|\leq\sqrt{\frac{\lambda_2}{\lambda_1}}\|z(t_0)\|\exp\left[-\frac{\zeta}{2\lambda_2}(t-t_0)\right]$, $\forall t$. From (18), (21), and (23), $u_s,\ u_{e_l},\ u_{e_h},\in\mathcal{L}_\infty$, $\forall t$. ■

V. Experiments

Due to the current Covid-19 pandemic, and the subsequent closing of university lab spaces, progress towards human subject testing was halted. Testing is expected to be completed after the university and labs have be reopened.

VI. Conclusion

The telerobotic hand-cycle system developed in this paper, in conjunction with the addition of variable motor assistance in unstimulated regions, was designed to improve FES rehabilitative cycling techniques by introducing coordinated motion between the upper and lower limbs in a way that enables the rider to obtain feedback of the leg cycling capability and to have direct control over the desired cadence and position. To maintain coordination, and positively influence a participant's neural plasticity, the telerobotic system was designed with variable motor assistance in the FES regions to ensure that it is strongly coupled. Lyapunov-based analysis techniques for switched systems were used to prove global exponential stability of the equilibrium point of the combined hand and leg-cycle systems provided that certain gain conditions are met.

Future work will seek to account for time-delays in communication within the telerobotic system, allowing a physical therapist to operate the hand-cycle remotely while experiencing force-feedback from the leg-cycle components, thus "feeling" where a patient might be experiencing difficulties in movement. The variable motor assistance provided through the FES regions could also be modified to be a function of the ratio between current stimulation intensity and maximum intensity. By doing so, the assistive motor effort will automatically be increased if a muscle group approaches saturation levels, ultimately slowing the onset of muscle fatigue and increasing the time of rehabilitation sessions by allowing the participant to adjust their cadence to reduce stimulation intensity and maintain their chosen level of comfort. Experimental results are pending.

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