

Mixed EW-QCD two-loop amplitudes for $q\bar{q} \rightarrow \ell^+\ell^-$ and γ_5 scheme independence of multi-loop corrections

Matthias Heller,^a Andreas von Manteuffel,^b Robert M. Schabinger^b
and Hubert Spiesberger^c

^a*PRISMA⁺ Cluster of Excellence, Institut für Kernphysik, Johannes Gutenberg Universität,
55099 Mainz, Deutschland*

^b*Department of Physics and Astronomy, Michigan State University,
East Lansing, Michigan 48824, U.S.A.*

^c*PRISMA⁺ Cluster of Excellence, Institut für Physik, Johannes Gutenberg Universität,
55099 Mainz, Deutschland*

E-mail: maheller@students.uni-mainz.de, vmante@msu.edu,
schabing@msu.edu, spiesber@uni-mainz.de

ABSTRACT: We perform a dedicated study of the $q\bar{q}$ -initiated two-loop electroweak-QCD Drell-Yan scattering amplitude in dimensional regularization schemes for vanishing light quark and lepton masses. For the relative order α and α_s one-loop Standard Model corrections, details of our comparison to the original literature are given. The infrared pole terms of the mixed two-loop amplitude are governed by a known generalization of the dipole formula and we show explicitly that exactly the same two-loop polarized hard scattering functions are obtained in both the standard 't Hooft-Veltman-Breitenlohner-Maison γ_5 scheme and Kreimer's anticommuting γ_5 scheme.

KEYWORDS: NLO Computations, QCD Phenomenology

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1 Introduction

A precise theoretical understanding of the neutral- and charged-current Drell-Yan processes [1] in Standard Model perturbation theory is essential to the success of the Large Hadron Collider physics program. It is therefore unsurprising that significant resources have been allocated, over the course of a multi-generational effort, to bring the higher-order corrections to the Drell-Yan scattering processes under control. The total cross section in pure Quantum Chromodynamics (QCD) was successfully calculated through to next-to-next-to-next-to-leading order in the strong coupling constant for both the neutral- and charged-current processes [2–8]. On the electroweak side, neutral gauge boson production and decay has been calculated through to next-to-next-to-leading order in pure Quantum Electrodynamics (QED) [9, 10], but full electroweak (EW) corrections are known only at next-to-leading order [11–19]. Due to the size of the next-to-leading order electroweak corrections, it has long been of interest to calculate the mixed EW-QCD corrections of relative order $\alpha\alpha_s$ as well. Indeed, many partial results in this direction have been put forth over the last decade [20–27], but a complete calculation in the regime of large lepton pair invariant mass is not available yet.

Despite the significant attention these two-loop EW-QCD calculations have received, it has long remained a challenge to include the non-factorizable two-loop virtual corrections. For many years, the primary technical problem was the evaluation of the massive two-loop box-type master integrals which emerge from the most complicated Feynman diagrams (see [28–30] for various results). Recently, some of us successfully evaluated the complete set of two-loop box-type master integrals relevant to the non-factorizable corrections as linear combinations of standard multiple polylogarithms in the physical regions of phase space [31]. In this paper, we take an important step towards complete relative order $\alpha\alpha_s$ corrections by elucidating the treatment of γ_5 for the box-type diagrams and deriving the most complicated unknown parts of the order $\alpha^2\alpha_s$ two-loop helicity amplitudes for $q\bar{q} \rightarrow \ell^+\ell^-$. Here, we do not include contributions involving the top-quark, light fermionic loops, or $b\bar{b}$ initial states, and focus on the region of phase space with center-of-mass energy above all two-particle thresholds.

As has been known since the earliest higher-order studies of the Drell-Yan process, it is important to have a solid understanding of the γ_5 problem of dimensional regularization [32–36] if one wishes to calculate *all* Feynman diagrams at a given order, including those where a W or Z connect the initial- and final-state fermion lines. As will be discussed in detail in what follows, care must be taken in practical calculations to consistently include scheme-dependent higher-order-in- ϵ terms, see also [37] for a recent review. In order to establish confidence in our results, we carried out two independent calculations, one where 't Hooft-Veltman-Breitenlohner-Maison's (HVBM's) γ_5 scheme [32–34, 38] was employed and one where Kreimer's γ_5 scheme [39–42] was employed. For our calculations, Kreimer's γ_5 scheme was practically superior because it does not require the introduction of finite counterterms to restore the chiral symmetry of the Standard Model.

This article is organized as follows. In sections 2.1–2.4, a general discussion of the subtleties involved with the application of dimensional regularization in the presence of chiral

couplings is given, both for HVBM's γ_5 scheme and Kreimer's γ_5 scheme. In sections 2.5–2.7 our renormalization scheme for the coupling constants, the wavefunctions, and the particle masses is described. In section 2.8, we discuss the finite renormalizations we perform in HVBM's γ_5 scheme. In section 3.1, we define the scattering process we consider more precisely. In section 3.2, supplemented by appendix A, we describe our general approach to the form factor decomposition of the amplitude, and, in sections 3.3 and 3.4, we give further technical details of the numerator algebra scripts we used to carry out our Feynman diagram calculations in the different γ_5 schemes. In section 3.5, we elucidate the infrared structure of the one- and two-loop scattering amplitudes we calculate and define *hard scattering functions* which, at leading order in the parameter of dimensional regularization, encode physical information about the kernel of the virtual corrections which remains after infrared subtraction. In sections 4.1–4.3, supplemented by appendices B.1 and B.2, explicit results for the one-loop Standard Model corrections to the Drell-Yan process are given, together with a detailed comparison to the original literature. In sections 5.1–5.3, we give details specifically relevant to the two-loop mixed EW-QCD calculation. In section 6.1, we provide a recipe to pass from our form factors to scattering amplitudes for states of definite helicity and, in section 6.2, visualizations of our final results for the order $\alpha\alpha_s$, order α^2 , and order $\alpha^2\alpha_s$ polarized hard scattering functions. Finally, in section 7, we summarize our findings.

2 Regularization and renormalization

2.1 Universal features of dimensional regularization

In this section, our aim is to review some background and motivation for the more technical, scheme-dependent discussions of chiral couplings in dimensional regularization which follow in sections 2.2 and 2.3. The simultaneous dimensional regularization of ultraviolet and infrared divergences [38, 43–46] has proven to be an essential tool for the calculation of higher-order perturbative scattering amplitudes in gauge theories. For a gauge theory Lagrange density defined in four spacetime dimensions, the method purports to provide a prescription for the continuation of all off-shell Feynman rules from four to d dimensions. All higher-order correlation functions and scattering amplitudes in the model under consideration are then taken to be meromorphic functions of d in a complex domain containing the point $d = 4$. Accordingly, the Fourier integrals which appear at higher orders in perturbation theory are continued from four dimensions to $d = 4 - 2\epsilon$ dimensions as

$$\int \frac{d^4 k_i}{(2\pi)^4} \longrightarrow (\mu^2)^\epsilon \int \frac{d^{4-2\epsilon} k_i}{(2\pi)^{4-2\epsilon}}, \quad (2.1)$$

where ϵ is the parameter of dimensional regularization and μ is the 't Hooft scale, a unit of mass introduced to maintain a dimensionless action. The ultraviolet and infrared singularities of higher-order correlation functions and scattering amplitudes manifest themselves as poles in ϵ which ultimately cancel only in higher-order corrections to physical observables.

While some of the prescribed continuations of the Lorentz and Dirac algebra such as

$$g^{\mu\nu} g_{\mu\nu} = 4 - 2\epsilon, \tag{2.2}$$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \mathbf{1}, \tag{2.3}$$

$$\text{and } \gamma^\mu \gamma_\mu = \frac{1}{2} g^{\mu\nu} \{\gamma_\mu, \gamma_\nu\} = g^{\mu\nu} g_{\mu\nu} = 4 - 2\epsilon \tag{2.4}$$

feel very natural, other aspects of the program are far more subtle and require careful explanation. Before proceeding to a discussion of these subtleties, we state a universal identity for the d -dimensional contraction of two Dirac matrices with n d -dimensional Dirac matrices interspersed which will be useful later on:

$$\gamma^\mu \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_\mu = 2 \sum_{i=1}^n (-1)^{i+n} \gamma_{\nu_i} \gamma_{\nu_1} \cdots \gamma_{\nu_{i-1}} \gamma_{\nu_{i+1}} \cdots \gamma_{\nu_n} + (4 - 2\epsilon) (-1)^n \gamma_{\nu_1} \cdots \gamma_{\nu_n}. \tag{2.5}$$

This identity is simply obtained by repeatedly applying eq. (2.3) to move the γ_μ factor to the left until it is finally possible to apply eq. (2.4). The reader can easily check for the first few values of n that the well-known textbook results (see e.g. [47]) are all encoded in eq. (2.5).

As has been clear from its inception [38, 44], it is technically challenging to handle chiral couplings in dimensional regularization. This is primarily because there is no canonical way to continue the usual four-dimensional definition of γ_5 ,

$$\gamma_5 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3, \tag{2.6}$$

into d dimensions. It was emphasized in [38] by 't Hooft and Veltman that one *cannot* expect to maintain the usual anticommutation relation

$$\{\gamma_\mu, \gamma_5\} = 0 \tag{2.7}$$

and, at the same time, work with traces of Dirac matrices which enjoy all of the usual mathematical properties. In particular, a cyclic Dirac trace was explicitly assumed alongside eq. (2.7) in the algebraic proof that the trace of $\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5$ must either vanish in dimensional regularization or be subject to a modified set of algebraic rules (see e.g. [48] for a review). 't Hooft and Veltman influenced most later authors to modify the γ_5 anticommutation relation, but, as was pointed out by Kreimer [39], retaining eq. (2.7) while abandoning the cyclicity of the Dirac trace is another path towards mathematical consistency. It would at this stage be reasonable to expect a concrete prescription for the d -dimensional continuation of classical trace formulae involving γ_5 such as

$$\text{tr} \{ \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5 \} = -4i \epsilon_{\mu\nu\rho\sigma} \tag{2.8}$$

in some refined dimensional regularization scheme.

In fact, there *is* no d -dimensional replacement for γ_5 or the Levi-Civita tensor, $\epsilon_{\mu\nu\rho\sigma}$, in the dimensional regularization schemes we consider; rather, one simply accepts that eq. (2.6), eq. (2.8), and the familiar properties

$$\gamma_5^2 = \mathbf{1} \quad \text{and} \quad \gamma_5^\dagger = \gamma_5 \tag{2.9}$$

remain unchanged in d dimensions. That γ_5 and $\varepsilon_{\mu\nu\rho\sigma}$ are intrinsically four-dimensional objects inevitably forces one to distinguish a four-dimensional subspace and sacrifice somewhat for what concerns the construction of a manifestly Lorentz-covariant formalism. In this work, it is frequently convenient to explicitly distinguish the four-dimensional and (-2ϵ) -dimensional subspaces of d -dimensional Lorentz indices in perturbative calculations. Following [48], we write

$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \tag{2.10}$$

$$\text{and } \gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu, \tag{2.11}$$

where bars denote four-dimensional objects and hats denote (-2ϵ) -dimensional objects. Similarly, we write the usual four-dimensional identities,

$$\varepsilon^{\mu\nu\rho\sigma} \varepsilon_{\mu\nu\rho\sigma} = -24, \tag{2.12}$$

$$\varepsilon^{\mu\nu\rho}{}_\alpha \varepsilon_{\mu\nu\rho\beta} = -6 \bar{g}_{\alpha\beta}, \tag{2.13}$$

$$\varepsilon^{\mu\nu}{}_{\alpha\delta} \varepsilon_{\mu\nu\beta\eta} = -2 (\bar{g}_{\alpha\beta} \bar{g}_{\delta\eta} - \bar{g}_{\alpha\eta} \bar{g}_{\beta\delta}), \tag{2.14}$$

$$\begin{aligned} \varepsilon^\mu{}_{\alpha\delta\kappa} \varepsilon_{\mu\beta\eta\lambda} &= \bar{g}_{\alpha\lambda} \bar{g}_{\beta\kappa} \bar{g}_{\delta\eta} - \bar{g}_{\alpha\lambda} \bar{g}_{\beta\delta} \bar{g}_{\eta\kappa} - \bar{g}_{\alpha\eta} \bar{g}_{\beta\kappa} \bar{g}_{\delta\lambda} \\ &\quad + \bar{g}_{\alpha\beta} \bar{g}_{\delta\lambda} \bar{g}_{\eta\kappa} + \bar{g}_{\alpha\eta} \bar{g}_{\beta\delta} \bar{g}_{\eta\kappa} - \bar{g}_{\alpha\beta} \bar{g}_{\delta\eta} \bar{g}_{\eta\kappa}, \end{aligned} \tag{2.15}$$

$$\begin{aligned} \varepsilon_{\alpha\delta\kappa\tau} \varepsilon_{\beta\eta\lambda\chi} &= -\bar{g}_{\alpha\chi} \bar{g}_{\beta\tau} \bar{g}_{\delta\lambda} \bar{g}_{\eta\kappa} + \bar{g}_{\alpha\chi} \bar{g}_{\beta\tau} \bar{g}_{\delta\eta} \bar{g}_{\kappa\lambda} + \bar{g}_{\alpha\lambda} \bar{g}_{\beta\kappa} \bar{g}_{\delta\lambda} \bar{g}_{\eta\tau} - \bar{g}_{\alpha\chi} \bar{g}_{\beta\delta} \bar{g}_{\eta\tau} \bar{g}_{\kappa\lambda} \\ &\quad - \bar{g}_{\alpha\chi} \bar{g}_{\beta\kappa} \bar{g}_{\delta\eta} \bar{g}_{\lambda\tau} + \bar{g}_{\alpha\chi} \bar{g}_{\beta\delta} \bar{g}_{\eta\kappa} \bar{g}_{\lambda\tau} + \bar{g}_{\alpha\lambda} \bar{g}_{\beta\tau} \bar{g}_{\delta\chi} \bar{g}_{\eta\kappa} - \bar{g}_{\alpha\eta} \bar{g}_{\beta\tau} \bar{g}_{\delta\chi} \bar{g}_{\eta\kappa} \\ &\quad - \bar{g}_{\alpha\lambda} \bar{g}_{\beta\kappa} \bar{g}_{\delta\chi} \bar{g}_{\eta\tau} + \bar{g}_{\alpha\beta} \bar{g}_{\delta\chi} \bar{g}_{\eta\tau} \bar{g}_{\kappa\lambda} + \bar{g}_{\alpha\eta} \bar{g}_{\beta\kappa} \bar{g}_{\delta\lambda} \bar{g}_{\eta\tau} - \bar{g}_{\alpha\beta} \bar{g}_{\delta\lambda} \bar{g}_{\eta\tau} \bar{g}_{\kappa\lambda} \\ &\quad - \bar{g}_{\alpha\lambda} \bar{g}_{\beta\tau} \bar{g}_{\delta\eta} \bar{g}_{\kappa\chi} + \bar{g}_{\alpha\eta} \bar{g}_{\beta\tau} \bar{g}_{\delta\lambda} \bar{g}_{\kappa\chi} + \bar{g}_{\alpha\lambda} \bar{g}_{\beta\delta} \bar{g}_{\eta\tau} \bar{g}_{\kappa\chi} - \bar{g}_{\alpha\beta} \bar{g}_{\delta\lambda} \bar{g}_{\eta\tau} \bar{g}_{\kappa\chi} \\ &\quad - \bar{g}_{\alpha\eta} \bar{g}_{\beta\delta} \bar{g}_{\kappa\chi} \bar{g}_{\lambda\tau} + \bar{g}_{\alpha\beta} \bar{g}_{\delta\eta} \bar{g}_{\kappa\chi} \bar{g}_{\lambda\tau} + \bar{g}_{\alpha\lambda} \bar{g}_{\beta\kappa} \bar{g}_{\delta\eta} \bar{g}_{\tau\chi} - \bar{g}_{\alpha\lambda} \bar{g}_{\beta\delta} \bar{g}_{\eta\kappa} \bar{g}_{\tau\chi} \\ &\quad - \bar{g}_{\alpha\eta} \bar{g}_{\beta\kappa} \bar{g}_{\delta\lambda} \bar{g}_{\tau\chi} + \bar{g}_{\alpha\beta} \bar{g}_{\delta\lambda} \bar{g}_{\eta\kappa} \bar{g}_{\tau\chi} + \bar{g}_{\alpha\eta} \bar{g}_{\beta\delta} \bar{g}_{\kappa\lambda} \bar{g}_{\tau\chi} - \bar{g}_{\alpha\beta} \bar{g}_{\delta\eta} \bar{g}_{\kappa\lambda} \bar{g}_{\tau\chi}, \end{aligned} \tag{2.16}$$

using $\bar{g}_{\mu\nu}$ to denote the four-dimensional metric tensor.

In both schemes we consider, eqs. (2.12)–(2.16) force us to apply eq. (2.10) to *split indices*, $\bar{g}_{\mu\nu} k_i^\mu k_j^\nu = k_i \cdot k_j - \hat{k}_i \cdot \hat{k}_j$. Split loop momenta lead to well-known technical complications: in sufficiently-complicated Feynman diagrams, Dirac traces produce d -dimensional Feynman integrals with numerator insertions of $\hat{k}_i \cdot \hat{k}_j$. Fortunately, general approaches which involve the introduction of dimensionally-shifted Feynman integrals [49–51] have been worked out at one loop [52] and at two loops [53]. For the convenience of the reader, we provide details of a modern formulation of these developments in section 2.4 below.

As reviewed in [54], many authors have in the past adopted the so-called naive γ_5 scheme of [35] (see also [55]) for one-loop electroweak calculations. The naive scheme amounts to a set of *ad hoc* rules for γ_5 manipulations designed to lead to correct results while avoiding the technical complications which might arise in a more mathematically-rigorous handling of chiral couplings. Historically, this was justified at the one-loop level by checking *a posteriori* that the relevant Ward identities check out. Naturally, in order to calculate with confidence at higher orders in Standard Model perturbation theory, we feel that it is of considerable importance to adopt a mathematically consistent framework. It should be stressed that the naive γ_5 scheme is equivalent to Kreimer’s γ_5 scheme (introduced

below) for what concerns the calculation of the fermion self-energies and vertex form factors considered in this paper. As we shall see in what follows, the Feynman diagrams of box type are really the only ones in $2 \rightarrow 2$ scattering processes with sensitivity to the γ_5 problem.

HVBM's γ_5 scheme [32–34, 38] and Kreimer's γ_5 scheme [39–42] are the two variants of dimensional regularization we work with in this paper. The precise treatment of γ_5 in these schemes has a profound impact on both the Feynman rules and the handling of Dirac traces which we discuss at greater length in sections 2.2 and 2.3.¹ Indeed, four-dimensional Dirac trace identities which rely on γ_5 manipulations must be, depending on the γ_5 scheme, either carefully reformulated or abandoned. Consider, for example, the algebraic proof given in [47] that the trace of an odd number of Dirac matrices must be zero. For n odd, we have

$$\begin{aligned} \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} &= \text{tr} \{ \gamma_5 \gamma_5 \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} && (\gamma_5^2 = \mathbf{1}) \\ &= (-1)^n \text{tr} \{ \gamma_5 \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_5 \} && (\text{anticommutation relation (2.7)}) \\ &= -\text{tr} \{ \gamma_5 \gamma_5 \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} && (n \text{ odd, trace cyclicity}) \\ &= -\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} && (\gamma_5^2 = \mathbf{1}), \end{aligned} \tag{2.17}$$

which implies $\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} = 0$ as desired. As should be now be clear, the above proof is valid *only* in four spacetime dimensions; different lines of reasoning must be adopted in $d = 4 - 2\epsilon$ dimensions. As we shall see, in HVBM's γ_5 scheme, eq. (2.5) facilitates a simple proof of the proposition by induction, whereas, in Kreimer's γ_5 scheme, the trace of an odd number of Dirac matrices is argued to be zero in d dimensions by appealing to a mathematical property of the infinite-dimensional Dirac algebra [39].

2.2 HVBM's prescription for γ_5 in dimensional regularization

In HVBM's scheme, chiral couplings are treated in a consistent way in d dimensions by retaining the four-dimensional definition of γ_5 , eq. (2.6), at the expense of its anticommutation relation with the standard Dirac matrices. In place of eq. (2.7), one can write

$$\{ \gamma_\mu, \gamma_5 \} = 0 \quad \text{for } \mu = 0, 1, 2, \text{ and } 3 \tag{2.18}$$

$$\text{and} \quad [\gamma_\mu, \gamma_5] = 0 \quad \text{in the } (-2\epsilon)\text{-dimensional subspace,} \tag{2.19}$$

with the understanding that, as reviewed in e.g. [48], the (-2ϵ) -dimensional subspace is in fact infinite dimensional. For our purposes, we find an equivalent formulation which utilizes eq. (2.11) to express the content of eqs. (2.18) and (2.19) as

$$\{ \bar{\gamma}_\mu, \gamma_5 \} = 0 \tag{2.20}$$

$$\text{and} \quad [\hat{\gamma}_\mu, \gamma_5] = 0, \tag{2.21}$$

$$\text{where} \quad \{ \bar{\gamma}^\mu, \hat{\gamma}^\nu \} = 0, \tag{2.22}$$

to be much more transparent and practically useful.

¹It actually turns out that the *algebraic form* of the standard family of Dirac trace identities which generalizes eq. (2.8) is the same in both schemes, but this requires some explanation.

As advertised above, this modification of the anticommutation relation has an immediate impact on the form of the Standard Model Feynman rules in dimensional regularization. As an example of particular relevance to this work, let us consider the axial part of the coupling of the Z boson to the light quarks and leptons in four dimensions:

$$-e a_f Z^\mu \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f, \quad (2.23)$$

where a_f is a function of the electroweak gauge boson masses and the isospin of fermion f . Suppose now that we wish to consider interaction (2.23) in d dimensions. Due to the fact that we can no longer avail ourselves of eq. (2.7), (2.23) will *fail* to be Hermitian in d dimensions. We can, however, preserve the Hermiticity of the interaction if we first rewrite (2.23) as

$$-e a_f Z^\mu \bar{\psi}_f \frac{1}{2} [\gamma_\mu, \gamma_5] \psi_f \quad (2.24)$$

in four dimensions *before* giving up eq. (2.7) in favor of eqs. (2.18) and (2.19) [48].

HVBM's γ_5 scheme allows for a very familiar treatment of the usual families of Dirac traces. As pointed out already in [38], the trace of an even number of Dirac matrices with d dimensional indices may be conveniently evaluated using the recursive formula

$$\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} = \sum_{i=1}^{n-1} (-1)^{i+1} g_{\nu_1 \nu_{i+1}} \text{tr} \{ \gamma_{\nu_2} \cdots \gamma_{\nu_i} \gamma_{\nu_{i+2}} \cdots \gamma_{\nu_n} \}, \quad (2.25)$$

valid for even n , with termination criterion

$$\text{tr} \{ \mathbf{1} \} = 4. \quad (2.26)$$

Eq. (2.25) is derived in a manner very similar to that of eq. (2.5), but by repeatedly applying eq. (2.3) to move the γ_{ν_1} factor all the way to the right. Finally, the cyclic property of the trace is required to identify the final term, $-\text{tr} \{ \gamma_{\nu_2} \cdots \gamma_{\nu_n} \gamma_{\nu_1} \}$, with the original expression on the left-hand side.

The trace

$$\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_5 \} \quad (2.27)$$

for even n may be evaluated in much the same way by replacing γ_5 by the covariant version of its definition,

$$\gamma_5 = -\frac{i}{4!} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma, \quad (2.28)$$

and then evaluating the resulting trace of $n+4$ Dirac matrices using eqs. (2.25) and (2.26).

As expected, the trace of an odd number of Dirac matrices is still zero in d dimensions. As mentioned above, this can be proven by induction on the trace of eq. (2.5). Let us consider the argument for $n=1$. On the one hand, by trace cyclicity and eq. (2.4), we have

$$\text{tr} \{ \gamma^\mu \gamma_{\nu_1} \gamma_\mu \} = (4 - 2\epsilon) \text{tr} \{ \gamma_{\nu_1} \}. \quad (2.29)$$

On the other hand, applying eq. (2.3) before eq. (2.4), we have

$$\text{tr} \{ \gamma^\mu \gamma_{\nu_1} \gamma_\mu \} = (-2 + 2\epsilon) \text{tr} \{ \gamma_{\nu_1} \}. \quad (2.30)$$

Subtracting eqs. (2.29) and (2.30), we see that, indeed, $\text{tr} \{ \gamma_{\nu_1} \}$ must be zero.

Now, trace cyclicity and eq. (2.4) generically reduce the trace of the left-hand side of eq. (2.5):

$$\text{tr} \{ \gamma^\mu \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_\mu \} = (4 - 2\epsilon) \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \}. \quad (2.31)$$

However, we must simplify the trace of the summand of the first term on the right-hand side of eq. (2.5) using the induction hypothesis before we can finish the proof: it is easy to see that

$$\text{tr} \{ \gamma_{\nu_i} \gamma_{\nu_1} \cdots \gamma_{\nu_{i-1}} \gamma_{\nu_{i+1}} \cdots \gamma_{\nu_n} \} = (-1)^{i-1} \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} \quad (\text{odd } n) \quad (2.32)$$

is a valid identity under the assumption $\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_{n-2}} \} = 0$. eq. (2.32) provides the desired reduction of the right-hand side of the trace of eq. (2.5),

$$\begin{aligned} \text{tr} \{ \gamma^\mu \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_\mu \} &= 2 \sum_{i=1}^n \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} - (4 - 2\epsilon) \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} \\ &= (2n - 4 + 2\epsilon) \text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} \quad (\text{odd } n). \end{aligned} \quad (2.33)$$

Finally, the difference of eqs. (2.31) and (2.33) implies that

$$\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} = 0 \quad (\text{odd } n) \quad (2.34)$$

by induction.

Similarly, eq. (2.28) and the line of reasoning used to establish eq. (2.34) together reveal that

$$\text{tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_5 \} = 0 \quad (\text{odd } n). \quad (2.35)$$

HVBM's prescription for γ_5 has the very unfortunate drawback that the Standard Model's chiral symmetry is violated in all bare perturbative calculations involving chiral couplings. As was originally pointed out by Breitenlohner and Maison in [32–34] and reviewed in [48], it is possible to *restore* the chiral symmetry of the Standard Model as part of the renormalization program by introducing various finite counterterms on top of the usual divergent ones. Let us emphasize here that, in general, one expects *all* interactions in the Standard Model to require finite counterterms at some order in perturbation theory, not just those which explicitly involve γ_5 . Furthermore, these finite counterterms must generically be calculated beyond $\mathcal{O}(\epsilon^0)$ to consistently restore chiral symmetry at still higher orders. We discuss some elementary examples relevant to this article in section 2.8.

For what concerns the particular one- and two-loop calculations considered later on in this work, *Larin's principle* [56] allows us to bypass many finite counterterm computations for fermion self-energies and vertex form factors involving chiral couplings. Larin pointed

out that, for these building blocks, chiral symmetry implies the finite renormalizations do nothing but turn results obtained in HVBM's γ_5 scheme into analogous results obtained in an anticommuting γ_5 scheme. However, as we will discuss in greater detail in section 2.8, the HVBM scheme requires the insertion of explicit finite counterterms and a careful treatment also of the higher-order-in- ϵ terms if box-type Feynman diagrams with chiral couplings contribute to the two-loop or higher-order corrections of interest (see also [57] for a related recent study). Overall, one comes away with the feeling that there must exist a pure dimensional regularization scheme better suited to the study of higher-order perturbative corrections in the Standard Model. In fact, the arduous and subtle character of perturbative calculations carried out in HVBM's γ_5 scheme is what led to the development of Kreimer's prescription for γ_5 .

2.3 Kreimer's prescription for γ_5 in dimensional regularization

In Kreimer's γ_5 scheme, chiral couplings are treated in a consistent way in d dimensions by retaining the four-dimensional definition of γ_5 , eq. (2.6), at the expense of Dirac trace cyclicity. Kreimer argued in [39] that Dirac trace cyclicity is *not* a natural property to retain in d dimensions, due to the fact that the Dirac algebra necessarily becomes infinite dimensional. The standard trace of classical linear algebra, tr , is replaced in Kreimer's γ_5 scheme with one of several possible exotic trace operations which differ only in their *reading point prescriptions*. The reading point prescription effectively specifies with which Dirac matrix to start the non-cyclic trace. In our case, the Lorentz projectors we define in section 3.4 provide a natural anchor to start all Dirac traces involving the external fermion lines. In order to clearly distinguish the trace operations in what follows, we will write Tr rather than tr when using a reading point prescription. While not relevant to the specific higher-order calculations discussed in this paper,² we would average over all possible choices of first entry for internal closed fermion loops. Other reading point prescriptions, such as one which mandates an average over all possible choices of first entry in all cases, could have been chosen as well, but we find our choice particularly convenient.

Kreimer's γ_5 scheme has practical advantages over HVBM's γ_5 scheme. Crucially, it does *not* force a change of algebraic form for any Standard Model Feynman rule when considering interactions in d spacetime dimensions. Employing a consistent reading point prescription then *automatically* preserves the chiral symmetry of the Standard Model in bare perturbative calculations and the complete absence of finite counterterms makes renormalization straightforward. In addition, although box-type diagrams do ultimately force the introduction of split loop momenta also in Kreimer's γ_5 scheme, the bulk of the numerator algebra can be carried out *before* the split enters. This stands in stark contrast to HVBM's γ_5 scheme, where split loop momenta should be introduced immediately in order to progress with the necessary algebraic simplifications efficiently (this assertion will be clarified in section 3.3).

²As explained in section 3.1, we do not consider contributions involving closed fermion loops or top quarks here.

As alluded to above, the standard families of odd Dirac traces still vanish identically when tr is replaced by Tr ,

$$\text{Tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \} = 0 \tag{2.36}$$

$$\text{and} \quad \text{Tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_n} \gamma_5 \} = 0. \tag{2.37}$$

For the standard families of even Dirac traces, [40] defines

$$\text{Tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_{2n}} \} = 4 \sum_{\sigma} (-1)^{\text{sgn}(\sigma)} g_{\nu_{i_1} \nu_{j_1}} \cdots g_{\nu_{i_n} \nu_{j_n}} \quad \text{and} \tag{2.38}$$

$$\text{Tr} \{ \gamma_{\nu_1} \cdots \gamma_{\nu_{2n}} \gamma_5 \} = -\frac{i}{3!} \varepsilon_{\nu_{2n+1} \nu_{2n+2} \nu_{2n+3} \nu_{2n+4}} \sum_{\sigma} (-1)^{\text{sgn}(\sigma)} g_{\nu_{i_1} \nu_{j_1}} \cdots g_{\nu_{i_{n+2}} \nu_{j_{n+2}}} \tag{2.39}$$

where σ is the set of permutations of the $2m$ indices which appear in the sums on the right-hand sides ($m = n$ and $m = n + 2$ in (2.38) and (2.39), respectively), subject to the restrictions

$$1 = i_1 < \dots < i_m, \quad i_k < j_k, \quad \text{for all } k \in \{1 \dots m\}. \tag{2.40}$$

As is perhaps not immediate, these definitions are equivalent to the results one would obtain in HVBM's γ_5 scheme recursively using tr and eqs. (2.25), (2.26), and (2.28). Of course, eqs. (2.25) and (2.26) must ultimately yield the same answer as eq. (2.38) above, as a string of plain Dirac matrices knows nothing about γ_5 . As a corollary, due to eq. (2.28), one immediately arrives at the same conclusion for the Kreimer trace of a string of Dirac matrices with a γ_5 factor appended (see also [42] for a supplementary discussion).

Despite the fact that Kreimer's γ_5 scheme was introduced a long time ago [39–42], it has not been universally accepted and embraced as a superior alternative to HVBM's γ_5 scheme; researchers continue to express doubts about the applicability of Kreimer's γ_5 scheme beyond one loop (see e.g. [54] for a current review of the status of the γ_5 problem). In this article, we present an explicit application of Kreimer's γ_5 scheme to a non-trivial calculation of two-loop box-type diagrams involving chiral couplings and show that it *does* yield physically sensible results. Also worth mentioning is concurrent work by one of us and others, where the two-loop box contributions to $gg \rightarrow ZZ$ featuring a closed top-quark loop have been calculated in Kreimer's γ_5 scheme [58]. The results were compared against calculations in a more commonly used γ_5 scheme and found to agree after ultraviolet and infrared subtraction. While the exact top mass corrections to $gg \rightarrow ZZ$ in next-to-leading order QCD couple the ZZ pair to the massive top quark line and involve anomalous double triangle contributions, the γ_5 dependence of the mixed two-loop EW-QCD corrections to the Drell-Yan process at the level of Feynman diagrams is significantly more complicated.

2.4 d -dimensional Feynman integrals with four-dimensional scalar numerators

In this section, we describe how we deal with particularly awkward terms which often occur in the numerators of d -dimensional Feynman integrals in Standard Model perturbation theory: four-dimensional contractions of d -dimensional loop momenta, $\bar{g}_{\mu\nu} k_i^\mu k_j^\nu$. As mentioned above, we decompose our loop momenta into four-dimensional and (-2ϵ) -dimensional components, $k_i = \bar{k}_i + \hat{k}_i$. This decomposition maps the awkward four-dimensional numerators

to ordinary inverse propagators and “ μ -term” insertions of the form $\hat{k}_i \cdot \hat{k}_j$, as introduced long ago by Bern and Morgan [52]. Below, we review algorithms which allow for a direct treatment of μ -term insertions at one and two loops with dimensionally-shifted Feynman integrals. While the two-loop algorithm we review was, to our knowledge, the first to appear in the literature [53], other approaches such as that of [59] are certainly possible. It seems that some computational effort is simply unavoidable, as both approaches require the calculation of auxiliary integration by parts reductions [60–62].³ We found it very appealing to work with dimensionally-shifted Feynman integrals, as all of the necessary auxiliary integration by parts identities were already calculated by us some time ago for numerical cross-checks on our master integrals [29, 31], facilitated by passing to a finite basis of master integrals [64, 65] through Tarasov dimension shifts [51].

As has been clear at least since the appearance of [52], non-zero one-loop Feynman integrals with μ -term insertions are of the form

$$\mathcal{I}_n^{(1)} \left[(\hat{k}_1 \cdot \hat{k}_1)^r \right] \equiv \int \frac{d^d k_1}{i\pi^{d/2}} \frac{(\hat{k}_1 \cdot \hat{k}_1)^r}{D_1^{\nu_1} \dots D_n^{\nu_n}}, \quad (2.41)$$

where the D_i are the independent propagators of the integral family. Crucially, eq. (2.41) can be rewritten in terms of standard dimensionally-shifted one-loop Feynman integrals with the same propagator structure. By absorbing the $(\hat{k}_1 \cdot \hat{k}_1)^r$ numerator insertion into the integration measure, it is shown in [52] that

$$\mathcal{I}_n^{(1)} \left[(\hat{k}_1 \cdot \hat{k}_1)^r \right] = (-1)^r \frac{\Gamma(r - \epsilon)}{\Gamma(-\epsilon)} \int \frac{d^{d+2r} k_1}{i\pi^{(d+2r)/2}} \frac{1}{D_1^{\nu_1} \dots D_n^{\nu_n}}. \quad (2.42)$$

Although eq. (2.42) involves dimensionally-shifted integrals in an essential way, its form does not depend on the number of legs, the complexity of the kinematics, or the details of the ν_i , observations which suggest a connection to the dimension shift of Tarasov [51]. Since, to our knowledge, a manipulation of the Schwinger representation furnishes the simplest possible proof of Tarasov’s relation, it is natural to expect the Schwinger representation to also allow for an efficient alternative proof of eq. (2.42). To proceed, it is useful to recall that the Tarasov dimension shift is derived in the Schwinger parametrization by simply multiplying and dividing by the first Symanzik polynomial. Furthermore, for an integral topology with t propagator denominators, the first Symanzik polynomial at one loop has the especially simple, universal form

$$\mathcal{U}_t^{(1)} = \alpha_1 + \dots + \alpha_t, \quad (2.43)$$

due to the uniqueness and triviality of the one-loop vacuum (tadpole) integral topology.

The general Schwinger representation is most simply expressed if one first orders the Feynman propagators such that the $\epsilon \rightarrow 0$ limits of their exponents form a monotonically decreasing sequence. We assume that, in this limit, the first n^+ propagators appear raised

³Note that one can circumvent the discussion of this section altogether by performing a classical Passarino-Veltman tensor reduction [63] on all one- and two-loop integrals. In fact, we found it very productive to carry out such an analysis as a comprehensive cross-check of our μ -term implementation.

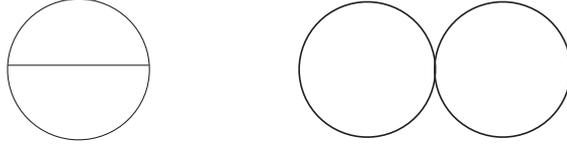


Figure 1. The unique (up to internal mass assignment) non-factorizable (left) and factorizable (right) two-loop vacuum integral topologies.

to positive powers and the remaining n^- propagators appear raised to negative powers (we may assume without loss of generality that the final n^- propagator exponents are in fact non-positive integers). In terms of the Symanzik polynomials of the integral family, $\bar{\mathcal{U}}$ and $\bar{\mathcal{F}}$, we have

$$\mathcal{I}_n^{(L)} = e^{-i\pi \sum_{j=1}^{n^+} \nu_j} \prod_{i=1}^{n^+} \left(\int_0^\infty d\alpha_i \frac{\alpha_i^{\nu_i-1}}{\Gamma(\nu_i)} \right) \prod_{\ell=n^++1}^n \frac{\partial^{|\nu_\ell|}}{\partial \alpha_\ell^{|\nu_\ell|}} \left(\bar{\mathcal{U}}^{-d_0/2+\epsilon} e^{-\bar{\mathcal{F}}/\bar{\mathcal{U}}} \right) \Big|_{\alpha_{n^++1}=\dots=\alpha_{n^++n^-}=0} \quad (2.44)$$

for $d = d_0 - 2\epsilon$, where d_0 is an even positive integer ($d_0 = 4$ is of course the main case of interest to us in this article). In the absence of inverse propagators, the usual result given in terms of the Symanzik polynomials of the integral topology under consideration, \mathcal{U} and \mathcal{F} , is recovered.

Now, as emphasized in [52], generic d -dimensional Feynman propagators at one loop have the form $\bar{k}_1 \cdot \bar{k}_1 + 2P_{ext} \cdot \bar{k}_1 + P_{ext} \cdot P_{ext} - m^2 + \hat{k}_1 \cdot \hat{k}_1$, where P_{ext} denotes some sum of four-dimensional external momenta and m some particle mass. This form motivates a split of the loop momentum integral in the derivation of the ordinary Schwinger parametrization into four-dimensional and (-2ϵ) -dimensional pieces. Due to the fact that, by construction, μ -term insertions are expected to lead to Feynman integrals explicitly suppressed by powers of ϵ , the authors of [53] deduced that the (-2ϵ) -dimensional part of the momentum integral must be proportional to the ϵ -dependent part of eq. (2.44),

$$\int d^{-2\epsilon} \hat{k}_1 e^{\bar{\mathcal{U}}_n^{(1)} \hat{k}_1 \cdot \hat{k}_1} \propto \left(\bar{\mathcal{U}}_n^{(1)} \right)^\epsilon, \quad (2.45)$$

and therefore serve as a generating function for all μ -term insertions via differentiation with respect to $\bar{\mathcal{U}}_n^{(1)}$. Indeed, by differentiating the right-hand side of the above relation r times with respect to $\bar{\mathcal{U}}_n^{(1)}$, we find

$$(-1)^r \frac{\Gamma(r-\epsilon)}{\Gamma(-\epsilon)} \left(\bar{\mathcal{U}}_n^{(1)} \right)^{\epsilon-r}, \quad (2.46)$$

and, by recognizing that the shift $\epsilon \rightarrow \epsilon - r$ corresponds precisely to a shift $d_0 \rightarrow d_0 + 2r$ in eq. (2.44), immediately recover the form of eq. (2.42) as expected.

Fortunately, the approach described above may be fruitfully applied to the study of generic μ integrals at the two-loop level as well. This is primarily because there is a unique non-factorizable two-loop tadpole topology (see figure 1). The main new feature at the two-loop level is that shifts in the propagator exponents appear alongside the various

dimension shifts. At two loops, generic non-zero μ -term insertions can be constructed by taking products of $\hat{k}_1 \cdot \hat{k}_1$, $\hat{k}_2 \cdot \hat{k}_2$, and $(\hat{k}_1 - \hat{k}_2)^2$. Due to the uniqueness of the non-factorizable two-loop tadpole topology, the first Symanzik polynomial of an arbitrary non-factorizable two-loop integral family is of the form

$$\bar{U}_n^{(2)} = T_{k_1^2} T_{k_1 \cdot k_2} + T_{k_2^2} T_{k_1 \cdot k_2} + T_{k_1^2} T_{k_2^2}, \tag{2.47}$$

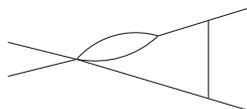
where $T_{k_1 \cdot k_2}$ is the sum of the Schwinger parameters associated to propagators which depend on the dot product $k_1 \cdot k_2$, $T_{k_1^2}$ is the sum of the Schwinger parameters associated to propagators which depend on k_1^2 but not on $k_1 \cdot k_2$, and $T_{k_2^2}$ is the sum of the Schwinger parameters associated to propagators which depend on k_2^2 but not on $k_1 \cdot k_2$. It follows that the two-loop analog of eq. (2.45) ought to be

$$\int d^{-2\epsilon} \hat{k}_1 d^{-2\epsilon} \hat{k}_2 e^{T_{k_1^2} \hat{k}_1 \cdot \hat{k}_1 + T_{k_2^2} \hat{k}_2 \cdot \hat{k}_2 + T_{k_1 \cdot k_2} (\hat{k}_1 - \hat{k}_2)^2} \propto \left(\bar{U}_n^{(2)}\right)^\epsilon, \tag{2.48}$$

a generating function via differentiation by $T_{k_1^2}$, $T_{k_2^2}$, and $T_{k_1 \cdot k_2}$. As at one-loop, the above relation suggests that arbitrary two-loop μ integrals may be rewritten as a linear combination of ordinary Feynman integrals by simply comparing the result of each term of the prescribed differentiation of the right-hand side of (2.48) with the form of eq. (2.44).

Starting at two loops, a subtle complication can arise in our formulation due to the fact that we prefer to write all numerator polynomials as linear combinations of power products of inverse propagators and particle masses. If a propagator depending on e.g. k_1 alone appears in the numerator of a Feynman integral alongside an explicit insertion of $\hat{k}_1 \cdot \hat{k}_1$, it will not work to use the above line of reasoning to eliminate the explicit $\hat{k}_1 \cdot \hat{k}_1$ factor. This is simply because the $k_1^2 = \bar{k}_1 \cdot \bar{k}_1 + \hat{k}_1 \cdot \hat{k}_1$ term of the propagator itself depends on $\hat{k}_1 \cdot \hat{k}_1$. In practice, we found it most convenient to temporarily delay rewriting all scalar products of the form $k_i \cdot p_j$ in terms of inverse propagators. Due to the fact that the $k_i \cdot p_j$ scalar products live in four dimensions, (2.48) may be applied to eliminate all explicit μ terms *before* rewriting the remaining scalar products as linear combinations of propagators.

To clarify the procedure described above, we consider a non-trivial two-loop μ integral from integral family A of [29, 31] which may be directly rewritten using the technology of [53]:



$[D_7 (\hat{k}_1 \cdot \hat{k}_1)^2]$

$$= \int \frac{d^d k_1 d^d k_2}{(i\pi^{d/2})^2} \frac{(k_2 - p_1 - p_2)^2 (\hat{k}_1 \cdot \hat{k}_1)^2}{k_1^2 k_2^2 (k_1 - k_2)^2 (k_1 - p_1)^2 (k_1 - p_1 - p_2)^2}, \tag{2.49}$$

where we have introduced the notation $D_7 \equiv (k_2 - p_1 - p_2)^2$ from [29, 31] on the left-hand side for later convenience. Note that, in this example, there is no clash between the explicit factor of $(\hat{k}_1 \cdot \hat{k}_1)^2$ in eq. (2.49) and the implicit factor of $\hat{k}_2 \cdot \hat{k}_2$ present in

$$D_7 = \bar{k}_2 \cdot \bar{k}_2 - 2(p_1 + p_2) \cdot \bar{k}_2 + (p_1 + p_2)^2 + \hat{k}_2 \cdot \hat{k}_2. \tag{2.50}$$

The Schwinger parameter subsets of interest are

$$T_{k_1^2} = \alpha_1 + \alpha_4 + \alpha_6, \quad T_{k_2^2} = \alpha_2 + \alpha_7, \quad \text{and} \quad T_{k_1 \cdot k_2} = \alpha_3, \quad (2.51)$$

and differentiating the right-hand side of eq. (2.48) twice with respect to $T_{k_1^2}$ gives $\epsilon(\epsilon - 1)$ times an insertion of

$$\frac{(T_{k_2^2} + T_{k_1 \cdot k_2})^2}{(\overline{U}_6^{(2)})^2} = \frac{\alpha_2^2 + \alpha_3^2 + \alpha_7^2 + 2\alpha_2\alpha_3 + 2\alpha_2\alpha_7 + 2\alpha_3\alpha_7}{(\overline{U}_6^{(2)})^2} \quad (2.52)$$

into eq. (2.44) specialized to our two-loop example.

The factor in the denominator induces the dimension shift $4 - 2\epsilon \rightarrow 8 - 2\epsilon$. Powers of Schwinger parameters associated to the propagator denominators induce positive shifts in the corresponding propagator exponents, and, for h powers of parameter i , an overall factor of $(-1)^h \Gamma(\nu_i + h) / \Gamma(\nu_i)$. Powers of the Schwinger parameter associated to the numerator also induce positive shifts in the exponent, but only an overall factor of $h!$ from the action of the derivatives in eq. (2.44). Due to the fact that α_7 is merely an auxiliary parameter set to zero at the end of the calculation, the insertion of α_7^2 gives a vanishing contribution for the specific example considered. We find:

$$\begin{aligned}
 & \left[D_7(\hat{k}_1 \cdot \hat{k}_1)^2 \right] = 2\epsilon(\epsilon - 1) \left[\begin{aligned} & \text{Diagram 1}^{8-2\epsilon} [D_7] \\ & + \text{Diagram 2}^{8-2\epsilon} [D_7] + \text{Diagram 3}^{8-2\epsilon} [D_7] \\ & - \text{Diagram 4}^{8-2\epsilon} [D_7] \end{aligned} \right]. \quad (2.53)
 \end{aligned}$$

The integrals on the right-hand side of eq. (2.53) can be straightforwardly evaluated in Feynman parameters to all orders in ϵ due to the fact that they are iteratively one loop. After some simplifications, we obtain

$$\begin{aligned}
 & \left[D_7(\hat{k}_1 \cdot \hat{k}_1)^2 \right] \\
 & = \frac{(5 - 2\epsilon)\Gamma^2(\epsilon)\Gamma^2(1 - \epsilon)\Gamma(2\epsilon)\Gamma(1 - 2\epsilon)\Gamma(2 - \epsilon)}{\Gamma(6 - 3\epsilon)\Gamma(3 - 2\epsilon)\Gamma^2(-1 + \epsilon)}, \quad (2.54)
 \end{aligned}$$

where we have set $(p_1 + p_2)^2 = -1$. To check eq. (2.54) and our treatment of the two-loop μ terms, we invite the reader to first integrate out k_2 in integral (2.49) above and then treat the μ -term insertion in the context of the remaining integration over k_1 (i.e. by applying eq. (2.42)). Finally, let us stress again that a numerator insertion of $D_7(\hat{k}_2 \cdot \hat{k}_2)^2$ would *not* be directly covered by our method.

To be clear, factorizable two-loop μ integrals also appear in our calculations. They could be evaluated as products of one-loop μ integrals, but we prefer to treat them along the lines described above. In the factorizable case, $(\hat{k}_1 - \hat{k}_2)^2 \rightarrow \hat{k}_1 \cdot \hat{k}_1 + \hat{k}_2 \cdot \hat{k}_2$ and we have

$$\int d^{-2\epsilon} \hat{k}_1 d^{-2\epsilon} \hat{k}_2 e^{T_{k_1^2} \hat{k}_1 \cdot \hat{k}_1 + T_{k_2^2} \hat{k}_2 \cdot \hat{k}_2} \propto (T_{k_1^2} T_{k_2^2})^\epsilon \quad (2.55)$$

for our generating function. This is nothing but a degenerate case of eq. (2.48) above with $T_{k_1 \cdot k_2}$ set to zero. Note that, while we believe that it should be possible to make a similar construction to handle μ -term insertions in d -dimensional Feynman integrals at higher loops, more effort would be required to identify the underlying vacuum topologies.

2.5 $\overline{\text{MS}}$ renormalization of α and α_s

In this work, we renormalize both α and α_s in the $\overline{\text{MS}}$ scheme. Due to our neglect of all fermion masses, this choice is particularly convenient for our calculation of the two-loop mixed EW-QCD corrections to the neutral-current Drell-Yan process. As explained in section 3.1, we neglect the gauge-invariant subset of Feynman diagrams with closed fermion loops and any contributions involving top quarks throughout this work. This includes in particular the two-loop Standard Model vacuum polarization diagrams featuring a single quark loop and a single gluon exchange. As a corollary, the form of the contributing two-loop Feynman diagrams (see section 5.1) immediately implies that the order $\alpha\alpha_s$ corrections to the electric charge renormalization constant must vanish identically.

Throughout this work, we follow the convention of [20] for fixed-order results. Therefore, quantities in Standard Model perturbation theory are written in the form

$$\mathbf{z} = \sum_{m,n=0}^{\infty} \mathbf{z}^{(m,n)} \left(\frac{\alpha}{4\pi}\right)^m \left(\frac{\alpha_s}{4\pi}\right)^n \quad (2.56)$$

to all orders in the coupling constants. In this notation, neglecting contributions with closed fermion loops which are proportional to the number of light leptons (n_l) or light quarks (n_q) or involve the mass of the top quark (m_t), we have

$$\delta Z_e^{(1,1)} \Big|_{n_\ell, n_q \rightarrow 0; m_t \rightarrow \infty} = 0, \quad (2.57)$$

for the order $\alpha\alpha_s$ corrections to the electric charge renormalization constant.

Since the tree-level Drell-Yan amplitude is of order α , the trivial renormalization of the strong coupling constant

$$(4\pi)^\epsilon e^{-\gamma_E \epsilon} \alpha_s^{\text{bare}} = \alpha_s \quad (2.58)$$

is all that is required for our order $\alpha^2\alpha_s$ two-loop calculation. On the other hand, the one-loop electric charge renormalization constant does make an appearance. It has been known since the work of [66] in the standard on-shell scheme where the charge renormalization

constant is fixed order-by-order in the Thomsen limit (see e.g. [67] for a review). In the $\overline{\text{MS}}$ scheme it is given by,

$$\delta Z_e^{(1,0)} \Big|_{n_\ell, n_q \rightarrow 0; m_t \rightarrow \infty} = -\frac{7}{2\epsilon}, \quad (2.59)$$

the pole part of eq. (5.40) of [66] once one discards the first, fermionic term inside the bracket. For notational convenience, we shall suppress the qualifiers provided on the left-hand sides of eqs. (2.57) and (2.59) throughout the rest of this work. When necessary, appropriate caveats will be provided in the text.

2.6 On-shell electroweak gauge boson wavefunction and mass renormalization

In this section, we review the on-shell renormalization of the electroweak gauge bosons in a generalized 't Hooft-Feynman gauge. Our conventions for the electroweak sector of the Standard Model Lagrange density and Feynman rules are identical to those of [66], upon which the useful resources [54, 67, 68] were modeled. Our electroweak gauge boson renormalization constants are also defined with exactly the same renormalization conditions, but we have chosen to factor the squared electroweak gauge boson mass, m_v^2 , out of the mass renormalization constants, $Z_{m_v^2}$, to render them dimensionless.

As usual, a particularly convenient choice for the gauge parameter renormalization constants is

$$Z_{\xi_\gamma} = Z_{\gamma\gamma}, \quad Z_{\xi_z} = Z_{ZZ}, \quad \text{and} \quad Z_{\xi_w} = Z_{W^\pm} \quad (2.60)$$

because it simplifies the all-orders form of the electroweak gauge boson kinetic terms in the renormalized Lagrange density. Subsequently, our generalized 't Hooft-Feynman gauge is defined by setting all of the renormalized gauge parameters to one at the outset,

$$\xi_\gamma = \xi_z = \xi_w = 1. \quad (2.61)$$

This is justified because, due to eqs. (2.60), the gauge parameters in this scheme receive no radiative corrections.

Apart from the exactly-transverse photon-photon self-energy,

$$-i \Sigma_{\gamma\gamma}^{\mu\nu}(q) = -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{\Sigma}_{\gamma\gamma}(q^2), \quad (2.62)$$

the electroweak gauge boson self-energies naturally decompose into a sum of transverse and longitudinal components:

$$-i \Sigma_{\gamma Z}^{\mu\nu}(q) = -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{\Sigma}_{\gamma Z}(q^2) - i \frac{q^\mu q^\nu}{q^2} \Sigma_{\gamma Z}^L(q^2), \quad (2.63)$$

$$-i \Sigma_{ZZ}^{\mu\nu}(q) = -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{\Sigma}_{ZZ}(q^2) - i \frac{q^\mu q^\nu}{q^2} \Sigma_{ZZ}^L(q^2), \quad (2.64)$$

$$\text{and} \quad -i \Sigma_{W^+W^-}^{\mu\nu}(q) = -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{\Sigma}_{W^+W^-}(q^2) - i \frac{q^\mu q^\nu}{q^2} \Sigma_{W^+W^-}^L(q^2). \quad (2.65)$$

The longitudinal components of the self-energies, $\Sigma_{VV'}^L$, are unphysical and need not be calculated explicitly.

Working in d dimensions, we see that the relevant Lorentz projectors for the transverse gauge boson self-energies can be cleanly summarized as

$$\left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} = \frac{i}{3-2\epsilon} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \quad (2.66)$$

For all of the Lorentz projection operators considered in this work, a sum over the available open Lorentz and spin indices is always implied for their action on Feynman diagrams. We find the following results for the transverse projections of the Fourier-transformed electroweak gauge boson kinetic terms, $\tilde{\mathcal{L}}_{VV'}^{\mu\nu}$, of order α :

$$\left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \tilde{\mathcal{L}}_{\gamma\gamma}^{\mu\nu} = \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \left(-i \delta Z_{\gamma\gamma}^{(1,0)} \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\right) = \delta Z_{\gamma\gamma}^{(1,0)} q^2, \quad (2.67)$$

$$\begin{aligned} \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \tilde{\mathcal{L}}_{\gamma Z}^{\mu\nu} &= \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \left(-\frac{i}{2} \left(\left(\delta Z_{\gamma Z}^{(1,0)} + \delta Z_{Z\gamma}^{(1,0)}\right) q^2 - m_z^2 \delta Z_{Z\gamma}^{(1,0)}\right) g^{\mu\nu}\right) \\ &= \frac{1}{2} \left(\left(\delta Z_{\gamma Z}^{(1,0)} + \delta Z_{Z\gamma}^{(1,0)}\right) q^2 - m_z^2 \delta Z_{Z\gamma}^{(1,0)}\right), \end{aligned} \quad (2.68)$$

$$\begin{aligned} \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \tilde{\mathcal{L}}_{ZZ}^{\mu\nu} &= \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \left(-i \left(\delta Z_{ZZ}^{(1,0)} \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)}\right) g^{\mu\nu}\right)\right) \\ &= \delta Z_{ZZ}^{(1,0)} q^2 - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)}\right), \quad \text{and} \end{aligned} \quad (2.69)$$

$$\begin{aligned} \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \tilde{\mathcal{L}}_{W^\pm}^{\mu\nu} &= \left(\mathbb{P}_{VV'}^T\right)_{\mu\nu} \left(-i \left(\delta Z_{W^\pm}^{(1,0)} \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) - m_w^2 \left(\delta Z_{W^\pm}^{(1,0)} + \delta Z_{m_w^2}^{(1,0)}\right) g^{\mu\nu}\right)\right) \\ &= \delta Z_{W^\pm}^{(1,0)} q^2 - m_w^2 \left(\delta Z_{W^\pm}^{(1,0)} + \delta Z_{m_w^2}^{(1,0)}\right). \end{aligned} \quad (2.70)$$

Analyzing eqs. (2.67)–(2.70) and taking into account the renormalization conditions of [66], we see immediately that

$$\begin{aligned} \delta Z_{\gamma\gamma}^{(1,0)} &= -\left.\frac{d\bar{\Sigma}_{\gamma\gamma}^{(1,0)}(q^2)}{dq^2}\right|_{q^2=0}, & \delta Z_{Z\gamma}^{(1,0)} &= \left.\frac{2\bar{\Sigma}_{\gamma Z}^{(1,0)}(q^2)}{m_z^2}\right|_{q^2=0}, \\ \delta Z_{\gamma Z}^{(1,0)} &= -\left.\frac{2\bar{\Sigma}_{\gamma Z}^{(1,0)}(q^2)}{m_z^2}\right|_{q^2=m_z^2}, & \delta Z_{ZZ}^{(1,0)} &= -\left.\frac{d\bar{\Sigma}_{ZZ}^{(1,0)}(q^2)}{dq^2}\right|_{q^2=m_z^2}, \\ \delta Z_{m_z^2}^{(1,0)} &= \left.\frac{\bar{\Sigma}_{ZZ}^{(1,0)}(q^2)}{m_z^2}\right|_{q^2=m_z^2}, & \delta Z_{m_w^2}^{(1,0)} &= \left.\frac{\bar{\Sigma}_{W^+W^-}^{(1,0)}(q^2)}{m_w^2}\right|_{q^2=m_w^2}. \end{aligned} \quad (2.71)$$

Apart from light fermion and top quark contributions which we consistently neglect throughout this work, explicit expressions for the one-loop self-energies and counterterms relevant to the higher-order corrections of interest to us will be provided to all orders in ϵ in section B.2. Note that we do not provide an explicit expression for $\delta Z_{W^\pm}^{(1,0)}$ because it plays no role in the renormalization of any of the Feynman diagrams we calculate.

2.7 On-shell wavefunction renormalization for massless fermion fields

In this section, we review the systematics of on-shell renormalization for massless fermion fields in the Standard Model. The on-shell scheme for massless fermions is widely used

in QCD because, due to the scalelessness of the contributing gluon-exchange diagram, the one-loop quark wavefunction counterterm vanishes identically in dimensional regularization. Although we will ultimately take the $q^2 \rightarrow 0$ limit, it is nevertheless instructive to begin with the off-shell setup of [66], where the momentum transfer q^2 in the Lorentz decomposition of the fermion self energy

$$i(\bar{\Sigma}_f(q))_{\alpha\beta} = i\bar{\Sigma}_{V,f}(q^2) (\not{q})_{\alpha\beta} + i\bar{\Sigma}_{A,f}(q^2) (\not{q}\gamma_5)_{\alpha\beta} \quad (2.72)$$

is taken to be different from zero.⁴ In writing eq. (2.72), we have implicitly assumed Kreimer’s γ_5 scheme; analogous calculations in HVBM’s γ_5 scheme will be discussed in section 2.8 and, accordingly, we have written $\bar{\Sigma}_f(q)$ instead of $\Sigma_f(q)$ in order to allow for a side-by-side comparison in the next section.

We employ the notation of [66] for the interactions of the electroweak gauge bosons with matter. The Z interaction is parametrized by flavor-dependent axial vector and vector coupling coefficients,

$$a_f = \frac{m_z^2 I_f^3}{2m_w \sqrt{m_z^2 - m_w^2}} \quad \text{and} \quad v_f = \frac{m_z^2}{2m_w \sqrt{m_z^2 - m_w^2}} \left(I_f^3 - 2 \frac{m_z^2 - m_w^2}{m_z^2} Q_f \right). \quad (2.73)$$

The W interaction, on the other hand, is parametrized by a universal coupling coefficient

$$a_w = v_w = \frac{m_z}{2\sqrt{2}\sqrt{m_z^2 - m_w^2}}. \quad (2.74)$$

We will make extensive use of these aliases throughout the rest of this paper.

At first sight, it might seem counterproductive not to employ eq. (2.74) to eliminate a_w and v_w , as they are equal and have no dependence on the fermion flavor. However, as will become clear later on in this section, it is useful to retain the dependence on a_w and v_w at intermediate stages of fermion self-energy and vertex calculations because it generically allows for a determination of axial vector components from vector components through chiral symmetry. Actually, the fermion self-energies considered in this section are particularly simple and have no independent W -exchange Feynman diagrams; all W -exchange diagrams can be obtained from the Z -exchange diagrams for free by making the replacements $a_f \rightarrow a_w$ and $v_f \rightarrow v_w$ (see figure 2).

Thus, for the relatively low perturbative orders of interest to us, it suffices to consider the calculation of the Z -exchange contributions to the vector component of the fermion self-energy. At this stage, one is faced with a conceptual hurdle. Taken at face value, the self-energy in the on-shell scheme for massless fermions would seem to be most appropriately defined as the $q^2 \rightarrow 0$ limit of the fermion self-energy calculated off-shell.⁵ Of course, it also seems that one ought to be able to simply set $q^2 = 0$ in all Feynman diagrams at

⁴Throughout this section, we suppress the color indices of the quarks to streamline our discussion and allow for a unified description of propagator corrections at one loop. The two-loop mixed EW-QCD scattering amplitudes of primary interest to us in this work always come with a color factor of C_F , due to the fact that all contributing Feynman diagrams feature exactly one gluon exchange between quark lines.

⁵Here, the $q^2 \rightarrow 0$ limit should be taken *before* expanding in ϵ .

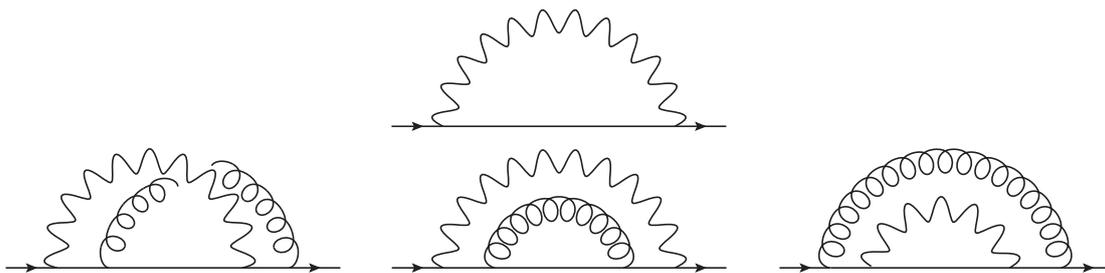


Figure 2. Independent Z exchange diagrams at one and two loops. Due to the absence of fermion masses, diagrams which feature only gluon and/or photon exchanges vanish identically.

the very beginning, significantly simplifying all subsequent calculations. In fact, as we now demonstrate at the one-loop level, both approaches are possible and yield the same answer.

If we begin with q^2 generic, one can immediately verify that the Lorentz projector of interest is

$$\left(\mathbb{P}_f^V\right)_{\beta\alpha} \Big|_{\text{off-shell}} = -\frac{i}{4q^2} (\not{q})_{\beta\alpha} \quad (2.75)$$

and our Kreimer reading point prescription instructs us to begin reading our Dirac traces from the projector insertion. Therefore, using the Feynman rules of [66] and the anti-commutation relation satisfied by Kreimer's γ_5 , eq. (2.7), the contribution of the one-loop Z -exchange diagram to the vector component of the fermion self-energy is

$$\bar{\Sigma}_{V,f}^{(1,0)}(q^2) \Big|_{Z \text{ exchange}} = -\frac{c_1(\epsilon, \mu^2)(a_f^2 + v_f^2)}{4q^2} \int d^d k_1 \frac{\text{Tr}\{\not{q}\gamma^\nu(\not{q} + \not{k}_1)\gamma_\nu\}}{(q+k_1)^2(k_1^2 - m_z^2)}, \quad (2.76)$$

for⁶

$$c_1(\epsilon, \mu^2) = \frac{e^{\gamma_E \epsilon} (\mu^2)^\epsilon}{i\pi^{2-\epsilon}}. \quad (2.77)$$

As eq. (2.76) does not depend on γ_5 , it can be trivially simplified using the contraction identity and even trace formulae of section 2.2. After carrying out the numerator algebra, we obtain:

$$\bar{\Sigma}_{V,f}^{(1,0)}(q^2) \Big|_{Z \text{ exchange}} = \frac{2c_1(\epsilon, \mu^2)(a_f^2 + v_f^2)(1-\epsilon)}{q^2} \int d^d k_1 \frac{q^2 + q \cdot k_1}{(q+k_1)^2(k_1^2 - m_z^2)}, \quad (2.78)$$

which can immediately be rewritten as a linear combination of standard scalar integrals using Passarino-Veltman reduction [63]; the result is

$$\begin{aligned} \bar{\Sigma}_{V,f}^{(1,0)}(q^2) \Big|_{Z \text{ exchange}} &= \frac{c_1(\epsilon, \mu^2)(a_f^2 + v_f^2)(1-\epsilon)}{q^2} \left((q^2 - m_z^2) \int \frac{d^d k_1}{(q+k_1)^2(k_1^2 - m_z^2)} \right. \\ &\quad \left. + \int \frac{d^d k_1}{k_1^2 - m_z^2} \right), \end{aligned} \quad (2.79)$$

⁶As explained in section 2.5, α is renormalized in the $\overline{\text{MS}}$ scheme.

where, using Feynman/Schwinger parameters,

$$\int \frac{d^d k_1}{k_1^2 - m_z^2} = -\frac{e^{\gamma_E \epsilon}}{c_1(\epsilon, \mu^2)} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon m_z^2 \Gamma(-1 + \epsilon) \quad \text{and} \quad (2.80)$$

$$\int \frac{d^d k_1}{(q + k_1)^2 (k_1^2 - m_z^2)} = 2 \frac{e^{\gamma_E \epsilon}}{c_1(\epsilon, \mu^2)} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon \frac{\Gamma(\epsilon) \Gamma(2 - 2\epsilon)}{\Gamma(3 - 2\epsilon)} {}_2F_1\left(1, \epsilon; 2 - \epsilon; \frac{q^2}{m_z^2}\right). \quad (2.81)$$

In writing eq. (2.81), q^2 was assumed to be less than m_z^2 . Modulo minor typos,⁷ our results agree as expected with those of [66] through to $\mathcal{O}(\epsilon^0)$ in the limit of vanishing fermion mass.

From eqs. (2.80) and (2.81) it is a simple matter to work out the $q^2 \rightarrow 0$ limit of eq. (2.79). From l'Hôpital's rule, we find:

$$\lim_{q^2 \rightarrow 0} \left(\bar{\Sigma}_{V,f}^{(1,0)}(q^2) \Big|_{Z \text{ exchange}} \right) = \frac{2(a_f^2 + v_f^2)(1 - \epsilon) \Gamma(\epsilon) e^{\gamma_E \epsilon}}{2 - \epsilon} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon. \quad (2.82)$$

The above derivation is certainly valid, but let us stress that attempting to calculate the $q^2 \rightarrow 0$ limit of $\bar{\Sigma}_{V,f}(q^2)$ in this way is not recommended at higher orders in perturbation theory; while we were able to confirm numerically that the correct $q^2 \rightarrow 0$ limit exists at order α_s for fixed, finite ϵ , it is not amenable to analytic evaluation.

In general, our opinion is that it is far simpler to implement the on-shell condition $q^2 = 0$ from the get-go. Of course, eq. (2.75) is ill-defined at $q^2 = 0$; a valid projector when $q^2 = 0$ is given by

$$\left(\mathbb{P}_f^V\right)_{\beta\alpha} \Big|_{\text{on-shell}} = -\frac{i}{4\eta \cdot q} (\not{\eta})_{\beta\alpha}, \quad (2.83)$$

where η is an arbitrary four-vector such that $\eta \cdot q \neq 0$.

Setting $q^2 = 0$ from the beginning of the calculation, we have

$$\begin{aligned} \bar{\Sigma}_{V,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} &= -\frac{c_1(\epsilon, \mu^2)(a_f^2 + v_f^2)}{4\eta \cdot q} \int d^d k_1 \frac{\text{Tr}\{\not{\eta} \gamma^\nu (\not{q} + \not{k}_1) \gamma_\nu\}}{(q + k_1)^2 (k_1^2 - m_z^2)} \\ &= \frac{2c_1(\epsilon, \mu^2)(a_f^2 + v_f^2)(1 - \epsilon)}{\eta \cdot q} \int d^d k_1 \frac{\eta \cdot q + \eta \cdot k_1}{(q + k_1)^2 (k_1^2 - m_z^2)}. \end{aligned} \quad (2.84)$$

Proceeding directly from Feynman/Schwinger parameters for the integral evaluations, we find

$$\int d^d k_1 \frac{1}{(q + k_1)^2 (k_1^2 - m_z^2)} \Big|_{q^2=0} = -\frac{e^{\gamma_E \epsilon}}{c_1(\epsilon, \mu^2)} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon \Gamma(-1 + \epsilon) \quad (2.85)$$

$$\text{and} \quad \int d^d k_1 \frac{\eta \cdot k_1}{(q + k_1)^2 (k_1^2 - m_z^2)} \Big|_{q^2=0} = \frac{\eta \cdot q e^{\gamma_E \epsilon}}{c_1(\epsilon, \mu^2)} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon \frac{\Gamma(-1 + \epsilon)}{2 - \epsilon}. \quad (2.86)$$

⁷While comparing to [66], we noticed a typo in the expression for $\Sigma_A^{i\sigma}$ on the 3rd line of eq. (5.27): the first term inside the bracket should actually read $-2v_{i\sigma} a_{i\sigma} (2B_1(k^2; m_{i\sigma}, M_z) + 1)$. Additional typos were discovered in eq. (B.2); the tensor reduction formula needed for $B_1(k^2; m_{i\sigma}, M_z)$ should read

$$2k^2 B_1(k^2; M_1, M_2) = A(M_1) - A(M_2) + (M_2^2 - M_1^2 - k^2) B_0(k^2; M_1, M_2).$$

Note that, in the degenerate $q^2 = 0$ kinematics under consideration, *all* contributing one-loop Feynman integrals must ultimately turn out to be proportional to the one-loop tadpole.

Finally, we obtain

$$\bar{\Sigma}_{V,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} = \frac{2(a_f^2 + v_f^2)(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon \quad (2.87)$$

and see immediately that

$$\lim_{q^2 \rightarrow 0} \left(\bar{\Sigma}_{V,f}^{(1,0)}(q^2) \Big|_{Z \text{ exchange}} \right) = \bar{\Sigma}_{V,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} \quad (2.88)$$

as claimed.

We now consider the ramifications of chiral symmetry. In the Standard Model, the right- and left-handed components of the Z -exchange contribution to the one-loop fermion self-energy are exchanged under $\gamma_5 \rightarrow -\gamma_5$. This implies that the axial vector component of the Z -exchange contribution to the fermion self-energy ought to be obtained by a replacement of the form $v_f^2 + a_f^2 \rightarrow \pm 2a_f v_f$ in eq. (2.87). In order to fix the overall sign, recall that the coupling of the W to matter is exactly left-handed, that the W -exchange contribution is obtained by making the replacements $a_f \rightarrow a_w$ and $v_f \rightarrow v_w$, and that $a_w = v_w$. Taken together, these considerations allow us to conclude that

$$\bar{\Sigma}_{A,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} = -\frac{4a_f v_f(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon \quad (2.89)$$

with no additional calculation.

In the on-shell scheme, massless fermion wavefunction counterterms are defined to exactly cancel the perturbative corrections to the corresponding fermion self-energies order-by-order. Therefore, we find

$$\begin{aligned} \delta Z_{V,f}^{(1,0)} &= -\bar{\Sigma}_{V,f}^{(1,0)}(0) \quad (2.90) \\ &= -\frac{2(a_f^2 + v_f^2)(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon - \frac{2(a_w^2 + v_w^2)(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_w^2}\right)^\epsilon \end{aligned}$$

$$\begin{aligned} \delta Z_{A,f}^{(1,0)} &= -\bar{\Sigma}_{A,f}^{(1,0)}(0) \quad (2.91) \\ &= \frac{4a_f v_f(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_z^2}\right)^\epsilon + \frac{4a_w v_w(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{2-\epsilon} \left(\frac{\mu^2}{m_w^2}\right)^\epsilon \end{aligned}$$

for the vector and axial vector components of the one-loop fermion wavefunction counterterm.

We now turn to the vector and axial vector components of the two-loop quark self-energy of order $\alpha\alpha_s$. To our knowledge, these mixed two-loop corrections to the self-energy were first considered in [69], though not in the on-shell scheme with massless fermions. It is straightforward to calculate the desired quantities by acting with the appropriate Lorentz projector in Kreimer's γ_5 scheme, eq. (2.83), on the eight non-zero two-loop Feynman diagrams summarized by figure 2. Remarkable simplifications are achieved by setting

$q^2 = 0$ at the beginning of the calculation. In the end, we find that the vector component of the quark wavefunction counterterm is given by

$$\begin{aligned} \delta Z_{V,q}^{(1,1)} &= -\bar{\Sigma}_{V,q}^{(1,1)}(0) \\ &= -\frac{2(a_q^2 + v_q^2)C_F(1-\epsilon)(1-3\epsilon)(3-2\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)\Gamma(-1+2\epsilon)e^{2\gamma_E\epsilon}}{\Gamma(3-\epsilon)}\left(\frac{\mu^2}{m_z^2}\right)^{2\epsilon} \\ &\quad -\frac{2(a_w^2 + v_w^2)C_F(1-\epsilon)(1-3\epsilon)(3-2\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)\Gamma(-1+2\epsilon)e^{2\gamma_E\epsilon}}{\Gamma(3-\epsilon)}\left(\frac{\mu^2}{m_w^2}\right)^{2\epsilon} \end{aligned} \quad (2.92)$$

and the axial vector component of the quark wavefunction counterterm is given by

$$\begin{aligned} \delta Z_{A,q}^{(1,1)} &= -\bar{\Sigma}_{A,q}^{(1,1)}(0) \\ &= \frac{4a_q v_q C_F(1-\epsilon)(1-3\epsilon)(3-2\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)\Gamma(-1+2\epsilon)e^{2\gamma_E\epsilon}}{\Gamma(3-\epsilon)}\left(\frac{\mu^2}{m_z^2}\right)^{2\epsilon} \\ &\quad + \frac{4a_w v_w C_F(1-\epsilon)(1-3\epsilon)(3-2\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)\Gamma(-1+2\epsilon)e^{2\gamma_E\epsilon}}{\Gamma(3-\epsilon)}\left(\frac{\mu^2}{m_w^2}\right)^{2\epsilon}. \end{aligned} \quad (2.93)$$

As at one loop, the simple form of eqs. (2.92) and (2.93) results from the fact that only the one-mass two-loop tadpole integral can survive in the $q^2 \rightarrow 0$ limit.

2.8 Chiral symmetry restoration in HVBM's γ_5 scheme and Larin's principle

As mentioned in section 2.2, HVBM's γ_5 scheme is especially challenging to work with in practice due to the fact that, in general, a plethora of finite counterterms must be introduced to restore the chiral symmetry of the Standard Model. We shall see that Larin's principle [56] provides a practical recipe which helps tremendously to minimize the number of finite counterterms which must be computed explicitly for a given application. While we will be most interested in the standard one-loop example of the order α_s gluon-exchange correction to the $Z\bar{q}q$ vertex shown in figure 4 [48], let us first revisit the one-loop fermion self-energy calculation of section 2.7 in HVBM's γ_5 scheme to illustrate the utility of Larin's principle with minimal computational effort.

In HVBM's γ_5 scheme, the Lorentz decomposition for the massless fermion self-energy reads

$$i\left(\Sigma_f(q)\right)_{\alpha\beta} = i\Sigma_{V,f}(0)\left(\not{q}\right)_{\alpha\beta} + i\Sigma_{A,f}(0)\frac{1}{2}\left[\not{q}, \gamma_5\right]_{\alpha\beta}, \quad (2.94)$$

where we have assumed the $q^2 = 0$ Lorentz projector, eq. (2.83), will be employed to compute $\Sigma_{V,f}(0)$ and, as in section 2.7, the color indices are suppressed. It is once again sufficient to consider the Z -exchange contribution:

$$\begin{aligned} \Sigma_{V,f}^{(1,0)}(0)\Big|_{Z \text{ exchange}} &= -\frac{c_1(\epsilon, \mu^2)v_f^2}{4\eta \cdot q} \int \frac{d^d k_1}{(q+k_1)^2(k_1^2 - m_z^2)} \text{tr}\left\{\not{\eta}\gamma^\nu(\not{q} + \not{k}_1)\gamma_\nu\right\} \\ &\quad - \frac{c_1(\epsilon, \mu^2)a_f^2}{16\eta \cdot q} \int \frac{d^d k_1}{(q+k_1)^2(k_1^2 - m_z^2)} \text{tr}\left\{\not{\eta}[\gamma^\nu, \gamma_5](\not{q} + \not{k}_1)[\gamma_\nu, \gamma_5]\right\}. \end{aligned} \quad (2.95)$$

In order to simplify the term proportional to a_f^2 , it is useful to resolve the d -dimensional momenta and Dirac matrices into four-dimensional and (-2ϵ) -dimensional components using eqs. (2.10) and (2.11). Crucially, the linear dependence on k_1 in the numerator of eq. (2.95) implies that the Dirac traces *cannot* produce non-zero μ integrals. This observation allows for the immediate neglect of all terms involving \hat{k}_1 .

Using the HVBM γ_5 anticommutation relations of section 2.2, eq. (2.95) simplifies to

$$\begin{aligned} \Sigma_{V,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} &= -\frac{c_1(\epsilon, \mu^2) v_f^2}{4\eta \cdot q} \int \frac{d^d k_1}{(q+k_1)^2(k_1^2 - m_z^2)} \text{tr} \left\{ \not{\eta} \gamma^\nu (\not{q} + \not{k}_1) \gamma_\nu \right\} \\ &\quad - \frac{c_1(\epsilon, \mu^2) a_f^2}{4\eta \cdot q} \int \frac{d^d k_1}{(q+k_1)^2(k_1^2 - m_z^2)} \text{tr} \left\{ \not{\eta} \bar{\gamma}^\nu (\not{q} + \not{k}_1) \bar{\gamma}_\nu \right\}. \end{aligned} \quad (2.96)$$

Finally, the trace on the second line of eq. (2.96) is nothing but the $\epsilon \rightarrow 0$ limit of the trace on the first line of eq. (2.96); from the analysis of section 2.7, we see that

$$\Sigma_{V,f}^{(1,0)}(0) \Big|_{Z \text{ exchange}} = \frac{2 \left(v_f^2(1 - \epsilon) + a_f^2 \right) \Gamma(\epsilon) e^{\gamma_E \epsilon}}{2 - \epsilon} \left(\frac{\mu^2}{m_z^2} \right)^\epsilon. \quad (2.97)$$

Starting at $\mathcal{O}(\epsilon^0)$, eq. (2.97) clearly exhibits a *chiral mismatch*, indicating that the vector components of the fermion kinetic terms of the Standard Model Lagrange density receive finite counterterm corrections already at order α . In other words, a correction must be made because the coefficient of a_f^2 is not equal to the coefficient of v_f^2 in eq. (2.97) at $\mathcal{O}(\epsilon^0)$. As Larin emphasized in [56], the form of the finite counterterm may be deduced by simply *demanding* that the a_f^2 and v_f^2 coefficients be equal. For the present calculation, Larin’s principle dictates that the finite counterterm is given to all orders in ϵ by the difference

$$\bar{\Sigma}_{V,f}^{(1,0)}(0) - \Sigma_{V,f}^{(1,0)}(0) = -\frac{2 a_f^2 \Gamma(1 + \epsilon) e^{\gamma_E \epsilon}}{2 - \epsilon} \left(\frac{\mu^2}{m_z^2} \right)^\epsilon - \frac{2 a_w^2 \Gamma(1 + \epsilon) e^{\gamma_E \epsilon}}{2 - \epsilon} \left(\frac{\mu^2}{m_w^2} \right)^\epsilon. \quad (2.98)$$

In fact, we could have simply skipped the calculation of the bare self-energy in HVBM’s γ_5 scheme *altogether*; requiring that our result respect the chiral symmetry of the Standard Model to all orders in ϵ *uniquely* leads to the result obtained in Kreimer’s γ_5 scheme in section 2.7 after finite renormalization.⁸

Of course, Larin’s principle can be successfully applied to the fermion self-energies and vertex form factors because they have a direct correspondence to terms in the renormalized Lagrange density. For more complicated contributions to higher-multiplicity scattering amplitudes at two loops and beyond, it is expected that non-trivial corrections will arise from the finite counterterms, beginning with insertions into lower-loop box-type diagrams.

⁸In [56], it is suggested that one ought to perform additional renormalizations for the purpose of chiral symmetry restoration only after the removal of all ultraviolet divergences with ordinary counterterms. In our view, this is not necessarily the most transparent approach. However, at higher orders in perturbation theory, it may be that restoring the chiral symmetry first in the way we propose leads to divergent “finite” counterterms. Therefore, in our approach, “finite” may not be the best moniker to generally describe the counterterms responsible for the restoration of the Standard Model chiral symmetry in HVBM’s γ_5 scheme.

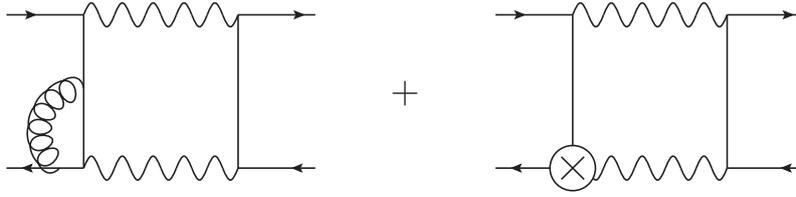


Figure 3. A two-loop γZ box-type diagram for the mixed EW-QCD corrections of relative order $\alpha\alpha_s$ to the neutral-current Drell-Yan process which receives a correction from the insertion of a finite one-loop counterterm ($\delta Z_{Z\bar{q}q}^{(0,1)}$ of eq. (2.113)) into a corresponding one-loop box diagram.

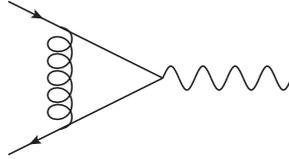


Figure 4. The order α_s gluon-exchange correction to the $Z\bar{q}q$ vertex form factor.

Indeed, as we shall see later on in our discussion of the two-loop mixed EW-QCD corrections to the neutral-current Drell-Yan process, certain two-loop box-type diagrams which involve axial vector couplings to quarks receive corrections from finite counterterm insertions into corresponding one-loop box diagrams (see figure 3). The bare calculation in HVBM's γ_5 scheme needed to derive the finite counterterm featured in figure 3 is a familiar one [48, 56] and, fortunately, the *only* one relevant to this work which we cannot bypass.

In HVBM's γ_5 scheme, the $Z\bar{q}q$ vertex has the Lorentz decomposition

$$ie \left(\mathcal{F}_\mu^{Z\bar{q}q}(p_1, p_2) \right)_{kj} = ie \mathcal{V}_{Z\bar{q}q}(s) \bar{v}_k(p_2) \gamma_\mu u_j(p_1) + ie \mathcal{A}_{Z\bar{q}q}(s) \bar{v}_k(p_2) \frac{1}{2} [\gamma_\mu, \gamma_5] u_j(p_1), \quad (2.99)$$

where $s = (p_1 + p_2)^2$ denotes the virtuality of the Z and color indices are suppressed for brevity. Here, we have assumed an incoming massless quark of momentum p_1 and an incoming massless antiquark of momentum p_2 for later convenience. The Lorentz projectors

$$\left(\mathbb{P}_q^\mu \right)_{jk} = \frac{i}{4eN_c s(1-\epsilon)} \bar{u}_j(p_1) \gamma^\mu v_k(p_2) \quad (2.100)$$

$$\text{and} \quad \left(\mathbb{P}_{\mathcal{A}_q}^\mu \right)_{jk} = \frac{i}{4eN_c s} \bar{u}_j(p_1) \frac{1}{2} [\gamma^\mu, \gamma_5] v_k(p_2) \quad (2.101)$$

deliver $\mathcal{V}_{Z\bar{q}q}(s)$ and $\mathcal{A}_{Z\bar{q}q}(s)$ respectively when acting on the sum of all $Z\bar{q}q$ vertex Feynman diagrams with an implicit sum on the open Lorentz and spin indices.

For the axial vector component of the order α_s correction to the $Z\bar{q}q$ vertex calculated in HVBM's γ_5 scheme, Larin's principle is applied by demanding equality to all orders in ϵ between its a_q coefficient and minus the v_q coefficient of the *vector* component of the order α_s correction to the $Z\bar{q}q$ vertex form factor, a condition which would hold automatically in Kreimer's γ_5 scheme. In other words, in the notation of [66], chiral symmetry

guarantees that

$$\bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s) = -\frac{a_q}{v_q} \bar{\mathcal{V}}_{Z\bar{q}q}^{(0,1)}(s) \quad (2.102)$$

where $\bar{\mathcal{V}}_{Z\bar{q}q}^{(0,1)}(s) = \mathcal{V}_{Z\bar{q}q}^{(0,1)}(s)$ is nothing but v_q times the bare time-like one-loop quark vertex form factor of massless QCD:

$$\begin{aligned} \bar{\mathcal{V}}_{Z\bar{q}q}^{(0,1)}(s) &= v_q \bar{\mathcal{F}}_1^q(\epsilon) e^{i\pi\epsilon} \left(\frac{\mu^2}{s}\right)^\epsilon \\ &= -v_q \frac{(2-\epsilon+2\epsilon^2)\Gamma^2(1-\epsilon)\Gamma(\epsilon)e^{\gamma_E\epsilon}}{\epsilon\Gamma(2-2\epsilon)} C_F e^{i\pi\epsilon} \left(\frac{\mu^2}{s}\right)^\epsilon. \end{aligned} \quad (2.103)$$

The derivation of the analytic expression for $\bar{\mathcal{F}}_1^q(\epsilon)$ given above is reviewed in many places in the literature, see e.g. [70]. Thus, $\bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s)$ is determined through eq. (2.102) and, to fix the finite counterterm of interest, it remains only to directly calculate $\mathcal{A}_{Z\bar{q}q}^{(0,1)}(s)$.

Applying eq. (2.101), the axial vector component of the order α_s correction to the $Z\bar{q}q$ vertex form factor calculated in HVBM's γ_5 scheme is

$$\begin{aligned} \mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) &= -\frac{a_q c_1(\epsilon, \mu^2) C_F}{16s} \int \frac{d^d k_1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2} \\ &\quad \text{tr} \left\{ [\gamma^\rho, \gamma_5] \not{p}_2 \gamma^\sigma (\not{p}_2 + \not{k}_1) [\gamma_\rho, \gamma_5] (\not{p}_1 - \not{k}_1) \gamma_\sigma \not{p}_1 \right\}. \end{aligned} \quad (2.104)$$

Splitting d -dimensional indices with eqs. (2.10) and (2.11) and eliminating all γ_5 matrices using the HVBM anticommutation relations, (2.20) and (2.21), we find that eq. (2.104) simplifies to

$$\begin{aligned} \mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) &= -\frac{a_q c_1(\epsilon, \mu^2) C_F}{4s} \int \frac{d^d k_1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2} \\ &\quad \left[\text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \bar{\gamma}^\sigma (\not{p}_2 + \not{k}_1) \bar{\gamma}_\rho (\not{p}_1 - \not{k}_1) \bar{\gamma}_\sigma \not{p}_1 \right\} - \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \hat{\gamma}^\sigma (\not{p}_2 + \not{k}_1) \bar{\gamma}_\rho (\not{p}_1 - \not{k}_1) \hat{\gamma}_\sigma \not{p}_1 \right\} \right. \\ &\quad \left. + \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \bar{\gamma}^\sigma \not{k}_1 \bar{\gamma}_\rho \not{k}_1 \bar{\gamma}_\sigma \not{p}_1 \right\} - \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \hat{\gamma}^\sigma \not{k}_1 \bar{\gamma}_\rho \not{k}_1 \hat{\gamma}_\sigma \not{p}_1 \right\} \right]. \end{aligned} \quad (2.105)$$

Using eq. (2.22) and the relation

$$\hat{\gamma}^\mu \hat{\gamma}_\mu = -2\epsilon, \quad (2.106)$$

we can rewrite eq. (2.105) solely in terms of four-dimensional Dirac traces:

$$\begin{aligned} \mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) &= -\frac{a_q c_1(\epsilon, \mu^2) C_F}{4s} \int \frac{d^d k_1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2} \\ &\quad \left[\text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \bar{\gamma}^\sigma (\not{p}_2 + \not{k}_1) \bar{\gamma}_\rho (\not{p}_1 - \not{k}_1) \bar{\gamma}_\sigma \not{p}_1 \right\} - 2\epsilon \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 (\not{p}_2 + \not{k}_1) \bar{\gamma}_\rho (\not{p}_1 - \not{k}_1) \not{p}_1 \right\} \right. \\ &\quad \left. - \hat{k}_1 \cdot \hat{k}_1 \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \bar{\gamma}^\sigma \bar{\gamma}_\rho \bar{\gamma}_\sigma \not{p}_1 \right\} + 2\epsilon \hat{k}_1 \cdot \hat{k}_1 \text{tr} \left\{ \bar{\gamma}^\rho \not{p}_2 \bar{\gamma}_\rho \not{p}_1 \right\} \right]. \end{aligned} \quad (2.107)$$

Clearly, the remaining Dirac traces in eq. (2.107) may be evaluated in a straightforward manner by applying eqs. (2.5), (2.25), and (2.26) with ϵ set to zero and the d -dimensional metric tensor replaced by the four-dimensional metric tensor throughout. Discarding all contributions which vanish due to scalelessness and setting $\bar{k}_1 \cdot \bar{k}_1 = k_1^2 - \hat{k}_1 \cdot \hat{k}_1$, we find

$$\mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) = \frac{2 a_q c_1(\epsilon, \mu^2) C_F}{s} \int d^d k_1 \frac{2s k_1^2 + (1 + \epsilon)k_1^4 + s^2 + (1 + \epsilon)s \hat{k}_1 \cdot \hat{k}_1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2}. \quad (2.108)$$

From eq. (2.42), we have

$$\int \frac{d^d k_1}{i\pi^{d/2}} \frac{\hat{k}_1 \cdot \hat{k}_1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2} = \epsilon \int \frac{d^{d+2} k_1}{i\pi^{(d+2)/2}} \frac{1}{(p_1 - k_1)^2 k_1^2 (p_2 + k_1)^2}. \quad (2.109)$$

It is worth pointing out that, by construction, such explicit μ terms could *not* occur in a direct calculation of $\bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s)$ in Kreimer's γ_5 scheme. Finally, taking into account the algorithms of [64] and their implementation in `Reduze 2` [71–73] to treat the dimensionally-shifted integral on the right-hand side of eq. (2.109), it follows that

$$\mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) = \frac{a_q c_1(\epsilon, \mu^2) C_F (2 - 3\epsilon - \epsilon^2)}{(1 - \epsilon)\epsilon} \int \frac{d^d k_1}{(p_1 - k_1)^2 (p_2 + k_1)^2} \quad (2.110)$$

after a straightforward integration by parts reduction. As is well-known (see e.g. [74]),

$$\int \frac{d^d k_1}{(p_1 - k_1)^2 (p_2 + k_1)^2} = \frac{e^{\gamma_E \epsilon}}{c_1(\epsilon, \mu^2)} e^{i\pi\epsilon} \left(\frac{\mu^2}{s}\right)^\epsilon \frac{\Gamma^2(1 - \epsilon)\Gamma(\epsilon)}{\Gamma(2 - 2\epsilon)} \quad (2.111)$$

and therefore

$$\mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) = a_q \frac{(2 - 3\epsilon - \epsilon^2)\Gamma^2(1 - \epsilon)\Gamma(\epsilon)e^{\gamma_E \epsilon}}{(1 - \epsilon)\epsilon\Gamma(2 - 2\epsilon)} C_F e^{i\pi\epsilon} \left(\frac{\mu^2}{s}\right)^\epsilon. \quad (2.112)$$

Now, as explained above, the finite counterterm of interest is obtained as the difference between the Kreimer's γ_5 scheme expression deduced above and eq. (2.112):

$$\begin{aligned} \delta Z_{Z\bar{q}q}^{(0,1)} &= \bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s) - \mathcal{A}_{Z\bar{q}q}^{(0,1)}(s) \\ &= 2 a_q \frac{(2 - \epsilon)\Gamma^2(1 - \epsilon)\Gamma(1 + \epsilon)e^{\gamma_E \epsilon}}{(1 - \epsilon)\Gamma(2 - 2\epsilon)} C_F e^{i\pi\epsilon} \left(\frac{\mu^2}{s}\right)^\epsilon. \end{aligned} \quad (2.113)$$

eq. (2.113) is manifestly finite at $\epsilon = 0$; as expected, we see that $\delta Z_{Z\bar{q}q}^{(0,1)} = 4a_q C_F + \mathcal{O}(\epsilon)$, in complete agreement with eq. (9) of [56].⁹

Before leaving this section, let us stress that we do *not* employ in this paper what is commonly referred to as ‘‘Larin’s scheme’’ in the QCD literature. It is clear that the goal of [56] was to provide a recipe in FORM for the consistent treatment of multi-loop QCD corrections to the axial vector components of Standard Model vertex form factors

⁹In making this comparison, note that eq. (3) of [56] was not written down for the $Z\bar{q}q$ interaction verbatim and therefore does not have the overall factor of $-a_q$ present in interaction (2.24). Note also that the cited equation numbers refer to the journal version of [56].

in HVBM's γ_5 scheme; Larin did not discuss a prescription for the treatment of multi-loop box-type diagrams, for example. Indeed, in contrast to the prescription of [56] for the treatment of multi-loop QCD corrections to Standard Model vertex form factors, we found it essential for the restoration of chiral symmetry in the two-loop box-type diagrams treated below to consistently keep also higher-order terms of the ϵ expansions of our finite counterterms in order to avoid any premature truncation.

3 Calculational details

3.1 Process definition

In this work, we study lepton pair production in quark-antiquark annihilation,

$$q(p_1) + \bar{q}(p_2) \rightarrow \ell^-(p_3) + \ell^+(p_4), \tag{3.1}$$

at one and two loops. We obtain complete results for scattering amplitudes at order $\alpha\alpha_s$, order α^2 , and order $\alpha^2\alpha_s$ in the approximation of vanishing light quark and lepton masses, i.e. $p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$. As the present paper represents only a first decisive step towards the assembly of phenomenological results for Drell-Yan lepton production including complete two-loop EW-QCD effects in all channels, we defer the inclusion of the gauge-invariant contributions proportional to the number of light quark or lepton flavors and all top quark mass corrections to future work. The most important two-loop corrections we omit are those which involve a closed fermion loop; these contributions arise due to electroweak gauge boson self-energy insertions and are already known, see e.g. [26, 75]. We also do not consider $b\bar{b}$ initial states, which lead to double W exchange diagrams involving the top quark; in other words, we restrict ourselves to four light flavors in the initial state.

As usual, the $2 \rightarrow 2$ kinematics of (3.1) is conveniently described by the Mandelstam variables

$$s = (p_1 + p_2)^2 \quad \text{and} \quad t = (p_1 - p_3)^2; \quad u = (p_2 - p_3)^2 = -s - t. \tag{3.2}$$

In total, five independent kinematic scales, s , t , m_w , m_z , and m_h , appear in our results. Here, m_w is the mass of the W boson, m_z is the mass of the Z boson, and m_h is the mass of the Higgs boson. For simplicity, we present our final results in a physical phase-space region where t is allowed to take arbitrary physical values while s is constrained to be above all two-particle thresholds present in the contributing Feynman diagrams. This amounts to the condition

$$s > (m_h + m_z)^2 \quad (-s < t < 0) \tag{3.3}$$

imposed by the Z - Z self-energy. Working in the kinematic region defined by (3.3) allows us to do without the complex mass scheme, thereby avoiding further technical complications for our proof-of-concept study. As discussed in [31], this restriction also simplifies the explicit expressions for the two-loop master integrals.

3.2 Form factor decomposition

We decompose the one- and two-loop scattering amplitudes for our process into basic Lorentz structures (“standard matrix elements”) multiplied by form factors, which depend only on the kinematic invariants. Since we are considering massless external fermions in the Standard Model, the amplitude in conventional dimensional regularization can be written up to two loops in terms of the Lorentz structures [76]

$$\bar{\mathcal{T}}_1 = \bar{v}(p_2)\gamma^\mu u(p_1) \bar{u}(p_3)\gamma_\mu v(p_4), \tag{3.4}$$

$$\bar{\mathcal{T}}_2 = \bar{v}(p_2)\not{p}_3 u(p_1) \bar{u}(p_3)\not{p}_1 v(p_4), \tag{3.5}$$

$$\bar{\mathcal{T}}_3 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\gamma^\rho u(p_1) \bar{u}(p_3)\gamma_\mu\gamma_\nu\gamma_\rho v(p_4), \tag{3.6}$$

$$\bar{\mathcal{T}}_4 = \bar{v}(p_2)\gamma^\mu\not{p}_3\gamma^\nu u(p_1) \bar{u}(p_3)\gamma_\mu\not{p}_1\gamma_\nu v(p_4), \tag{3.7}$$

$$\bar{\mathcal{T}}_5 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau u(p_1) \bar{u}(p_3)\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau v(p_4), \tag{3.8}$$

$$\text{and } \bar{\mathcal{T}}_6 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\not{p}_3\gamma^\rho\gamma^\sigma u(p_1) \bar{u}(p_3)\gamma_\mu\gamma_\nu\not{p}_1\gamma_\rho\gamma_\sigma v(p_4), \tag{3.9}$$

where we suppressed color and spin indices and ignored insertions of γ_5 . For our EW-QCD corrections, structures $\bar{\mathcal{T}}_5$ and $\bar{\mathcal{T}}_6$ can be omitted even in a general R_ξ gauge because the gluon couples only to the initial-state quark line.

Allowing also for insertions of γ_5 at the end of each Dirac chain, we obtain sixteen different Lorentz structures. For the sake of our argument, let us consider an anticommuting γ_5 to illustrate the general principle of our projector method. The described form of the amplitude can be obtained either using Passarino-Veltman reduction of the tensor integrals or using projectors. In $d = 4$ dimensions, only four out of the sixteen Lorentz structures are independent, as can be seen using Schouten and Fierz identities. In order to facilitate a smooth $d \rightarrow 4$ limit, we can define a d -dimensional basis of the four structures derived from $\bar{\mathcal{T}}_1$ by inserting γ_5 matrices:

$$\bar{\mathcal{T}}_{\text{VV}} = \bar{v}(p_2)\gamma^\mu u(p_1) \bar{u}(p_3)\gamma_\mu v(p_4), \tag{3.10}$$

$$\bar{\mathcal{T}}_{\text{AA}} = \bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1) \bar{u}(p_3)\gamma_\mu\gamma_5 v(p_4), \tag{3.11}$$

$$\bar{\mathcal{T}}_{\text{AV}} = \bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1) \bar{u}(p_3)\gamma_\mu v(p_4), \tag{3.12}$$

$$\bar{\mathcal{T}}_{\text{VA}} = \bar{v}(p_2)\gamma^\mu u(p_1) \bar{u}(p_3)\gamma_\mu\gamma_5 v(p_4), \tag{3.13}$$

and twelve further structures (derived from $\bar{\mathcal{T}}_2$, $\bar{\mathcal{T}}_3$, and $\bar{\mathcal{T}}_4$ in a similar way), which are strictly “orthogonal” in d dimensions and smoothly vanish for $d \rightarrow 4$. The d dimensional projection onto the structures (3.10)–(3.13) obtained in this way are identical to those calculated by ignoring the additional directions at the outset. This construction is described in detail in appendix A, see also [77–81] for related work. We note that the four independent structures in $d = 4$ are of course directly related to the four independent helicity amplitudes; alternatively, for a given γ_5 scheme, we could have written polarized Lorentz structures by replacing e.g. $u(p_i)$ with $\frac{1}{2}(1 \pm \gamma_5)u(p_i)$.

For finite, renormalized and infrared-subtracted scattering amplitudes, $\mathcal{O}(\epsilon)$ deviations from the four-dimensional projectors will be irrelevant for the $\mathcal{O}(\epsilon^0)$ result. For infrared divergent amplitudes, the $\mathcal{O}(\epsilon^0)$ term depends on the exact choice of the projector. However,

as long as we use the exact same prescription for the projector for all divergent contributions, those ambiguities will exactly cancel for a finite sum. This argument assumes that we use an analytic continuation which gives a well-defined meaning to the higher-order-in- ϵ terms of the divergent quantities and that we do not truncate their Laurent expansions in ϵ prematurely. In the schemes we consider here, for contributions involving Levi-Civita tensors, we expect that consistency is ensured because Levi-Civita tensors effectively truncate the higher-order-in- ϵ terms of finite quantities. We will discuss the details of how we construct our projectors for the γ_5 schemes we consider in the following.

3.3 Calculations in HVBM's γ_5 scheme

The Lorentz decomposition of the Drell-Yan scattering amplitude in HVBM's γ_5 scheme is

$$i\mathcal{A}_{\text{DY}} = i(\mathbf{C}_{\text{VV}} \mathcal{T}_{\text{VV}} + \mathbf{C}_{\text{AA}} \mathcal{T}_{\text{AA}} + \mathbf{C}_{\text{VA}} \mathcal{T}_{\text{VA}} + \mathbf{C}_{\text{AV}} \mathcal{T}_{\text{AV}} + \dots), \quad (3.14)$$

where the dots denote additional structures which decouple in the limit $d \rightarrow 4$ and can effectively be ignored in our projector based setup. The relevant Lorentz structures for our calculation are

$$\mathcal{T}_{\text{VV}} = \bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(p_3)\gamma_\mu v(p_4), \quad (3.15)$$

$$\mathcal{T}_{\text{AA}} = \bar{v}(p_2)\frac{1}{2}[\gamma^\mu, \gamma_5]u(p_1)\bar{u}(p_3)\frac{1}{2}[\gamma_\mu, \gamma_5]v(p_4), \quad (3.16)$$

$$\mathcal{T}_{\text{VA}} = \bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(p_3)\frac{1}{2}[\gamma_\mu, \gamma_5]v(p_4), \quad (3.17)$$

$$\mathcal{T}_{\text{AV}} = \bar{v}(p_2)\frac{1}{2}[\gamma^\mu, \gamma_5]u(p_1)\bar{u}(p_3)\gamma_\mu v(p_4), \quad (3.18)$$

In the manner described in section 3.2, we employ Lorentz projectors to calculate the form factors \mathbf{C}_{VV} , \mathbf{C}_{AA} , \mathbf{C}_{VA} , and \mathbf{C}_{AV} in d dimensions. We request

$$\mathbf{C} = \sum_{\text{spin,color}} \mathbb{P}_{\mathbf{C}} i\mathcal{A}_{\text{DY}}, \quad \text{for } \mathbf{C} = \mathbf{C}_{\text{VV}}, \mathbf{C}_{\text{AA}}, \mathbf{C}_{\text{VA}}, \mathbf{C}_{\text{AV}}, \quad (3.19)$$

to hold in d dimensions and obtain

$$\mathbb{P}_{\mathbf{C}_{\text{VV}}} = \frac{i(s^2 - 2tu)}{8N_c(\epsilon s^2(s^2 - 2tu) - 4t^2u^2)}\mathcal{T}_{\text{VV}}^\dagger - \frac{is(t-u)}{8N_c(\epsilon s^2(s^2 - 2tu) - 4t^2u^2)}\mathcal{T}_{\text{AA}}^\dagger, \quad (3.20)$$

$$\mathbb{P}_{\mathbf{C}_{\text{AA}}} = -\frac{is(t-u)}{8N_c(\epsilon s^2(s^2 - 2tu) - 4t^2u^2)}\mathcal{T}_{\text{VV}}^\dagger + \frac{i((1-\epsilon)s^2 - 2tu)}{8N_c(\epsilon s^2(s^2 - 2tu) - 4t^2u^2)}\mathcal{T}_{\text{AA}}^\dagger, \quad (3.21)$$

$$\mathbb{P}_{\mathbf{C}_{\text{VA}}} = -\frac{i(s^2 - 2tu)}{32N_c t^2 u^2}\mathcal{T}_{\text{VA}}^\dagger + \frac{is(t-u)}{32N_c t^2 u^2}\mathcal{T}_{\text{AV}}^\dagger, \quad (3.22)$$

$$\mathbb{P}_{\mathbf{C}_{\text{AV}}} = \frac{is(t-u)}{32N_c t^2 u^2}\mathcal{T}_{\text{VA}}^\dagger - \frac{i(s^2 - 2tu)}{32N_c t^2 u^2}\mathcal{T}_{\text{AV}}^\dagger. \quad (3.23)$$

In practice, we use these projectors only for box-type diagrams; for diagrams with self-energy or vertex insertions, we choose to sew effective propagators and vertices into appropriate tree-level diagrams. Note that, as we plan to perform a calculation of all Feynman diagrams in Kreimer's γ_5 scheme anyway, Larin's principle allows us to *completely bypass*

the direct calculation of almost all vertex Feynman diagrams in HVBM's γ_5 scheme; as explained in section 2.8, the vertex form factor projector defined in eq. (2.101) above was employed for the calculation of the finite counterterm $\delta Z_{Z\bar{q}q}^{(0,1)}$ of eq. (2.113), but that is all.¹⁰

After carrying out the renormalization program outlined in sections 2.5–2.8, the loop-level amplitude form factor coefficients still diverge in the infrared. These singularities in the virtual amplitudes manifest as poles in ϵ which are canceled by taking real radiative corrections into account and, in the end, considering perturbative corrections to physical observables. Fortunately, we have *a priori* knowledge of the infrared singularity structure of the scattering amplitudes we calculate in this work due to the well-known dipole formula [82–88] and a simple generalization thereof. In section 3.5, we review the two-loop mixed EW-QCD neutral-current Drell-Yan dipole singularity structure derived in [20] and show how subtraction functions may be defined which allow for the comparison of finite hard scattering functions in different γ_5 schemes.

We now turn to a technical discussion of our approach to Dirac trace manipulation and numerator algebra in `Mathematica`. First of all, note that all terms which involve an odd number of γ_5 matrices *in total* must vanish due to the fact that we consider a $2 \rightarrow 2$ scattering process with purely massless external momenta which cannot support a contraction with a Levi-Civita tensor. One possible approach to calculate the non-vanishing terms in HVBM's γ_5 scheme, popularized by [56], would be to make use of the identity

$$\frac{1}{2}[\gamma_\mu, \gamma_5] = -\frac{i}{3!} \epsilon_{\mu\nu\rho\sigma} \gamma^\nu \gamma^\rho \gamma^\sigma, \quad (3.24)$$

derived from eqs. (2.3) and (2.28), at the level of the d -dimensional Feynman rules. This prescription, however, has the very unpleasant side-effect that the lengths of *all* Dirac traces involving γ_5 are artificially inflated at intermediate stages.

While the associated performance penalty could be ameliorated to some extent by exploiting the total antisymmetry of the Levi-Civita tensor, we prefer instead to follow [52] and split indices from the very beginning. We use the anticommutation relations, eqs. (2.20) and (2.21), to write¹¹

$$\frac{1}{2}[\gamma_\mu, \gamma_5] = \bar{\gamma}_\mu \gamma_5. \quad (3.25)$$

Once all explicit γ_5 matrices have been anticommutated away and traces with an odd number of hatted variables have been discarded, we apply eq. (2.5) and its companion identity,

$$\not{q} \gamma_{\nu_1} \cdots \gamma_{\nu_n} \not{q} = 2 \sum_{i=1}^n (-1)^{i+n} q_{\nu_i} \not{q} \gamma_{\nu_1} \cdots \gamma_{\nu_{i-1}} \gamma_{\nu_{i+1}} \cdots \gamma_{\nu_n} + q^2 (-1)^n \gamma_{\nu_1} \cdots \gamma_{\nu_n}, \quad (3.26)$$

to shorten all Dirac traces at an early stage to the maximal extent possible.

In our view, this strategy is most sensible because, due to the presence of box-type diagrams, it is anyway inevitable that we must split indices at *some* stage. As a bonus,

¹⁰Strictly speaking, there is another finite counterterm which is needed as well, $\delta Z_{W^\pm \bar{q}q}^{(0,1)}$, but it may be trivially obtained from eq. (2.113) via the replacement $a_q \rightarrow a_w$.

¹¹To be clear, we distilled eq. (3.25) from [48]; axial vector couplings were not considered in [52].

eq. (3.26) also allows us to exploit the on-shell kinematics to discard vanishing terms at an early stage. While it may not be obvious at first sight, eqs. (2.5), (2.25), and (3.26) can be applied verbatim to traces which contain both barred and hatted variables, provided that mixed contractions of indices are discarded as soon as they are produced by the formulae. Although it was not really needed for the two-loop calculations considered in this work, it is worth mentioning that an aggressive use of memoization for the Dirac traces appearing in our calculations could have significantly decreased the runtime required by our code to process all Feynman diagrams.

Once all Dirac traces have been evaluated and all compatible Lorentz indices contracted, contractions and products of Levi-Civita tensors from the box-type diagrams must be dealt with using eqs. (2.12)–(2.16). Subsequently, all scalar products of the form $\bar{g}_{\mu\nu} k_i^\mu k_j^\nu$ must be processed using the methods reviewed in section 2.4. Finally, taking into account the algorithms of [64] and their implementation in `Reduze 2`, we obtain the amplitude form factor coefficients via integration by parts reduction as a linear combination of master integrals. At both one and two loops, we find it particularly convenient to rotate to *normal form bases* of Feynman integrals, characterized by the simple, ϵ -decoupled differential equations they satisfy [89, 90]. Our one- and two-loop normal form bases are defined in sections 4.2 and 5.2.

3.4 Calculations in Kreimer’s γ_5 scheme

The Lorentz decomposition of the Drell-Yan scattering amplitude in Kreimer’s γ_5 scheme is essentially eq. (B.10) of [66]. Suppressing the color indices, we have

$$i\bar{\mathcal{A}}_{\text{DY}} = i \left(\bar{\mathcal{C}}_{\text{VV}} \bar{\mathcal{T}}_{\text{VV}} + \bar{\mathcal{C}}_{\text{AA}} \bar{\mathcal{T}}_{\text{AA}} + \bar{\mathcal{C}}_{\text{VA}} \bar{\mathcal{T}}_{\text{VA}} + \bar{\mathcal{C}}_{\text{AV}} \bar{\mathcal{T}}_{\text{AV}} + \dots \right), \quad (3.27)$$

where the dots denote additional Lorentz structures which can be ignored in our projector based setup, and the Lorentz structures of relevance are defined in (3.10)–(3.13). The corresponding Lorentz projectors in Kreimer’s γ_5 scheme fulfill

$$\bar{\mathcal{C}} = \sum_{\text{spin,color}} \mathbb{P}_{\bar{\mathcal{C}}} i\bar{\mathcal{A}}_{\text{DY}}, \quad \text{for } \bar{\mathcal{C}} = \bar{\mathcal{C}}_{\text{VV}}, \bar{\mathcal{C}}_{\text{AA}}, \bar{\mathcal{C}}_{\text{VA}}, \bar{\mathcal{C}}_{\text{AV}}, \quad (3.28)$$

in d dimensions and are given by

$$\mathbb{P}_{\bar{\mathcal{C}}_{\text{VV}}} = -\frac{i((1-\epsilon)s^2 + 2t(s+t))}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{VV}}^\dagger + \frac{is(s+2t)}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{AA}}^\dagger, \quad (3.29)$$

$$\mathbb{P}_{\bar{\mathcal{C}}_{\text{AA}}} = \frac{is(s+2t)}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{VV}}^\dagger - \frac{i((1-\epsilon)s^2 + 2t(s+t))}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{AA}}^\dagger, \quad (3.30)$$

$$\mathbb{P}_{\bar{\mathcal{C}}_{\text{VA}}} = -\frac{i((1-\epsilon)s^2 + 2t(s+t))}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{VA}}^\dagger + \frac{is(s+2t)}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{AV}}^\dagger, \quad (3.31)$$

$$\mathbb{P}_{\bar{\mathcal{C}}_{\text{AV}}} = \frac{is(s+2t)}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{VA}}^\dagger - \frac{i((1-\epsilon)s^2 + 2t(s+t))}{8N_c(2t^2 - \epsilon s^2)D} \bar{\mathcal{T}}_{\text{AV}}^\dagger, \quad (3.32)$$

with $D \equiv (2-\epsilon)s^2 + 2t(2s+t)$.

Again, we employ eqs. (3.29)–(3.32) only for the box contributions. For the vertex corrections, the Kreimer’s γ_5 scheme analogs of eqs. (2.99), (2.100), and (2.101),

$$ie \left(\bar{\mathcal{F}}_{\mu}^{V\bar{q}q}(p_1, p_2) \right)_{kj} = ie \bar{\mathcal{V}}_{V\bar{q}q}(s) \bar{v}_k(p_2) \gamma_{\mu} u_j(p_1) \tag{3.33}$$

$$+ ie \bar{\mathcal{A}}_{V\bar{q}q}(s) \bar{v}_k(p_2) \gamma_{\mu} \gamma_5 u_j(p_1),$$

$$\left(\mathbb{P}_{\bar{\mathcal{V}}_q}^{\mu} \right)_{jk} = \frac{i}{4eN_c s(1-\epsilon)} \bar{u}_j(p_1) \gamma^{\mu} v_k(p_2), \tag{3.34}$$

$$\left(\mathbb{P}_{\bar{\mathcal{A}}_q}^{\mu} \right)_{jk} = \frac{i}{4eN_c s(1-\epsilon)} \bar{u}_j(p_1) \gamma^{\mu} \gamma_5 v_k(p_2), \tag{3.35}$$

are useful.¹² Of course, a decomposition completely analogous to (3.33) exists for the lepton vertex form factors,

$$ie \left(\bar{\mathcal{F}}_{\mu}^{V\bar{\ell}\ell}(p_3, p_4) \right)_{mn} = ie \bar{\mathcal{V}}_{V\bar{\ell}\ell}(s) \bar{u}_m(p_3) \gamma_{\mu} v_n(p_4) \tag{3.36}$$

$$+ ie \bar{\mathcal{A}}_{V\bar{\ell}\ell}(s) \bar{u}_m(p_3) \gamma_{\mu} \gamma_5 v_n(p_4).$$

$\left(\mathbb{P}_{\bar{\mathcal{V}}_{\ell}}^{\mu} \right)_{nm}$ and $\left(\mathbb{P}_{\bar{\mathcal{A}}_{\ell}}^{\mu} \right)_{nm}$ are obtained by setting $N_c = 1$ in eqs. (3.34) and (3.35),

$$\left(\mathbb{P}_{\bar{\mathcal{V}}_{\ell}}^{\mu} \right)_{nm} = \frac{i}{4es(1-\epsilon)} \bar{v}_n(p_4) \gamma^{\mu} u_m(p_3), \tag{3.37}$$

$$\left(\mathbb{P}_{\bar{\mathcal{A}}_{\ell}}^{\mu} \right)_{nm} = \frac{i}{4es(1-\epsilon)} \bar{v}_n(p_4) \gamma^{\mu} \gamma_5 u_m(p_3). \tag{3.38}$$

Note that the symmetric form of e.g. eqs. (3.34) and (3.35) is a reflection of the absence of chiral mismatch in Kreimer’s γ_5 scheme (contrast with the asymmetric form of eqs. (2.100) and (2.101)).

We calculate our one- and two-loop scattering amplitudes in **Mathematica**, using a code pipeline similar to that described in section 3.3; as mentioned in section 2.3, the algebraic form of the non-vanishing families of even and odd Dirac traces is the same in Kreimer’s γ_5 scheme as in HVBM’s γ_5 scheme. A key advantage of Kreimer’s γ_5 scheme, however, is that, due to the standard anticommutation relation, eq. (2.7), Dirac traces need not be calculated with split indices. Rather, one can first evaluate all traces and introduce scalar products of the form $\bar{g}_{\mu\nu} k_i^{\mu} k_j^{\nu}$ only at a later stage, when contractions and products of Levi-Civita tensors are eliminated using eqs. (2.12)–(2.16). As a cross-check, we also calculated the interference of the one- and two-loop scattering amplitudes with the tree-level Drell-Yan amplitude in Kreimer’s γ_5 scheme using an independent setup in **FORM 4** [91] which, in particular, employed Passarino-Veltman reduction [63] instead of the technology of section 2.4 to simplify scalar products of the form $\bar{g}_{\mu\nu} k_i^{\mu} k_j^{\nu}$. Finding complete agreement between our independent implementations served as a crucial cross-check on our calculations.

¹²In eqs. (3.33) and (3.36), V may be either a photon or a Z boson.

3.5 Infrared dipole singularity structure and subtraction functions

The infrared singularities of general scattering amplitudes in two-loop QCD can be predicted in a process independent way [53, 92–96]. The dipole formula [82–88] provides a particularly concise and straightforward recipe for the generation of infrared subtraction terms at two loops.¹³ While, historically, the singularity structures of scattering amplitudes in particularly simple models such as massless QED, massless QCD, and $\mathcal{N} = 4$ super Yang-Mills theory were discussed most frequently in the literature, broader applications to theories with massive particles or mixed gauge groups are certainly possible and have long been known, see e.g. [20, 107]. In particular, it was shown in [20] that with straightforward modifications, the dipole formula can be extended to describe the singularity structure of two-loop mixed QED-QCD Drell-Yan scattering amplitudes. This setup covers also the case of the mixed EW-QCD corrections discussed in this paper.

We begin by introducing the building blocks required to describe the infrared singularities of the order $\alpha\alpha_s$, order α^2 , and order $\alpha^2\alpha_s$ neutral-current Drell-Yan scattering amplitudes. As has long been clear, the leading infrared singularities of gauge theory scattering amplitudes are governed by cusp anomalous dimensions [108]. The one-loop quark cusp anomalous dimension of massless QCD may be extracted, for example, from the expression for the one-loop quark form factor of massless QCD given in eq. (2.103). In QCD, we have

$$\Gamma_q^{(0,1)} = 4 C_F, \tag{3.39}$$

whereas in QED we obtain the result from eq. (3.39) by replacing the quadratic Casimir invariant C_F with the squared charge of the fermion flavor,

$$\Gamma_f^{(1,0)} = 4 Q_f^2. \tag{3.40}$$

However, as pointed out in [20], the mixed quark cusp anomalous dimension vanishes:

$$\Gamma_q^{(1,1)} = 0. \tag{3.41}$$

The next-to-leading infrared singularities are more complicated and receive contributions from both soft anomalous dimensions and resummation functions derived from the ϵ^{-1} poles of massless vertex form factors with an appropriate number of gluon and/or photon exchanges. The one-loop Drell-Yan soft anomalous dimensions are well-known and have been calculated in many places in the literature, see e.g. [83, 84] for a thorough discussion. Note that, for general QCD scattering processes, the soft anomalous dimension becomes a mixing matrix in color space. For processes with just two partons in the initial state such as the Drell-Yan process, the color space structure trivializes and all matrices in the dipole formula may be replaced with functions. Rewriting the results to make all

¹³At three loops and beyond, an extension of the formalism is necessary [97–106].

imaginary parts explicit in the physical kinematic region of interest, we find

$$S_{\text{DY}}^{(0,1)} = \left(-4 \ln \left(\frac{\mu^2}{s} \right) - 4i\pi \right) C_F \quad (3.42)$$

$$\text{and } S_{\text{DY}}^{(1,0)} = \left(-4 \ln \left(\frac{\mu^2}{s} \right) - 4i\pi \right) (Q_\ell^2 + Q_q^2) + 8Q_\ell Q_q \ln \left(\frac{t}{u} \right). \quad (3.43)$$

Once again, as shown in [20], the two-loop mixed EW-QCD Drell-Yan soft anomalous dimension vanishes identically:

$$S_{\text{DY}}^{(1,1)} = 0. \quad (3.44)$$

We define our massless QCD resummation functions in the framework of [109]. In practice, following [20], all results may be derived from the one- and two-loop quark resummation functions of massless QCD by making simple replacements. In what follows, $f[k]$ denotes the coefficient of ϵ^k in the Taylor series expansion of $f(\epsilon)$,

$$f(\epsilon) = \sum_{k=0}^{\infty} f[k] \epsilon^k. \quad (3.45)$$

To the relevant ϵ orders, we have

$$G_q^{(0,1)}[0] = 6 C_F, \quad (3.46)$$

$$G_q^{(0,1)}[1] = \left(16 - 2\zeta_2 \right) C_F, \quad (3.47)$$

$$G_q^{(0,1)}[2] = \left(32 - 3\zeta_2 - \frac{28}{3}\zeta_3 \right) C_F, \quad (3.48)$$

$$G_q^{(0,1)}[3] = \left(64 - 8\zeta_2 - 14\zeta_3 - \frac{47}{10}\zeta_2^2 \right) C_F, \quad (3.49)$$

$$G_q^{(0,2)}[0] = \left(3 - 24\zeta_2 + 48\zeta_3 \right) C_F^2 + \left(\frac{2545}{27} + \frac{44}{3}\zeta_2 - 52\zeta_3 \right) C_A C_F, \quad (3.50)$$

$$\text{and } G_q^{(0,2)}[1] = \left(\frac{1}{2} - 116\zeta_2 + 120\zeta_3 + \frac{176}{5}\zeta_2^2 \right) C_F^2 + \left(\frac{70165}{162} + \frac{575}{9}\zeta_2 - \frac{520}{3}\zeta_3 - \frac{176}{5}\zeta_2^2 \right) C_A C_F \quad (3.51)$$

from eqs. (3.10) and (3.11) of [109] after discarding all contributions proportional to the number of light quarks.

As with the cusp anomalous dimensions, the one-loop QED results may be obtained by making the replacement $C_F \rightarrow Q_f^2$ in eqs. (3.46)–(3.49). Explicitly, we have

$$G_f^{(1,0)}[0] = 6 Q_f^2, \quad (3.52)$$

$$G_f^{(1,0)}[1] = \left(16 - 2\zeta_2 \right) Q_f^2, \quad (3.53)$$

$$G_f^{(1,0)}[2] = \left(32 - 3\zeta_2 - \frac{28}{3}\zeta_3\right) Q_f^2, \quad (3.54)$$

$$\text{and } G_f^{(1,0)}[3] = \left(64 - 8\zeta_2 - 14\zeta_3 - \frac{47}{10}\zeta_2^2\right) Q_f^2. \quad (3.55)$$

Finally, the two-loop mixed EW-QCD quark resummation functions are obtained from eqs. (3.50) and (3.51) by setting C_A to zero and replacing C_F^2 with $Q_q^2 C_F$:

$$G_q^{(1,1)}[0] = \left(3 - 24\zeta_2 + 48\zeta_3\right) Q_q^2 C_F \quad (3.56)$$

$$\text{and } G_q^{(1,1)}[1] = \left(\frac{1}{2} - 116\zeta_2 + 120\zeta_3 + \frac{176}{5}\zeta_2^2\right) Q_q^2 C_F. \quad (3.57)$$

Given the ingredients discussed above, we are now in a position to present the predictions of the generalized dipole formula for the ultraviolet-renormalized, infrared-divergent scattering amplitudes \mathcal{A}_{DY} and $\bar{\mathcal{A}}_{\text{DY}}$ in, respectively, HVBM's γ_5 scheme and Kreimer's γ_5 scheme. At tree level we have

$$\overleftrightarrow{\mathcal{A}}_{\text{DY}} = 4\pi\alpha \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)}[0] + \dots, \quad (3.58)$$

where the dots stand for terms of higher order in the coupling constants. As the notation suggests, the tree-level hard functions, $\mathcal{H}_{\text{DY}}^{(0,0)}[0]$ and $\bar{\mathcal{H}}_{\text{DY}}^{(0,0)}[0]$, are defined to be independent of the coupling constants. We have

$$\overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)}[0] = \left(\frac{Q_q Q_\ell}{s} + \frac{v_q v_\ell}{s - m_z^2}\right) \overleftrightarrow{\mathcal{T}}_{\text{VV}} - \frac{a_\ell v_q}{s - m_z^2} \overleftrightarrow{\mathcal{T}}_{\text{VA}} - \frac{a_q v_\ell}{s - m_z^2} \overleftrightarrow{\mathcal{T}}_{\text{AV}} + \frac{a_q a_\ell}{s - m_z^2} \overleftrightarrow{\mathcal{T}}_{\text{AA}} \quad (3.59)$$

in the notation of [66] for the couplings of the Z boson to matter, see eqs. (2.73).

As is clear from the form of eqs. (3.59), the tree-level amplitudes in the two γ_5 schemes we consider trivially coincide in the four-dimensional limit because all of the differences between the schemes disappear in four spacetime dimensions:

$$\lim_{\epsilon \rightarrow 0} \left\{ \mathcal{H}_{\text{DY}}^{(0,0)}[0] \right\} = \lim_{\epsilon \rightarrow 0} \left\{ \bar{\mathcal{H}}_{\text{DY}}^{(0,0)}[0] \right\}. \quad (3.60)$$

For the remainder of this work, it will sometimes be convenient to view the hard scattering functions in the two schemes we consider as vectors of coefficients with respect to $\overleftrightarrow{\mathcal{T}}_{\text{VV}}$, $\overleftrightarrow{\mathcal{T}}_{\text{VA}}$, $\overleftrightarrow{\mathcal{T}}_{\text{AV}}$, and $\overleftrightarrow{\mathcal{T}}_{\text{AA}}$. This minor abuse of notation will allow us to rewrite e.g. eq. (3.60) as

$$\mathcal{H}_{\text{DY}}^{(0,0)}[0] = \bar{\mathcal{H}}_{\text{DY}}^{(0,0)}[0]. \quad (3.61)$$

At higher perturbative orders, we have

$$\overleftrightarrow{\mathcal{A}}_{\text{DY}} = 4\pi\alpha \left(\overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(0,0)} + \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(0,1)} \left(\frac{\alpha_s}{4\pi}\right) + \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(1,0)} \left(\frac{\alpha}{4\pi}\right) + \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(1,1)} \left(\frac{\alpha}{4\pi}\right) \left(\frac{\alpha_s}{4\pi}\right) + \dots \right), \quad (3.62)$$

where

$$\overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(0,0)} = \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)}, \quad (3.63)$$

$$\begin{aligned}
 \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(0,1)} &= \frac{1}{\epsilon^2} \left(-\frac{1}{2} \Gamma_q^{(0,1)} \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) + \frac{1}{\epsilon} \left(\frac{1}{2} [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) \\
 &\quad - \frac{1}{2} G_q^{(0,1)} [1] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [0] + \epsilon \left(-\frac{1}{2} G_q^{(0,1)} [2] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [1] \right) \\
 &\quad + \epsilon^2 \left(-\frac{1}{2} G_q^{(0,1)} [3] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [2] \right) + \dots, \tag{3.64}
 \end{aligned}$$

$$\begin{aligned}
 \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(1,0)} &= \frac{1}{\epsilon^2} \left(-\frac{1}{2} [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) + \frac{1}{\epsilon} \left(\frac{1}{2} [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] - G_q^{(1,0)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) \\
 &\quad - \frac{1}{2} [G_\ell^{(1,0)} [1] + G_q^{(1,0)} [1]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [0] + \epsilon \left(-\frac{1}{2} [G_\ell^{(1,0)} [2] + G_q^{(1,0)} [2]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right. \\
 &\quad \left. + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [1] \right) + \epsilon^2 \left(-\frac{1}{2} [G_\ell^{(1,0)} [3] + G_q^{(1,0)} [3]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [2] \right) + \dots, \tag{3.65}
 \end{aligned}$$

and

$$\begin{aligned}
 \overleftrightarrow{\mathcal{A}}_{\text{DY}}^{(1,1)} &= \frac{1}{\epsilon^4} \left(\frac{1}{4} \Gamma_q^{(0,1)} [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) + \frac{1}{\epsilon^3} \left(-\frac{1}{4} \left(\Gamma_q^{(0,1)} [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] \right. \right. \\
 &\quad \left. \left. - G_q^{(1,0)} [0] + [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \right) \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] \right) + \frac{1}{\epsilon^2} \left(\frac{1}{4} \left(\Gamma_q^{(0,1)} [G_\ell^{(1,0)} [1] \right. \right. \\
 &\quad \left. \left. + G_q^{(1,0)} [1] + [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] G_q^{(0,1)} [1] + [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] \right. \right. \\
 &\quad \left. \left. - G_q^{(1,0)} [0] \right) \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] - \frac{1}{2} \Gamma_q^{(0,1)} \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [0] - \frac{1}{2} [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [0] \right) \\
 &\quad + \frac{1}{\epsilon} \left(\frac{1}{4} \left(\Gamma_q^{(0,1)} [G_\ell^{(1,0)} [2] + G_q^{(1,0)} [2]] + [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] G_q^{(0,1)} [2] \right. \right. \\
 &\quad \left. \left. - G_q^{(0,1)} [1] [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] - G_q^{(1,0)} [0]] - [G_\ell^{(1,0)} [1] + G_q^{(1,0)} [1]] [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \right. \right. \\
 &\quad \left. \left. - 2G_q^{(1,1)} [0] \right) \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] - \frac{1}{2} \Gamma_q^{(0,1)} \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [1] - \frac{1}{2} [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [1] \right) \\
 &\quad + \frac{1}{2} [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] - G_q^{(1,0)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [0] + \frac{1}{2} [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [0] \\
 &\quad + \frac{1}{4} \left(\Gamma_q^{(0,1)} [G_\ell^{(1,0)} [3] + G_q^{(1,0)} [3]] + [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] G_q^{(0,1)} [3] \right. \\
 &\quad \left. - G_q^{(0,1)} [2] [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] - G_q^{(1,0)} [0]] - [G_\ell^{(1,0)} [2] + G_q^{(1,0)} [2]] [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \right. \\
 &\quad \left. + G_q^{(0,1)} [1] [G_\ell^{(1,0)} [1] + G_q^{(1,0)} [1]] - 2G_q^{(1,1)} [1] \right) \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,0)} [0] - \frac{1}{2} G_q^{(0,1)} [1] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [0] \\
 &\quad - \frac{1}{2} [G_\ell^{(1,0)} [1] + G_q^{(1,0)} [1]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [0] + \frac{1}{2} [S_{\text{DY}}^{(0,1)} - G_q^{(0,1)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [1] \\
 &\quad + \frac{1}{2} [S_{\text{DY}}^{(1,0)} - G_\ell^{(1,0)} [0] - G_q^{(1,0)} [0]] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [1] - \frac{1}{2} \Gamma_q^{(0,1)} \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,0)} [2] \\
 &\quad - \frac{1}{2} [\Gamma_\ell^{(1,0)} + \Gamma_q^{(1,0)}] \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(0,1)} [2] + \overleftrightarrow{\mathcal{H}}_{\text{DY}}^{(1,1)} [0] + \dots. \tag{3.66}
 \end{aligned}$$

The one- and two-loop hard scattering functions which appear above are determined by demanding equality between eqs. (3.62) and the corresponding explicit calculations at one and two loops.

All other things being equal, it is generally believed that HVBM's γ_5 scheme and Kreimer's γ_5 scheme produce identical results through to one loop and $\mathcal{O}(\epsilon^0)$ [54]. This proposition was implicitly affirmed at order $\alpha\alpha_s$ in section 2.8. At order α^2 , our explicit one-loop calculations, discussed in detail in the next section, do indeed show that

$$\mathcal{H}_{\text{DY}}^{(1,0)}[0] = \bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[0]. \tag{3.67}$$

However, we also find that

$$\mathcal{H}_{\text{DY}}^{(1,0)}[k] \neq \bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[k] \quad \text{for } k > 0. \tag{3.68}$$

Thus, in light of the fact that the higher-order-in- ϵ one-loop hard functions first enter eqs. (3.62) at the level of the ϵ^{-1} poles of the relative order $\alpha\alpha_s$ expressions, it is not obvious that an analog of eq. (3.67) continues to hold at higher orders in perturbation theory:

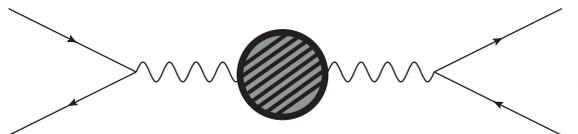
$$\mathcal{H}_{\text{DY}}^{(1,1)}[0] \stackrel{?}{=} \bar{\mathcal{H}}_{\text{DY}}^{(1,1)}[0]. \tag{3.69}$$

While it is generally expected that all terms of virtual and real radiative corrections sensitive to ambiguities in the treatment of γ_5 eventually cancel out of combinations which furnish complete fixed-order perturbative corrections to physical observables, hard partonic scattering functions by themselves are not physical observables. We will test whether (3.69) actually *is* an equality with our explicit calculations below.

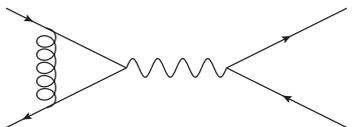
4 One-loop scattering amplitudes

4.1 Diagrammatic structure

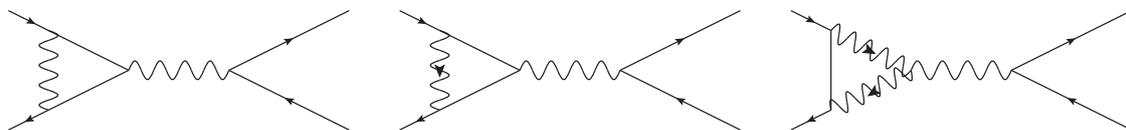
In this section, we present the diagrammatic structure of the one-loop perturbative corrections to the neutral-current Drell-Yan process. Due to their familiarity, the renormalized self-energy corrections to the photon and Z propagators,



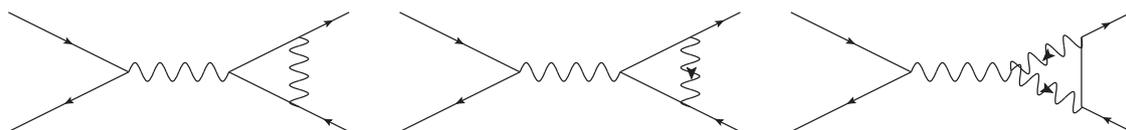
will not be visualized in explicit detail. In what follows, we will also use a generic wavy line for both the photon and the Z , keeping in mind that some contributions involving e.g. a coupling to neutrinos can occur only for the Z . As is well-known, just a single class of diagram contributes to the one-loop corrections of relative order α_s ,



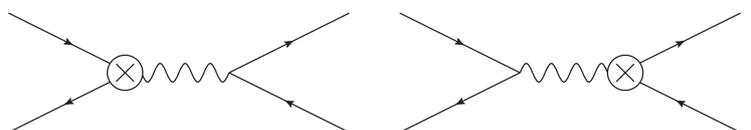
The remaining one-loop corrections of relative order α fall into three categories: initial state vertex diagrams, final state vertex diagrams, and box diagrams. Both the initial state vertex diagrams



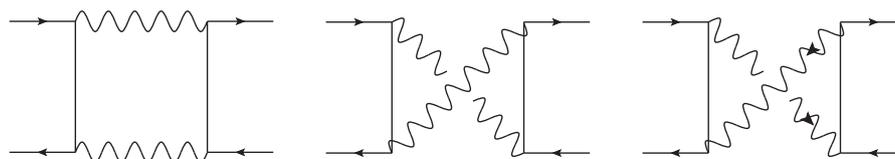
and the final state vertex diagrams



require renormalization,



due to the presence of ultraviolet divergences. The box diagrams



have infrared divergences only.

4.2 One-loop integral definitions

All of the one-loop integrals defined below are pure functions taken in the standard $\overline{\text{MS}}$ normalization. Therefore, all of our one-loop Feynman integrals are understood to have a multiplicative factor of

$$c_1(\epsilon, \mu^2) = \frac{e^{\gamma_E \epsilon} (\mu^2)^\epsilon}{i\pi^{2-\epsilon}} \tag{4.1}$$

in the integration measure. In what follows, thin solid lines denote massless propagators or massless external momenta, thick, dotted lines denote propagators of mass m_w , thick, dashed lines denote propagators of mass m_z , and thick solid lines denote propagators of mass m_h . In total, 31 linearly-independent one-loop integrals appear in the one-loop calculations of section 4.3 (supplemented by appendix B):

$$I_1 = \epsilon \left[\text{Diagram: a circle with a thick dotted line and a dot, and five thin solid lines entering from the left} \right] (m_w^2) \tag{4.2}$$

$$I_2 = \sqrt{s(s - 4m_w^2)} \epsilon \left[\text{Diagram: a circle with a thick dotted line and a dot, and four thin solid lines entering from the left} \right] (s, m_w^2) \tag{4.3}$$

$$I_3 = \sqrt{m_z^2(4m_w^2 - m_z^2)} \epsilon \text{ --- } \text{---} \left(m_w^2, m_z^2 \right) \quad (4.4)$$

$$I_4 = \epsilon \text{ ---} \left(m_z^2 \right) \quad (4.5)$$

$$I_5 = \epsilon \text{ ---} \left(m_h^2 \right) \quad (4.6)$$

$$I_6 = \sqrt{\lambda(s, m_z^2, m_h^2)} \epsilon \left(\text{---} + \text{---} \right) (s, m_z^2, m_h^2) \quad (4.7)$$

$$I_7 = \sqrt{m_h^2(4m_z^2 - m_h^2)} \epsilon \left(\text{---} + \text{---} \right) (m_z^2, m_h^2) \quad (4.8)$$

$$I_8 = s \epsilon \text{ ---} (s, m_w^2) \quad (4.9)$$

$$I_9 = \sqrt{\lambda(s, m_w^2, m_z^2)} \epsilon \left(\text{---} + \text{---} \right) (s, m_w^2, m_z^2) \quad (4.10)$$

$$I_{10} = \sqrt{\lambda(s, m_w^2, m_h^2)} \epsilon \left(\text{---} + \text{---} \right) (s, m_w^2, m_h^2) \quad (4.11)$$

$$I_{11} = \sqrt{m_z^2(4m_w^2 - m_z^2)} \epsilon \left(\text{---} + \text{---} \right) (m_w^2, m_z^2) \quad (4.12)$$

$$I_{12} = \sqrt{m_h^2(4m_w^2 - m_h^2)} \epsilon \left(\text{---} + \text{---} \right) (m_w^2, m_h^2) \quad (4.13)$$

$$I_{13} = s \epsilon \text{ ---} (s) \quad (4.14)$$

$$I_{14} = s \epsilon^2 \text{ ---} (s, m_z^2) \quad (4.15)$$

$$I_{15} = s \epsilon^2 \text{ ---} (s, m_w^2) \quad (4.16)$$

$$I_{16} = s \epsilon^2 \text{ ---} (s, m_w^2) \quad (4.17)$$

$$I_{17} = t \epsilon \quad \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} (t) \quad (4.18)$$

$$I_{18} = u \epsilon \quad \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} (u) \quad (4.19)$$

$$I_{19} = s t \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, t) \quad (4.20)$$

$$I_{20} = s u \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, u) \quad (4.21)$$

$$I_{21} = s \epsilon \quad \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} (s, m_z^2) \quad (4.22)$$

$$I_{22} = t \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \end{array} (t, m_z^2) \quad (4.23)$$

$$I_{23} = u \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \end{array} (u, m_z^2) \quad (4.24)$$

$$I_{24} = t (s - m_z^2) \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, t, m_z^2) \quad (4.25)$$

$$I_{25} = u (s - m_z^2) \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, u, m_z^2) \quad (4.26)$$

$$I_{26} = \sqrt{s(s - 4m_z^2)} \epsilon \quad \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} (s, m_z^2) \quad (4.27)$$

$$I_{27} = s \epsilon^2 \quad \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \end{array} (s, m_z^2) \quad (4.28)$$

$$I_{28} = \sqrt{s t (s t - 4m_z^2 (t + m_z^2))} \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, t, m_z^2) \quad (4.29)$$

$$I_{29} = \sqrt{s u (s u - 4m_z^2 (u + m_z^2))} \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, u, m_z^2) \quad (4.30)$$

$$I_{30} = u \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ \diagdown \quad \diagup \\ \text{---} \text{---} \text{---} \\ \diagup \quad \diagdown \end{array} (u, m_w^2) \quad (4.31)$$

$$I_{31} = \sqrt{s u (s u - 4m_w^2 (u + m_w^2))} \epsilon^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \\ | \quad | \\ \text{---} \text{---} \text{---} \end{array} (s, u, m_w^2) , \quad (4.32)$$

where $\lambda(x, y, z)$ is the Källén function,

$$\lambda(x, y, z) = x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2. \quad (4.33)$$

For clarity and later convenience, we have made all dependence on the kinematic variables $s, t, u = -s - t, m_w^2, m_z^2,$ and m_h^2 completely explicit on the right-hand side. Moreover, we have included integrals for all relevant permutations of the kinematic variables separately. In order to actually calculate the master integrals, it is enough to find explicit expressions for $I_1, I_2, I_3, I_6, I_7, I_8, I_{13}, I_{14}, I_{16}, I_{17}, I_{19}, I_{22}, I_{24},$ and I_{28} , as all other integrals may be obtained by making simple replacements. For example, I_{31} is obtained from I_{28} by replacing m_z^2 with m_w^2 and t with u . Due to the fact that we consider the physical region $s > (m_z + m_h)^2$ (see (3.3)), the same form of the analytic result can be used in a straightforward way.

Our calculation requires knowledge of the most complicated one-loop box-type master integrals (i.e. I_{24} and I_{28}) expanded through to fourth order in ϵ . We could not find suitable analytic solutions for all integrals in the literature. Several of the explicit results we could locate in the `Loopedia` database [110] were either not expanded to a sufficiently high order in ϵ for our purposes or not provided in a form convenient for numerical evaluations in the physical region of interest to us.

We therefore computed all of the integrals from scratch, either by direct integration for generic ϵ followed by an expansion¹⁴ in ϵ or by using the method of differential equations [49, 50, 112–116]. The integral definitions given above lead to a ϵ -decoupled form for the differential equations [89, 90], which we integrate in terms of multiple polylogarithms. To proceed, we construct real-valued multiple polylogarithms in the region of phase space of interest to us, employing the functional basis presented in [29, 31], together with a few additional logarithms and polylogarithms required to integrate e.g. I_6 to a sufficiently high order in ϵ . We provide our results in the ancillary file `oneloopmasters.m` attached to the `arXiv` submission of this paper.

4.3 Assembly of one-loop results

In this section, we give our results for the renormalized one-loop amplitude coefficients in both HVBM's γ_5 scheme (3.14) and Kreimer's γ_5 scheme (3.27). Assembling the components provided in appendices B.1 and B.2, we find

$$\mathbf{C}_{\text{VV}}^{(0,1)} = -\frac{Q_\ell}{s} \bar{\mathcal{V}}_{\gamma\bar{q}q}^{(0,1)}(s) + \frac{v_\ell}{s - m_z^2} \bar{\mathcal{V}}_{Z\bar{q}q}^{(0,1)}(s), \quad (4.34)$$

$$\mathbf{C}_{\text{VA}}^{(0,1)} = -\frac{a_\ell}{s - m_z^2} \bar{\mathcal{V}}_{Z\bar{q}q}^{(0,1)}(s), \quad (4.35)$$

$$\mathbf{C}_{\text{AV}}^{(0,1)} = \frac{v_\ell}{s - m_z^2} \bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s), \quad (4.36)$$

$$\mathbf{C}_{\text{AA}}^{(0,1)} = -\frac{a_\ell}{s - m_z^2} \bar{\mathcal{A}}_{Z\bar{q}q}^{(0,1)}(s), \quad (4.37)$$

¹⁴For some of the more complicated expansions, we used the `Mathematica` package `HypExp` [111].

$$\begin{aligned}
 \mathbf{C}_{\text{VV}}^{(1,0)} = & -\frac{Q_q Q_\ell}{s^2} \left(\bar{\Sigma}_{\gamma\gamma}^{(1,0)}(s) - s \delta Z_{\gamma\gamma}^{(1,0)} \right) \\
 & + \frac{1}{s(s-m_z^2)} (Q_q v_\ell + Q_\ell v_q) \left(\bar{\Sigma}_{\gamma Z}^{(1,0)}(s) + \frac{1}{2} \left(s \left(\delta Z_{\gamma Z}^{(1,0)} + \delta Z_{Z\gamma}^{(1,0)} \right) - m_z^2 \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & - \frac{v_q v_\ell}{(s-m_z^2)^2} \left(\bar{\Sigma}_{ZZ}^{(1,0)}(s) + s \delta Z_{ZZ}^{(1,0)} - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)} \right) \right) \\
 & - \frac{Q_q}{s} \left(\bar{\mathcal{V}}_{\gamma\ell\ell}^{(1,0)}(s) - Q_\ell \left(\delta Z_e^{(1,0)} + \delta Z_{V,\ell}^{(1,0)} + \frac{1}{2} \delta Z_{\gamma\gamma}^{(1,0)} - \frac{v_\ell}{2Q_\ell} \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & - \frac{Q_\ell}{s} \left(\bar{\mathcal{V}}_{\gamma\bar{q}q}^{(1,0)}(s) - Q_q \left(\delta Z_e^{(1,0)} + \delta Z_{V,q}^{(1,0)} + \frac{1}{2} \delta Z_{\gamma\gamma}^{(1,0)} - \frac{v_q}{2Q_q} \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & + \frac{v_q}{s-m_z^2} \left(\bar{\mathcal{V}}_{Z\ell\ell}^{(1,0)}(s) + v_\ell \left(\delta Z_e^{(1,0)} + \delta Z_{V,\ell}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) - a_\ell \delta Z_{A,\ell}^{(1,0)} \right. \\
 & \quad \left. - \frac{Q_\ell}{2} \delta Z_{\gamma Z}^{(1,0)} + \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(v_\ell - \frac{2m_w^2}{m_z^2} a_\ell \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) \\
 & + \frac{v_\ell}{s-m_z^2} \left(\bar{\mathcal{V}}_{Z\bar{q}q}^{(1,0)}(s) + v_q \left(\delta Z_e^{(1,0)} + \delta Z_{V,q}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) - a_q \delta Z_{A,q}^{(1,0)} \right. \\
 & \quad \left. - \frac{Q_q}{2} \delta Z_{\gamma Z}^{(1,0)} + \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(v_q - \frac{2m_w^2}{m_z^2} a_q \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) + \mathcal{B}_{\text{VV}}^{(1,0)}, \tag{4.38}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{C}_{\text{VA}}^{(1,0)} = & -\frac{Q_q a_\ell}{s(s-m_z^2)} \left(\bar{\Sigma}_{\gamma Z}^{(1,0)}(s) + \frac{1}{2} \left(s \left(\delta Z_{\gamma Z}^{(1,0)} + \delta Z_{Z\gamma}^{(1,0)} \right) - m_z^2 \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & + \frac{a_\ell v_q}{(s-m_z^2)^2} \left(\bar{\Sigma}_{ZZ}^{(1,0)}(s) + s \delta Z_{ZZ}^{(1,0)} - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)} \right) \right) \\
 & - \frac{Q_q}{s} \left(\bar{\mathcal{A}}_{\gamma\ell\ell}^{(1,0)}(s) - Q_\ell \left(\delta Z_{A,\ell}^{(1,0)} + \frac{a_\ell}{2Q_\ell} \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & + \frac{v_q}{s-m_z^2} \left(\bar{\mathcal{A}}_{Z\ell\ell}^{(1,0)}(s) + v_\ell \delta Z_{A,\ell}^{(1,0)} - a_\ell \left(\delta Z_e^{(1,0)} + \delta Z_{V,\ell}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) \right. \\
 & \quad \left. - a_\ell \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(1 - \frac{2m_w^2}{m_z^2} \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) \\
 & - \frac{a_\ell}{s-m_z^2} \left(\bar{\mathcal{V}}_{Z\bar{q}q}^{(1,0)}(s) + v_q \left(\delta Z_e^{(1,0)} + \delta Z_{V,q}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) - a_q \delta Z_{A,q}^{(1,0)} \right. \\
 & \quad \left. - \frac{Q_q}{2} \delta Z_{\gamma Z}^{(1,0)} + \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(v_q - \frac{2m_w^2}{m_z^2} a_q \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) + \mathcal{B}_{\text{VA}}^{(1,0)}, \tag{4.39}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{C}_{\text{AV}}^{(1,0)} = & -\frac{Q_\ell a_q}{s(s-m_z^2)} \left(\bar{\Sigma}_{\gamma Z}^{(1,0)}(s) + \frac{1}{2} \left(s \left(\delta Z_{\gamma Z}^{(1,0)} + \delta Z_{Z\gamma}^{(1,0)} \right) - m_z^2 \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & + \frac{a_q v_\ell}{(s-m_z^2)^2} \left(\bar{\Sigma}_{ZZ}^{(1,0)}(s) + s \delta Z_{ZZ}^{(1,0)} - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)} \right) \right) \\
 & - \frac{Q_\ell}{s} \left(\bar{\mathcal{A}}_{\gamma\bar{q}q}^{(1,0)}(s) - Q_q \left(\delta Z_{A,q}^{(1,0)} + \frac{a_q}{2Q_q} \delta Z_{Z\gamma}^{(1,0)} \right) \right) \\
 & - \frac{a_q}{s-m_z^2} \left(\bar{\mathcal{V}}_{Z\ell\ell}^{(1,0)}(s) + v_\ell \left(\delta Z_e^{(1,0)} + \delta Z_{V,\ell}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) - a_\ell \delta Z_{A,\ell}^{(1,0)} \right. \\
 & \quad \left. - \frac{Q_\ell}{2} \delta Z_{\gamma Z}^{(1,0)} + \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(v_\ell - \frac{2m_w^2}{m_z^2} a_\ell \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) \\
 & + \frac{v_\ell}{s-m_z^2} \left(\bar{\mathcal{A}}_{Z\bar{q}q}^{(1,0)}(s) + v_q \delta Z_{A,q}^{(1,0)} - a_q \left(\delta Z_e^{(1,0)} + \delta Z_{V,q}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) \right. \\
 & \quad \left. - a_q \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(1 - \frac{2m_w^2}{m_z^2} \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) + \mathcal{B}_{\text{AV}}^{(1,0)}, \tag{4.40}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{C}_{\text{AA}}^{(1,0)} = & -\frac{a_q a_\ell}{(s - m_z^2)^2} \left(\bar{\Sigma}_{ZZ}^{(1,0)}(s) + s \delta Z_{ZZ}^{(1,0)} - m_z^2 \left(\delta Z_{ZZ}^{(1,0)} + \delta Z_{m_z^2}^{(1,0)} \right) \right) \\
 & - \frac{a_q}{s - m_z^2} \left(\bar{\mathcal{A}}_{Z\ell\ell}^{(1,0)}(s) + v_\ell \delta Z_{A,\ell}^{(1,0)} - a_\ell \left(\delta Z_e^{(1,0)} + \delta Z_{V,\ell}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) \right. \\
 & \quad \left. - a_\ell \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(1 - \frac{2m_w^2}{m_z^2} \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) \\
 & - \frac{a_\ell}{s - m_z^2} \left(\bar{\mathcal{A}}_{Zq\bar{q}}^{(1,0)}(s) + v_q \delta Z_{A,q}^{(1,0)} - a_q \left(\delta Z_e^{(1,0)} + \delta Z_{V,q}^{(1,0)} + \frac{1}{2} \delta Z_{ZZ}^{(1,0)} \right) \right. \\
 & \quad \left. - a_q \frac{m_z^2}{2(m_z^2 - m_w^2)} \left(1 - \frac{2m_w^2}{m_z^2} \right) \left(\delta Z_{m_z^2}^{(1,0)} - \delta Z_{m_w^2}^{(1,0)} \right) \right) + \mathcal{B}_{\text{AA}}^{(1,0)} \quad (4.41)
 \end{aligned}$$

for the renormalized one-loop scattering amplitude coefficients in HVBM's γ_5 scheme. In writing the above expressions, we made use of the tabulated one-loop vertex counterterms collected in appendix A of [67]. Results in Kreimer's γ_5 scheme may be simply obtained from eqs. (4.34)–(4.41):

$$\bar{\mathbf{C}}_{\text{VV}}^{(0,1)} = \mathbf{C}_{\text{VV}}^{(0,1)}, \quad (4.42)$$

$$\bar{\mathbf{C}}_{\text{VA}}^{(0,1)} = \mathbf{C}_{\text{VA}}^{(0,1)}, \quad (4.43)$$

$$\bar{\mathbf{C}}_{\text{AV}}^{(0,1)} = \mathbf{C}_{\text{AV}}^{(0,1)}, \quad (4.44)$$

$$\bar{\mathbf{C}}_{\text{AA}}^{(0,1)} = \mathbf{C}_{\text{AA}}^{(0,1)}, \quad (4.45)$$

$$\bar{\mathbf{C}}_{\text{VV}}^{(1,0)} = \mathbf{C}_{\text{VV}}^{(1,0)} / \mathcal{B}_{\text{VV}}^{(1,0)} \rightarrow \bar{\mathcal{B}}_{\text{VV}}^{(1,0)}, \quad (4.46)$$

$$\bar{\mathbf{C}}_{\text{VA}}^{(1,0)} = \mathbf{C}_{\text{VA}}^{(1,0)} / \mathcal{B}_{\text{VA}}^{(1,0)} \rightarrow \bar{\mathcal{B}}_{\text{VA}}^{(1,0)}, \quad (4.47)$$

$$\bar{\mathbf{C}}_{\text{AV}}^{(1,0)} = \mathbf{C}_{\text{AV}}^{(1,0)} / \mathcal{B}_{\text{AV}}^{(1,0)} \rightarrow \bar{\mathcal{B}}_{\text{AV}}^{(1,0)}, \quad (4.48)$$

$$\bar{\mathbf{C}}_{\text{AA}}^{(1,0)} = \mathbf{C}_{\text{AA}}^{(1,0)} / \mathcal{B}_{\text{AA}}^{(1,0)} \rightarrow \bar{\mathcal{B}}_{\text{AA}}^{(1,0)}, \quad (4.49)$$

where $/.$ denotes the replacement operator.

Eqs. (4.34)–(4.49) may be directly matched onto eqs. (3.62) to determine $\mathcal{H}_{\text{DY}}^{(0,1)}[0]$, $\mathcal{H}_{\text{DY}}^{(0,1)}[1]$, $\mathcal{H}_{\text{DY}}^{(0,1)}[2]$, $\bar{\mathcal{H}}_{\text{DY}}^{(0,1)}[0]$, $\bar{\mathcal{H}}_{\text{DY}}^{(0,1)}[1]$, $\bar{\mathcal{H}}_{\text{DY}}^{(0,1)}[2]$, $\mathcal{H}_{\text{DY}}^{(1,0)}[0]$, $\mathcal{H}_{\text{DY}}^{(1,0)}[1]$, $\mathcal{H}_{\text{DY}}^{(1,0)}[2]$, $\bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[0]$, $\bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[1]$, and $\bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[2]$. We observe analytically that the $1/\epsilon^2$ and $1/\epsilon$ poles cancel in the hard scattering functions, leaving a finite remainder, as expected.

Let us emphasize that, upon substituting the explicit expressions for the master integrals discussed in the previous section (see also appendix B.2), we find

$$\mathcal{H}_{\text{DY}}^{(1,0)}[0] = \bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[0] \quad (4.50)$$

but¹⁵

$$\mathcal{H}_{\text{DY}}^{(1,0)}[k] \neq \bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[k] \quad \text{for } k > 0. \quad (4.51)$$

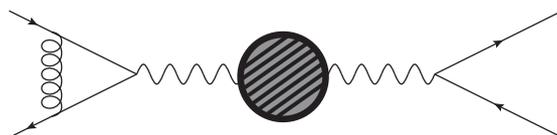
In section 6.1, it is explained in detail how one-loop polarized hard scattering functions may be derived from $\mathcal{H}_{\text{DY}}^{(0,1)}[0]$ and $\mathcal{H}_{\text{DY}}^{(1,0)}[0]$ or, equivalently, from $\bar{\mathcal{H}}_{\text{DY}}^{(0,1)}[0]$ and $\bar{\mathcal{H}}_{\text{DY}}^{(1,0)}[0]$.

¹⁵Due to the fact that the relative order α_s one-loop corrections are comprised of simple vertex form factor diagrams with no sensitivity to the γ_5 problem, we have $\mathcal{H}_{\text{DY}}^{(0,1)}[k] = \bar{\mathcal{H}}_{\text{DY}}^{(0,1)}[k]$ for all k .

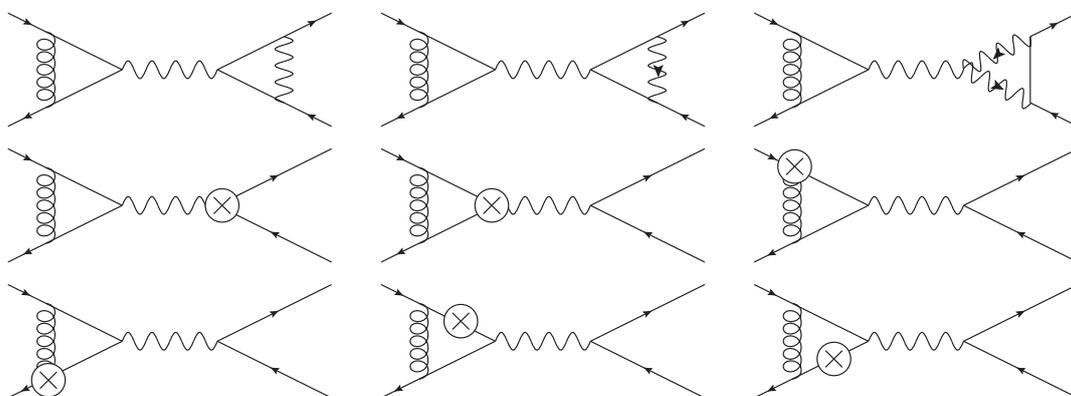
5 Two-loop scattering amplitudes

5.1 Diagrammatic structure

In this section, we present the diagrammatic structure of the two-loop perturbative corrections to the neutral-current Drell-Yan process of relative order $\alpha\alpha_s$. As at one loop, we do not explicitly identify the photon and Z . Due to their familiarity, diagrams with one-loop renormalized self-energy insertions of the form

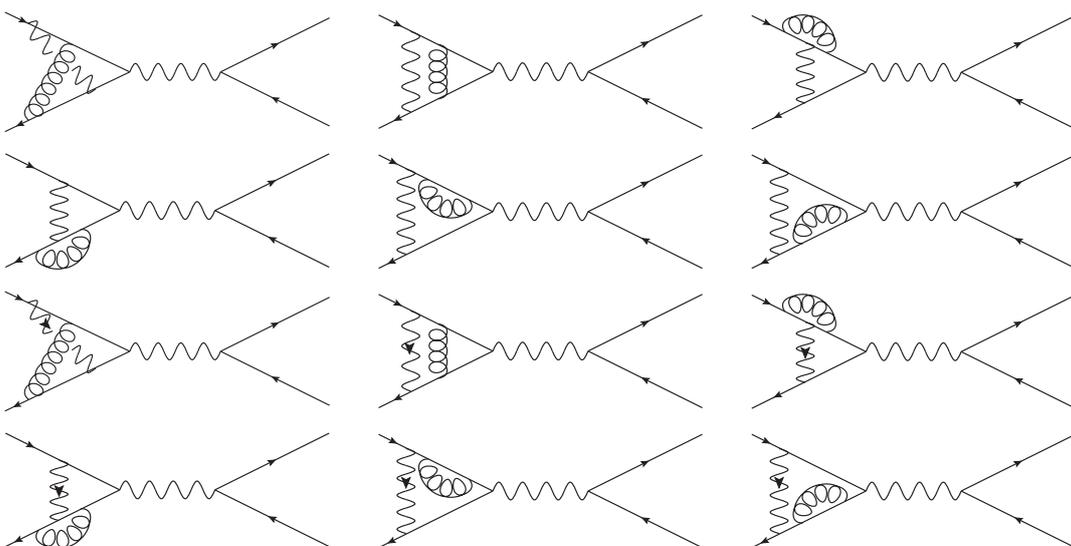


will not be visualized in explicit detail. Several other two-loop diagrams are also trivial, in that they essentially consist of simple products of one-loop contributions:



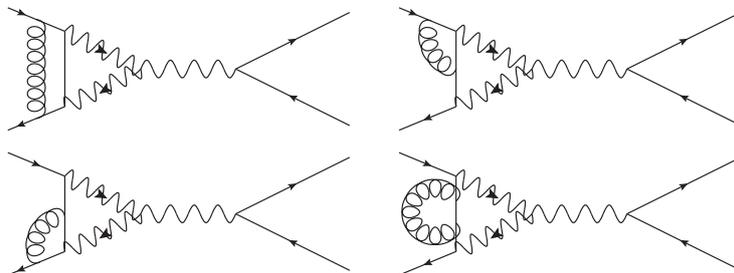
Note that, in the above, the sum of the final four diagrams vanishes identically.

For the sake of brevity, we draw only one half of the two-loop vertex diagrams with a single massive vector boson stretched across the quark line:

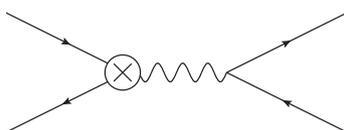


The remaining diagrams of this type may be trivially recovered from the above by exchanging the gluon and the electroweak gauge boson attached to the quark line.

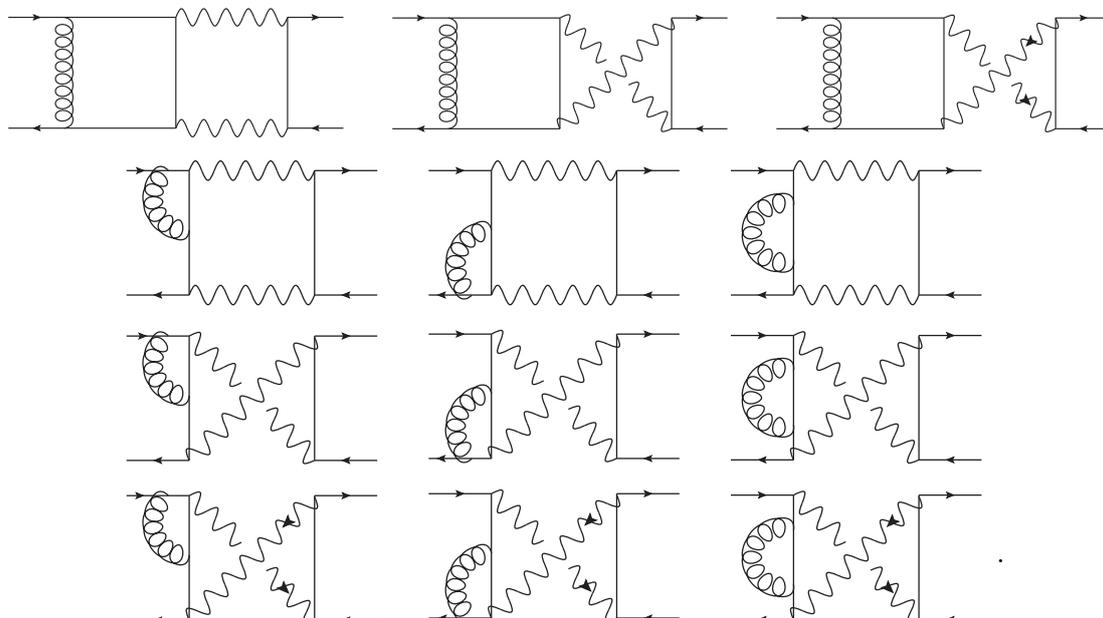
Of course, there are also diagrams with $\gamma W^+ W^-$ and $Z W^+ W^-$ interactions:



Finally, the order $\alpha\alpha_s$ vertex receives a correction from a two-loop vertex counterterm insertion,

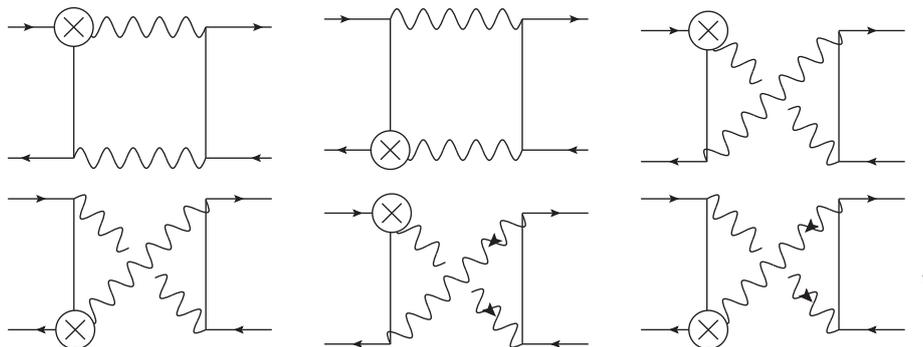


The most complicated two-loop diagrams are those of box type:



Our evaluation of these box-type diagrams is one of the most important new results of this

work. In HVBM's γ_5 scheme, further diagrams,



need to be included to ensure that the order $\alpha^2\alpha_s$ hard scattering function in HVBM's γ_5 scheme respects the chiral symmetry of the Standard Model (see section 2.8 for more details).

5.2 Two-loop integral definitions

All of the two-loop integrals defined below are pure functions, converted from the idiosyncratic notation of [29, 31], where they were originally evaluated in the physical region above all two-particle thresholds, to standard $\overline{\text{MS}}$ normalization. To be explicit, the m_i integrals from [29] and the \mathbf{m}_i integrals from [31] must be multiplied by

$$c_2(\epsilon, \mu^2, s) = \frac{e^{2\gamma_E\epsilon}}{\Gamma^2(1-\epsilon)} \left(\frac{\mu^2}{s}\right)^{2\epsilon}. \quad (5.1)$$

That is to say, the integral measure of our two-loop integrals is exactly:

$$\left(\frac{e^{\gamma_E\epsilon}(\mu^2)^\epsilon}{i\pi^{2-\epsilon}}\right)^2 \int d^d k_1 \int d^d k_2. \quad (5.2)$$

For brevity, we suppress the factors of $c_2(\epsilon, \mu^2, s)$ which belong in the definitions of all of the non-factorizable two-loop integrals given below. In total, 133 linearly-independent two-loop integrals appear in our calculations:

$J_1 = m_{17}(m_z^2)$	$J_2 = m_{17}(m_w^2)$	$J_3 = m_1(s)$
$J_4 = (I_{13})^2$	$J_5 = m_5(s)$	$J_6 = m_{12}(s)$
$J_7 = I_4 I_{13}$	$J_8 = m_{18}(s, m_z^2)$	$J_9 = m_{19}(s, m_z^2)$
$J_{10} = m_{28}(s, m_z^2)$	$J_{11} = m_{31}(s, m_z^2)$	$J_{12} = m_{32}(s, m_z^2)$
$J_{13} = I_{13} I_{14}$	$J_{14} = m_{37}(s, m_z^2)$	$J_{15} = m_{39}(s, m_z^2)$
$J_{16} = m_{42}(s, m_z^2)$	$J_{17} = m_{43}(s, m_z^2)$	$J_{18} = m_{44}(s, m_z^2)$
$J_{19} = m_{56}(s, m_z^2)$	$J_{20} = m_{57}(s, m_z^2)$	$J_{21} = m_{58}(s, m_z^2)$
$J_{22} = m_{15}(s, m_w^2)$	$J_{23} = m_{18}(s, m_w^2)$	$J_{24} = m_{19}(s, m_w^2)$

$J_{25} = m_{28} (s, m_w^2)$	$J_{26} = m_{31} (s, m_w^2)$	$J_{27} = m_{32} (s, m_w^2)$
$J_{28} = I_{13}I_{15}$	$J_{29} = m_{37} (s, m_w^2)$	$J_{30} = m_{39} (s, m_w^2)$
$J_{31} = m_{42} (s, m_w^2)$	$J_{32} = m_{43} (s, m_w^2)$	$J_{33} = m_{44} (s, m_w^2)$
$J_{34} = m_{56} (s, m_w^2)$	$J_{35} = m_{57} (s, m_w^2)$	$J_{36} = m_{58} (s, m_w^2)$
$J_{37} = I_2I_{13}$	$J_{38} = \mathbf{m}_{10} (s, m_w^2)$	$J_{39} = \mathbf{m}_{11} (s, m_w^2)$
$J_{40} = \mathbf{m}_{12} (s, m_w^2)$	$J_{41} = \mathbf{m}_{23} (s, m_w^2)$	$J_{42} = \mathbf{m}_{24} (s, m_w^2)$
$J_{43} = \mathbf{m}_{25} (s, m_w^2)$	$J_{44} = m_2 (t)$	$J_{45} = m_2 (u)$
$J_{46} = m_6 (t)$	$J_{47} = m_6 (u)$	$J_{48} = m_7 (s, t)$
$J_{49} = m_7 (s, u)$	$J_{50} = m_8 (s, t)$	$J_{51} = m_8 (s, u)$
$J_{52} = m_9 (s, t)$	$J_{53} = m_9 (s, u)$	$J_{54} = m_{10} (s, t)$
$J_{55} = m_{10} (s, u)$	$J_{56} = m_{11} (s, t)$	$J_{57} = m_{11} (s, u)$
$J_{58} = I_{13}I_{21}$	$J_{59} = m_{25} (s, m_z^2)$	$J_{60} = m_{26} (s, m_z^2)$
$J_{61} = m_{27} (t, m_z^2)$	$J_{62} = m_{37} (t, m_z^2)$	$J_{63} = m_{38} (t, m_z^2)$
$J_{64} = m_{40} (s, m_z^2)$	$J_{65} = m_{45} (s, t, m_z^2)$	$J_{66} = m_{46} (s, t, m_z^2)$
$J_{67} = m_{47} (s, t, m_z^2)$	$J_{68} = m_{48} (s, t, m_z^2)$	$J_{69} = m_{49} (s, t, m_z^2)$
$J_{70} = m_{50} (s, t, m_z^2)$	$J_{71} = m_{51} (s, t, m_z^2)$	$J_{72} = m_{52} (s, t, m_z^2)$
$J_{73} = m_{53} (s, t, m_z^2)$	$J_{74} = m_{54} (s, t, m_z^2)$	$J_{75} = m_{55} (s, t, m_z^2)$
$J_{76} = m_{27} (u, m_z^2)$	$J_{77} = m_{37} (u, m_z^2)$	$J_{78} = m_{38} (u, m_z^2)$
$J_{79} = m_{45} (s, u, m_z^2)$	$J_{80} = m_{46} (s, u, m_z^2)$	$J_{81} = m_{47} (s, u, m_z^2)$
$J_{82} = m_{48} (s, u, m_z^2)$	$J_{83} = m_{49} (s, u, m_z^2)$	$J_{84} = m_{50} (s, u, m_z^2)$
$J_{85} = m_{51} (s, u, m_z^2)$	$J_{86} = m_{52} (s, u, m_z^2)$	$J_{87} = m_{53} (s, u, m_z^2)$
$J_{88} = m_{54} (s, u, m_z^2)$	$J_{89} = m_{55} (s, u, m_z^2)$	$J_{90} = I_{13}I_{26}$
$J_{91} = \mathbf{m}_{10} (s, m_z^2)$	$J_{92} = \mathbf{m}_{11} (s, m_z^2)$	$J_{93} = \mathbf{m}_{12} (s, m_z^2)$
$J_{94} = \mathbf{m}_{23} (s, m_z^2)$	$J_{95} = \mathbf{m}_{24} (s, m_z^2)$	$J_{96} = \mathbf{m}_{25} (s, m_z^2)$
$J_{97} = I_{13}I_{27}$	$J_{98} = \mathbf{m}_{26} (s, t, m_z^2)$	$J_{99} = \mathbf{m}_{27} (s, t, m_z^2)$
$J_{100} = \mathbf{m}_{30} (s, m_z^2)$	$J_{101} = \mathbf{m}_{31} (s, t, m_z^2)$	$J_{102} = \mathbf{m}_{32} (s, t, m_z^2)$
$J_{103} = \mathbf{m}_{33} (s, t, m_z^2)$	$J_{104} = \mathbf{m}_{34} (s, t, m_z^2)$	$J_{105} = \mathbf{m}_{35} (s, t, m_z^2)$
$J_{106} = \mathbf{m}_{36} (s, t, m_z^2)$	$J_{107} = \mathbf{m}_{26} (s, u, m_z^2)$	$J_{108} = \mathbf{m}_{27} (s, u, m_z^2)$
$J_{109} = \mathbf{m}_{31} (s, u, m_z^2)$	$J_{110} = \mathbf{m}_{32} (s, u, m_z^2)$	$J_{111} = \mathbf{m}_{33} (s, u, m_z^2)$
$J_{112} = \mathbf{m}_{34} (s, u, m_z^2)$	$J_{113} = \mathbf{m}_{35} (s, u, m_z^2)$	$J_{114} = \mathbf{m}_{36} (s, u, m_z^2)$

$$\begin{aligned}
J_{115} &= m_{27} \left(u, m_w^2 \right) & J_{116} &= m_{37} \left(u, m_w^2 \right) & J_{117} &= m_{38} \left(u, m_w^2 \right) \\
J_{118} &= m_{46} \left(s, u, m_w^2 \right) & J_{119} &= m_{47} \left(s, u, m_w^2 \right) & J_{120} &= m_{48} \left(s, u, m_w^2 \right) \\
J_{121} &= m_{49} \left(s, u, m_w^2 \right) & J_{122} &= m_{52} \left(s, u, m_w^2 \right) & J_{123} &= m_{53} \left(s, u, m_w^2 \right) \\
J_{124} &= I_{13} I_{16} & J_{125} &= \mathbf{m}_{26} \left(s, u, m_w^2 \right) & J_{126} &= \mathbf{m}_{27} \left(s, u, m_w^2 \right) \\
J_{127} &= \mathbf{m}_{30} \left(s, m_w^2 \right) & J_{128} &= \mathbf{m}_{31} \left(s, u, m_w^2 \right) & J_{129} &= \mathbf{m}_{32} \left(s, u, m_w^2 \right) \\
J_{130} &= \mathbf{m}_{33} \left(s, u, m_w^2 \right) & J_{131} &= \mathbf{m}_{34} \left(s, u, m_w^2 \right) & J_{132} &= \mathbf{m}_{35} \left(s, u, m_w^2 \right) \\
J_{133} &= \mathbf{m}_{36} \left(s, u, m_w^2 \right). & & & &
\end{aligned} \tag{5.3}$$

In writing the above definitions, we have made all dependence on the kinematic variables $s, t, u = -s - t, m_w^2, m_z^2$, and m_h^2 explicit on the right-hand side for all functions not expressed as a simple product of the one-loop integrals from section 4.2. The attentive reader will note that, out of 124 non-factorizable integrals, only 57 actually need to be evaluated due to the fact that the rest may subsequently be accessed by considering simple permutations of the kinematic invariants.

One noteworthy fact is that, apart from J_{14} , all non-factorizable crossed two-loop integrals not treated explicitly in references [29] or [31] may be directly obtained through to weight four from the integral evaluations carried out in [29, 31] via the replacement $t \rightarrow u$. For the sake of completeness, we present a simple expression for J_{14} in the physical region above its one-mass threshold. Indeed, the integral admits a compact evaluation in Feynman/Schwinger parameters to all orders in ϵ :

$$\begin{aligned}
J_{14} &= \frac{\epsilon \Gamma^2(1 - \epsilon) \Gamma(2 + 2\epsilon) e^{2\gamma_E \epsilon}}{\Gamma(1 - 2\epsilon)} \left(\frac{\mu^2}{s} \right)^{2\epsilon} \\
&\quad \left\{ \left(\frac{m_z^2}{s} \right)^{-1-2\epsilon} \frac{\Gamma(-1 - 2\epsilon) \Gamma(1 + \epsilon)}{(1 - \epsilon) \Gamma(-\epsilon)} {}_2F_1 \left(1, 1; 2 - \epsilon; -\frac{s}{m_z^2} \right) \right. \\
&\quad \left. - \frac{\Gamma(1 - 2\epsilon) \Gamma(-2 - 2\epsilon) \Gamma(2 + \epsilon) e^{2i\pi\epsilon}}{\Gamma(1 - 3\epsilon)} {}_2F_1 \left(1, -2\epsilon; 1 - 3\epsilon; -\frac{s}{m_z^2} \right) \right\}. \tag{5.4}
\end{aligned}$$

Expanding eq. (5.4) in ϵ , we find results equivalent to those obtained from J_{62} by performing a crossing from t to s and then carefully rewriting the result in terms of the basis of multiple polylogarithms suitable for the ϵ expansion of J_{13} in the physical region above the one-particle threshold.

5.3 Assembly of two-loop results

In this section, we provide further details relevant to the assembly of our renormalized two-loop scattering amplitudes for the mixed EW-QCD corrections to $q\bar{q} \rightarrow \ell^+ \ell^-$ and discuss some implications of our results. As alluded to in section 2.5, we have

$$\delta Z_{VV'}^{(1,1)} = 0 \quad \text{and} \quad \delta Z_{m_v^2}^{(1,1)} = 0, \tag{5.5}$$

due to our neglect of all contributions proportional to the number of light fermion flavors or involving the top quark. Together with eq. (2.57), eqs. (5.5) imply exceptionally-simple forms for the order $\alpha\alpha_s \gamma\bar{q}q$ and $Z\bar{q}q$ vertex counterterms:

$$\delta Z_{V,\gamma\bar{q}q}^{(1,1)} = -Q_q \delta Z_{V,q}^{(1,1)}, \tag{5.6}$$

$$\delta Z_{A,\gamma\bar{q}q}^{(1,1)} = -Q_q \delta Z_{A,q}^{(1,1)}, \tag{5.7}$$

$$\delta Z_{V,Z\bar{q}q}^{(1,1)} = v_q \delta Z_{V,q}^{(1,1)} - a_q \delta Z_{A,q}^{(1,1)}, \tag{5.8}$$

$$\text{and} \quad \delta Z_{A,Z\bar{q}q}^{(1,1)} = -a_q \delta Z_{V,q}^{(1,1)} + v_q \delta Z_{A,q}^{(1,1)}, \tag{5.9}$$

where $\delta Z_{V,q}^{(1,1)}$ and $\delta Z_{A,q}^{(1,1)}$ are, respectively, the vector and axial vector components of the order $\alpha\alpha_s$ quark wavefunction counterterm.¹⁶ Eqs. (5.6)–(5.9) are in fact the *only* non-zero counterterms of order $\alpha\alpha_s$ which directly contribute to the renormalization of the scattering amplitudes we calculate.

Of course, it is crucial to bear in mind that the renormalization program in HVBM’s γ_5 scheme is complicated by the fact that insertions of the finite counterterms¹⁷ $\delta Z_{Z\bar{q}q}^{(0,1)}$ and $\delta Z_{W^\pm\bar{q}q}^{(0,1)}$ into one-loop box diagrams are required on top of all of the conventional renormalizations required to remove ultraviolet divergences. As emphasized in section 2.8, $\delta Z_{Z\bar{q}q}^{(0,1)}$ and $\delta Z_{W^\pm\bar{q}q}^{(0,1)}$ do *not* contribute to the renormalization of the two-loop scattering amplitudes in Kreimer’s γ_5 scheme. This fundamental difference between the systematics of renormalized Standard Model perturbation theory in the two γ_5 schemes we consider is what makes a direct two-loop comparison of them instructive and highly non-trivial.

After applying the computational methods and master integral evaluations described above to the Feynman diagrams of section 5.1, our explicit two-loop calculations confirm analytically that (3.69) does in fact hold:

$$\mathcal{H}_{\text{DY}}^{(1,1)}[0] = \bar{\mathcal{H}}_{\text{DY}}^{(1,1)}[0]. \tag{5.10}$$

That is to say, the hard scattering functions for the two-loop mixed EW-QCD corrections to the neutral-current Drell-Yan process we consider in HVBM’s γ_5 scheme and in Kreimer’s γ_5 scheme coincide. As expected, once all relations between crossed- and uncrossed-channel multiple polylogarithms were identified, we observed that the $1/\epsilon^4$ through $1/\epsilon$ poles cancelled analytically in the two-loop hard scattering functions we derived from eqs. (3.62). In our view, the rigorous establishment of eq. (5.10) constitutes a very important cross-check on our entire formulation and a significant theoretical result. In the future, our detailed comparison of HVBM and Kreimer at the two-loop level should benefit further calculations of still-unknown multi-loop perturbative corrections to more complicated Standard Model processes in pure dimensional regularization.

In future studies of Standard Model scattering processes, it would certainly be possible to perform two parallel calculations for each process in the two γ_5 schemes considered in

¹⁶We remind the reader that explicit all-orders-in- ϵ results for the components of the quark wavefunction counterterm at order $\alpha\alpha_s$ are provided in eqs. (2.92) and (2.93).

¹⁷Expressions for the finite counterterms relevant to this work are provided in unintegrated form in eqs. (B.5) and (B.6) and, due to the trivial ratio of $\delta Z_{Z\bar{q}q}^{(0,1)}$ and $\delta Z_{W^\pm\bar{q}q}^{(0,1)}$, in integrated form in eq. (2.113).

this work, but there may be a more practical way to rigorously cross-check perturbative calculations going forward. During the course of our investigation of $q\bar{q} \rightarrow \ell^+\ell^-$ in Kreimer's γ_5 scheme, we discovered that we could effectively check the correctness of the two-loop hard scattering functions within Kreimer's γ_5 scheme itself by setting up our code pipeline to allow the user to make different choices of reading point prescription for the Dirac traces. For example, we found that the fully-symmetrized reading point prescription mentioned in section 2.3 led to different higher-order-in- ϵ one-loop hard scattering functions, but to a unique order $\alpha^2\alpha_s$ hard scattering function at $\mathcal{O}(\epsilon^0)$.

Even when written as a linear combination of a minimal number of multiple polylogarithms such that the cancellation of infrared poles becomes manifest, the two-loop hard scattering functions calculated in this work are too long to be included in the article. Once we have implemented support for the remaining kinematic regions of the physical phase space, we plan to provide public access to an efficient C++ program for the evaluation of the polarized hard scattering functions discussed in the next section.

6 Numerical results

6.1 Helicity amplitudes

Having arrived at finite quantities in four dimensions, we can directly cast our results into expressions for the scattering of fermions with definite helicities using explicit four-dimensional representations.

Due to the form of the massless fermion propagator and the manner in which the electroweak gauge bosons couple to matter, the helicity quantum number is conserved along the initial and final fermion lines (see e.g. [117] for a review). This implies that there are just four non-vanishing helicity amplitudes, those where both the quark and the antiquark have opposite helicity and the lepton and antilepton have opposite helicity. We parametrize the center-of-momentum frame according to

$$\begin{aligned}
 p_1 &= \frac{\sqrt{s}}{2}(1, 0, 0, 1), & p_3 &= \frac{\sqrt{s}}{2}(1, \sin\theta, 0, \cos\theta), \\
 p_2 &= \frac{\sqrt{s}}{2}(1, 0, 0, -1), & p_4 &= \frac{\sqrt{s}}{2}(1, -\sin\theta, 0, -\cos\theta),
 \end{aligned}
 \tag{6.1}$$

with

$$-\frac{t}{s} = \frac{1 - \cos\theta}{2},
 \tag{6.2}$$

and use an explicit basis of spinor states in the chiral representation [47]

$$\begin{aligned}
 u_+(p_1) &= s^{1/4}(0, 0, 1, 0), & u_+(p_3) &= s^{-1/4}(0, 0, \sqrt{-u}, \sqrt{-t}), \\
 u_-(p_1) &= s^{1/4}(0, 1, 0, 0), & u_-(p_3) &= s^{-1/4}(-\sqrt{-t}, \sqrt{-u}, 0, 0), \\
 v_+(p_2) &= s^{1/4}(-1, 0, 0, 0), & v_+(p_4) &= s^{-1/4}(-\sqrt{-u}, -\sqrt{-t}, 0, 0), \\
 v_-(p_2) &= s^{1/4}(0, 0, 0, 1), & \text{and } v_-(p_4) &= s^{-1/4}(0, 0, -\sqrt{-t}, \sqrt{-u}).
 \end{aligned}
 \tag{6.3}$$

Plugging these representations¹⁸ into our Lorentz structures directly gives us results for the helicity amplitudes. Decomposing the hard functions (or finite remainders) according to

$$\mathcal{H}_{\text{DY}}^{(\zeta)(m,n)}[0] = \mathbf{C}_{\text{VV}}^{(\zeta)(m,n),\text{fin}} \mathcal{T}_{\text{VV}}^{(\zeta)} + \mathbf{C}_{\text{AA}}^{(\zeta)(m,n),\text{fin}} \mathcal{T}_{\text{AA}}^{(\zeta)} + \mathbf{C}_{\text{VA}}^{(\zeta)(m,n),\text{fin}} \mathcal{T}_{\text{VA}}^{(\zeta)} + \mathbf{C}_{\text{AV}}^{(\zeta)(m,n),\text{fin}} \mathcal{T}_{\text{AV}}^{(\zeta)}, \quad (6.4)$$

we obtain for the polarized hard scattering functions $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(m,n)}$:

$$\mathcal{H}_{+-+-}^{(m,n)} = -2(s+t) \left(\mathbf{C}_{\text{VV}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{AA}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{VA}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{AV}}^{(m,n),\text{fin}} \right), \quad (6.5)$$

$$\mathcal{H}_{-++-}^{(m,n)} = -2(s+t) \left(\mathbf{C}_{\text{VV}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{AA}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{VA}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{AV}}^{(m,n),\text{fin}} \right), \quad (6.6)$$

$$\mathcal{H}_{+--+}^{(m,n)} = -2t \left(\mathbf{C}_{\text{VV}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{AA}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{VA}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{AV}}^{(m,n),\text{fin}} \right), \quad (6.7)$$

$$\text{and } \mathcal{H}_{-+-+}^{(m,n)} = -2t \left(\mathbf{C}_{\text{VV}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{AA}}^{(m,n),\text{fin}} + \mathbf{C}_{\text{VA}}^{(m,n),\text{fin}} - \mathbf{C}_{\text{AV}}^{(m,n),\text{fin}} \right). \quad (6.8)$$

for coupling orders m and n . In eqs. (6.5)–(6.8), due to the observed γ_5 scheme-independence of the hard scattering functions at zeroth order in the ϵ expansion (see section 5.3), we drop the dual notation employed in eq. (6.4).

6.2 Final results

In this section we present visualizations of the polarized hard scattering functions $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(0,0)}$, $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(0,1)}$, $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1,0)}$, and $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1,1)}$ defined in section 6.1 for all four non-trivial helicity configurations. In the numerical analysis, we set the gauge boson and Higgs masses to their on-shell values as listed in [119],

$$m_w = 80.379 \text{ GeV}, \quad m_z = 91.1876 \text{ GeV}, \quad m_h = 125.10 \text{ GeV}, \quad (6.9)$$

and the renormalization scale $\mu_R = \sqrt{s}$. Since we keep powers of α and α_s factored out of the expressions we plot, we do not need to specify their values here. For illustrative purposes, we found it sufficient to consider up-type quarks in the initial state. Our figures do look rather different for down-type quarks, but our impression is that, on the whole, they do not introduce completely new features which would be of paramount importance to discuss here. We elected to plot only the real parts of our final results, as the real parts of $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1,1)}$ contain the most complicated weight four multiple polylogarithms appearing in our calculations.

In our numerical analysis, we focus on larger values of \sqrt{s} where the previously unknown non-factorizable two-loop box-type Feynman diagrams of section 5.1 are expected to be important. In order to compare the different orders in α and α_s , we find it convenient to include in our plots the 4π suppression factors taken out of each higher-order term in eq. (3.62); that is, we consider $\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(m,n)}/(4\pi)^{m+n}$, where the relative orders in the electroweak and strong couplings are given by (m, n) (see also eq. (2.56)).

¹⁸The phase conventions of (6.3) above are in line with the spinor helicity formalism, as in [118] where $u_{\pm}(p_i) = v_{\mp}(p_i)$ for all i . Let us stress, however, that we do *not* adopt the all-outgoing convention for the external four-momenta. We made use of eqs. (6.1) and (6.2) to derive eqs. (6.3).

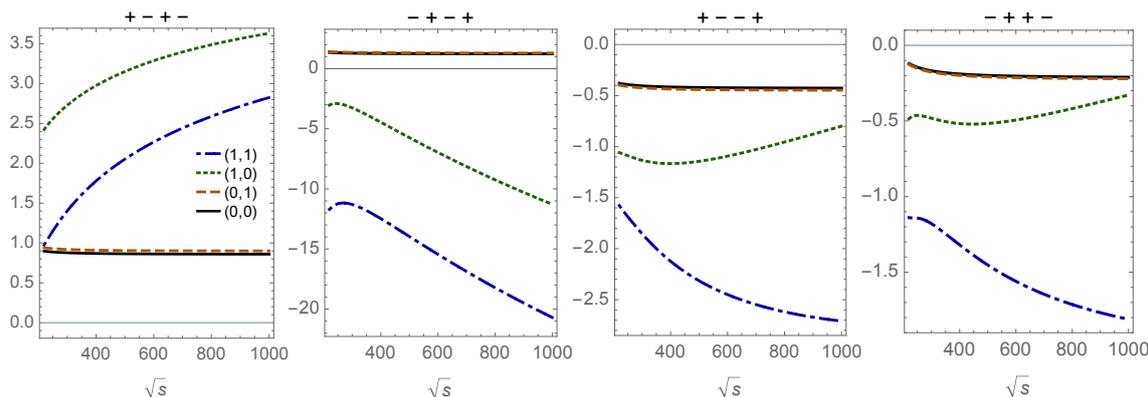


Figure 5. Helicity amplitudes for $u\bar{u} \rightarrow \ell^+\ell^-$ in dependence on the center-of-momentum energy for central scattering, $\cos\theta = 0$, and $\mu_R = \sqrt{s}$. The figure shows the real parts of the polarized hard scattering functions (finite remainders) $\text{Re } \mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(m,n)}/(4\pi)^{m+n}$, see eqs. (6.5)–(6.8), where the relative orders in the electroweak and strong couplings are denoted by (m, n) .

In all plots, we include the tree-level and relative order α_s results for reference, they show a rather simple behavior. The one-loop QCD corrections lie almost on top of the tree level results. Indeed, they fully factorize from the tree results; we have

$$\mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(0,1)}/(4\pi) = \left(\frac{\pi}{3} - i\right) \mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(0,0)} \quad (6.10)$$

for all helicity configurations and both up- and down-type quarks in the initial state. For the real part we see that $\pi/3 \approx 1.05$, thus explaining the observed similarity.

Figure 5 shows the dependence of the hard functions on the center-of-mass energy \sqrt{s} for fixed central scattering angle. We observe that the absolute values of all plotted real parts of relative order α_s increase as a function of \sqrt{s} , justifying the expectation that the calculation of the mixed two-loop EW-QCD corrections for the leptonic final state is well-motivated in the kinematic regime depicted in the figure. We note that the real parts of the one-loop EW and the two-loop EW-QCD corrections are not aligned for all helicity configurations.

Figure 6 shows the dependence of the hard functions on the cosine of the scattering angle for fixed center-of-mass energy \sqrt{s} . The electroweak corrections show a rather complex angular dependence. While the one-loop EW and the two-loop EW-QCD corrections show a similarity in their angular dependence, they differ in the details.

We also compared the curves of figure 6 to analogous ones for the pure QED-QCD model studied in [20].¹⁹ While maybe not instructive from the phenomenological point of view, it was interesting to see a somewhat similar qualitative angular dependence emerge when comparing the two-loop polarized hard scattering functions for QED-QCD and EW-QCD normalized by their respective tree level contributions.

¹⁹To our knowledge, the authors of [20] calculated unpolarized hard scattering functions only.

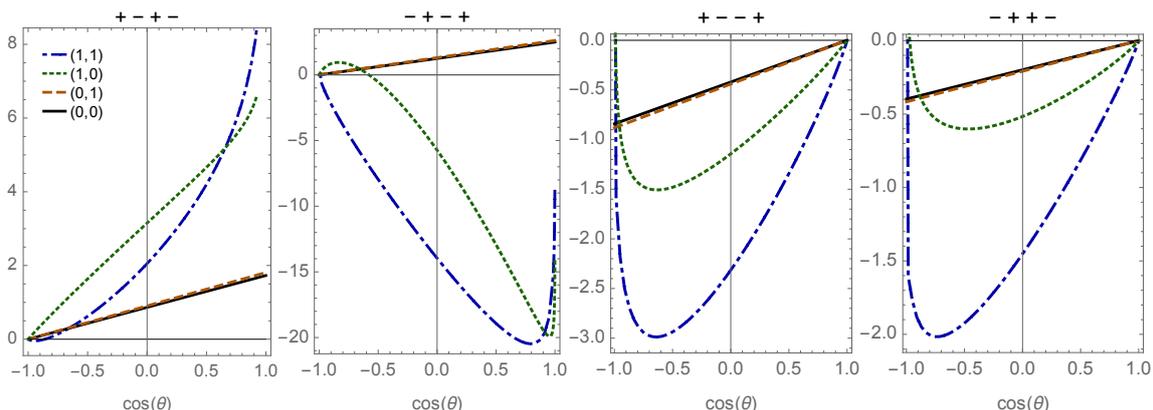


Figure 6. Helicity amplitudes for $u\bar{u} \rightarrow \ell^+\ell^-$ in dependence on the cosine of the scattering angle for $\sqrt{s} = 500$ GeV and $\mu_R = \sqrt{s}$. The figure shows the real parts of the polarized hard scattering functions (finite remainders) $\text{Re } \mathcal{H}_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(m,n)}/(4\pi)^{m+n}$, see eqs. (6.5)–(6.8), where the relative orders in the electroweak and strong couplings are denoted by (m, n) .

7 Summary and outlook

In this article, we calculated the relative order $\alpha\alpha_s$ mixed EW-QCD corrections to Drell-Yan lepton pair production, $q\bar{q} \rightarrow \ell^+\ell^-$. We performed the calculation in both the 't Hooft-Veltman-Breitenlohner-Maison γ_5 scheme and in the Kreimer γ_5 scheme using the projector method for chiral fermions. While our two-loop scattering amplitudes were found to be scheme dependent starting at $\mathcal{O}(\epsilon^{-1})$, unique polarized hard scattering functions in $d = 4$ were obtained after infrared subtraction. In the 't Hooft-Veltman-Breitenlohner-Maison scheme, we restored chiral symmetry using local counterterms, where we found it essential to consistently calculate our renormalization constants to higher orders in ϵ . To the best of our knowledge, our application of Kreimer's γ_5 scheme to the calculation of genuine two-loop $2 \rightarrow 2$ scattering amplitudes with non-trivial electroweak effects is the first of its kind.

Our calculation provides a major building block for the calculation of the relative order $\alpha\alpha_s$ corrections to off-shell Drell-Yan production in the high energy region, which is particularly relevant to new physics searches and the establishment of constraints on possible extensions of the Standard Model.

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Physics (MITP) of the Cluster of Excellence PRISMA⁺ for its hospitality and support. Our figures were generated using Jaxodraw [120], based on AxoDraw [121].

A Decoupling of irrelevant Lorentz structures

The helicity amplitudes presented in this article have been calculated using projectors involving only a set of Lorentz structures which are independent in four dimensions. In this appendix we show how the additional Lorentz structures in d dimensions can be cleanly separated in d dimensions such that the limit $d \rightarrow 4$ is smooth. Our construction is similar to the discussion in [80], but here we consider also explicit occurrences of γ_5 . We present the argument for the anticommuting γ_5 scheme.

The basic Lorentz structures relevant to our calculation are constructed from $\bar{\mathcal{T}}_1, \dots, \bar{\mathcal{T}}_4$ in (3.4)–(3.7), where we allow for additional insertions of γ_5 just before the last spinor of each Dirac chain. For each of the four Lorentz structures $\bar{\mathcal{T}}_i$ we obtain four structures in this way: $\bar{\mathcal{T}}_{i,VV}$, $\bar{\mathcal{T}}_{i,AA}$, $\bar{\mathcal{T}}_{i,AV}$, and $\bar{\mathcal{T}}_{i,VA}$. For the case of $\bar{\mathcal{T}}_1$ this was shown explicitly in (3.10)–(3.13), where we suppressed the index 1 for brevity.

We could collect the basic Lorentz structures according to

$$T_\alpha = (\bar{\mathcal{T}}_{1,VV}, \bar{\mathcal{T}}_{1,AA}, \bar{\mathcal{T}}_{2,VV}, \bar{\mathcal{T}}_{2,AA}, \dots, \bar{\mathcal{T}}_{1,AV}, \bar{\mathcal{T}}_{1,VA}, \bar{\mathcal{T}}_{2,AV}, \bar{\mathcal{T}}_{2,VA}, \dots) \quad (\text{A.1})$$

and decompose our two-loop amplitudes in d dimensions according to

$$i\mathcal{A}_{\text{DY}} = i \sum_{\alpha=1}^{16} \mathbf{C}_\alpha T_\alpha, \quad (\text{A.2})$$

either by explicit Passarino-Veltman reduction of the tensor integrals, or using a projector method. In fact, we could consider a matrix with elements

$$M_{\alpha\beta} = \sum_{\text{spin,color}} T_\alpha^\dagger T_\beta \quad (\text{A.3})$$

to construct projectors

$$\mathcal{P}_\alpha = -i \sum_{\beta=1}^{16} M_{\alpha\beta}^{-1} T_\beta^\dagger \quad (\text{A.4})$$

which fulfill

$$\mathbf{C}_\alpha = \sum_{\text{spin,color}} \mathcal{P}_\alpha i\mathcal{A}_{\text{DY}} \quad (\text{A.5})$$

using the standard spin summation of conventional dimensional regularization,

$$\sum_{\text{spin,color}} u(p_1)\bar{u}(p_1) = N_c \not{p}_1, \quad \sum_{\text{spin,color}} u(p_3)\bar{u}(p_3) = \not{p}_3, \quad (\text{A.6})$$

$$\sum_{\text{spin,color}} v(p_2)\bar{v}(p_2) = N_c \not{p}_2, \quad \sum_{\text{spin,color}} v(p_4)\bar{v}(p_4) = \not{p}_4, \quad (\text{A.7})$$

for the spinors of the initial state quarks and of the final state leptons. Here, the matrix $M = (M_{\alpha\beta})$ and its inverse are rational functions of s , t and ϵ . The problem with the

basis (A.1) is two-fold. First, M^{-1} has an overall factor $1/\epsilon$, which introduces spurious poles into the calculation and prevents a smooth limit $d \rightarrow 4$ at the level of the Lorentz structures. Second, even if one is interested only in the physical form factors $\bar{T}_{1,VV}$, $\bar{T}_{1,AA}$, $\bar{T}_{1,AV}$, $\bar{T}_{1,VA}$, which are independent in $d = 4$ dimensions, one can not ignore the other Lorentz structures when calculating the former.

Both problems can be cured by changing to a different basis of Lorentz structures. Before we discuss this new choice we note that the matrix M above is block diagonal, and the VV, AA block (indices $1 \dots 8$) is identical to the AV, VA block (indices $9 \dots 16$). It is therefore sufficient to restrict the following discussion to the VV, AA block and restrict indices to $1 \dots 8$. We consider a new basis of Lorentz structures $T'_\alpha = \sum_\beta R_{\alpha\beta} T_\beta$ such that

$$T'_1 = T_1, \quad T'_2 = T_2, \quad T'_\alpha = T_\alpha + \sum_{\beta=1}^2 R_{\alpha\beta} T_\beta \quad \text{for } \alpha = 3 \dots 8 \quad (\text{A.8})$$

or

$$(T'_\alpha) = (R_{\alpha\beta})(T_\beta) = \begin{pmatrix} \mathbb{1}_{2 \times 2} & 0_{2 \times 6} \\ R_{6 \times 2} & \mathbb{1}_{6 \times 6} \end{pmatrix} (T_\beta) \quad (\text{A.9})$$

and the first two structures are strictly orthogonal to the others in d dimensions,

$$(M'_{\alpha\beta}) = \sum_{\text{spin,color}} (T'^\dagger_\alpha T'_\beta) = (RMR^\dagger) = \begin{pmatrix} M'_{2 \times 2} & 0 \\ 0 & M'_{6 \times 6} \end{pmatrix}. \quad (\text{A.10})$$

The remaining part of R is obtained according to

$$R_{\alpha\beta} = - \sum_{\alpha=1}^2 (M_{2 \times 2}^{-1})_{\beta\gamma} M_{\gamma\alpha} \quad \text{for } \alpha = 3 \dots 8, \beta = 1 \dots 2. \quad (\text{A.11})$$

Note that only the inverse of the left upper 2×2 submatrix of M ,

$$M_{2 \times 2} = N_c \begin{pmatrix} 8(t^2 + u^2 - s^2\epsilon) & 8s(t - u) \\ 8s(t - u) & 8(t^2 + u^2 - s^2\epsilon) \end{pmatrix}, \quad (\text{A.12})$$

enters, which, unlike M^{-1} , is free of $1/\epsilon$ poles. For R we find explicitly

$$R_{31} = R_{42} = tu(t - u)(tu + \epsilon s^2/2)/d_R \quad (\text{A.13})$$

$$R_{41} = R_{32} = -stu(tu - \epsilon s^2/2)/d_R \quad (\text{A.14})$$

$$R_{51} = R_{62} = -2(20t^2u^2 - \epsilon(13s^4 + 26s^3t + 38s^2t^2 + 36st^3 + 12t^4) + 2\epsilon^2s^2(7s^2 + 9st + 6t^2) - 3\epsilon^3s^4)/d_R \quad (\text{A.15})$$

$$R_{61} = R_{52} = -2(12t^2u^2 - 3\epsilon s(s^3 + 2s^2t + 4st^2 + 4t^3) + 3\epsilon^2(t - u)s^3)/d_R \quad (\text{A.16})$$

$$R_{71} = R_{82} = tu(4t^2u + 2\epsilon(s^3 + 2s^2t + 5st^2 + 2t^3) - \epsilon^2s^2(3s + 2t))/d_R \quad (\text{A.17})$$

$$R_{81} = R_{72} = tu(4t^2u + 2\epsilon s(s^2 + t^2) - 3\epsilon^2s^3)/d_R, \quad (\text{A.18})$$

where

$$d_R = (2t^2 - \epsilon s^2)(2u^2 - \epsilon s^2). \quad (\text{A.19})$$

In the new basis, the matrix M' takes the form

$$M' = \begin{pmatrix} M_{2 \times 2} & 0 \\ 0 & M'_{6 \times 6} \end{pmatrix}, \tag{A.20}$$

where $M'_{6 \times 6}$ has an overall factor of ϵ and therefore vanishes for $d \rightarrow 4$. Since in the new basis the first two Lorentz structures are orthogonal to the other structures in d dimensions, the projectors for the first two form factors can be constructed without explicitly considering the other structures. Moreover, from the form of $M'_{6 \times 6}$ we see that contributions due to the other structures are suppressed by a power of ϵ . This motivates to consider only the first two Lorentz structures for the calculation of physical helicity amplitudes, just ignoring the other “irrelevant” Lorentz structures (in the terminology of [80]). As pointed out before, the same construction applies to the AV, VA structures as well, leaving us with a total of four Lorentz structures, matching the number of different helicity configurations.

B Explicit one-loop results

B.1 Relative order α_s results

Corrections to the neutral-current Drell-Yan process of relative order α_s are produced exclusively via gluon exchange across the initial quark line. As should be clear from the diagrams of section 4.1, the corresponding order α_s vertex form factors,

$$\bar{V}_{\gamma\bar{q}q}^{(0,1)}(s) = -Q_q C_F \mathbf{t}_1 I_{13}, \tag{B.1}$$

$$\bar{V}_{Z\bar{q}q}^{(0,1)}(s) = v_q C_F \mathbf{t}_1 I_{13}, \tag{B.2}$$

$$\text{and } \bar{A}_{Z\bar{q}q}^{(0,1)}(s) = -a_q C_F \mathbf{t}_1 I_{13}, \tag{B.3}$$

where

$$\mathbf{t}_1 = \frac{2 - \epsilon + 2\epsilon^2}{(1 - 2\epsilon)\epsilon^2}, \tag{B.4}$$

are nearly trivial.

In HVBM's γ_5 scheme, the finite counterterms relevant to the renormalization of the order $\alpha^2\alpha_s$ scattering amplitude,

$$\delta Z_{Z\bar{q}q}^{(0,1)} = -a_q C_F \mathbf{t}_2 I_{13} \tag{B.5}$$

$$\text{and } \delta Z_{W^\pm\bar{q}q}^{(0,1)} = -a_w C_F \mathbf{t}_2 I_{13}, \tag{B.6}$$

where

$$\mathbf{t}_2 = \frac{2(2 - \epsilon)}{(1 - \epsilon)(1 - 2\epsilon)}, \tag{B.7}$$

are also of order α_s (see section 2.8 for more details).

B.2 Relative order α results

Corrections to the neutral-current Drell-Yan process of relative order α are generated by electroweak gauge boson self-energy diagrams, vertex diagrams, and box diagrams. For the sake of completeness, we also present expressions for the counterterms of order α discussed in sections 2.6 and 2.7.

The fermion wavefunction counterterms, calculated to all orders in ϵ in section 2.7, are

$$\delta Z_{V,f}^{(1,0)} = (a_w^2 + v_w^2) \mathbf{p} I_1 + (a_f^2 + v_f^2) \mathbf{p} I_4 \quad (\text{B.8})$$

$$\text{and} \quad \delta Z_{A,f}^{(1,0)} = -2a_w v_w \mathbf{p} I_1 - 2a_f v_f \mathbf{p} I_4, \quad (\text{B.9})$$

where

$$\mathbf{p} = -\frac{2(1-\epsilon)}{(2-\epsilon)\epsilon}. \quad (\text{B.10})$$

Due to their excessive length, we abbreviate the integral coefficients of the electroweak gauge boson self-energies and associated counterterms as \mathbf{r}_i and present explicit expressions for them in the ancillary file `twopointcoeffs.m` attached to the arXiv submission of this paper. In the notation of section 2.6, we have

$$\bar{\Sigma}_{\gamma\gamma}^{(1,0)}(s) = \mathbf{r}_1 I_1 + \mathbf{r}_2 I_2, \quad (\text{B.11})$$

$$\bar{\Sigma}_{\gamma Z}^{(1,0)}(s) = \mathbf{r}_3 I_1 + \mathbf{r}_4 I_2, \quad (\text{B.12})$$

$$\bar{\Sigma}_{ZZ}^{(1,0)}(s) = \mathbf{r}_5 I_1 + \mathbf{r}_6 I_2 + \mathbf{r}_7 I_4 + \mathbf{r}_8 I_5 + \mathbf{r}_9 I_6, \quad (\text{B.13})$$

$$\text{and} \quad \bar{\Sigma}_{W^+W^-}^{(1,0)}(s) = \mathbf{r}_{10} I_1 + \mathbf{r}_{11} I_4 + \mathbf{r}_{12} I_5 + \mathbf{r}_{13} I_8 + \mathbf{r}_{14} I_9 + \mathbf{r}_{15} I_{10} \quad (\text{B.14})$$

for the transverse parts of the bare electroweak gauge boson self-energies of relative order α . Inserting explicit expressions for our master integrals, we find complete agreement through to $\mathcal{O}(\epsilon^0)$ with the results of [66, 67] after setting contributions proportional to the number of fermion flavors to zero.

Through eqs. (2.71), we subsequently find

$$\delta Z_{\gamma\gamma}^{(1,0)} = \mathbf{r}_{16} I_1, \quad (\text{B.15})$$

$$\delta Z_{Z\gamma}^{(1,0)} = \mathbf{r}_{17} I_1, \quad (\text{B.16})$$

$$\delta Z_{\gamma Z}^{(1,0)} = \mathbf{r}_{18} I_1 + \mathbf{r}_{19} I_3, \quad (\text{B.17})$$

$$\delta Z_{ZZ}^{(1,0)} = \mathbf{r}_{20} I_1 + \mathbf{r}_{21} I_3 + \mathbf{r}_{22} I_4 + \mathbf{r}_{23} I_5 + \mathbf{r}_{24} I_7, \quad (\text{B.18})$$

$$\delta Z_{m_z^2}^{(1,0)} = \mathbf{r}_{25} I_1 + \mathbf{r}_{26} I_3 + \mathbf{r}_{27} I_4 + \mathbf{r}_{28} I_5 + \mathbf{r}_{29} I_7, \quad (\text{B.19})$$

$$\text{and} \quad \delta Z_{m_w^2}^{(1,0)} = \mathbf{r}_{30} I_1 + \mathbf{r}_{31} I_4 + \mathbf{r}_{32} I_5 + \mathbf{r}_{33} I_{11} + \mathbf{r}_{34} I_{12} \quad (\text{B.20})$$

for the relative order α electroweak gauge boson mass and wavefunction counterterms appearing in our calculations. As it was not possible for us to find eqs. (B.11)–(B.20) in the literature verbatim, it may be that we present explicit all-orders-in- ϵ results for the above quantities for the first time.

Although the vertex form factor integral coefficients are all reasonably compact, we nevertheless abbreviate them as \mathbf{v}_i in what follows in order to highlight the strong constraints imposed on their structure by the chiral symmetry of the Standard Model. We present explicit expressions for the \mathbf{v}_i in the ancillary file `vertexcoeffs.m` attached to the arXiv submission of this paper. For the order α vertex form factors, we find

$$\begin{aligned} \bar{\mathcal{V}}_{\gamma\bar{q}q}^{(1,0)}(s) = & -Q_q^3 \mathbf{v}_1 I_{13} - Q_q (a_q^2 + v_q^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & - Q_{q'} (a_w^2 + v_w^2) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) - I_q^3 (a_w^2 + v_w^2) (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.21})$$

$$\begin{aligned} \bar{\mathcal{A}}_{\gamma\bar{q}q}^{(1,0)}(s) = & 2Q_q a_q v_q (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) + 2Q_{q'} a_w v_w (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & + 2I_q^3 a_w v_w (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.22})$$

$$\begin{aligned} \bar{\mathcal{V}}_{Z\bar{q}q}^{(1,0)}(s) = & Q_q^2 v_q \mathbf{v}_1 I_{13} + v_q (3a_q^2 + v_q^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & + (2a_w v_w a_{q'} + (a_w^2 + v_w^2) v_{q'}) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & + I_q^3 (a_w^2 + v_w^2) \frac{m_w}{\sqrt{m_z^2 - m_w^2}} (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.23})$$

$$\begin{aligned} \bar{\mathcal{A}}_{Z\bar{q}q}^{(1,0)}(s) = & -Q_q^2 a_q \mathbf{v}_1 I_{13} - a_q (a_q^2 + 3v_q^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & - (2a_w v_w v_{q'} + (a_w^2 + v_w^2) a_{q'}) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & - 2I_q^3 a_w v_w \frac{m_w}{\sqrt{m_z^2 - m_w^2}} (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.24})$$

$$\begin{aligned} \bar{\mathcal{V}}_{\gamma\bar{\ell}\ell}^{(1,0)}(s) = & -Q_\ell^3 \mathbf{v}_1 I_{13} - Q_\ell (a_\ell^2 + v_\ell^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & - Q_{\ell'} (a_w^2 + v_w^2) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) - I_\ell^3 (a_w^2 + v_w^2) (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.25})$$

$$\begin{aligned} \bar{\mathcal{A}}_{\gamma\bar{\ell}\ell}^{(1,0)}(s) = & 2Q_\ell a_\ell v_\ell (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) + 2Q_{\ell'} a_w v_w (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & + 2I_\ell^3 a_w v_w (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.26})$$

$$\begin{aligned} \bar{\mathcal{V}}_{Z\bar{\ell}\ell}^{(1,0)}(s) = & Q_\ell^2 v_\ell \mathbf{v}_1 I_{13} + v_\ell (3a_\ell^2 + v_\ell^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & + (2a_w v_w a_{\ell'} + (a_w^2 + v_w^2) v_{\ell'}) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & + I_\ell^3 (a_w^2 + v_w^2) \frac{m_w}{\sqrt{m_z^2 - m_w^2}} (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \quad \text{and} \end{aligned} \quad (\text{B.27})$$

$$\begin{aligned} \bar{\mathcal{A}}_{Z\bar{\ell}\ell}^{(1,0)}(s) = & -Q_\ell^2 a_\ell \mathbf{v}_1 I_{13} - a_\ell (a_\ell^2 + 3v_\ell^2) (\mathbf{v}_2 I_4 + \mathbf{v}_3 I_{13} + \mathbf{v}_4 I_{14}) \\ & - (2a_w v_w v_{\ell'} + (a_w^2 + v_w^2) a_{\ell'}) (\mathbf{v}_5 I_1 + \mathbf{v}_6 I_{13} + \mathbf{v}_7 I_{15}) \\ & - 2I_\ell^3 a_w v_w \frac{m_w}{\sqrt{m_z^2 - m_w^2}} (\mathbf{v}_8 I_1 + \mathbf{v}_9 I_2 + \mathbf{v}_{10} I_{16}), \end{aligned} \quad (\text{B.28})$$

where f' denotes the isospin partner of fermion f . It is worth pointing out that, in the above, \mathbf{v}_5 , \mathbf{v}_6 , and \mathbf{v}_7 may be trivially obtained from \mathbf{v}_2 , \mathbf{v}_3 , and \mathbf{v}_4 respectively by replacing m_z^2 with m_w^2 . For the infrared-finite contributions featuring massive electroweak vector bosons in the loop, our results for the one-loop vertex form factors are directly comparable to the original calculation of [66]. Indeed, upon inserting explicit expressions

for our master integrals, we find complete agreement through to $\mathcal{O}(\epsilon^0)$ with the relevant terms of eqs. (5.28) and (5.30) of [66] modulo minor typos.²⁰

Finally, for HVBM's γ_5 scheme, we let $\mathcal{B}_{VV}^{(1,0)}$, $\mathcal{B}_{VA}^{(1,0)}$, $\mathcal{B}_{AV}^{(1,0)}$, and $\mathcal{B}_{AA}^{(1,0)}$ denote the contributions from order α^2 box diagrams to $\mathbf{C}_{VV}^{(1,0)}$, $\mathbf{C}_{VA}^{(1,0)}$, $\mathbf{C}_{AV}^{(1,0)}$, and $\mathbf{C}_{AA}^{(1,0)}$ and, for Kreimer's γ_5 scheme, $\bar{\mathcal{B}}_{VV}^{(1,0)}$, $\bar{\mathcal{B}}_{VA}^{(1,0)}$, $\bar{\mathcal{B}}_{AV}^{(1,0)}$, and $\bar{\mathcal{B}}_{AA}^{(1,0)}$ denote the contributions from order α^2 box diagrams to $\bar{\mathbf{C}}_{VV}^{(1,0)}$, $\bar{\mathbf{C}}_{VA}^{(1,0)}$, $\bar{\mathbf{C}}_{AV}^{(1,0)}$, and $\bar{\mathbf{C}}_{AA}^{(1,0)}$. We find

$$\begin{aligned}
 \mathcal{B}_{VV}^{(1,0)} = & Q_q^2 Q_\ell^2 (\mathbf{a}_1 I_{13} + \mathbf{a}_2 I_{17} + \mathbf{a}_3 I_{18} + \mathbf{a}_4 I_{19} + \mathbf{a}_5 I_{20}) \\
 & + Q_q Q_\ell a_q a_\ell (\mathbf{a}_6 I_4 + \mathbf{a}_7 I_{17} + \mathbf{a}_8 I_{18} + \mathbf{a}_9 I_{21} + \mathbf{a}_{10} I_{22} + \mathbf{a}_{11} I_{23} + \mathbf{a}_{12} I_{24} + \mathbf{a}_{13} I_{25}) \\
 & + Q_q Q_\ell v_q v_\ell (\mathbf{a}_{14} I_4 + \mathbf{a}_{15} I_{17} + \mathbf{a}_{16} I_{18} + \mathbf{a}_{17} I_{21} + \mathbf{a}_{18} I_{22} + \mathbf{a}_{19} I_{23} + \mathbf{a}_{20} I_{24} + \mathbf{a}_{21} I_{25}) \\
 & + a_q^2 a_\ell^2 (\mathbf{a}_{22} I_4 + \mathbf{a}_{23} I_{17} + \mathbf{a}_{24} I_{18} + \mathbf{a}_{25} I_{22} + \mathbf{a}_{26} I_{23} + \mathbf{a}_{27} I_{26} + \mathbf{a}_{28} I_{27} + \mathbf{a}_{29} I_{28} + \mathbf{a}_{30} I_{29}) \\
 & + a_q^2 v_\ell^2 (\mathbf{a}_{31} I_4 + \mathbf{a}_{32} I_{17} + \mathbf{a}_{33} I_{18} + \mathbf{a}_{34} I_{22} + \mathbf{a}_{35} I_{23} + \mathbf{a}_{36} I_{26} + \mathbf{a}_{37} I_{27} + \mathbf{a}_{38} I_{28} + \mathbf{a}_{39} I_{29}) \\
 & + a_\ell^2 v_q^2 (\mathbf{a}_{40} I_4 + \mathbf{a}_{41} I_{17} + \mathbf{a}_{42} I_{18} + \mathbf{a}_{43} I_{22} + \mathbf{a}_{44} I_{23} + \mathbf{a}_{45} I_{26} + \mathbf{a}_{46} I_{27} + \mathbf{a}_{47} I_{28} + \mathbf{a}_{48} I_{29}) \\
 & + v_q^2 v_\ell^2 (\mathbf{a}_{49} I_4 + \mathbf{a}_{50} I_{17} + \mathbf{a}_{51} I_{18} + \mathbf{a}_{52} I_{22} + \mathbf{a}_{53} I_{23} + \mathbf{a}_{54} I_{26} + \mathbf{a}_{55} I_{27} + \mathbf{a}_{56} I_{28} + \mathbf{a}_{57} I_{29}) \\
 & + a_q a_\ell v_q v_\ell (\mathbf{a}_{58} I_4 + \mathbf{a}_{59} I_{17} + \mathbf{a}_{60} I_{18} + \mathbf{a}_{61} I_{22} + \mathbf{a}_{62} I_{23} + \mathbf{a}_{63} I_{26} + \mathbf{a}_{64} I_{27} + \mathbf{a}_{65} I_{28} + \mathbf{a}_{66} I_{29}) \\
 & + a_w^4 (\mathbf{a}_{67} I_1 + \mathbf{a}_{68} I_2 + \mathbf{a}_{69} I_{16} + \mathbf{a}_{70} I_{18} + \mathbf{a}_{71} I_{30} + \mathbf{a}_{72} I_{31}) \\
 & + v_w^4 (\mathbf{a}_{73} I_1 + \mathbf{a}_{74} I_2 + \mathbf{a}_{75} I_{16} + \mathbf{a}_{76} I_{18} + \mathbf{a}_{77} I_{30} + \mathbf{a}_{78} I_{31}) \\
 & + a_w^2 v_w^2 (\mathbf{a}_{79} I_1 + \mathbf{a}_{80} I_2 + \mathbf{a}_{81} I_{16} + \mathbf{a}_{82} I_{18} + \mathbf{a}_{83} I_{30} + \mathbf{a}_{84} I_{31}), \tag{B.29}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{B}_{VA}^{(1,0)} = & Q_q Q_\ell a_q v_\ell (\mathbf{b}_1 I_4 + \mathbf{b}_2 I_{17} + \mathbf{b}_3 I_{18} + \mathbf{b}_4 I_{21} + \mathbf{b}_5 I_{22} + \mathbf{b}_6 I_{23} + \mathbf{b}_7 I_{24} + \mathbf{b}_8 I_{25}) \\
 & + Q_q Q_\ell a_\ell v_q (\mathbf{b}_9 I_4 + \mathbf{b}_{10} I_{17} + \mathbf{b}_{11} I_{18} + \mathbf{b}_{12} I_{21} + \mathbf{b}_{13} I_{22} + \mathbf{b}_{14} I_{23} + \mathbf{b}_{15} I_{24} + \mathbf{b}_{16} I_{25}) \\
 & + a_q^2 a_\ell v_\ell (\mathbf{b}_{17} I_4 + \mathbf{b}_{18} I_{17} + \mathbf{b}_{19} I_{18} + \mathbf{b}_{20} I_{22} + \mathbf{b}_{21} I_{23} + \mathbf{b}_{22} I_{26} + \mathbf{b}_{23} I_{27} + \mathbf{b}_{24} I_{28} + \mathbf{b}_{25} I_{29}) \\
 & + a_q a_\ell^2 v_q (\mathbf{b}_{26} I_4 + \mathbf{b}_{27} I_{17} + \mathbf{b}_{28} I_{18} + \mathbf{b}_{29} I_{22} + \mathbf{b}_{30} I_{23} + \mathbf{b}_{31} I_{26} + \mathbf{b}_{32} I_{27} + \mathbf{b}_{33} I_{28} + \mathbf{b}_{34} I_{29}) \\
 & + a_q v_q v_\ell^2 (\mathbf{b}_{35} I_4 + \mathbf{b}_{36} I_{17} + \mathbf{b}_{37} I_{18} + \mathbf{b}_{38} I_{22} + \mathbf{b}_{39} I_{23} + \mathbf{b}_{40} I_{26} + \mathbf{b}_{41} I_{27} + \mathbf{b}_{42} I_{28} + \mathbf{b}_{43} I_{29}) \\
 & + a_\ell v_q^2 v_\ell (\mathbf{b}_{44} I_4 + \mathbf{b}_{45} I_{17} + \mathbf{b}_{46} I_{18} + \mathbf{b}_{47} I_{22} + \mathbf{b}_{48} I_{23} + \mathbf{b}_{49} I_{26} + \mathbf{b}_{50} I_{27} + \mathbf{b}_{51} I_{28} + \mathbf{b}_{52} I_{29}) \\
 & + a_w^3 v_w (\mathbf{b}_{53} I_{16} + \mathbf{b}_{54} I_{30} + \mathbf{b}_{55} I_{31}) \\
 & + a_w v_w^3 (\mathbf{b}_{56} I_{16} + \mathbf{b}_{57} I_{30} + \mathbf{b}_{58} I_{31}), \tag{B.30}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{B}_{AV}^{(1,0)} = & Q_q Q_\ell a_q v_\ell (\mathbf{c}_1 I_4 + \mathbf{c}_2 I_{17} + \mathbf{c}_3 I_{18} + \mathbf{c}_4 I_{21} + \mathbf{c}_5 I_{22} + \mathbf{c}_6 I_{23} + \mathbf{c}_7 I_{24} + \mathbf{c}_8 I_{25}) \\
 & + Q_q Q_\ell a_\ell v_q (\mathbf{c}_9 I_4 + \mathbf{c}_{10} I_{17} + \mathbf{c}_{11} I_{18} + \mathbf{c}_{12} I_{21} + \mathbf{c}_{13} I_{22} + \mathbf{c}_{14} I_{23} + \mathbf{c}_{15} I_{24} + \mathbf{c}_{16} I_{25}) \\
 & + a_q^2 a_\ell v_\ell (\mathbf{c}_{17} I_4 + \mathbf{c}_{18} I_{17} + \mathbf{c}_{19} I_{18} + \mathbf{c}_{20} I_{22} + \mathbf{c}_{21} I_{23} + \mathbf{c}_{22} I_{26} + \mathbf{c}_{23} I_{27} + \mathbf{c}_{24} I_{28} + \mathbf{c}_{25} I_{29}) \\
 & + a_q a_\ell^2 v_q (\mathbf{c}_{26} I_4 + \mathbf{c}_{27} I_{17} + \mathbf{c}_{28} I_{18} + \mathbf{c}_{29} I_{22} + \mathbf{c}_{30} I_{23} + \mathbf{c}_{31} I_{26} + \mathbf{c}_{32} I_{27} + \mathbf{c}_{33} I_{28} + \mathbf{c}_{34} I_{29}) \\
 & + a_q v_q v_\ell^2 (\mathbf{c}_{35} I_4 + \mathbf{c}_{36} I_{17} + \mathbf{c}_{37} I_{18} + \mathbf{c}_{38} I_{22} + \mathbf{c}_{39} I_{23} + \mathbf{c}_{40} I_{26} + \mathbf{c}_{41} I_{27} + \mathbf{c}_{42} I_{28} + \mathbf{c}_{43} I_{29}) \\
 & + a_\ell v_q^2 v_\ell (\mathbf{c}_{44} I_4 + \mathbf{c}_{45} I_{17} + \mathbf{c}_{46} I_{18} + \mathbf{c}_{47} I_{22} + \mathbf{c}_{48} I_{23} + \mathbf{c}_{49} I_{26} + \mathbf{c}_{50} I_{27} + \mathbf{c}_{51} I_{28} + \mathbf{c}_{52} I_{29}) \\
 & + a_w^3 v_w (\mathbf{c}_{53} I_{16} + \mathbf{c}_{54} I_{30} + \mathbf{c}_{55} I_{31}) \\
 & + a_w v_w^3 (\mathbf{c}_{56} I_{16} + \mathbf{c}_{57} I_{30} + \mathbf{c}_{58} I_{31}), \tag{B.31}
 \end{aligned}$$

²⁰While comparing to [66], we noticed two typos in eq. (B.6), the definition of the form factor $\Lambda_3(k^2, M)$ appearing in eqs. (5.28) and (5.30): the sixth and seventh lines of eq. (B.6) should read

$$+ \frac{2}{3} w(w+2) \left[\ln^2 \left(\frac{1 + \sqrt{1-4w}}{1 - \sqrt{1-4w}} \right) - \pi^2 \right] - i\pi \left[\frac{2w+1}{3} \sqrt{1-4w} + \frac{4}{3} w(w+2) \ln \left(\frac{1 + \sqrt{1-4w}}{1 - \sqrt{1-4w}} \right) \right].$$

$$\begin{aligned}
 \mathcal{B}_{AA}^{(1,0)} = & Q_q^2 Q_\ell^2 (\mathbf{d}_1 I_{13} + \mathbf{d}_2 I_{17} + \mathbf{d}_3 I_{18} + \mathbf{d}_4 I_{19} + \mathbf{d}_5 I_{20}) \\
 & + Q_q Q_\ell a_q a_\ell (\mathbf{d}_6 I_4 + \mathbf{d}_7 I_{17} + \mathbf{d}_8 I_{18} + \mathbf{d}_9 I_{21} + \mathbf{d}_{10} I_{22} + \mathbf{d}_{11} I_{23} + \mathbf{d}_{12} I_{24} + \mathbf{d}_{13} I_{25}) \\
 & + Q_q Q_\ell v_q v_\ell (\mathbf{d}_{14} I_4 + \mathbf{d}_{15} I_{17} + \mathbf{d}_{16} I_{18} + \mathbf{d}_{17} I_{21} + \mathbf{d}_{18} I_{22} + \mathbf{d}_{19} I_{23} + \mathbf{d}_{20} I_{24} + \mathbf{d}_{21} I_{25}) \\
 & + a_q^2 a_\ell^2 (\mathbf{d}_{22} I_4 + \mathbf{d}_{23} I_{17} + \mathbf{d}_{24} I_{18} + \mathbf{d}_{25} I_{22} + \mathbf{d}_{26} I_{23} + \mathbf{d}_{27} I_{26} + \mathbf{d}_{28} I_{27} + \mathbf{d}_{29} I_{28} + \mathbf{d}_{30} I_{29}) \\
 & + a_q^2 v_\ell^2 (\mathbf{d}_{31} I_4 + \mathbf{d}_{32} I_{17} + \mathbf{d}_{33} I_{18} + \mathbf{d}_{34} I_{22} + \mathbf{d}_{35} I_{23} + \mathbf{d}_{36} I_{26} + \mathbf{d}_{37} I_{27} + \mathbf{d}_{38} I_{28} + \mathbf{d}_{39} I_{29}) \\
 & + a_\ell^2 v_q^2 (\mathbf{d}_{40} I_4 + \mathbf{d}_{41} I_{17} + \mathbf{d}_{42} I_{18} + \mathbf{d}_{43} I_{22} + \mathbf{d}_{44} I_{23} + \mathbf{d}_{45} I_{26} + \mathbf{d}_{46} I_{27} + \mathbf{d}_{47} I_{28} + \mathbf{d}_{48} I_{29}) \\
 & + v_q^2 v_\ell^2 (\mathbf{d}_{49} I_4 + \mathbf{d}_{50} I_{17} + \mathbf{d}_{51} I_{18} + \mathbf{d}_{52} I_{22} + \mathbf{d}_{53} I_{23} + \mathbf{d}_{54} I_{26} + \mathbf{d}_{55} I_{27} + \mathbf{d}_{56} I_{28} + \mathbf{d}_{57} I_{29}) \\
 & + a_q a_\ell v_q v_\ell (\mathbf{d}_{58} I_4 + \mathbf{d}_{59} I_{17} + \mathbf{d}_{60} I_{18} + \mathbf{d}_{61} I_{22} \\
 & \quad + \mathbf{d}_{62} I_{23} + \mathbf{d}_{63} I_{26} + \mathbf{d}_{64} I_{27} + \mathbf{d}_{65} I_{28} + \mathbf{d}_{66} I_{29}) \\
 & + a_w^4 (\mathbf{d}_{67} I_1 + \mathbf{d}_{68} I_2 + \mathbf{d}_{69} I_{16} + \mathbf{d}_{70} I_{18} + \mathbf{d}_{71} I_{30} + \mathbf{d}_{72} I_{31}) \\
 & + v_w^4 (\mathbf{d}_{73} I_1 + \mathbf{d}_{74} I_2 + \mathbf{d}_{75} I_{16} + \mathbf{d}_{76} I_{18} + \mathbf{d}_{77} I_{30} + \mathbf{d}_{78} I_{31}) \\
 & + a_w^2 v_w^2 (\mathbf{d}_{79} I_1 + \mathbf{d}_{80} I_2 + \mathbf{d}_{81} I_{16} + \mathbf{d}_{82} I_{18} + \mathbf{d}_{83} I_{30} + \mathbf{d}_{84} I_{31}) , \tag{B.32}
 \end{aligned}$$

$$\begin{aligned}
 \bar{\mathcal{B}}_{VV}^{(1,0)} = & Q_q^2 Q_\ell^2 (\bar{\mathbf{a}}_1 I_{13} + \bar{\mathbf{a}}_2 I_{17} + \bar{\mathbf{a}}_3 I_{18} + \bar{\mathbf{a}}_4 I_{19} + \bar{\mathbf{a}}_5 I_{20}) \\
 & + Q_q Q_\ell a_q a_\ell (\bar{\mathbf{a}}_6 I_4 + \bar{\mathbf{a}}_7 I_{17} + \bar{\mathbf{a}}_8 I_{18} + \bar{\mathbf{a}}_9 I_{21} + \bar{\mathbf{a}}_{10} I_{22} + \bar{\mathbf{a}}_{11} I_{23} + \bar{\mathbf{a}}_{12} I_{24} + \bar{\mathbf{a}}_{13} I_{25}) \\
 & + Q_q Q_\ell v_q v_\ell (\bar{\mathbf{a}}_{14} I_4 + \bar{\mathbf{a}}_{15} I_{17} + \bar{\mathbf{a}}_{16} I_{18} + \bar{\mathbf{a}}_{17} I_{21} + \bar{\mathbf{a}}_{18} I_{22} + \bar{\mathbf{a}}_{19} I_{23} + \bar{\mathbf{a}}_{20} I_{24} + \bar{\mathbf{a}}_{21} I_{25}) \\
 & + a_q^2 a_\ell^2 (\bar{\mathbf{a}}_{22} I_4 + \bar{\mathbf{a}}_{23} I_{17} + \bar{\mathbf{a}}_{24} I_{18} + \bar{\mathbf{a}}_{25} I_{22} + \bar{\mathbf{a}}_{26} I_{23} + \bar{\mathbf{a}}_{27} I_{26} + \bar{\mathbf{a}}_{28} I_{27} + \bar{\mathbf{a}}_{29} I_{28} + \bar{\mathbf{a}}_{30} I_{29}) \\
 & + a_q^2 v_\ell^2 (\bar{\mathbf{a}}_{31} I_4 + \bar{\mathbf{a}}_{32} I_{17} + \bar{\mathbf{a}}_{33} I_{18} + \bar{\mathbf{a}}_{34} I_{22} + \bar{\mathbf{a}}_{35} I_{23} + \bar{\mathbf{a}}_{36} I_{26} + \bar{\mathbf{a}}_{37} I_{27} + \bar{\mathbf{a}}_{38} I_{28} + \bar{\mathbf{a}}_{39} I_{29}) \\
 & + a_\ell^2 v_q^2 (\bar{\mathbf{a}}_{40} I_4 + \bar{\mathbf{a}}_{41} I_{17} + \bar{\mathbf{a}}_{42} I_{18} + \bar{\mathbf{a}}_{43} I_{22} + \bar{\mathbf{a}}_{44} I_{23} + \bar{\mathbf{a}}_{45} I_{26} + \bar{\mathbf{a}}_{46} I_{27} + \bar{\mathbf{a}}_{47} I_{28} + \bar{\mathbf{a}}_{48} I_{29}) \\
 & + v_q^2 v_\ell^2 (\bar{\mathbf{a}}_{49} I_4 + \bar{\mathbf{a}}_{50} I_{17} + \bar{\mathbf{a}}_{51} I_{18} + \bar{\mathbf{a}}_{52} I_{22} + \bar{\mathbf{a}}_{53} I_{23} + \bar{\mathbf{a}}_{54} I_{26} + \bar{\mathbf{a}}_{55} I_{27} + \bar{\mathbf{a}}_{56} I_{28} + \bar{\mathbf{a}}_{57} I_{29}) \\
 & + a_q a_\ell v_q v_\ell (\bar{\mathbf{a}}_{58} I_4 + \bar{\mathbf{a}}_{59} I_{17} + \bar{\mathbf{a}}_{60} I_{18} + \bar{\mathbf{a}}_{61} I_{22} + \bar{\mathbf{a}}_{62} I_{23} + \bar{\mathbf{a}}_{63} I_{26} + \bar{\mathbf{a}}_{64} I_{27} + \bar{\mathbf{a}}_{65} I_{28} + \bar{\mathbf{a}}_{66} I_{29}) \\
 & + a_w^4 (\bar{\mathbf{a}}_{67} I_1 + \bar{\mathbf{a}}_{68} I_2 + \bar{\mathbf{a}}_{69} I_{16} + \bar{\mathbf{a}}_{70} I_{18} + \bar{\mathbf{a}}_{71} I_{30} + \bar{\mathbf{a}}_{72} I_{31}) \\
 & + v_w^4 (\bar{\mathbf{a}}_{73} I_1 + \bar{\mathbf{a}}_{74} I_2 + \bar{\mathbf{a}}_{75} I_{16} + \bar{\mathbf{a}}_{76} I_{18} + \bar{\mathbf{a}}_{77} I_{30} + \bar{\mathbf{a}}_{78} I_{31}) \\
 & + a_w^2 v_w^2 (\bar{\mathbf{a}}_{79} I_1 + \bar{\mathbf{a}}_{80} I_2 + \bar{\mathbf{a}}_{81} I_{16} + \bar{\mathbf{a}}_{82} I_{18} + \bar{\mathbf{a}}_{83} I_{30} + \bar{\mathbf{a}}_{84} I_{31}) , \tag{B.33}
 \end{aligned}$$

$$\begin{aligned}
 \bar{\mathcal{B}}_{VA}^{(1,0)} = & Q_q Q_\ell a_q v_\ell (\bar{\mathbf{b}}_1 I_4 + \bar{\mathbf{b}}_2 I_{17} + \bar{\mathbf{b}}_3 I_{18} + \bar{\mathbf{b}}_4 I_{21} + \bar{\mathbf{b}}_5 I_{22} + \bar{\mathbf{b}}_6 I_{23} + \bar{\mathbf{b}}_7 I_{24} + \bar{\mathbf{b}}_8 I_{25}) \\
 & + Q_q Q_\ell a_\ell v_q (\bar{\mathbf{b}}_9 I_4 + \bar{\mathbf{b}}_{10} I_{17} + \bar{\mathbf{b}}_{11} I_{18} + \bar{\mathbf{b}}_{12} I_{21} + \bar{\mathbf{b}}_{13} I_{22} + \bar{\mathbf{b}}_{14} I_{23} + \bar{\mathbf{b}}_{15} I_{24} + \bar{\mathbf{b}}_{16} I_{25}) \\
 & + a_q^2 a_\ell v_\ell (\bar{\mathbf{b}}_{17} I_4 + \bar{\mathbf{b}}_{18} I_{17} + \bar{\mathbf{b}}_{19} I_{18} + \bar{\mathbf{b}}_{20} I_{22} + \bar{\mathbf{b}}_{21} I_{23} + \bar{\mathbf{b}}_{22} I_{26} + \bar{\mathbf{b}}_{23} I_{27} + \bar{\mathbf{b}}_{24} I_{28} + \bar{\mathbf{b}}_{25} I_{29}) \\
 & + a_q a_\ell^2 v_q (\bar{\mathbf{b}}_{26} I_4 + \bar{\mathbf{b}}_{27} I_{17} + \bar{\mathbf{b}}_{28} I_{18} + \bar{\mathbf{b}}_{29} I_{22} + \bar{\mathbf{b}}_{30} I_{23} + \bar{\mathbf{b}}_{31} I_{26} + \bar{\mathbf{b}}_{32} I_{27} + \bar{\mathbf{b}}_{33} I_{28} + \bar{\mathbf{b}}_{34} I_{29}) \\
 & + a_q v_q v_\ell^2 (\bar{\mathbf{b}}_{35} I_4 + \bar{\mathbf{b}}_{36} I_{17} + \bar{\mathbf{b}}_{37} I_{18} + \bar{\mathbf{b}}_{38} I_{22} + \bar{\mathbf{b}}_{39} I_{23} + \bar{\mathbf{b}}_{40} I_{26} + \bar{\mathbf{b}}_{41} I_{27} + \bar{\mathbf{b}}_{42} I_{28} + \bar{\mathbf{b}}_{43} I_{29}) \\
 & + a_\ell v_q^2 v_\ell (\bar{\mathbf{b}}_{44} I_4 + \bar{\mathbf{b}}_{45} I_{17} + \bar{\mathbf{b}}_{46} I_{18} + \bar{\mathbf{b}}_{47} I_{22} + \bar{\mathbf{b}}_{48} I_{23} + \bar{\mathbf{b}}_{49} I_{26} + \bar{\mathbf{b}}_{50} I_{27} + \bar{\mathbf{b}}_{51} I_{28} + \bar{\mathbf{b}}_{52} I_{29}) \\
 & + a_w^3 v_w (\bar{\mathbf{b}}_{53} I_1 + \bar{\mathbf{b}}_{54} I_2 + \bar{\mathbf{b}}_{55} I_{16} + \bar{\mathbf{b}}_{56} I_{18} + \bar{\mathbf{b}}_{57} I_{30} + \bar{\mathbf{b}}_{58} I_{31}) \\
 & + a_w v_w^3 (\bar{\mathbf{b}}_{59} I_1 + \bar{\mathbf{b}}_{60} I_2 + \bar{\mathbf{b}}_{61} I_{16} + \bar{\mathbf{b}}_{62} I_{18} + \bar{\mathbf{b}}_{63} I_{30} + \bar{\mathbf{b}}_{64} I_{31}) , \tag{B.34}
 \end{aligned}$$

$$\begin{aligned}
 \bar{B}_{AV}^{(1,0)} = & Q_q Q_\ell a_q v_\ell (\bar{c}_1 I_4 + \bar{c}_2 I_{17} + \bar{c}_3 I_{18} + \bar{c}_4 I_{21} + \bar{c}_5 I_{22} + \bar{c}_6 I_{23} + \bar{c}_7 I_{24} + \bar{c}_8 I_{25}) \\
 & + Q_q Q_\ell a_\ell v_q (\bar{c}_9 I_4 + \bar{c}_{10} I_{17} + \bar{c}_{11} I_{18} + \bar{c}_{12} I_{21} + \bar{c}_{13} I_{22} + \bar{c}_{14} I_{23} + \bar{c}_{15} I_{24} + \bar{c}_{16} I_{25}) \\
 & + a_q^2 a_\ell v_\ell (\bar{c}_{17} I_4 + \bar{c}_{18} I_{17} + \bar{c}_{19} I_{18} + \bar{c}_{20} I_{22} + \bar{c}_{21} I_{23} + \bar{c}_{22} I_{26} + \bar{c}_{23} I_{27} + \bar{c}_{24} I_{28} + \bar{c}_{25} I_{29}) \\
 & + a_q a_\ell^2 v_q (\bar{c}_{26} I_4 + \bar{c}_{27} I_{17} + \bar{c}_{28} I_{18} + \bar{c}_{29} I_{22} + \bar{c}_{30} I_{23} + \bar{c}_{31} I_{26} + \bar{c}_{32} I_{27} + \bar{c}_{33} I_{28} + \bar{c}_{34} I_{29}) \\
 & + a_q v_q v_\ell^2 (\bar{c}_{35} I_4 + \bar{c}_{36} I_{17} + \bar{c}_{37} I_{18} + \bar{c}_{38} I_{22} + \bar{c}_{39} I_{23} + \bar{c}_{40} I_{26} + \bar{c}_{41} I_{27} + \bar{c}_{42} I_{28} + \bar{c}_{43} I_{29}) \\
 & + a_\ell v_q^2 v_\ell (\bar{c}_{44} I_4 + \bar{c}_{45} I_{17} + \bar{c}_{46} I_{18} + \bar{c}_{47} I_{22} + \bar{c}_{48} I_{23} + \bar{c}_{49} I_{26} + \bar{c}_{50} I_{27} + \bar{c}_{51} I_{28} + \bar{c}_{52} I_{29}) \\
 & + a_w^3 v_w (\bar{c}_{53} I_1 + \bar{c}_{54} I_2 + \bar{c}_{55} I_{16} + \bar{c}_{56} I_{18} + \bar{c}_{57} I_{30} + \bar{c}_{58} I_{31}) \\
 & + a_w v_w^3 (\bar{c}_{59} I_1 + \bar{c}_{60} I_2 + \bar{c}_{61} I_{16} + \bar{c}_{62} I_{18} + \bar{c}_{63} I_{30} + \bar{c}_{64} I_{31}), \quad \text{and} \quad (\text{B.35})
 \end{aligned}$$

$$\begin{aligned}
 \bar{B}_{AA}^{(1,0)} = & Q_q^2 Q_\ell^2 (\bar{d}_1 I_{13} + \bar{d}_2 I_{17} + \bar{d}_3 I_{18} + \bar{d}_4 I_{19} + \bar{d}_5 I_{20}) \\
 & + Q_q Q_\ell a_q a_\ell (\bar{d}_6 I_4 + \bar{d}_7 I_{17} + \bar{d}_8 I_{18} + \bar{d}_9 I_{21} + \bar{d}_{10} I_{22} + \bar{d}_{11} I_{23} + \bar{d}_{12} I_{24} + \bar{d}_{13} I_{25}) \\
 & + Q_q Q_\ell v_q v_\ell (\bar{d}_{14} I_4 + \bar{d}_{15} I_{17} + \bar{d}_{16} I_{18} + \bar{d}_{17} I_{21} + \bar{d}_{18} I_{22} + \bar{d}_{19} I_{23} + \bar{d}_{20} I_{24} + \bar{d}_{21} I_{25}) \\
 & + a_q^2 a_\ell^2 (\bar{d}_{22} I_4 + \bar{d}_{23} I_{17} + \bar{d}_{24} I_{18} + \bar{d}_{25} I_{22} + \bar{d}_{26} I_{23} + \bar{d}_{27} I_{26} + \bar{d}_{28} I_{27} + \bar{d}_{29} I_{28} + \bar{d}_{30} I_{29}) \\
 & + a_q^2 v_\ell^2 (\bar{d}_{31} I_4 + \bar{d}_{32} I_{17} + \bar{d}_{33} I_{18} + \bar{d}_{34} I_{22} + \bar{d}_{35} I_{23} + \bar{d}_{36} I_{26} + \bar{d}_{37} I_{27} + \bar{d}_{38} I_{28} + \bar{d}_{39} I_{29}) \\
 & + a_\ell^2 v_q^2 (\bar{d}_{40} I_4 + \bar{d}_{41} I_{17} + \bar{d}_{42} I_{18} + \bar{d}_{43} I_{22} + \bar{d}_{44} I_{23} + \bar{d}_{45} I_{26} + \bar{d}_{46} I_{27} + \bar{d}_{47} I_{28} + \bar{d}_{48} I_{29}) \\
 & + v_q^2 v_\ell^2 (\bar{d}_{49} I_4 + \bar{d}_{50} I_{17} + \bar{d}_{51} I_{18} + \bar{d}_{52} I_{22} + \bar{d}_{53} I_{23} + \bar{d}_{54} I_{26} + \bar{d}_{55} I_{27} + \bar{d}_{56} I_{28} + \bar{d}_{57} I_{29}) \\
 & + a_q a_\ell v_q v_\ell (\bar{d}_{58} I_4 + \bar{d}_{59} I_{17} + \bar{d}_{60} I_{18} + \bar{d}_{61} I_{22} \\
 & \quad + \bar{d}_{62} I_{23} + \bar{d}_{63} I_{26} + \bar{d}_{64} I_{27} + \bar{d}_{65} I_{28} + \bar{d}_{66} I_{29}) \\
 & + a_w^4 (\bar{d}_{67} I_1 + \bar{d}_{68} I_2 + \bar{d}_{69} I_{16} + \bar{d}_{70} I_{18} + \bar{d}_{71} I_{30} + \bar{d}_{72} I_{31}) \\
 & + v_w^4 (\bar{d}_{73} I_1 + \bar{d}_{74} I_2 + \bar{d}_{75} I_{16} + \bar{d}_{76} I_{18} + \bar{d}_{77} I_{30} + \bar{d}_{78} I_{31}) \\
 & + a_w^2 v_w^2 (\bar{d}_{79} I_1 + \bar{d}_{80} I_2 + \bar{d}_{81} I_{16} + \bar{d}_{82} I_{18} + \bar{d}_{83} I_{30} + \bar{d}_{84} I_{31}), \quad (\text{B.36})
 \end{aligned}$$

where, due to their excessive length, the \mathbf{a}_i , \mathbf{b}_i , \mathbf{c}_i , \mathbf{d}_i , $\bar{\mathbf{a}}_i$, $\bar{\mathbf{b}}_i$, $\bar{\mathbf{c}}_i$, and $\bar{\mathbf{d}}_i$ are provided in the ancillary files `HVBMboxcoeffs.m` and `Kreimerboxcoeffs.m` attached to the arXiv submission of this paper.

In fact, it is possible to compare our results for the one-loop box diagrams to the original calculation of [66], where a photon mass λ was employed to regulate infrared singularities, by making the identification $\ln(\lambda^2/s) \rightarrow 1/\epsilon$. Inserting explicit expressions for our master integrals into eqs. (B.29)–(B.36), we find, modulo typos, complete agreement through to $\mathcal{O}(\epsilon^0)$ with eqs. (B.10) and (B.12)–(B.14) of [66]. To find agreement with [66], we first set the factor of $\alpha/(2\pi s)$ in eq. (B.10) to one, the factors of $\alpha/(2\pi)$ in eq. (B.12) to $2/s$, the factors of $\alpha/(2\pi)$ in eq. (B.13) to $2/(s - M_Z^2)$, and the factors of $\alpha/(2\pi)$ in eq. (B.14) to $2/s$. In addition, we find that the term inside the brackets on the second line of eq. (B.12) should read

$$-2 \left[\ln\left(\frac{-t}{s}\right) - \ln\left(\frac{-u}{s}\right) \right] \ln\left(\frac{-s - i\epsilon}{\lambda^2}\right),$$

that the term inside the parentheses on the right-hand side of the second equality of eq. (B.13) should read²¹

$$\begin{aligned} & \text{Sp} \left(\frac{M_Z^2 + t}{t} \right) - \text{Sp} \left(\frac{M_Z^2 + u}{u} \right) + \frac{1}{2} \left[\ln \left(\frac{t^2}{s^2} \right) - \ln \left(\frac{u^2}{s^2} \right) \right] \ln \left(\frac{M_Z^2}{M_Z^2 - s - i\epsilon} \right) \\ & - \left[\ln \left(\frac{-t}{s} \right) - \ln \left(\frac{-u}{s} \right) \right] \ln \left(\frac{M_Z^2 - s - i\epsilon}{\lambda^2} \right), \end{aligned}$$

and that the factor of $x_2 + x_1$ in the denominator on the third line of eq. (B.14) should be replaced by $x_2 - x_1$.

From the ingredients presented above, we distill complete results for $\mathbf{C}_{VV}^{(0,1)}$, $\mathbf{C}_{VA}^{(0,1)}$, $\mathbf{C}_{AV}^{(0,1)}$, $\mathbf{C}_{AA}^{(0,1)}$, $\bar{\mathbf{C}}_{VV}^{(0,1)}$, $\bar{\mathbf{C}}_{VA}^{(0,1)}$, $\bar{\mathbf{C}}_{AV}^{(0,1)}$, $\bar{\mathbf{C}}_{AA}^{(0,1)}$, $\mathbf{C}_{VV}^{(1,0)}$, $\mathbf{C}_{VA}^{(1,0)}$, $\mathbf{C}_{AV}^{(1,0)}$, $\mathbf{C}_{AA}^{(1,0)}$, $\bar{\mathbf{C}}_{VV}^{(1,0)}$, $\bar{\mathbf{C}}_{VA}^{(1,0)}$, $\bar{\mathbf{C}}_{AV}^{(1,0)}$, and $\bar{\mathbf{C}}_{AA}^{(1,0)}$ in section 4.3.

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References

- [1] S.D. Drell and T.-M. Yan, *Massive Lepton Pair Production in Hadron-Hadron Collisions at High-Energies*, *Phys. Rev. Lett.* **25** (1970) 316 [Erratum *ibid.* **25** (1970) 902] [[INSPIRE](#)].
- [2] G. Altarelli, R.K. Ellis and G. Martinelli, *Large Perturbative Corrections to the Drell-Yan Process in QCD*, *Nucl. Phys. B* **157** (1979) 461 [[INSPIRE](#)].
- [3] R. Hamberg, W.L. van Neerven and T. Matsuura, *A Complete calculation of the order α_s^2 correction to the Drell-Yan K factor*, *Nucl. Phys. B* **359** (1991) 343 [Erratum *ibid.* **644** (2002) 403] [[INSPIRE](#)].
- [4] R.V. Harlander and W.B. Kilgore, *Next-to-next-to-leading order Higgs production at hadron colliders*, *Phys. Rev. Lett.* **88** (2002) 201801 [[hep-ph/0201206](#)] [[INSPIRE](#)].
- [5] C. Anastasiou, L.J. Dixon, K. Melnikov and F. Petriello, *High precision QCD at hadron colliders: Electroweak gauge boson rapidity distributions at NNLO*, *Phys. Rev. D* **69** (2004) 094008 [[hep-ph/0312266](#)] [[INSPIRE](#)].
- [6] K. Melnikov and F. Petriello, *The W boson production cross section at the LHC through $\mathcal{O}(\alpha_s^2)$* , *Phys. Rev. Lett.* **96** (2006) 231803 [[hep-ph/0603182](#)] [[INSPIRE](#)].
- [7] C. Duhr, F. Dulat and B. Mistlberger, *The Drell-Yan cross section to third order in the strong coupling constant*, *Phys. Rev. Lett.* **125** (2020) 172001 [[arXiv:2001.07717](#)] [[INSPIRE](#)].
- [8] C. Duhr, F. Dulat and B. Mistlberger, *Charged current Drell-Yan production at N^3LO* , *JHEP* **11** (2020) 143 [[arXiv:2007.13313](#)] [[INSPIRE](#)].
- [9] F.A. Berends, W.L. van Neerven and G.J.H. Burgers, *Higher Order Radiative Corrections at LEP Energies*, *Nucl. Phys. B* **297** (1988) 429 [Erratum *ibid.* **304** (1988) 921] [[INSPIRE](#)].

²¹The Spence function of [66], $\text{Sp}(x)$, is exactly the dilogarithm function, $\text{Li}_2(x)$.

- [10] J. Blümlein, A. De Freitas, C.G. Raab and K. Schönwald, *The $\mathcal{O}(\alpha^2)$ Initial State QED Corrections to e^+e^- Annihilation to a Neutral Vector Boson Revisited*, *Phys. Lett. B* **791** (2019) 206 [[arXiv:1901.08018](#)] [[INSPIRE](#)].
- [11] U. Baur, S. Keller and D. Wackerth, *Electroweak radiative corrections to W boson production in hadronic collisions*, *Phys. Rev. D* **59** (1999) 013002 [[hep-ph/9807417](#)] [[INSPIRE](#)].
- [12] U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackerth, *Electroweak radiative corrections to neutral current Drell-Yan processes at hadron colliders*, *Phys. Rev. D* **65** (2002) 033007 [[hep-ph/0108274](#)] [[INSPIRE](#)].
- [13] S. Dittmaier and M. Krämer, *Electroweak radiative corrections to W boson production at hadron colliders*, *Phys. Rev. D* **65** (2002) 073007 [[hep-ph/0109062](#)] [[INSPIRE](#)].
- [14] U. Baur and D. Wackerth, *Electroweak radiative corrections to $p\bar{p} \rightarrow W^\pm \rightarrow \ell^\pm \nu$ beyond the pole approximation*, *Phys. Rev. D* **70** (2004) 073015 [[hep-ph/0405191](#)] [[INSPIRE](#)].
- [15] A. Arbuzov et al., *One-loop corrections to the Drell-Yan process in SANC. I. The charged current case*, *Eur. Phys. J. C* **46** (2006) 407 [*Erratum ibid.* **50** (2007) 505] [[hep-ph/0506110](#)] [[INSPIRE](#)].
- [16] C.M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, *Precision electroweak calculation of the charged current Drell-Yan process*, *JHEP* **12** (2006) 016 [[hep-ph/0609170](#)] [[INSPIRE](#)].
- [17] A.B. Arbuzov and R.R. Sadykov, *Inverse bremsstrahlung contributions to Drell-Yan like processes*, *J. Exp. Theor. Phys.* **106** (2008) 488 [[arXiv:0707.0423](#)] [[INSPIRE](#)].
- [18] C.M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, *Precision electroweak calculation of the production of a high transverse-momentum lepton pair at hadron colliders*, *JHEP* **10** (2007) 109 [[arXiv:0710.1722](#)] [[INSPIRE](#)].
- [19] S. Dittmaier and M. Huber, *Radiative corrections to the neutral-current Drell-Yan process in the Standard Model and its minimal supersymmetric extension*, *JHEP* **01** (2010) 060 [[arXiv:0911.2329](#)] [[INSPIRE](#)].
- [20] W.B. Kilgore and C. Sturm, *Two-Loop Virtual Corrections to Drell-Yan Production at order $\alpha_s\alpha^3$* , *Phys. Rev. D* **85** (2012) 033005 [[arXiv:1107.4798](#)] [[INSPIRE](#)].
- [21] S. Dittmaier, A. Huss and C. Schwinn, *Mixed QCD-electroweak $\mathcal{O}(\alpha_s\alpha)$ corrections to Drell-Yan processes in the resonance region: pole approximation and non-factorizable corrections*, *Nucl. Phys. B* **885** (2014) 318 [[arXiv:1403.3216](#)] [[INSPIRE](#)].
- [22] S. Dittmaier, A. Huss and C. Schwinn, *Dominant mixed QCD-electroweak $\mathcal{O}(\alpha_s\alpha)$ corrections to Drell-Yan processes in the resonance region*, *Nucl. Phys. B* **904** (2016) 216 [[arXiv:1511.08016](#)] [[INSPIRE](#)].
- [23] M. Delto, M. Jaquier, K. Melnikov and R. Röntsch, *Mixed QCD \otimes QED corrections to on-shell Z boson production at the LHC*, *JHEP* **01** (2020) 043 [[arXiv:1909.08428](#)] [[INSPIRE](#)].
- [24] F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov and R. Röntsch, *Mixed QCD-electroweak corrections to on-shell Z production at the LHC*, *Phys. Lett. B* **811** (2020) 135969 [[arXiv:2005.10221](#)] [[INSPIRE](#)].
- [25] R. Bonciani, F. Buccioni, N. Rana and A. Vicini, *NNLO QCD \times EW corrections to on-shell Z production*, *Phys. Rev. Lett.* **125** (2020) 232004 [[arXiv:2007.06518](#)] [[INSPIRE](#)].

- [26] S. Dittmaier, T. Schmidt and J. Schwarz, *Mixed NNLO QCD \times electroweak corrections of $\mathcal{O}(N_f\alpha_s\alpha)$ to single- W/Z production at the LHC*, *JHEP* **12** (2020) 201 [[arXiv:2009.02229](#)] [[INSPIRE](#)].
- [27] A. Behring et al., *Mixed QCD-electroweak corrections to W -boson production in hadron collisions*, *Phys. Rev. D* **103** (2021) 013008 [[arXiv:2009.10386](#)] [[INSPIRE](#)].
- [28] R. Bonciani, S. Di Vita, P. Mastrolia and U. Schubert, *Two-Loop Master Integrals for the mixed EW-QCD virtual corrections to Drell-Yan scattering*, *JHEP* **09** (2016) 091 [[arXiv:1604.08581](#)] [[INSPIRE](#)].
- [29] A. von Manteuffel and R.M. Schabinger, *Numerical Multi-Loop Calculations via Finite Integrals and One-Mass EW-QCD Drell-Yan Master Integrals*, *JHEP* **04** (2017) 129 [[arXiv:1701.06583](#)] [[INSPIRE](#)].
- [30] S.M. Hasan and U. Schubert, *Master Integrals for the mixed QCD-QED corrections to the Drell-Yan production of a massive lepton pair*, *JHEP* **11** (2020) 107 [[arXiv:2004.14908](#)] [[INSPIRE](#)].
- [31] M. Heller, A. von Manteuffel and R.M. Schabinger, *Multiple polylogarithms with algebraic arguments and the two-loop EW-QCD Drell-Yan master integrals*, *Phys. Rev. D* **102** (2020) 016025 [[arXiv:1907.00491](#)] [[INSPIRE](#)].
- [32] P. Breitenlohner and D. Maison, *Dimensional Renormalization and the Action Principle*, *Commun. Math. Phys.* **52** (1977) 11 [[INSPIRE](#)].
- [33] P. Breitenlohner and D. Maison, *Dimensionally Renormalized Green's Functions for Theories with Massless Particles. 1*, *Commun. Math. Phys.* **52** (1977) 39 [[INSPIRE](#)].
- [34] P. Breitenlohner and D. Maison, *Dimensionally Renormalized Green's Functions for Theories with Massless Particles. 2*, *Commun. Math. Phys.* **52** (1977) 55 [[INSPIRE](#)].
- [35] M.S. Chanowitz, M. Furman and I. Hinchliffe, *The Axial Current in Dimensional Regularization*, *Nucl. Phys. B* **159** (1979) 225 [[INSPIRE](#)].
- [36] J.G. Körner, G. Schuler, G. Kramer and B. Lampe, *γ_5 and Infrared Dimensional Regularization*, *Phys. Lett. B* **164** (1985) 136 [[INSPIRE](#)].
- [37] C. Gnendiger et al., *To d , or not to d : recent developments and comparisons of regularization schemes*, *Eur. Phys. J. C* **77** (2017) 471 [[arXiv:1705.01827](#)] [[INSPIRE](#)].
- [38] G. 't Hooft and M.J.G. Veltman, *Regularization and Renormalization of Gauge Fields*, *Nucl. Phys. B* **44** (1972) 189 [[INSPIRE](#)].
- [39] D. Kreimer, *The γ_5 Problem and Anomalies: A Clifford Algebra Approach*, *Phys. Lett. B* **237** (1990) 59 [[INSPIRE](#)].
- [40] J.G. Körner, D. Kreimer and K. Schilcher, *A practicable γ_5 scheme in dimensional regularization*, *Z. Phys. C* **54** (1992) 503 [[INSPIRE](#)].
- [41] D. Kreimer, *Dimensionale Regularisierung im Standard Modell*, Ph.D. Thesis, Faculty of Physics, Johannes Gutenberg University, Germany (1992).
- [42] D. Kreimer, *The role of γ_5 in dimensional regularization*, [hep-ph/9401354](#) [[INSPIRE](#)].
- [43] E.R. Speer and M.J. Westwater, *Generic Feynman amplitudes*, *Ann. Inst. Henri Poincaré A* **14** (1971) 1.
- [44] J.F. Ashmore, *A Method of Gauge Invariant Regularization*, *Lett. Nuovo Cim.* **4** (1972) 289 [[INSPIRE](#)].

- [45] C.G. Bollini and J.J. Giambiagi, *Dimensional Renormalization: The Number of Dimensions as a Regularizing Parameter*, *Nuovo Cim. B* **12** (1972) 20 [INSPIRE].
- [46] W.J. Marciano and A. Sirlin, *Dimensional Regularization of Infrared Divergences*, *Nucl. Phys. B* **88** (1975) 86 [INSPIRE].
- [47] M.E. Peskin and D.V. Schroeder, *An introduction to quantum field theory*, Addison-Wesley, Reading, U.S.A. (1995).
- [48] J.C. Collins, *Renormalization: An Introduction to Renormalization, The Renormalization Group, and the Operator Product Expansion*, vol. 26 of *Cambridge Monographs on Mathematical Physics*, Cambridge University Press, Cambridge (1986), [DOI] [INSPIRE].
- [49] Z. Bern, L.J. Dixon and D.A. Kosower, *Dimensionally regulated one-loop integrals*, *Phys. Lett. B* **302** (1993) 299 [Erratum *ibid.* **318** (1993) 649] [hep-ph/9212308] [INSPIRE].
- [50] Z. Bern, L.J. Dixon and D.A. Kosower, *Dimensionally regulated pentagon integrals*, *Nucl. Phys. B* **412** (1994) 751 [hep-ph/9306240] [INSPIRE].
- [51] O.V. Tarasov, *Connection between Feynman integrals having different values of the space-time dimension*, *Phys. Rev. D* **54** (1996) 6479 [hep-th/9606018] [INSPIRE].
- [52] Z. Bern and A.G. Morgan, *Massive loop amplitudes from unitarity*, *Nucl. Phys. B* **467** (1996) 479 [hep-ph/9511336] [INSPIRE].
- [53] Z. Bern, A. De Freitas and L.J. Dixon, *Two loop helicity amplitudes for gluon-gluon scattering in QCD and supersymmetric Yang-Mills theory*, *JHEP* **03** (2002) 018 [hep-ph/0201161] [INSPIRE].
- [54] A. Denner and S. Dittmaier, *Electroweak Radiative Corrections for Collider Physics*, *Phys. Rept.* **864** (2020) 1 [arXiv:1912.06823] [INSPIRE].
- [55] T.L. Trueman, *Spurious anomalies in dimensional renormalization*, *Z. Phys. C* **69** (1996) 525 [hep-ph/9504315] [INSPIRE].
- [56] S.A. Larin, *The renormalization of the axial anomaly in dimensional regularization*, *Phys. Lett. B* **303** (1993) 113 [hep-ph/9302240] [INSPIRE].
- [57] T. Ahmed, W. Bernreuther, L. Chen and M. Czakon, *Polarized $q\bar{q} \rightarrow Z + \text{Higgs}$ amplitudes at two loops in QCD: the interplay between vector and axial vector form factors and a pitfall in applying a non-anticommuting γ_5* , *JHEP* **07** (2020) 159 [arXiv:2004.13753] [INSPIRE].
- [58] B. Agarwal, S.P. Jones and A. von Manteuffel, *Two-loop helicity amplitudes for $gg \rightarrow ZZ$ with full top-quark mass effects*, arXiv:2011.15113 [INSPIRE].
- [59] R. Boughezal, F. Petriello, U. Schubert and H. Xing, *Spin-dependent quark beam function at NNLO*, *Phys. Rev. D* **96** (2017) 034001 [arXiv:1704.05457] [INSPIRE].
- [60] F.V. Tkachov, *A Theorem on Analytical Calculability of Four Loop Renormalization Group Functions*, *Phys. Lett. B* **100** (1981) 65 [INSPIRE].
- [61] K.G. Chetyrkin and F.V. Tkachov, *Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops*, *Nucl. Phys. B* **192** (1981) 159 [INSPIRE].
- [62] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, *Int. J. Mod. Phys. A* **15** (2000) 5087 [hep-ph/0102033] [INSPIRE].
- [63] G. Passarino and M.J.G. Veltman, *One Loop Corrections for e^+e^- Annihilation Into $\mu^+\mu^-$ in the Weinberg Model*, *Nucl. Phys. B* **160** (1979) 151 [INSPIRE].

- [64] A. von Manteuffel, E. Panzer and R.M. Schabinger, *A quasi-finite basis for multi-loop Feynman integrals*, *JHEP* **02** (2015) 120 [[arXiv:1411.7392](#)] [[INSPIRE](#)].
- [65] A. von Manteuffel, E. Panzer and R.M. Schabinger, *On the Computation of Form Factors in Massless QCD with Finite Master Integrals*, *Phys. Rev. D* **93** (2016) 125014 [[arXiv:1510.06758](#)] [[INSPIRE](#)].
- [66] M. Böhm, H. Spiesberger and W. Hollik, *On the One Loop Renormalization of the Electroweak Standard Model and Its Application to Leptonic Processes*, *Fortsch. Phys.* **34** (1986) 687 [[INSPIRE](#)].
- [67] A. Denner, *Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200*, *Fortsch. Phys.* **41** (1993) 307 [[arXiv:0709.1075](#)] [[INSPIRE](#)].
- [68] M. Böhm, A. Denner and H. Joos, *Gauge theories of the strong and electroweak interaction*, B.G. Teubner, Stuttgart/Leipzig/Wiesbaden, Germany (2001).
- [69] J. Franzkowski, *Virtuelle Strahlungskorrekturen im Standardmodell der Elementarteilchenphysik*, Ph.D. Thesis, Faculty of Physics, Johannes Gutenberg University, Germany (1997).
- [70] T. Gehrmann, T. Huber and D. Maître, *Two-loop quark and gluon form-factors in dimensional regularisation*, *Phys. Lett. B* **622** (2005) 295 [[hep-ph/0507061](#)] [[INSPIRE](#)].
- [71] C.W. Bauer, A. Frink and R. Kreckel, *Introduction to the GiNaC framework for symbolic computation within the C++ programming language*, *J. Symb. Comput.* **33** (2002) 1 [[cs/0004015](#)] [[INSPIRE](#)].
- [72] C. Studerus, *Reduze-Feynman Integral Reduction in C++*, *Comput. Phys. Commun.* **181** (2010) 1293 [[arXiv:0912.2546](#)] [[INSPIRE](#)].
- [73] A. von Manteuffel and C. Studerus, *Reduze 2 — Distributed Feynman Integral Reduction*, [arXiv:1201.4330](#) [[INSPIRE](#)].
- [74] V.A. Smirnov, *Evaluating Feynman integrals*, *Springer Tracts Mod. Phys.* **211** (2004).
- [75] A. Djouadi and P. Gambino, *Electroweak gauge bosons self-energies: Complete QCD corrections*, *Phys. Rev. D* **49** (1994) 3499 [*Erratum ibid.* **53** (1996) 4111] [[hep-ph/9309298](#)] [[INSPIRE](#)].
- [76] E.W.N. Glover, *Two loop QCD helicity amplitudes for massless quark quark scattering*, *JHEP* **04** (2004) 021 [[hep-ph/0401119](#)] [[INSPIRE](#)].
- [77] L. Chen, *A prescription for projectors to compute helicity amplitudes in D dimensions*, [arXiv:1904.00705](#) [[INSPIRE](#)].
- [78] T. Ahmed, A.H. Ajjath, L. Chen, P.K. Dhani, P. Mukherjee and V. Ravindran, *Polarised Amplitudes and Soft-Virtual Cross Sections for $b\bar{b} \rightarrow ZH$ at NNLO in QCD*, *JHEP* **01** (2020) 030 [[arXiv:1910.06347](#)] [[INSPIRE](#)].
- [79] T. Peraro and L. Tancredi, *Physical projectors for multi-leg helicity amplitudes*, *JHEP* **07** (2019) 114 [[arXiv:1906.03298](#)] [[INSPIRE](#)].
- [80] T. Peraro and L. Tancredi, *Tensor decomposition for bosonic and fermionic scattering amplitudes*, *Phys. Rev. D* **103** (2021) 054042 [[arXiv:2012.00820](#)] [[INSPIRE](#)].
- [81] F. Caola, A. Von Manteuffel and L. Tancredi, *Di-photon amplitudes in three-loop Quantum Chromodynamics*, *Phys. Rev. Lett.* **126** (2021) 112004 [[arXiv:2011.13946](#)] [[INSPIRE](#)].

- [82] G.F. Sterman and M.E. Tejeda-Yeomans, *Multiloop amplitudes and resummation*, *Phys. Lett. B* **552** (2003) 48 [[hep-ph/0210130](#)] [[INSPIRE](#)].
- [83] S.M. Aybat, L.J. Dixon and G.F. Sterman, *The Two-loop anomalous dimension matrix for soft gluon exchange*, *Phys. Rev. Lett.* **97** (2006) 072001 [[hep-ph/0606254](#)] [[INSPIRE](#)].
- [84] S.M. Aybat, L.J. Dixon and G.F. Sterman, *The Two-loop soft anomalous dimension matrix and resummation at next-to-next-to leading pole*, *Phys. Rev. D* **74** (2006) 074004 [[hep-ph/0607309](#)] [[INSPIRE](#)].
- [85] T. Becher and M. Neubert, *Infrared singularities of scattering amplitudes in perturbative QCD*, *Phys. Rev. Lett.* **102** (2009) 162001 [Erratum *ibid.* **111** (2013) 199905] [[arXiv:0901.0722](#)] [[INSPIRE](#)].
- [86] E. Gardi and L. Magnea, *Factorization constraints for soft anomalous dimensions in QCD scattering amplitudes*, *JHEP* **03** (2009) 079 [[arXiv:0901.1091](#)] [[INSPIRE](#)].
- [87] T. Becher and M. Neubert, *On the Structure of Infrared Singularities of Gauge-Theory Amplitudes*, *JHEP* **06** (2009) 081 [Erratum *ibid.* **11** (2013) 024] [[arXiv:0903.1126](#)] [[INSPIRE](#)].
- [88] L.J. Dixon, *Matter Dependence of the Three-Loop Soft Anomalous Dimension Matrix*, *Phys. Rev. D* **79** (2009) 091501 [[arXiv:0901.3414](#)] [[INSPIRE](#)].
- [89] A.V. Kotikov, *The Property of maximal transcendentality in the $\mathcal{N} = 4$ Supersymmetric Yang-Mills*, in D. Diakonov ed., *Subtleties in quantum field theory*, (2010) [[arXiv:1005.5029](#)] [[INSPIRE](#)].
- [90] J.M. Henn, *Multi-loop integrals in dimensional regularization made simple*, *Phys. Rev. Lett.* **110** (2013) 251601 [[arXiv:1304.1806](#)] [[INSPIRE](#)].
- [91] J. Kuipers, T. Ueda, J.A.M. Vermaseren and J. Vollinga, *FORM version 4.0*, *Comput. Phys. Commun.* **184** (2013) 1453 [[arXiv:1203.6543](#)] [[INSPIRE](#)].
- [92] S. Catani, *The singular behavior of QCD amplitudes at two loop order*, *Phys. Lett. B* **427** (1998) 161 [[hep-ph/9802439](#)] [[INSPIRE](#)].
- [93] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to the scattering of massless distinct quarks*, *Nucl. Phys. B* **601** (2001) 318 [[hep-ph/0010212](#)] [[INSPIRE](#)].
- [94] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two loop QCD corrections to massless identical quark scattering*, *Nucl. Phys. B* **601** (2001) 341 [[hep-ph/0011094](#)] [[INSPIRE](#)].
- [95] C. Anastasiou, E.W.N. Glover and M.E. Tejeda-Yeomans, *Two loop QED and QCD corrections to massless fermion boson scattering*, *Nucl. Phys. B* **629** (2002) 255 [[hep-ph/0201274](#)] [[INSPIRE](#)].
- [96] Z. Bern, L.J. Dixon and D.A. Kosower, *Two-loop $g \rightarrow gg$ splitting amplitudes in QCD*, *JHEP* **08** (2004) 012 [[hep-ph/0404293](#)] [[INSPIRE](#)].
- [97] S. Caron-Huot, *When does the gluon reggeize?*, *JHEP* **05** (2015) 093 [[arXiv:1309.6521](#)] [[INSPIRE](#)].
- [98] O. Almeldid, C. Duhr and E. Gardi, *Three-loop corrections to the soft anomalous dimension in multileg scattering*, *Phys. Rev. Lett.* **117** (2016) 172002 [[arXiv:1507.00047](#)] [[INSPIRE](#)].

- [99] J.M. Henn and B. Mistlberger, *Four-Gluon Scattering at Three Loops, Infrared Structure, and the Regge Limit*, *Phys. Rev. Lett.* **117** (2016) 171601 [[arXiv:1608.00850](#)] [[INSPIRE](#)].
- [100] S. Moch, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *Four-Loop Non-Singlet Splitting Functions in the Planar Limit and Beyond*, *JHEP* **10** (2017) 041 [[arXiv:1707.08315](#)] [[INSPIRE](#)].
- [101] S. Moch, B. Ruijl, T. Ueda, J.A.M. Vermaseren and A. Vogt, *On quartic colour factors in splitting functions and the gluon cusp anomalous dimension*, *Phys. Lett. B* **782** (2018) 627 [[arXiv:1805.09638](#)] [[INSPIRE](#)].
- [102] S. Catani, D. De Florian and M. Grazzini, *Soft-gluon effective coupling and cusp anomalous dimension*, *Eur. Phys. J. C* **79** (2019) 685 [[arXiv:1904.10365](#)] [[INSPIRE](#)].
- [103] T. Becher and M. Neubert, *Infrared singularities of scattering amplitudes and N^3LL resummation for n -jet processes*, *JHEP* **01** (2020) 025 [[arXiv:1908.11379](#)] [[INSPIRE](#)].
- [104] J.M. Henn, G.P. Korchemsky and B. Mistlberger, *The full four-loop cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills and QCD*, *JHEP* **04** (2020) 018 [[arXiv:1911.10174](#)] [[INSPIRE](#)].
- [105] T. Huber, A. von Manteuffel, E. Panzer, R.M. Schabinger and G. Yang, *The four-loop cusp anomalous dimension from the $N = 4$ Sudakov form factor*, *Phys. Lett. B* **807** (2020) 135543 [[arXiv:1912.13459](#)] [[INSPIRE](#)].
- [106] A. von Manteuffel, E. Panzer and R.M. Schabinger, *Cusp and collinear anomalous dimensions in four-loop QCD from form factors*, *Phys. Rev. Lett.* **124** (2020) 162001 [[arXiv:2002.04617](#)] [[INSPIRE](#)].
- [107] T. Becher and M. Neubert, *Infrared singularities of QCD amplitudes with massive partons*, *Phys. Rev. D* **79** (2009) 125004 [*Erratum ibid.* **80** (2009) 109901] [[arXiv:0904.1021](#)] [[INSPIRE](#)].
- [108] G.P. Korchemsky and A.V. Radyushkin, *Loop Space Formalism and Renormalization Group for the Infrared Asymptotics of QCD*, *Phys. Lett. B* **171** (1986) 459 [[INSPIRE](#)].
- [109] S. Moch, J.A.M. Vermaseren and A. Vogt, *The Quark form-factor at higher orders*, *JHEP* **08** (2005) 049 [[hep-ph/0507039](#)] [[INSPIRE](#)].
- [110] C. Bogner et al., *Loopedia, a Database for Loop Integrals*, *Comput. Phys. Commun.* **225** (2018) 1 [[arXiv:1709.01266](#)] [[INSPIRE](#)].
- [111] T. Huber and D. Maître, *HypExp: A Mathematica package for expanding hypergeometric functions around integer-valued parameters*, *Comput. Phys. Commun.* **175** (2006) 122 [[hep-ph/0507094](#)] [[INSPIRE](#)].
- [112] A.V. Kotikov, *Differential equations method: New technique for massive Feynman diagrams calculation*, *Phys. Lett. B* **254** (1991) 158 [[INSPIRE](#)].
- [113] A.V. Kotikov, *Differential equations method: The calculation of vertex type Feynman diagrams*, *Phys. Lett. B* **259** (1991) 314 [[INSPIRE](#)].
- [114] A.V. Kotikov, *Differential equation method: The calculation of N point Feynman diagrams*, *Phys. Lett. B* **267** (1991) 123 [*Erratum ibid.* **295** (1992) 409] [[INSPIRE](#)].
- [115] E. Remiddi, *Differential equations for Feynman graph amplitudes*, *Nuovo Cim. A* **110** (1997) 1435 [[hep-th/9711188](#)] [[INSPIRE](#)].

- [116] T. Gehrmann and E. Remiddi, *Differential equations for two-loop four-point functions*, *Nucl. Phys. B* **580** (2000) 485 [[hep-ph/9912329](#)] [[INSPIRE](#)].
- [117] L.J. Dixon, *A brief introduction to modern amplitude methods*, in *Theoretical Advanced Study Institute in Elementary Particle Physics: Particle Physics: The Higgs Boson and Beyond*, (2013), DOI [[arXiv:1310.5353](#)] [[INSPIRE](#)].
- [118] L.J. Dixon, *Calculating scattering amplitudes efficiently*, in *Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 95): QCD and Beyond*, (1996) [[hep-ph/9601359](#)] [[INSPIRE](#)].
- [119] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [120] D. Binosi and L. Theussl, *JaxoDraw: A graphical user interface for drawing Feynman diagrams*, *Comput. Phys. Commun.* **161** (2004) 76 [[hep-ph/0309015](#)] [[INSPIRE](#)].
- [121] J.A.M. Vermaseren, *Axodraw*, *Comput. Phys. Commun.* **83** (1994) 45 [[INSPIRE](#)].