Janus solutions in three-dimensional $\mathcal{N} = 8$ gauged supergravity

Kevin Chen and Michael Gutperle

Mani L. Bhaumik Institute for Theoretical Physics,
Department of Physics and Astronomy, University of California,
315 Portola Plaza, Los Angeles, CA 90095, U.S.A.

E-mail: kevinddchen@gmail.com, gutperle@physics.ucla.edu

ABSTRACT: Janus solutions are constructed in $d = 3$, $\mathcal{N} = 8$ gauged supergravity. We find explicit half-BPS solutions where two scalars in the $SO(8,1)/SO(8)$ coset have a nontrivial profile. These solutions correspond on the CFT side to an interface with a position-dependent expectation value for a relevant operator and a source which jumps across the interface for a marginal operator.

Contents

1 Introduction

Janus configurations are solutions of supergravity theories which are dual to interface CFTs. The original solution [1] was obtained by considering a deformation of $\text{AdS}_5 \times S^5$ in type IIB supergravity where the dilaton has a nontrivial profile with respect to the slicing coordinate of an AdS_4 slicing of AdS_5. Subsequently, many more Janus solutions have been found in many different settings. One may distinguish two kinds of solutions: first, there are top-down constructions of Janus solutions in ten-dimensional type IIB or eleven-dimensional M-theory which preserve half of the supersymmetry. Such solutions are generically constructed by considering a warped product of AdS and sphere factors over a two-dimensional Riemann surface with boundary (see e.g. [2–5]). Second, there are solutions of gauged supergravities in lower dimensions with various amounts of broken and unbroken supersymmetries (see e.g. [6–14]). Solutions of the second kind are useful since holographic calculations of quantities such as the entanglement entropy, sources and expectation values of operators, and correlation functions in the Janus background are easier to perform in the lower-dimensional supergravity. In many cases, such solutions can be constructed as consistent truncations, which can be lifted to solutions of ten- or eleven-dimensional supergravity.

In the present paper, we consider a particular example of the second approach. We construct Janus solutions in three-dimensional $\mathcal{N} = 8$ gauged supergravity. Such theories are naturally related to $\text{AdS}_3 \times S^3 \times M_4$ compactifications of type IIB, where M_4 is either

T_4 or $K3$. We consider one of the simplest nontrivial settings where we find solutions which preserve eight of the sixteen supersymmetries of the AdS_3 vacuum, where only two scalars in the coset have a nontrivial profile. One interesting feature of these solutions is that one scalar is dual to a marginal operator with dimension $\Delta = 2$ where the source terms have different values on the two sides of the interface. This behavior is the main feature of the original Janus solution [1, 15]. On the other hand, the second scalar is dual to a relevant operator with dimension $\Delta = 1$ with a vanishing source term and a position-dependent expectation value. This behavior is a feature of the Janus solution in M-theory [5].

The structure of the paper is as follows: in section 2 we review $\mathcal{N} = 8$ gauged supergravity in three dimensions, and in section 3 we construct the half-BPS Janus solutions and investigate some of their properties using the AdS/CFT dictionary, including the calculation of the holographic entanglement entropy. We discuss some generalizations and directions for future research in section 4. Some technical details are relegated to appendix A.

2 $d = 3$, $\mathcal{N} = 8$ gauged supergravity

In the following, we will use the notation and conventions of [16]. The scalar fields of $d = 3$, $\mathcal{N} = 8$ gauged supergravity are parameterized by a $G/H = \mathrm{SO}(8, n)/(\mathrm{SO}(8) \times \mathrm{SO}(n))$ coset, which has $8n$ independent scalar degrees of freedom. This theory can be obtained by a truncation of six-dimensional $\mathcal{N} = (2, 0)$ supergravity on $AdS_3 \times S^3$ coupled to $n_T \geq 1$ tensor multiplets, where $n_T = n - 3$. The cases $n_T = 5$ and 21 correspond to compactifications of ten-dimensional type IIB on T^3 and $K3$, respectively. See [17] for a discussion of consistent truncations of six-dimensional $\mathcal{N} = (1, 1)$ and $\mathcal{N} = (2, 0)$ using exceptional field theory.

For future reference, we use the following index conventions:

- $I, J, \ldots = 1, 2, \ldots, 8$ for $\mathrm{SO}(8)$.

- $r, s, \ldots = 9, 10, \ldots, n + 8$ for $\mathrm{SO}(n)$.

- $\bar{I}, \bar{J}, \ldots = 1, 2, \ldots, n + 8$ for $\mathrm{SO}(8, n)$.

- $\mathcal{M}, \mathcal{N}, \ldots$ for generators of $\mathrm{SO}(8, n)$.

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the noncompact generators. Explicitly, the generators of the vector representation are given by

$$(t^{\bar{I}\bar{J}})^{\bar{K}}{}_{\bar{L}} = \eta^{\bar{I}\bar{K}} \delta^{\bar{J}}_{\bar{L}} - \eta^{\bar{J}\bar{K}} \delta^{\bar{I}}_{\bar{L}} \tag{2.1}$$

where $\eta^{\bar{I}\bar{J}} = \mathrm{diag}(+ + + + + + + + - \cdots)$ is the $\mathrm{SO}(8, n)$-invariant tensor. These generators satisfy the following commutation relations,

$$[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}] = 2\left(\eta^{\bar{I}[\bar{K}} t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}} t^{\bar{L}]\bar{I}}\right) \tag{2.2}$$

The scalars fields can be parametrized by a G-valued matrix $L(x)$ in the vector representation, which transforms under H and the gauge group $G_0 \subseteq G$ by

$$L(x) \longrightarrow g_0(x) L(x) h^{-1}(x) \tag{2.3}$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations. We can pick a $SO(8) \times SO(n)$ gauge to put the coset representative into symmetric gauge,

$$L = \exp\left(\phi_{Ir} Y^{Ir}\right) \tag{2.4}$$

for scalar fields ϕ_{Ir}. The $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ tensors are defined by

$$L^{-1} t^{\mathcal{M}} L = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} t^{\mathcal{A}} = \frac{1}{2} \mathcal{V}^{\mathcal{M}}{}_{IJ} X^{IJ} + \frac{1}{2} \mathcal{V}^{\mathcal{M}}{}_{rs} X^{rs} + \mathcal{V}^{\mathcal{M}}{}_{Ir} Y^{Ir} \tag{2.5}$$

The gauging of the supergravity is accomplished by introducing Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$ and choosing an embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ (which has to satisfy various identities [18]) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry transformations and action depending on the gauge couplings. In the following, we will make one of the simplest choices and gauge a $G_0 = SO(4)$ subset of $SO(8)$. Explicitly, we further divide the I, J indices into

- $i, j, \ldots = 1, 2, 3, 4$ for $G_0 = SO(4)$.

- $\bar{\imath}, \bar{\jmath}, \ldots = 5, 6, 7, 8$ for the remaining ungauged $SO(4) \subset SO(8)$.

The embedding tensor we will employ in the following has the non-zero entries

$$\Theta_{IJ,KL} = \varepsilon_{ijk\ell} \tag{2.6}$$

As this is totally antisymmetric, the trace is $\theta = 0$. As discussed in [16], this choice of embedding tensor produces a supersymmetric AdS_3 ground state with

$$SU(2|1,1)_L \times SU(2|1,1)_R \tag{2.7}$$

super-algebra of isometries. From the embedding tensor, the G_0-covariant currents can be obtained,

$$L^{-1}(\partial_\mu + g\Theta_{\mathcal{M}\mathcal{N}} B_\mu^{\mathcal{M}} t^{\mathcal{N}}) L = \frac{1}{2} \mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2} \mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir} \tag{2.8}$$

It is convenient to define the T-tensor,

$$T_{\mathcal{A}|\mathcal{B}} = \Theta_{\mathcal{M}\mathcal{N}} \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \tag{2.9}$$

as well as the tensors $A_{1,2,3}$ which will appear in the potential and the supersymmetry transformations.

$$A_1^{AB} = -\frac{1}{48} \Gamma_{AB}^{IJKL} T_{IJ|KL}$$

$$A_2^{A\dot{A}r} = -\frac{1}{12} \Gamma_{A\dot{A}}^{IJK} T_{IJ|Kr}$$

$$A_3^{\dot{A}r\dot{B}s} = \frac{1}{48} \delta^{rs} \Gamma_{\dot{A}\dot{B}}^{IJKL} T_{IJ|KL} + \frac{1}{2} \Gamma_{\dot{A}\dot{B}}^{IJ} T_{IJ|rs} \tag{2.10}$$

A, B and $\dot{A}, \dot{B}$ are $SO(8)$-spinor indices and our conventions for the $SO(8)$ Gamma matrices are presented in appendix A.1.

We take the spacetime signature $\eta^{ab} = \text{diag}(+--)$ to be mostly negative. The bosonic Lagrangian is

$$e^{-1}\mathcal{L} = -\frac{1}{4}R + \frac{1}{4}\mathcal{P}_\mu^{Ir}\mathcal{P}^{\mu\,Ir} + W - \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho}g\Theta_{\mathcal{MN}}B_\mu^{\mathcal{M}}\left(\partial_\nu B_\rho^{\mathcal{N}} + \frac{1}{3}g\Theta_{\mathcal{KL}}f^{\mathcal{NK}}{}_{\mathcal{P}}B_\nu^{\mathcal{L}}B_\rho^{\mathcal{P}}\right)$$

$$W = \frac{1}{4}g^2\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\right) \tag{2.11}$$

The SUSY transformations are

$$\delta\chi^{\dot{A}r} = \frac{1}{2}i\Gamma_{A\dot{A}}^I\gamma^\mu\varepsilon^A\mathcal{P}_\mu^{Ir} + gA_2^{A\dot{A}r}\varepsilon^A$$

$$\delta\psi_\mu^A = \left(\partial_\mu\varepsilon^A + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\varepsilon^A + \frac{1}{4}\mathcal{Q}_\mu^{IJ}\Gamma_{AB}^{IJ}\varepsilon^B\right) + igA_1^{AB}\gamma_\mu\varepsilon^B \tag{2.12}$$

2.1 The $n = 1$ case

In this section we will consider the $n = 1$ theory, i.e. the scalar fields lie in a $\text{SO}(8,1)/\text{SO}(8)$ coset. The reason for this is that the resulting expressions for the supersymmetry transformations and BPS conditions are compact and everything can be worked out in detail. Furthermore, we believe that this case illustrates the important features of more general solutions.

As the index $r = 9$ takes only one value in this case, the scalar fields in the coset representative (2.4) are denoted by $\phi_i \equiv \phi_{i9}$ for $i = 1, 2, \ldots, 8$. We define the following quantities for notational convenience,

$$\Phi^2 \equiv \phi_I\phi_I = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 + \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2$$

$$\phi^2 \equiv \phi_i\phi_i = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$$

$$\bar{\phi}^2 \equiv \phi_{\bar{\imath}}\phi_{\bar{\imath}} = \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2 \tag{2.13}$$

The components of the $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ tensor are, with no summation over repeated indices and I, J, K, L being unique indices,

$$\mathcal{V}^{IJ}{}_{IJ} = 1 + (\phi_I^2 + \phi_J^2)\frac{\cosh\Phi - 1}{\Phi^2} \qquad\qquad \mathcal{V}^{IJ}{}_{IK} = \phi_J\phi_K\frac{\cosh\Phi - 1}{\Phi^2}$$

$$\mathcal{V}^{IJ}{}_{KL} = 0 \qquad\qquad \mathcal{V}^{I9}{}_{I9} = \cosh\Phi - \phi_I^2\frac{\cosh\Phi - 1}{\Phi^2}$$

$$\mathcal{V}^{I9}{}_{J9} = -\phi_I\phi_J\frac{\cosh\Phi - 1}{\Phi^2} \qquad\qquad \mathcal{V}^{IJ}{}_{I9} = \mathcal{V}^{I9}{}_{IJ} = \phi_J\frac{\sinh\Phi}{\Phi}$$

$$\mathcal{V}^{IJ}{}_{K9} = \mathcal{V}^{K9}{}_{IJ} = 0 \tag{2.14}$$

The u-components of the $\mathcal{Q}_\mu^{IJ}$ and $\mathcal{P}_\mu^I$ tensors are

$$\mathcal{Q}_u^{IJ} = (\phi_I'\phi_J - \phi_I\phi_J')\frac{\cosh\Phi - 1}{\Phi^2} + g\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}^{\mathcal{N}}{}_{IJ}$$

$$\mathcal{P}_u^I = \phi_I'\frac{\sinh\Phi}{\Phi} - \phi_I\Phi'\frac{\sinh\Phi - \Phi}{\Phi^2} + g\Theta_{\mathcal{MN}}B_u^{\mathcal{M}}\mathcal{V}^{\mathcal{N}}{}_{I9} \tag{2.15}$$

where the prime $' \equiv \partial/\partial u$ denotes the derivative with respect to u. The terms involving the gauge field have different forms depending on whether I, J are in i or $\bar{\imath}$.

$$\Theta_{\mathcal{MN}} B_u^{\mathcal{M}} \mathcal{V}_{ij}^{\mathcal{N}} = \varepsilon_{ijk\ell} \left[\frac{1}{2} B_u^{k\ell} \left(1 + (\phi_i^2 + \phi_j^2) \frac{\cosh \Phi - 1}{\Phi^2} \right) + \left(\phi_i B_u^{ik} \phi_\ell + \phi_j B_u^{jk} \phi_\ell \right) \frac{\cosh \Phi - 1}{\Phi^2} \right]$$

$$\Theta_{\mathcal{MN}} B_u^{\mathcal{M}} \mathcal{V}_{i\bar{\imath}}^{\mathcal{N}} = \frac{1}{2} \varepsilon_{ijk\ell} \phi_{\bar{\imath}} \phi_j B_u^{k\ell} \frac{\cosh \Phi - 1}{\Phi^2}$$

$$\Theta_{\mathcal{MN}} B_u^{\mathcal{M}} \mathcal{V}_{\bar{\imath}\bar{\jmath}}^{\mathcal{N}} = 0$$

$$\Theta_{\mathcal{MN}} B_u^{\mathcal{M}} \mathcal{V}_{i9}^{\mathcal{N}} = \frac{1}{2} \varepsilon_{ijk\ell} \phi_j B_u^{k\ell} \frac{\sinh \Phi}{\Phi}$$

$$\Theta_{\mathcal{MN}} B_u^{\mathcal{M}} \mathcal{V}_{\bar{\imath}9}^{\mathcal{N}} = 0 \tag{2.16}$$

The T-tensor has non-zero components

$$T_{ij|k\ell} = \varepsilon_{ijk\ell} \left(\phi^2 \frac{\cosh \Phi - 1}{\Phi^2} + 1 \right)$$

$$T_{ij|k\bar{\imath}} = \varepsilon_{ijk\ell} \phi_\ell \phi_{\bar{\imath}} \frac{\cosh \Phi - 1}{\Phi^2}$$

$$T_{ij|k9} = \varepsilon_{ijk\ell} \phi_\ell \frac{\sinh \Phi}{\Phi} \tag{2.17}$$

Taking $\varepsilon_{1234} = 1$, we can use the T-tensor to compute

$$A_1^{AB} = -\frac{1}{2} \Gamma_{AC}^{1234} \left[\left(\phi^2 \frac{\cosh \Phi - 1}{\Phi^2} + 1 \right) \delta_{CB} + (\Gamma_{C\dot{A}}^i \phi_i)(\Gamma_{\dot{A}B}^{\bar{\imath}} \phi_{\bar{\imath}}) \frac{\cosh \Phi - 1}{\Phi^2} \right]$$

$$A_2^{A\dot{A}} = -\frac{1}{2} \Gamma_{AB}^{1234} (\Gamma_{B\dot{A}}^i \phi_i) \frac{\sinh \Phi}{\Phi}$$

$$A_3^{\dot{A}\dot{B}} = -A_1^{AB} \delta_{A\dot{A}} \delta_{B\dot{B}} \tag{2.18}$$

Note that $A_1^{AB} = A_1^{BA}$ and

$$A_1^{AC} A_1^{BC} = \frac{1}{4} \delta_{AB} \left(\frac{\phi^2 \sinh^2 \Phi}{\Phi^2} + 1 \right)$$

$$A_2^{A\dot{A}} A_2^{B\dot{A}} = \frac{1}{4} \delta_{AB} \frac{\phi^2 \sinh^2 \Phi}{\Phi^2} \tag{2.19}$$

so the scalar potential (2.11) becomes

$$W = \frac{g^2}{4} \left(\frac{\phi^2 \sinh^2 \Phi}{\Phi^2} + 2 \right) \tag{2.20}$$

3   Half-BPS Janus solutions

In this section, we construct Janus solutions which preserve eight of the sixteen supersymmetries of the AdS$_3$ vacuum. Our strategy is to use an AdS$_2$ slicing of AdS$_3$ and make the scalar fields as well as the metric functions only dependent on the slicing coordinate. One complication is given by the presence of the gauge fields; due to the Chern-Simons action, the only consistent Janus solution will have vanishing field strength. We show that the gauge fields can be consistently set to zero for our solutions.

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3.1 Janus ansatz

We take the Janus ansatz for the metric, scalar fields and Chern-Simons gauge fields,

$$ds^2 = e^{2B(u)}\left(\frac{dt^2 - dz^2}{z^2}\right) - e^{2D(u)}\,du^2$$

$$\phi_I = \phi_I(u)$$
$$B^{\mathcal{M}} = B^{\mathcal{M}}(u)\,du \tag{3.1}$$

The AdS$_3$ vacuum solution given by $\phi_I \equiv 0$ and $e^B = e^D = L \sec u$ has a curvature radius related to the coupling constant by $L^{-1} = g$. The spin connection 1-forms are

$$\omega^{01} = \frac{dt}{z} \qquad \omega^{02} = -\frac{B'e^{B-D}}{z}\,dt \qquad \omega^{12} = -\frac{B'e^{B-D}}{z}\,dz \tag{3.2}$$

so the gravitino supersymmetry variation $\delta\psi_\mu^A = 0$ is

$$0 = \partial_t \varepsilon + \frac{1}{2z}\gamma_0\Big(\gamma_1 - B'e^{B-D}\gamma_2 + 2ige^B A_1\Big)\varepsilon$$
$$0 = \partial_z \varepsilon + \frac{1}{2z}\gamma_1\Big(-B'e^{B-D}\gamma_2 + 2ige^B A_1\Big)\varepsilon$$
$$0 = \partial_u \varepsilon + \frac{1}{4}\mathcal{Q}_u^{IJ}\Gamma^{IJ}\varepsilon + ige^D\gamma_2 A_1\varepsilon \tag{3.3}$$

where we have suppressed the SO(8)-spinor indices. As shown in appendix A.2, the integrability conditions are

$$0 = \Big(1 - (2ge^B A_1)^2 + (B'e^{B-D})^2\Big)\varepsilon$$
$$0 = 2ige^B\left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}_u^{IJ}\Gamma^{IJ}]\right)\varepsilon + \left(-\frac{d}{du}\big(B'e^{B-D}\big) + (2ge^B A_1)^2 e^{D-B}\right)\gamma_2\varepsilon \tag{3.4}$$

The first integrability condition gives a first-order equation which must be true for all ε, using the replacement for A_1^2 in (2.19),

$$0 = 1 - g^2 e^{2B}\left(\frac{\phi^2 \sinh^2\Phi}{\Phi^2} + 1\right) + (B'e^{B-D})^2 \tag{3.5}$$

The derivative of this simplifies the second integrability condition to

$$0 = \left(A_1' - \frac{1}{4}[A_1, \mathcal{Q}_u^{IJ}\Gamma^{IJ}]\right)\varepsilon + \frac{ige^D}{4B'}\frac{d}{du}\left(\frac{\phi^2 \sinh^2\Phi}{\Phi^2}\right)\gamma_2\varepsilon \tag{3.6}$$

The BPS equation $\delta\chi^{\dot{A}} = 0$ is

$$\left(-\frac{i}{2}e^{-D}\Gamma^I \mathcal{P}_u^I \gamma_2 + gA_2\right)_{A\dot{A}}\varepsilon^A = 0 \tag{3.7}$$

When $gA_2^2 \neq 0$, this equation can be rearranged into the form of a projector

$$0 = (iM_{AB}\gamma_2 + \delta_{AB})\varepsilon^A \tag{3.8}$$

where M_{AB} is given by

$$M_{AB} = \frac{e^{-D}}{g} \frac{\Phi}{\phi^2 \sinh \Phi} (\Gamma^I_{A\dot{A}} \mathcal{P}^I_u)(\Gamma^i_{\dot{A}C} \phi_i) \Gamma^{1234}_{CB} \tag{3.9}$$

For consistency of the projector, we must have

$$M_{AB} M_{BC} = \delta_{AC} \tag{3.10}$$

As $M^2 = 1$, every generalized eigenvector of rank ≥ 2 is automatically an eigenvector, so M is diagonalizable and has eight eigenvectors with eigenvalues ± 1. M is traceless as it is a sum of products of 2 or 4 Gamma matrices, so it has an equal number of $+1$ and -1 eigenvectors. The operator $iM_{AB}\gamma_2$ in the projector (3.8) squares to one and is traceless, and projects onto an eight-dimensional space of unbroken supersymmetry generators. If this is the only projection imposed on the solution, it will be half-BPS and hence preserve eight of the sixteen supersymmetries of the vacuum.

The condition $M^2 = 1$ gives an equation first-order in derivatives of scalars.

$$M^2 = \left(\frac{e^{-D}\Phi}{g\phi^2 \sinh \Phi}\right)^2 \Big(\phi^2(-\mathcal{P}^i_u \mathcal{P}^i_u + \mathcal{P}^{\bar{\imath}}_u \mathcal{P}^{\bar{\imath}}_u) - 2\phi^2(\Gamma^{\bar{\imath}} \mathcal{P}^{\bar{\imath}}_u)(\Gamma^i \mathcal{P}^i_u)$$
$$+ 2(\mathcal{P}^j_u \phi_j)(\Gamma^{\bar{\imath}} \mathcal{P}^{\bar{\imath}}_u + \Gamma^i \mathcal{P}^i_u)(\Gamma^k \phi_k)\Big) \tag{3.11}$$

For this to be proportional to the identity, we need all $\Gamma^{\bar{\imath}}\Gamma^i$ and $\Gamma^i\Gamma^j$ terms to vanish. Vanishing of the latter requires us to impose the condition

$$\mathcal{P}^i_u \phi_j = \mathcal{P}^j_u \phi_i \tag{3.12}$$

As the ratio $\mathcal{P}^i_u/\phi_i$ is the same for all i, this implies

$$\sum_i \mathcal{P}^i_u \phi_i = \sum_i \frac{\mathcal{P}^i_u}{\phi_i}\phi_i^2 = \frac{\mathcal{P}^1_u}{\phi_1}\phi^2 \qquad \Longrightarrow \qquad -\phi^2 \mathcal{P}^i_u + \phi_i \sum_j \mathcal{P}^j_u \phi_j = 0 \tag{3.13}$$

This means that imposing eq. (3.12) also ensures that the $\Gamma^{\bar{\imath}}\Gamma^i$ terms vanish. Note that

$$\sum_i \mathcal{P}^i_u \mathcal{P}^i_u = \sum_i \frac{\mathcal{P}^i_u}{\phi_i}\frac{\mathcal{P}^i_u}{\phi_i}\phi_i^2 = \left(\frac{\mathcal{P}^1_u}{\phi_1}\right)^2 \phi^2 \tag{3.14}$$

so the $M^2 = 1$ condition becomes

$$M^2 = \left(\frac{e^{-D}\Phi}{g\phi^2 \sinh \Phi}\right)^2 \phi^2(\mathcal{P}^i_u \mathcal{P}^i_u + \mathcal{P}^{\bar{\imath}}_u \mathcal{P}^{\bar{\imath}}_u) = 1 \tag{3.15}$$

We now give the argument why the Chern-Simons gauge fields can be set to zero. Since we demand that the $B^{\mathcal{M}}_\mu$ only has a component along the u direction and only depends on u, the field strength vanishes, consistent with the equation of motion coming from the variation of the Chern-Simons term in the action (2.11) with respect to the gauge field.

However, there is another term which contains the gauge field, namely the kinetic term of the scalars via (2.15). For the gauge field to be consistently set to zero, we have to impose

$$\frac{\delta \mathcal{L}}{\delta B_u^{k\ell}}\bigg|_{B_u^{\mathcal{M}}=0} = 0 \tag{3.16}$$

For the Janus ansatz, we find

$$\frac{\delta \mathcal{L}}{\delta B_u^{k\ell}}\bigg|_{B_u^{\mathcal{M}}=0} = eg\varepsilon_{ijk\ell}\mathcal{P}^{i\,u}\phi_j \frac{\sinh\Phi}{\Phi} \tag{3.17}$$

which indeed vanishes due to eq. (3.12) imposed by the half-BPS condition.

For a half-BPS solution, the second integrability condition (3.6) should be identical to the projector (3.8). Indeed, we have the simplification

$$A_1' - \frac{1}{4}[A_1, \mathcal{Q}_u^{IJ}\Gamma^{IJ}] = -\frac{1}{2}\frac{\phi^2 \sinh^2\Phi}{\Phi^2}M^\top \tag{3.18}$$

so the Gamma matrix structures of the two equations match. Equating the remaining scalar magnitude gives us an equation for the metric factor e^B,

$$-B' = \frac{\mathrm{d}}{\mathrm{d}u}\ln\frac{\phi\sinh\Phi}{\Phi} \tag{3.19}$$

We can now solve for the metric. Let us define

$$\alpha(u) \equiv \frac{\phi\sinh\Phi}{\Phi} \tag{3.20}$$

and set the integration constant for B to be

$$e^B = \frac{|C|}{g\alpha} \tag{3.21}$$

Plugging this into the first integrability condition (3.5) and picking the gauge $e^{-D} \equiv g$, we have a first-order equation for α,

$$0 = \alpha^2 - C^2(\alpha^2 + 1 - \alpha'^2/\alpha^2) \tag{3.22}$$

The solution depends on the value of $C \in [0,1]$ and up to translations in u is

$$\begin{aligned} \alpha &= e^{\pm u} & \text{if } C = 1 \\ \alpha &= \frac{|C|}{\sqrt{1-C^2}}\operatorname{sech} u & \text{if } 0 \le C < 1 \end{aligned} \tag{3.23}$$

We will take the case $0 \le C < 1$. This implies that the metric is

$$\mathrm{d}s^2 = g^{-2}\left[(1-C^2)\cosh^2 u\left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2\right] \tag{3.24}$$

The choice $C = 0$ corresponds to the AdS$_3$ vacuum.

3.2 ϕ_4, ϕ_5 truncation

We have yet to fully solve the half-BPS conditions (3.12) and (3.15). For simplicity, let us consider the case where only ϕ_4, ϕ_5 are non-zero and the other scalars are identically zero, which trivially satisfies eq. (3.12). It turns out that the important features of the Janus solution are captured by this truncation.

We introduce the following abbreviations

$$\Phi^2 = \phi_4^2 + \phi_5^2 \qquad \phi = |\phi_4| \qquad \bar{\phi} = |\phi_5| \tag{3.25}$$

Let us define

$$\beta(u) \equiv \frac{\phi_5 \sinh \Phi}{\Phi} \tag{3.26}$$

so that

$$\alpha^2 + \beta^2 = \sinh^2 \Phi$$
$$\mathcal{P}_u^4 = \alpha' + \alpha \Phi' \frac{1 - \cosh \Phi}{\sinh \Phi}$$
$$\mathcal{P}_u^5 = \beta' + \beta \Phi' \frac{1 - \cosh \Phi}{\sinh \Phi} \tag{3.27}$$

Plugging these into eq. (3.15) simplifies to

$$\alpha'^2 + \beta'^2 - \frac{(\alpha'\alpha + \beta'\beta)^2}{1 + \alpha^2 + \beta^2} = \alpha^2 \tag{3.28}$$

This can be rearranged into a first-order equation in $f \equiv \beta/\sqrt{1 + \alpha^2}$,

$$f' = \frac{\alpha^2/C}{1 + \alpha^2} \sqrt{1 + f^2} \tag{3.29}$$

where a sign ambiguity from taking a square-root has been absorbed into C, which is now extended to $C \in (-1, 1)$. Using the explicit solution (3.23) for α, by noting that

$$\frac{\mathrm{d}}{\mathrm{d}u} \tanh^{-1}(C \tanh u) = \frac{C \operatorname{sech}^2 u}{1 - C^2 \tanh^2 u} = \frac{\alpha^2/C}{1 + \alpha^2} \tag{3.30}$$

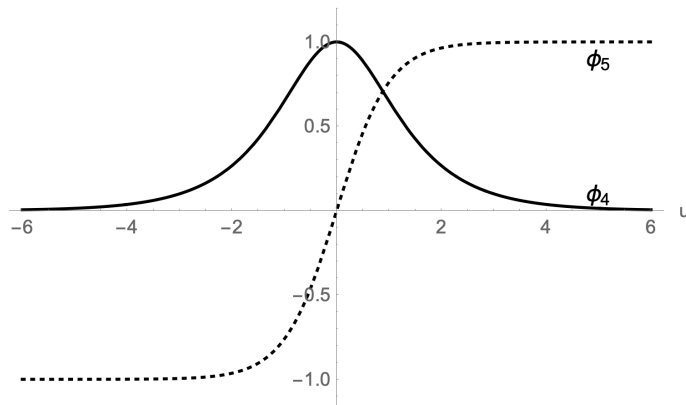the general solution is

$$f(u) = \frac{\sinh p + C \cosh p \tanh u}{\sqrt{1 - C^2 \tanh^2 u}}$$
$$\beta(u) = \frac{1}{\sqrt{1 - C^2}}(\sinh p + C \cosh p \tanh u) \tag{3.31}$$

for some constant $p \in \mathbb{R}$. For later convenience, we also redefine $C = \tanh q$ for $q \in \mathbb{R}$.

In summary, we have solved for the scalars ϕ_4, ϕ_5 implicitly through the functions α, β,

$$\frac{|\phi_4| \sinh \Phi}{\Phi} = |\sinh q| \operatorname{sech} u$$
$$\frac{\phi_5 \sinh \Phi}{\Phi} = \sinh p \cosh q + \cosh p \sinh q \tanh u \tag{3.32}$$

Figure 1. Plot of ϕ_4 and ϕ_5 for $(p, q) = (0, 1)$.

for real constants p, q. Note that the reflection $\phi_4 \to -\phi_4$ also gives a valid solution. We have explicitly checked that the Einstein equation and scalar equations of motion are satisfied.

The ϕ_4 scalar goes to zero at $u = \pm\infty$ as it is a massive scalar degree of freedom, and has a sech-like profile near the defect. The ϕ_5 scalar interpolates between two boundary values at $u = \pm\infty$, and has a tanh-like profile. The constant p is related to the boundary values of the ϕ_5 scalar, as we can note that

$$\phi_5(\pm\infty) = p \pm q \tag{3.33}$$

The constant q is then related to the jump value of the ϕ_5 scalar. The defect location $u = 0$ can also be freely translated to any point along the axis. Below is a plot of the solution for the choice $(p, q) = (0, 1)$.

3.3 Holography

In our AdS-sliced coordinates, the boundary is given by the two AdS_2 components at $u = \pm\infty$, which are joined together at the $z = 0$ interface. Using $C = \tanh q$, the metric (3.24) becomes

$$\mathrm{d}s^2 = g^{-2}\left[\mathrm{sech}^2 q \cosh^2 u \left(\frac{\mathrm{d}t^2 - \mathrm{d}z^2}{z^2}\right) - \mathrm{d}u^2\right] \tag{3.34}$$

Note that this is not AdS_3 unless $q = 0$, which corresponds to the vacuum solution with all scalars vanishing. The spacetime is, however, asymptotically AdS_3. In the limit of $u \to \pm\infty$, the $\mathrm{sech}^2 q$ can be eliminated from the leading $e^{\pm 2u}$ term in the metric (3.34) by a coordinate shift. We will present the asymptotic mapping to a Fefferman-Graham (FG) coordinate system below. In the following, we will set the AdS length scale to unity for notational simplicity, i.e. $g \equiv 1$.

According to the AdS/CFT correspondence, the mass m^2 of a supergravity scalar field in $d = 3$ is related to the scaling dimension Δ of the dual CFT operator by

$$m^2 = \Delta(\Delta - 2) \tag{3.35}$$

This relation comes from the linearized equations of motion for the scalar field near the asymptotic AdS$_3$ boundary. Expanding the supergravity action (2.11) to quadratic order around the AdS$_3$ vacuum shows that the ϕ_4 field has mass $m^2 = -1$, so the dual operator is relevant with $\Delta = 1$ and saturates the Breitenlohner-Freedman (BF) bound [19]. Note that we choose the standard quantization [20], which is the correct one for a supersymmetric solution. The ϕ_5 field is massless, so the dual CFT operator is marginal with scaling dimension $\Delta = 2$.

In FG coordinates,[1] the general expansion for a scalar field near the asymptotic AdS$_3$ boundary at $\rho = 0$ is

$$\begin{aligned}
\phi_{\Delta=1} &\sim \psi_0 \, \rho \ln \rho + \phi_0 \, \rho + \cdots \\
\phi_{\Delta\neq1} &\sim \tilde{\phi}_0 \, \rho^{2-\Delta} + \tilde{\phi}_2 \, \rho^\Delta + \cdots
\end{aligned} \tag{3.36}$$

Since $\phi_{\Delta=1}$ saturates the BF bound, holographic renormalization and the holographic dictionary are subtle due to the presence of the logarithm [21]. As we show below for the solution (3.32), there is no logarithmic term present and ϕ_0 can be identified with the expectation value of the dual operator [21, 22]. For the massless field $\phi_{\Delta=2}$, we can identify $\tilde{\phi}_0$ with the source and $\tilde{\phi}_2$ with the expectation value of the dual operator.

It is difficult to find a global map which puts the metric (3.34) in FG form. Here, we limit our discussion to the coordinate region away from the defect, where we take $u \to \pm\infty$ and keep z finite [23, 24]. This limit probes the region away from the interface on the boundary. The coordinate change suitable for the $u \to \infty$ limit can be expressed as a power series,

$$\begin{aligned}
z &= x + \frac{\rho^2}{2x} + \mathcal{O}(\rho^4) \\
e^u &= \cosh q \left(\frac{2x}{\rho} + \frac{\rho}{2x} + \mathcal{O}(\rho^3) \right)
\end{aligned} \tag{3.37}$$

The metric becomes

$$\mathrm{d}s^2 = \frac{1}{\rho^2} \left[-\mathrm{d}\rho^2 + \left(1 - \frac{\rho^2 \tanh^2 q}{2x^2} \right)(\mathrm{d}t^2 - \mathrm{d}x^2) + \mathcal{O}(\rho^3) \right] \tag{3.38}$$

In the $u \to -\infty$ limit, the asymptotic form of the metric is the same and the coordinate change is (3.37) with the replacements $e^u \to e^{-u}$ and $x \to -x$.

Using this coordinate change, the expansions of the scalar fields near the boundary is

$$|\phi_4| = |\tanh q| \frac{p + \tilde{q}}{\sinh(p + \tilde{q})} \cdot \frac{\rho}{|x|} + \mathcal{O}(\rho^3)$$

$$\phi_5 = (p + \tilde{q}) - \frac{1}{2\sinh(p + \tilde{q})} \left(\frac{p + \tilde{q}}{\sinh(p + \tilde{q})} \tanh^2 q + \frac{\sinh p \tanh \tilde{q}}{\cosh q} \right) \cdot \frac{\rho^2}{x^2} + \mathcal{O}(\rho^4) \tag{3.39}$$
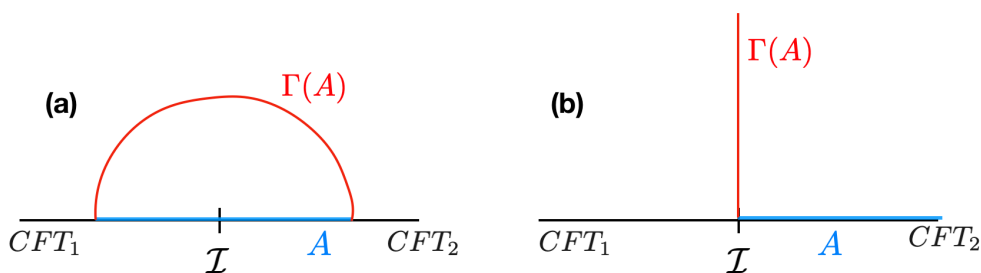
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[1]The AdS$_3$ metric in Poincaré coordinates is

$$\mathrm{d}s^2 = \frac{-\mathrm{d}\rho^2 + \mathrm{d}t^2 - \mathrm{d}x^2}{\rho^2}$$

and is related to the AdS-sliced metric by the coordinate change

$$z = \sqrt{x^2 + \rho^2} \qquad \sinh u = x/\rho.$$

Figure 2. (a) The entagling surface A is symmetric around the interface $\mathcal{I}$, (b) The entagleing surface A is ends at the interface $\mathcal{I}$.

where $\tilde{q} \equiv qx/|x|$ (see appendix A.3 for details). The defect is located on the boundary at $x = 0$. We can see that the relevant operator corresponding to ϕ_4 has no term proportional to $\rho \ln \rho$ in the expansion. This implies that the source is zero and the dual operator has a position-dependent expectation value. The marginal operator corresponding to ϕ_5 has a source term which takes different values on the two sides of the defect, corresponding to a Janus interface where the modulus associated with the marginal operator jumps across the interface.

Another quantity which can be calculated holographically is the entanglement entropy for an interval A using the Ryu-Takanayagi prescription [25],

$$S_{\text{EE}} = \frac{\text{Length}(\Gamma_A)}{4G_N^{(3)}} \tag{3.40}$$

where Γ_A is the minimal curve in the bulk which ends on ∂A.

There are two qualitatively different choices for location of the interval in an interface CFT, as shown in figure 2. First, the interval can be chosen symmetrically around the defect [26, 27]. The minimal surface for such a symmetric interval is particularly simple in the AdS-sliced coordinates (3.34), and is given by $z = z_0$ and $u \in (-\infty, \infty)$. The regularized length is given by

$$\text{Length}(\Gamma_A) = \int \mathrm{d}u = u_\infty - u_{-\infty} \tag{3.41}$$

We can use (3.37) to relate the FG cutoff $\rho = \varepsilon$, which furnishes the UV cutoff on the CFT side, to the cutoff $u_{\pm\infty}$ in the AdS-sliced metric,

$$u_{\pm\infty} = \pm(-\log \varepsilon + \log(2z_0) + \log(\cosh q)) \tag{3.42}$$

Putting this together and using the expression for the central charge in terms of $G_N^{(3)}$ gives

$$S_{\text{EE}} = \frac{c}{3} \log \frac{2z_0}{\varepsilon} + \frac{c}{3} \log(\cosh q) \tag{3.43}$$

Note that the first logarithmically divergent term is the standard expression for the entanglement entropy for a CFT without an interface present [28], since $2z_0$ is the length of

the interval. The constant term is universal in the presence of an interface and can be interpreted as the defect entropy (sometimes called g-factor [29]) associated with the interface.

Second, we can consider an interval which lies on one side of the interface and borders the interface [30, 31]. As shown in [32], the entangling surface is located at $u = 0$ and the entanglement entropy for an interval of length l bordering the interface is given by

$$S'_{\text{EE}} = \frac{c}{6} \operatorname{sech} q \log \frac{l}{\varepsilon} \tag{3.44}$$

3.4 All scalars

For completeness, we also present the general solution with all ϕ_I scalars turned on. Let us define

$$\alpha_i(u) \equiv \frac{\phi_i \sinh \Phi}{\Phi} \qquad\qquad i = 1, 2, 3, 4$$

$$\beta_{\bar{\imath}}(u) \equiv \frac{\phi_{\bar{\imath}} \sinh \Phi}{\Phi} \qquad\qquad \bar{\imath} = 5, 6, 7, 8 \tag{3.45}$$

As a consequence of eq. (3.12), the ratio ϕ'_i/ϕ_i is the same for all i so all the ϕ_i scalars are proportional to each other. In other words, we have $\alpha_i = n_i \alpha$ for constants n_i satisfying $n_i n_i = 1$, where α is given in eq. (3.23). Then eq. (3.15) becomes

$$\alpha'^2 + \beta'_{\bar{\imath}} \beta'_{\bar{\imath}} - \frac{(\alpha' \alpha + \beta'_{\bar{\imath}} \beta_{\bar{\imath}})^2}{1 + \alpha^2 + \beta_{\bar{\imath}} \beta_{\bar{\imath}}} = \alpha^2 \tag{3.46}$$

We can note that there exists a family of solutions where all $\beta_{\bar{\imath}}$ functions satisfy

$$\beta_{\bar{\imath}} = n_{\bar{\imath}} \beta \tag{3.47}$$

for some function β and constants $n_{\bar{\imath}}$ satisfying $n_{\bar{\imath}} n_{\bar{\imath}} = 1$. When this is the case, eq. (3.46) then further simplifies to

$$\alpha'^2 + \beta'^2 - \frac{(\alpha' \alpha + \beta' \beta)^2}{1 + \alpha^2 + \beta^2} = \alpha^2 \tag{3.48}$$

which has already been solved in the previous section. We can prove that these are the only solutions to eq. (3.46) which satisfy the equations of motion. The scalar dependence of the Lagrangian is

$$e^{-1}\mathcal{L} \supset -\frac{g^2}{4} \mathcal{P}_u^I \mathcal{P}_u^I + W$$

$$= -\frac{g^2}{4} \left(\alpha'^2 + \beta'_{\bar{\imath}} \beta'_{\bar{\imath}} - \frac{(\alpha' \alpha + \beta'_{\bar{\imath}} \beta_{\bar{\imath}})^2}{1 + \alpha^2 + \beta_{\bar{\imath}} \beta_{\bar{\imath}}} - (\alpha^2 + 2) \right) \tag{3.49}$$

If we write the $\beta_{\bar{\imath}}$ in spherical coordinates, where we call the radius β, this becomes

$$= -\frac{g^2}{4} \left(\alpha'^2 + \beta'^2 + \beta^2 K^2 - \frac{(\alpha' \alpha + \beta' \beta)^2}{1 + \alpha^2 + \beta^2} - (\alpha^2 + 2) \right) \tag{3.50}$$

where K^2 is the kinetic energy of the angular coordinates.[2] We can treat α, β, and the three angles as the coordinates of this Lagrangian. The equation of motion from varying

---

[2] Explicitly, let $K^2 = \theta'^2 + \sin^2 \theta \, \phi'^2 + \sin^2 \theta \sin^2 \phi \, \psi'^2$.

the Lagrangian with respect to α will only involve α and β and their derivatives. Plugging-in (3.23) for α, satisfying this equation of motion fixes the form of β to be what was found previously in eq. (3.31). This means that eq. (3.46) simplifies to $\beta^2 K^2 = 0$ and the three angles must be constant.

Therefore, the general solution is

$$\frac{\phi \sinh \Phi}{\Phi} = |\sinh q| \operatorname{sech} u$$

$$\beta = \sinh p \cosh q + \cosh p \sinh q \tanh u$$

$$\phi_i = n_i \phi, \qquad n_i n_i = 1$$

$$\frac{\phi_{\bar{\imath}} \sinh \Phi}{\Phi} = n_{\bar{\imath}} \beta, \qquad n_{\bar{\imath}} n_{\bar{\imath}} = 1 \tag{3.51}$$

4 Discussion

In this paper, we have presented Janus solutions for $d = 3$, $\mathcal{N} = 8$ gauged supergravity. We constructed the simplest solutions with the smallest number of scalars, namely the $\mathrm{SO}(n,1)/\mathrm{SO}(8)$ coset. The solutions we found have only two scalars displaying a nontrivial profile. One scalar is dual to a marginal operator O_2 with scaling dimension $\Delta = 2$ and the other scalar is dual to a relevant operator O_1 with scaling dimension $\Delta = 1$. We used the holographic correspondence to find the dual CFT interpretation of these solutions. It is given by a superconformal interface, with a constant source of the operator O_2 which jumps across the interface. For the operator O_1, the source vanishes but there is an expectation value which depends on the distance from the interface. It would be interesting to study whether half-BPS Janus interfaces which display these characteristics can be constructed in the two-dimensional $\mathcal{N} = (4,4)$ SCFTs.

We considered solutions for the $\mathrm{SO}(n,1)/\mathrm{SO}(8)$ coset, but these solutions can be trivially embedded into the $\mathrm{SO}(8,n)/(\mathrm{SO}(8) \times \mathrm{SO}(n))$ cosets with $n > 1$. Constructing solutions with more scalars with nontrivial profiles is in principle possible, but the explicit expressions for the quantities involved in the BPS equations are becoming very complicated. We also believe that the $n = 1$ case already illustrates the important features of the more general $n > 1$ cosets. Another possible generalization is given by considering more general gaugings. One important example is given by replacing the embedding tensor (2.6) with

$$\Theta_{IJ,KL} = \varepsilon_{ijk\ell} + \alpha \varepsilon_{\bar{\imath}\bar{\jmath}\bar{k}\bar{\ell}} \tag{4.1}$$

This is a deformation produces an AdS$_3$ vacuum which is dual to a SCFT with a large $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ superconformal algebra. As discussed in [16], this gauging is believed to be a truncation type II supergravity compactified on AdS$_3 \times S^3 \times S^3 \times S^1$ [33, 34]. It should be straightforward to adapt the methods for finding solutions developed in the present paper to this case.

We calculated the holographic defect entropy for our solution. It would be interesting to investigate whether this quantity can be related to the Calabi diastasis function following [35, 36]. For this identification to work we would have to consider the case $n = 2$ for which the scalar coset is a Kähler manifold.

We leave these interesting questions for future work.

Acknowledgments

A Technical details

In this appendix, we present various technical details which are used in the main part of the paper.

A.1 SO(8) Gamma matrices

We are working with 8×8 Gamma matrices $\Gamma^I_{A\dot{A}}$ and their transposes $\Gamma^I_{\dot{A}A}$, which satisfy the Clifford algebra,

$$\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} + \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B} = 2\delta^{IJ}\delta_{AB} \tag{A.1}$$

Explicitly, we use the basis in [37],

$$\begin{aligned}
\Gamma^8_{A\dot{A}} &= 1 \otimes 1 \otimes 1 & \Gamma^1_{A\dot{A}} &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2 \\
\Gamma^2_{A\dot{A}} &= 1 \otimes \sigma_1 \otimes i\sigma_2 & \Gamma^3_{A\dot{A}} &= 1 \otimes \sigma_3 \otimes i\sigma_2 \\
\Gamma^4_{A\dot{A}} &= \sigma_1 \otimes i\sigma_2 \otimes 1 & \Gamma^5_{A\dot{A}} &= \sigma_3 \otimes i\sigma_2 \otimes 1 \\
\Gamma^6_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_1 & \Gamma^7_{A\dot{A}} &= i\sigma_2 \otimes 1 \otimes \sigma_3
\end{aligned} \tag{A.2}$$

The matrices Γ^{IJ}_{AB}, $\Gamma^{IJ}_{\dot{A}\dot{B}}$ and similar are defined as unit-weight antisymmetrized products of Gamma matrices with the appropriate indices contracted. For instance,

$$\Gamma^{IJ}_{AB} \equiv \frac{1}{2}(\Gamma^I_{A\dot{A}}\Gamma^J_{\dot{A}B} - \Gamma^J_{A\dot{A}}\Gamma^I_{\dot{A}B}) \tag{A.3}$$

A.2 Integrability conditions

For BPS equations of the form

$$\begin{aligned}
\partial_t \varepsilon &= -\frac{1}{2z}\gamma_0(\gamma_1 + f(u) + g(u)\gamma_2)\varepsilon \\
\partial_z \varepsilon &= -\frac{1}{2z}\gamma_1(f(u) + g(u)\gamma_2)\varepsilon \\
\partial_u \varepsilon &= (F(u) + G(u)\gamma_2)\varepsilon
\end{aligned}$$

where f, g, F, G are matrices acting on ε that commute with γ_a, the integrability conditions are

$$\begin{aligned}
t, z: \quad & 0 = (1 + f^2 + g^2)\varepsilon + [f, g]\gamma_2\varepsilon \\
t, u: \quad & 0 = (f' + [f, F] - \{g, G\})\varepsilon + (g' + [g, F] + \{f, G\})\gamma_2\varepsilon \\
z, u: \quad & \text{same as for } t, u
\end{aligned}$$

A.3 Scalar asymptotics

The asymptotic expansions of the ϕ_4 and ϕ_5 scalar fields, as given in (3.32), in the limits $u \to \pm\infty$ are

$$
|\phi_4| = 2|\sinh q| \frac{p \pm q}{\sinh(p \pm q)} e^{\mp u}
$$
$$
- \frac{2|\sinh q|}{\sinh^2(p \pm q)} \left(\frac{p \pm q}{\sinh(p \pm q)} (\sinh^2 p + \sinh^2 q) \pm 2\sinh p \sinh q \right) e^{\mp 3u} + \mathcal{O}(e^{\mp 5u})
$$
$$
\phi_5 = (p \pm q) - \frac{2}{\sinh(p \pm q)} \left(\frac{p \pm q}{\sinh(p \pm q)} \sinh^2 q \pm \sinh p \sinh q \right) e^{\mp 2u} + \mathcal{O}(e^{\mp 4u}) \quad (A.4)
$$

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