



Investigation of three-dimensional vibration measurement by three scanning laser Doppler vibrometers in a continuously and synchronously scanning mode

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ABSTRACT

This paper presents for the first time an investigation of three-dimensional (3D) vibration measurement by three scanning laser Doppler vibrometers (SLDV) in a continuously and synchronously scanning mode. Three laser spots from the three SLDVs continuously move along the same scan trajectory and synchronously measure vibration of the same point on a surface of a structure. The three SLDVs, i.e., Top, Left, and Right SLDVs, are from a Polytec PSV-500-3D system and input signals to scan mirrors of each SLDV are controlled by an external dSPACE MicroLabBox control unit. The whole system is called a 3D continuously SLDV (3D-CSLDV) system. Methodologies are developed in this investigation to achieve the continuously and synchronously scanning mode by the system. First, the geometrical model of a CSLDV is built to obtain locations of the three CSLDVs with respect to a specified measurement coordinate system (MCS). Second, a scan trajectory on the measured surface is defined and rotation angles of scan mirrors of the Top CSLDV are obtained by scanning the trajectory. Third, since locations of the three CSLDVs with respect to the MCS have been obtained in the first step, rotation angles of scan mirrors of Left and Right CSLDVs to scan the defined trajectory can be obtained from those of the Top CSLDV based on spatial relations among the three CSLDVs. Experiments to scan a beam and a plate were conducted using the 3D-CSLDV system. Operating deflection shapes (ODSs) of the beam and plate in three directions of the MCS were obtained and they were in good agreement with those obtained by traditional step scanning. The results demonstrate the feasibility of 3D vibration measurement by the 3D-CSLDV system in a continuously and synchronously scanning mode.

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1. Introduction

A scanning laser Doppler vibrometer (SLDV) is widely used in the vibration community because it provides non-contact and spatially dense vibration measurement. In a conventional measurement method, a surface of a structure is divided by a measurement grid, and the laser spot from a SLDV stays at one point long enough to acquire sufficient vibration data of this point and then moves to the next one. This step scanning measurement mode usually takes a long acquisition time, especially when the surface is large and the measurement grid is dense. An improved method is that the laser spot contin-

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uously moves over the surface of a structure [1]. Since the laser spot continuously moves instead of staying at one point, an operating deflection shape (ODS) in the laser line-of-sight direction can be obtained from velocity response measured by a continuously SLDV (CSLDV). Several methods to extract ODSs and modal characteristics from velocity response have been developed. Stanbridge and Ewins [2] developed two CSLDV measurement methods to obtain ODSs of a structure under sinusoidal excitation. One measurement method is the demodulation method, where velocity response is multiplied by a sinusoidal signal at the excitation frequency and a low-pass filter is applied to obtain an ODS. The other one is the polynomial method, where an ODS is represented by a polynomial whose coefficients are obtained by processing the discrete Fourier transform of velocity response. Allen and Sracic [3] proposed a lifting method to treat velocity response of a structure measured by a CSLDV as free response of a linear time-periodic system and decomposed it into a set of frequency response functions, from which natural frequencies, mode shapes, and modal damping ratios of the structure can be obtained using conventional curve fitting methods. Methods mentioned above can be applied with different scan trajectories, such as line scans, circular scans, and area scans. Due to their non-contact, rapid, and dense measurement features, CSLDVs have gained attention from researchers and many applications, such as angular and radial vibration measurement, structural damage detection, and rotating structure vibration measurement, have been investigated. Halkon et al. [4,5] proposed a velocity sensitivity model to analyze vibration components in CSLDV measurement. The model shows existence of additional components at integer multiples of the scan frequency when using a CSLDV to measure a rotating target with a circular scan. They found that the additional components are related to dual mirror arrangement and misalignment between the scanning system and target rotation axes. Halkon and Rothberg [6] further applied the velocity sensitivity model to rotor applications. They analyzed the effect of laser speckles on measured data and described details on how to interpret additional components in the measurement. Rothberg et al. [7] used a SLDV to measure radial vibration and investigated effects of surface roughness and misalignment on the measurement. Yang and Allen [8] developed an output-only modal analysis method from CSLDV measurement and applied it to identify mode shapes of a parked 20 kW wind turbine. Chen et al. [9] applied a CSLDV to damage detection of beams based on its dense measurement feature. Damage was successfully identified by using differences between curvatures of ODSs associated with ODSs obtained by the demodulation method and polynomial method. Xu et al. [10] used free response shapes obtained from free response of a damaged beam measured by a CSLDV to identify damage. Chen et al. [11] extended the damage detection methodology by use of a CSLDV to plates with development of a general two-dimensional scan pattern.

A single SLDV in a step scanning mode is usually used to measure out-of-plane vibration of a surface. There is an angle between the laser line-of-sight direction and out-of-plane direction while scanning. This angle can be neglected when the surface is small. However, for a large surface and a short stand-off distance of the SLDV, this assumption can amplify measurement error and out-of-plane vibration may not completely agree with vibration measured by the SLDV [12]. Also, a single SLDV cannot simultaneously measure in-plane and out-of-plane vibrations. In order to overcome the drawback of a single SLDV, a 3D-SLDV system has been developed and commercialized (e.g., a Polytec PSV-400-3D or PSV-500-3D system), where three laser spots from three SLDVs are synchronously directed to a point to measure its vibration and then move to the next point. Vibration information of a structure in a specified measurement coordinate system (MCS) can be obtained by a 3D-SLDV system. Vuye et al. [13] measured the dynamic strain field of a fan blade by using a Polytec PSV-400-3D system and indicated that a normal strain far below one microstrain could be measured. Weisbecker et al. [14] used 3D displacement data from a Polytec PSV-400-3D system to estimate the dynamic in-plane strain of a planar structure. Ihrle et al. [15] used three Polytec OFV-534 vibrometers to measure velocities and displacements of several points on two ossicles connected by a biological joint and reconstruct relative motion of two ossicles. Staszewski et al. [16] used a Polytec PSV-400-3D system to measure lamb wave response of metallic structures and identified fatigue cracks from in-plane and out-of-plane vibrations. Since the cost of a commercial 3D-SLDV system is quite high, using a single SLDV to measure 3D vibration has also been investigated. Weekes and Ewins [17] used a single CSLDV combined with a Microsoft Kinect to obtain 3D ODSs of a blade under multi-frequency excitation. The Microsoft Kinect is used for range-finding to measure distances from the single CSLDV to the blade. Chen and Zhu [18] measured 3D vibration of a point on a beam under sinusoidal excitation by sequentially placing a single SLDV at three different positions. The main challenge of using a single CSLDV for 3D ODS measurement is to ensure that the scan trajectory is the same when the single CSLDV is sequentially placed at three different positions. Also, methods by using a single SLDV or CSLDV for 3D vibration measurement cannot deal with transient vibration measurement.

The aim of this work is to investigate the feasibility of 3D continuous and synchronous scanning with three CSLDVs. A configured 3D-CSLDV system, which includes Top, Left, and Right CSLDVs, inherits fast and dense measurement features of 1D continuous scanning, while provides more vibration information in three directions than an ODS obtained by a single CSLDV. This work addresses the main challenge of a 3D-CSLDV system, which is to ensure that three laser spots from the three CSLDVs continuously move along the same scan trajectory and synchronously measure vibration of the same points on a surface.

The remaining part of the paper is outlined as follows. The geometrical model of a CSLDV is presented in Section 2.1, and continuous and synchronous scanning relations among three CSLDVs are presented in Section 2.2. Experimental investigation of the proposed methodology to obtain 3D ODSs of a beam and a plate is presented in Section 3. Conclusion of the work is presented in Section 4.

2. Methodology

2.1. Geometrical model of a CSLDV

The laser spot from a CSLDV can be directed to any visible point on a surface of a structure by rotating two orthogonal scan mirrors called X and Y mirrors, which control horizontal and vertical movements of the laser spot, respectively. Rotations of scan mirrors are driven by corresponding stepper motors. To obtain 3D ODSs of a structure, a MCS $o-xyz$ needs to be first defined. Note that the 3D ODSs in this work mean the ODSs in x , y , and z directions of the MCS. A Polytec PSV-A-450 reference object that has some points with known coordinates is used in this work to define the MCS, as shown in Fig. 1(a). In order to obtain the location of a CSLDV with respect to the MCS, the geometrical model of a CSLDV is built, as shown in Fig. 1(b). Besides the MCS, another coordinate system $o'-x'y'z'$, which is fixed on the CSLDV and called a vibrometer coordinate system (VCS), is defined to describe the position of the CSLDV. The VCS $o'-x'y'z'$ is specified as follows: x' and y'' axes are along rotation axes of X and Y mirrors, respectively. Positive directions of x' and y'' axes are directed to sides of corresponding stepper motors. The y' axis is parallel to the y'' axis and has an intersection point o' with the x' axis. The z' axis is perpendicular to the plane formed by x' and y' axes; it goes through the point o' and has an intersection point o'' with the y'' axis. The length of $o'o''$, denoted by d , is defined as the separation distance between the two mirrors, which is equal to 29.2 mm for three CSLDVs used in this work. Variables α and β are rotation angles of X and Y mirrors measured from their initial positions, respectively.

Coordinates of a point P on the reference object in the MCS are $\mathbf{P}_{\text{MCS}} = [x, y, z]^T$. To direct the laser spot from the CSLDV to the point P , rotation angles of X and Y mirrors are α and β , respectively. Based on the geometrical model of the CSLDV in Fig. 1(b), coordinates of the point P in the VCS of the CSLDV can be expressed as

$$\mathbf{P}_{\text{VCS}} = [-d \tan(\beta) - r \sin(\beta), -r \cos(\alpha) \cos(\beta), -r \sin(\alpha) \cos(\beta)]^T \quad (1)$$

where r is the distance from the point P' , which is the incident point of the X mirror, to the point P . The coordinate relation between $\mathbf{P}_{\text{MCS}}$ and $\mathbf{P}_{\text{VCS}}$ can be established by

$$\mathbf{P}_{\text{MCS}} = \mathbf{T} + \mathbf{R}\mathbf{P}_{\text{VCS}} = \begin{bmatrix} x_{o'} \\ y_{o'} \\ z_{o'} \end{bmatrix} + \begin{bmatrix} \cos(x, x') & \cos(x, y') & \cos(x, z') \\ \cos(y, x') & \cos(y, y') & \cos(y, z') \\ \cos(z, x') & \cos(z, y') & \cos(z, z') \end{bmatrix} \mathbf{P}_{\text{VCS}} \quad (2)$$

where $\mathbf{T} = [x_{o'}, y_{o'}, z_{o'}]^T$ is a translation vector that specifies coordinates of the origin o' in the MCS, $\mathbf{R}$ is the direction cosine matrix from the VCS to the MCS. The translation vector $\mathbf{T}$ together with the direction cosine matrix $\mathbf{R}$ can determine the location of the CSLDV with respect to the MCS. In order to obtain $\mathbf{T}$ and $\mathbf{R}$, coordinates of at least four points in the MCS

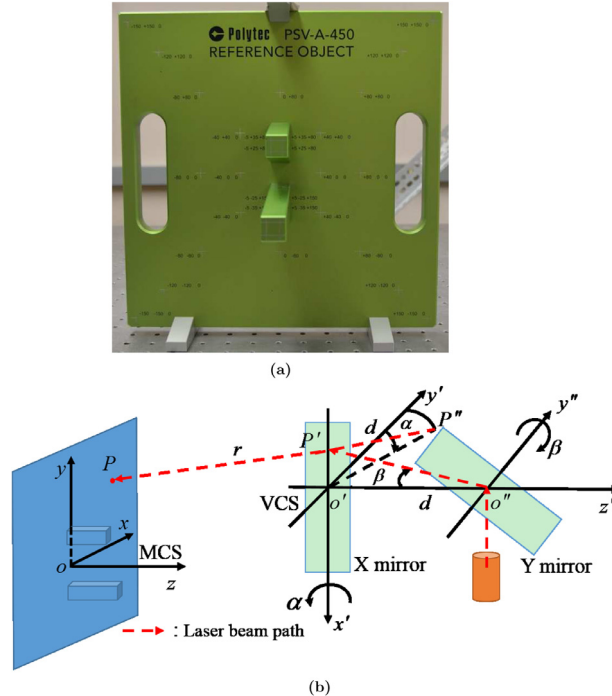


Fig. 1. (a) PSV-A-450 reference object and (b) the geometrical model of the CSLDV.

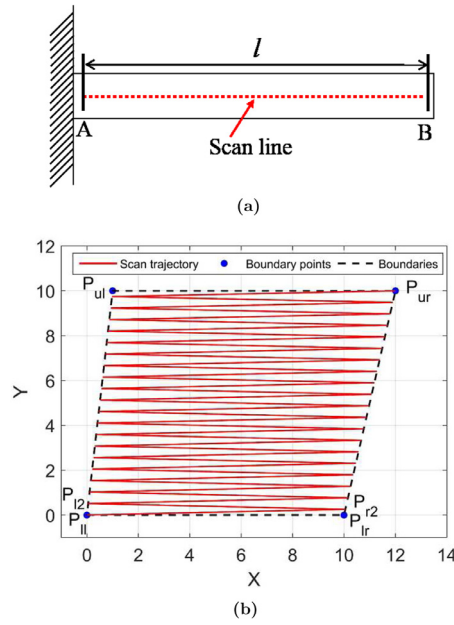
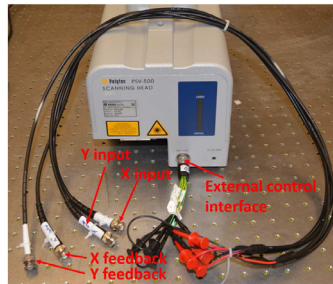


Fig. 2. (a) Line scan for a beam-like structure and (b) zigzag scan for a plate-like structure.

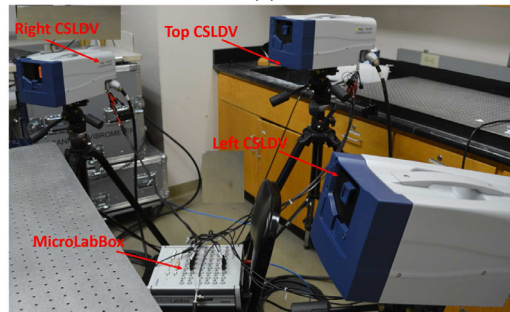
and VCS must be known. The reference object in Fig. 1(a) can provide coordinates of points in the MCS and corresponding rotation angles of scan mirrors can be determined by directing the laser spot to each point. Since the distance between two points P^m and P^n remains the same when their coordinates are expressed in the MCS and VCS, the variable r in Eq. (1) can be calculated by

$$|\mathbf{P}_{MCS}^m - \mathbf{P}_{MCS}^n| = |\mathbf{P}_{VCS}^m - \mathbf{P}_{VCS}^n| \quad (3)$$

where m is from 1 to $H - 1$ with H being the number of points on the reference object and n is from $m + 1$ to H . One can see that with H points one has H unknown distances and can form $H \times (H - 1)$ equations like Eq. (3). This is an over-



(a)



(b)

Fig. 3. (a) External control interface in the PSV-500-3D system and (b) the 3D-CSLDV system developed in this work.

determined nonlinear problem when $H \geq 4$, which can be solved by a nonlinear least squares method. The initial value of r can be obtained by measuring the distance from the X mirror to the point P .

After coordinates of H points in the MCS and VCS are obtained, $\mathbf{T}$ and $\mathbf{R}$ can be calculated by solving an optimization problem [18,19]:

$$F(\mathbf{T}, \mathbf{R}) = \delta = \min \sum_{m=1}^H |\mathbf{P}_{\text{MCS}}^m - (\mathbf{T} + \mathbf{R}\mathbf{P}_{\text{VCS}}^m)| \quad (4)$$

Let $\bar{\mathbf{P}}_{\text{MCS}} = (\sum_{m=1}^H \mathbf{P}_{\text{MCS}}^m)/H$, $\bar{\mathbf{P}}_{\text{VCS}} = (\sum_{m=1}^H \mathbf{P}_{\text{VCS}}^m)/H$, $\mathbf{q}_{\text{MCS}}^m = \mathbf{P}_{\text{MCS}}^m - \bar{\mathbf{P}}_{\text{MCS}}$, $\mathbf{q}_{\text{VCS}}^m = \mathbf{P}_{\text{VCS}}^m - \bar{\mathbf{P}}_{\text{VCS}}$, $\mathbf{Q}_{\text{MCS}} = [\mathbf{q}_{\text{MCS}}^1, \mathbf{q}_{\text{MCS}}^2, \dots, \mathbf{q}_{\text{MCS}}^H]$, $\mathbf{Q}_{\text{VCS}} = [\mathbf{q}_{\text{VCS}}^1, \mathbf{q}_{\text{VCS}}^2, \dots, \mathbf{q}_{\text{VCS}}^H]$, and $\mathbf{A} = \mathbf{Q}_{\text{VCS}}\mathbf{Q}_{\text{MCS}}^T$; by singular value decomposition, $\mathbf{A}$ can be decomposed as

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T \quad (5)$$

where $\mathbf{U}$ is a 3×3 orthogonal matrix, $\mathbf{D}$ is a 3×3 diagonal matrix with non-negative real values in the descending order on the diagonal, and $\mathbf{V}$ is a 3×3 orthogonal matrix. Hence, $\mathbf{R}$ and $\mathbf{T}$ can be calculated by

$$\mathbf{R} = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(\mathbf{V}\mathbf{U}^T) \end{bmatrix} \mathbf{U}^T \quad (6)$$

and

$$\mathbf{T} = \bar{\mathbf{P}}_{\text{MCS}} - \mathbf{R}\bar{\mathbf{P}}_{\text{VCS}} \quad (7)$$

respectively, where $\det$ denotes the determinant of a square matrix. With the geometrical model of a CSLDV, locations of three CSLDV's in a 3D-CSLDV system with respect to the MCS can be obtained.

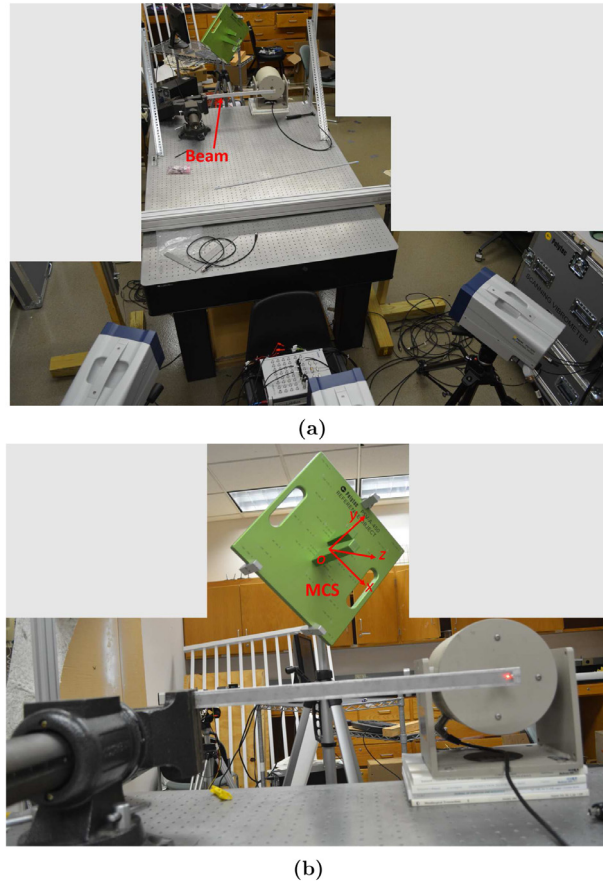


Fig. 4. (a) Experiment setup for the 3D vibration measurement of the beam and (b) the definition of the MCS in the beam experiment.

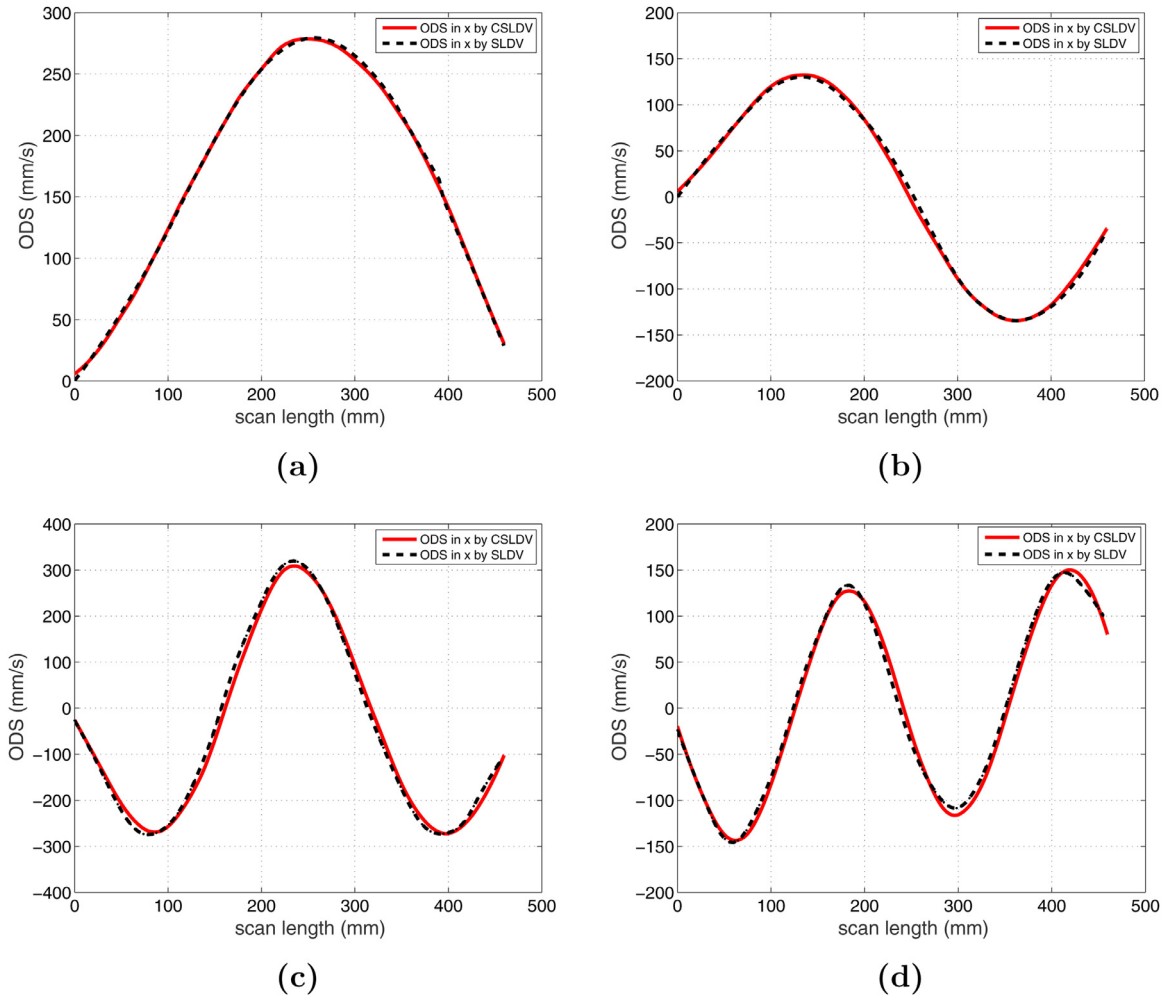


Fig. 5. 3D ODSs of the beam in the x direction of the MCS obtained by continuous scanning and step scanning at excitation frequencies of (a) 98 Hz, (b) 317 Hz, (c) 640 Hz, and (d) 1094 Hz.

2.2. Continuously and synchronously scanning relations among three CSLDV's

A scan trajectory is defined based on the geometry of a structure. A line scan and a zigzag scan can be applied to a beam-like structure and a plate-like structure, as shown in Fig. 2(a) and (b), respectively. Calculation of input signals to conduct the zigzag scan can be seen in Chen et al. [11]. The calculation method of a 3D-CSLDV system proposed in this work requires three CSLDV's to be placed at proper locations. The longitudinal axis of the scan head of the Top CSLDV needs to be perpendicular to the scan surface and the laser spot needs to point to the center of the scan area when input voltages to scan mirrors are zero. Left and Right CSLDV's can be placed at arbitrary locations as long as they are not too close to the Top CSLDV. Once locations of the Left and Right CSLDV's are set, one needs to adjust their scan head angles to make sure that the laser spot can cover the complete area to be scanned in the input voltage range. It is recommended that the angle between the longitudinal axis of the scan head of the Left or Right CSLDV and the normal direction of the scan surface is in the range between 30° and 60° . After three CSLDV's are placed at proper locations, input signals to scan mirrors of the Top CSLDV are adjusted to obtain a proper scan trajectory for the structure to be measured. Corresponding rotation angles of scan mirrors of the Top CSLDV to direct the laser spot to the point P^k on the prescribed scan trajectory are obtained thereafter. The third variable r_{Top}^k in Eq. (1), which is the distance from the point P' on the X mirror of the Top CSLDV to the point P^k , can be calculated by

$$r_{Top}^k = \frac{r_{Top}^0}{\cos(\alpha_{Top}^k) \cos(\beta_{Top}^k)} \quad (8)$$

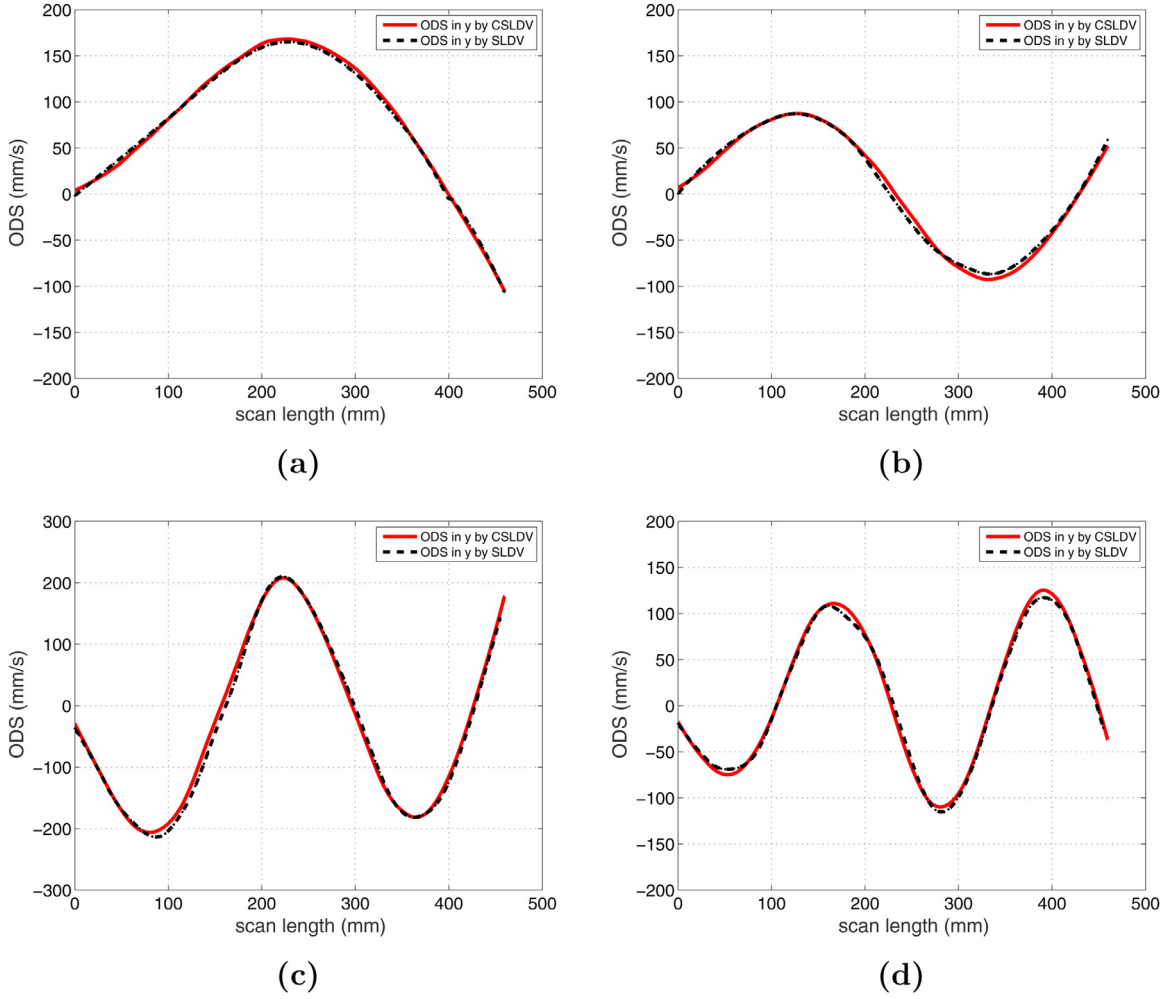


Fig. 6. 3D ODSs of the beam in the y direction of the MCS obtained by continuous scanning and step scanning at excitation frequencies of (a) 98 Hz, (b) 317 Hz, (c) 640 Hz, and (d) 1094 Hz.

where r_{Top}^0 is the distance from the point P' to the point on the prescribed scan trajectory with $\alpha_{\text{Top}} = 0$ and $\beta_{\text{Top}} = 0$ and can be obtained by the geometrical scan unit of the Top CSLDV, and α_{Top}^k and β_{Top}^k are corresponding rotation angles of X and Y mirrors of the Top CSLDV for the point P^k , respectively. Hence, coordinates of the point P^k in the VCS of the Top CSLDV can be obtained by Eq. (1). Note that Eq. (8) is derived based on the assumption that the longitudinal axis of the scan head of the Top CSLDV is perpendicular to the scan surface and the laser spot points to the center of the scan area when input voltages are zero. Since the location of the Top CSLDV with respect to the MCS is obtained by the method described in Section 2.1, coordinates of the point P^k in the MCS can be obtained by

$$\mathbf{P}_{\text{MCS}}^k = \mathbf{T}_{\text{Top}} + \mathbf{R}_{\text{Top}} \mathbf{P}_{\text{VCS}(\text{Top})}^k \quad (9)$$

where $\mathbf{T}_{\text{Top}}$ and $\mathbf{R}_{\text{Top}}$ are the translation vector and direction cosine matrix of the Top CSLDV with respect to the MCS, respectively. Since coordinates of the point P^k is constant in the MCS, in order to let laser spots from Left and Right CSLDVs direct to the same point P^k , synchronously scanning relations among the three CSLDVs for the point P^k are established by

$$\mathbf{T}_{\text{Top}} + \mathbf{R}_{\text{Top}} \mathbf{P}_{\text{VCS}(\text{Top})}^k = \mathbf{T}_{\text{Left}} + \mathbf{R}_{\text{Left}} \mathbf{P}_{\text{VCS}(\text{Left})}^k = \mathbf{T}_{\text{Right}} + \mathbf{R}_{\text{Right}} \mathbf{P}_{\text{VCS}(\text{Right})}^k \quad (10)$$

where $\mathbf{T}_{\text{Left}}$ and $\mathbf{R}_{\text{Left}}$ are the translation vector and direction cosine matrix of the Left CSLDV with respect to the MCS, respectively, and $\mathbf{T}_{\text{Right}}$ and $\mathbf{R}_{\text{Right}}$ are the translation vector and direction cosine matrix of the Right CSLDV with respect to the MCS, respectively. Hence, coordinates of the point P^k in the VCSs of Left and Right CSLDVs can be obtained by

$$\mathbf{P}_{\text{VCS}(\text{Left})}^k = \mathbf{R}_{\text{Left}}^{-1} (\mathbf{T}_{\text{Top}} - \mathbf{T}_{\text{Left}}) + \mathbf{R}_{\text{Left}}^{-1} \mathbf{R}_{\text{Top}} \mathbf{P}_{\text{VCS}(\text{Top})}^k \quad (11)$$

and

$$\mathbf{P}_{\text{VCS}(\text{Right})}^k = \mathbf{R}_{\text{Right}}^{-1} (\mathbf{T}_{\text{Top}} - \mathbf{T}_{\text{Right}}) + \mathbf{R}_{\text{Right}}^{-1} \mathbf{R}_{\text{Top}} \mathbf{P}_{\text{VCS}(\text{Top})}^k \quad (12)$$

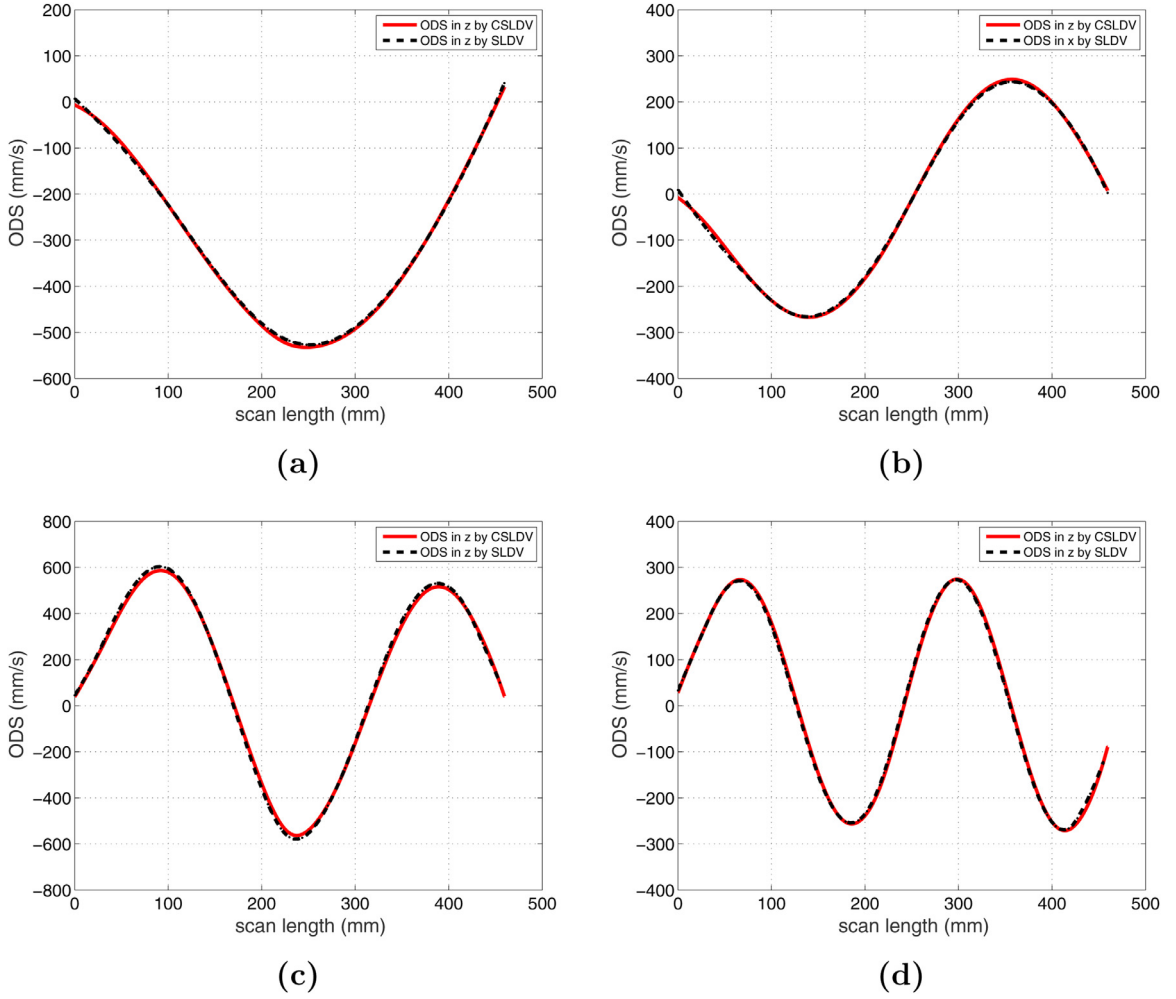


Fig. 7. 3D ODSs of the beam in the z direction of the MCS obtained by continuous scanning and step scanning at excitation frequencies of (a) 98 Hz, (b) 317 Hz, (c) 640 Hz, and (d) 1094 Hz.

respectively. Further, rotation angles of X and Y mirrors of Left and Right CSLDVs for the point P^k can be obtained by

$$\alpha_{\text{Left}}^k = \arctan(z_{\text{VCS(Left)}}^k / y_{\text{VCS(Left)}}^k), \beta_{\text{Left}}^k = \arctan(x_{\text{VCS(Left)}}^k / (y_{\text{VCS(Left)}}^k / \cos(\alpha_{\text{Left}}^k) - d)) \quad (13)$$

and

$$\alpha_{\text{Right}}^k = \arctan(z_{\text{VCS(Right)}}^k / y_{\text{VCS(Right)}}^k), \beta_{\text{Right}}^k = \arctan(x_{\text{VCS(Right)}}^k / (y_{\text{VCS(Right)}}^k / \cos(\alpha_{\text{Right}}^k) - d)) \quad (14)$$

respectively.

With the proposed methodology described above, three laser spots from three CSLDVs can continuously and synchronously scan the same point on the prescribed scan trajectory. After velocity response of the point P^k is measured by the three CSLDVs, vibration information of the point P^k in three laser line-of-sight directions can be obtained by the demodulation method. By assuming that the structure is under sinusoidal excitation at a frequency ω , velocity response v_{pk} can be expressed by

$$v_{pk} = V_{pk} \cos(\omega t) \quad (15)$$

where V_{pk} is vibration information at the point P^k at the excitation frequency ω . In order to obtain V_{pk} from velocity response, v_{pk} in Eq. (15) is multiplied by $\cos(\omega t)$, which gives

$$v_{pk} \cos \omega t = V_{pk} \cos \omega t \cos \omega t = 0.5V_{pk} + 0.5V_{pk} \cos 2\omega t \quad (16)$$

The term $0.5V_{pk} \cos 2\omega t$ in Eq. (16) can be eliminated by applying a low-pass filter to $v_{pk} \cos \omega t$ to yield $0.5V_{pk}$. Further, V_{pk} can be obtained by multiplying the filtered signal by two, which completes the demodulation method. The process described above is repeated for the three CSLDVs to obtain V_{Top}^k , V_{Left}^k , and V_{Right}^k .

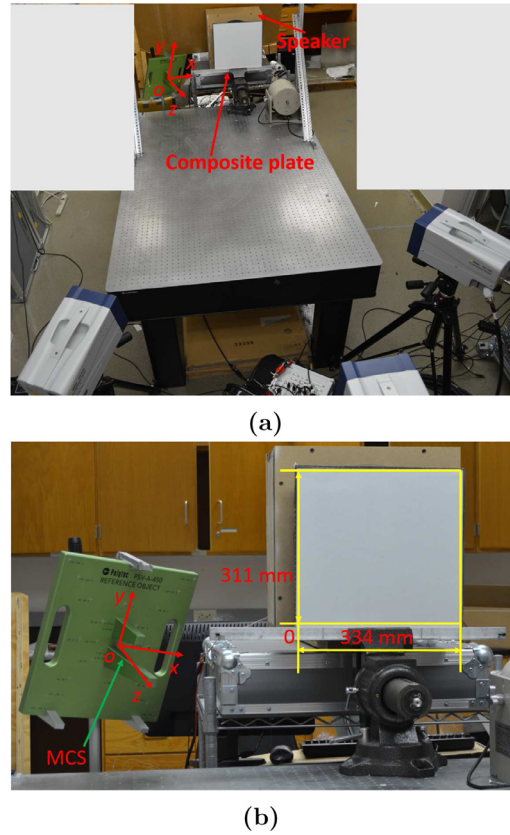


Fig. 8. (a) Experiment setup for the 3D vibration measurement of the composite plate and (b) the definition of the MCS in the plate experiment.

Vibration information of the point P^k in x , y , and z directions of the MCS can then be obtained by

$$[V_x^k, V_y^k, V_z^k]^T = \left[[\mathbf{R}_{\text{Top}} \mathbf{e}_{\text{Top}}^k, \mathbf{R}_{\text{Left}} \mathbf{e}_{\text{Left}}^k, \mathbf{R}_{\text{Right}} \mathbf{e}_{\text{Right}}^k]^T \right]^{-1} [V_{\text{Top}}^k, V_{\text{Left}}^k, V_{\text{Right}}^k]^T \quad (17)$$

where $\mathbf{e}_X^k = [\sin(\beta_X^k), -\cos(\alpha_X^k) \cos(\beta_X^k), -\sin(\alpha_X^k) \cos(\beta_X^k)]^T$, in which $X = \text{Top, Left, and Right}$, is a unit vector in the laser beam direction in the VCS of each CSLDV when the laser spot is directed to the point P^k . The process described above is repeated for each point on the prescribed scan trajectory. Finally, 3D ODSs of the structure in the MCS can be obtained by the 3D-CSLDV system.

3. Experimental investigation

3.1. Configuration of the 3D-CSLDV system

The Polytec PSV-500-3D system in the authors' lab includes three SLDVs that are named as Top SLDV, Left SLDV, and Right SLDV. The Top SLDV also has a vision camera that is used to monitor the position accuracy of the three laser spots during step scanning measurement. Continuous scanning is not directly provided in the Polytec software. However, each SLDV has an interface connector that provides the option to control scan mirrors by an external control unit. With this interface, continuous rotation of X and Y scan mirrors of each SLDV can be achieved by input signals from the external control unit using BNC cables, as shown in Fig. 3(a). A dSPACE MicroLabBox is used as the external control unit in this work to generate input signals to all the six scan mirrors in the configured 3D-CSLDV system, as shown in Fig. 3(b). The proposed methodology to develop the 3D-CSLDV system in this work is derived based on mathematical formulation and the vision camera in the commercial Polytec PSV-500-3D system is not used. Since input voltages to X and Y mirrors of each CSLDV directly determine corresponding rotation angles, each scan mirror needs to be first calibrated to obtain a more accurate relation between the input voltage and rotation angle. Nominal values from the Polytec manual for X and Y mirrors are $5.2^\circ/\text{V}$ and $4.2^\circ/\text{V}$, respectively. After calibration, the more accurate relation corresponding to each scan mirror can be obtained, as shown in Table 1. Steps to calculate the location of each CSLDV with respect to a MCS are similar to those for the commercial Polytec system, where the PSV-A-450 reference object is used. However, since the more accurate relation between the input voltage and rotation angle of each scan mirror has been obtained, the vision camera is not needed in the

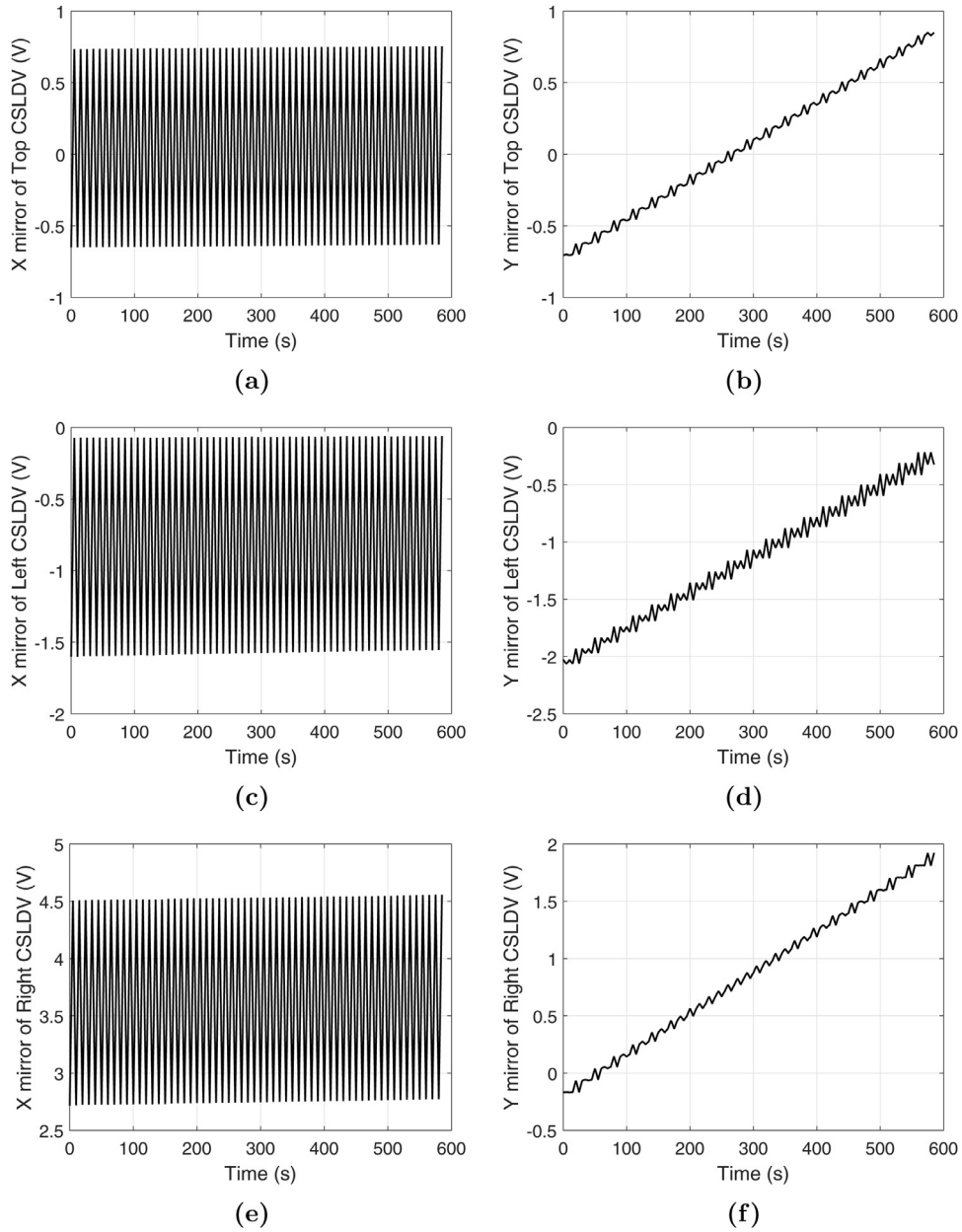


Fig. 9. Input signals to the (a) X mirror of the Top CSLDV, (b) Y mirror of the Top CSLDV, (c) X mirror of the Left CSLDV, (d) Y mirror of the Left CSLDV, (e) X mirror of the Right CSLDV, and (f) Y mirror of the Right CSLDV corresponding to the scan frequency of 0.1 Hz..

Table 1

Relation between the input voltage and rotation angle of each scan mirror for the configured 3D-CSLDV system.

	Mirror	k ($^{\circ}/V$)	Δ ($^{\circ}$)
Top CSLDV	X	5.215	0.032
	Y	4.204	0.052
Left CSLDV	X	5.197	0.075
	Y	4.193	0.058
Right CSLDV	X	5.151	0.086
	Y	4.195	0.025

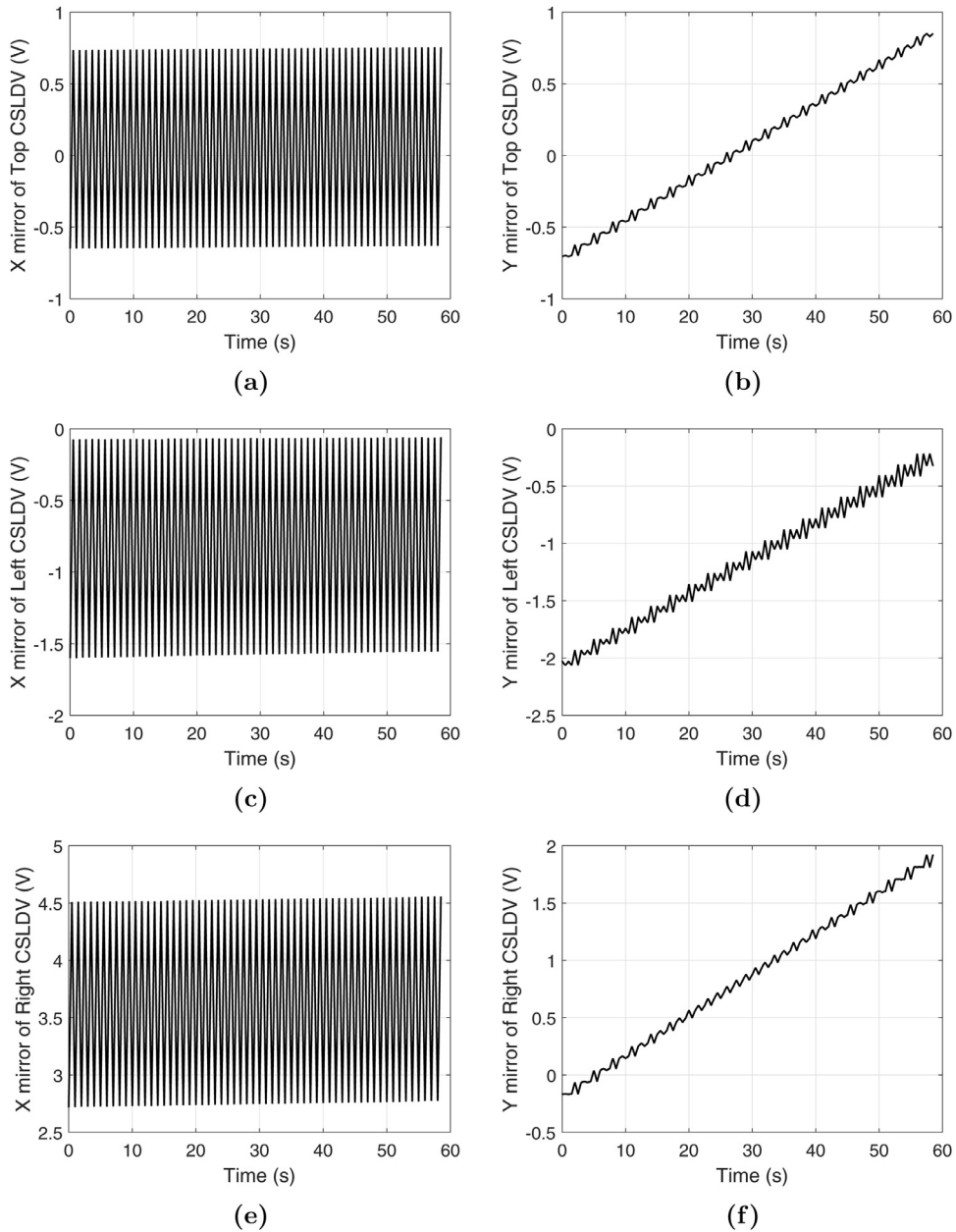


Fig. 10. Input signals to the (a) X mirror of the Top CSLDV, (b) Y mirror of the Top CSLDV, (c) X mirror of the Left CSLDV, (d) Y mirror of the Left CSLDV, (e) X mirror of the Right CSLDV, and (f) Y mirror of the Right CSLDV corresponding to the scan frequency of 1 Hz.

proposed methodology. Accuracy of geometrical calibration is ultimately evaluated by whether three laser spots from three CSLDVs can continuously and synchronously scan the same point along a prescribed scan trajectory. With the proposed methodology, it is recommended that accuracy is good enough for 3D-CSLDV measurement when only a larger laser spot is seen along the prescribed scan trajectory.

3.2. Beam experiment

The first experiment conducted by the 3D-CSLDV system was to measure 3D vibration of a beam. The experimental setup is shown in Fig. 4(a). A bench vice was used to clamp the left end of the beam to simulate a fixed boundary. A MB Dynamics MODAL-50 shaker was connected to the right end of the beam through a shaker screw to excite the beam. The PSV-A-450 reference object was arbitrarily mounted on a tripod and a MCS was defined thereafter, as shown in Fig. 4(b). The final 3D ODSs were calculated in this MCS. The three CSLDVs were placed at proper locations to measure vibration of the beam by

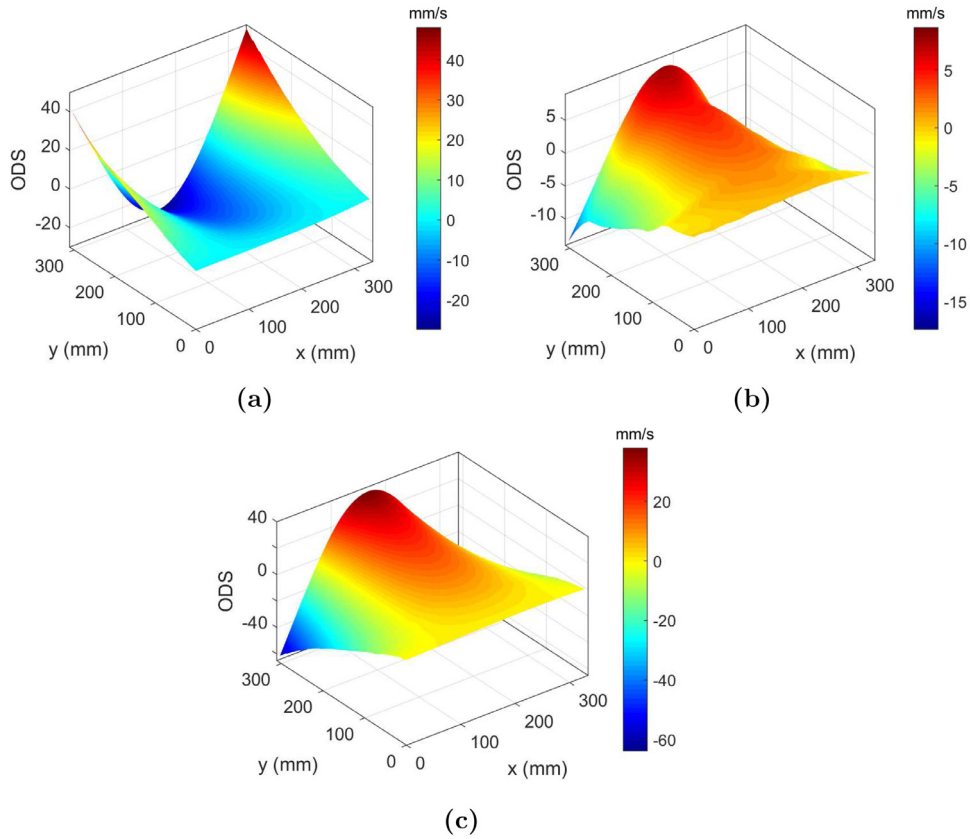


Fig. 11. 3D ODSs of the plate in (a) x, (b) y, and (c) z directions of the MCS obtained by the 3D-CSLDV system with the scan frequency of 0.1 Hz.

Table 2
Coordinates of six points in the MCS to obtain positions of three CSLDV.

Point index	x (mm)	y (mm)	z (mm)
1	80	80	0
2	80	80	0
3	120	120	0
4	80	80	0
5	-5	35	80
6	5	35	150

following the instruction in Section 2.2. Steps to conduct 3D continuous and synchronous scanning along the length of the beam by the 3D-CSLDV system are as follows. First, control scan mirrors of the Top CSLDV to direct the laser spot to six reference points on the PSV-A-450 reference object. Coordinates of the six reference points in the MCS are shown in Table 2. Corresponding rotation angles are recorded. A study in Chen and Zhu [18] shows that the direction cosine matrix has little change when the number of reference points changes from four to twelve. In this work, six reference points including four corner points and one reference point on either of two poles are selected to represent the basic shape of the PSV-A-450 reference object. By using the method described in Section 2.1, the translation vector and direction cosine matrix of the Top CSLDV with respect to the MCS are calculated. Repeat the same step for the Left CSLDV and Right CSLDV. Second, adjust input signals to scan mirrors of the Top CSLDV to achieve a line scan of the beam. A series of rotation angles corresponding to the line scan are obtained. By using the method described in Section 2.2, coordinates of each measurement point on the scan line in the MCS are obtained. Since the location of each CSLDV in the MCS has been obtained in the first step, rotation angles of scan mirrors corresponding to the Left and Right CSLDV are calculated to scan the same line as with the Top CSLDV. Third, feed input signals to three CSLDV and visually observe the scan trajectory. Since the Top CSLDV cannot be perfectly perpendicular to a scan surface of a structure in practice, in some cases, three laser spots may not completely overlap at a boundary of a scan trajectory. In that situation, parts of input voltages to the Left and Right CSLDV that correspond to boundaries of the scan trajectory may need to be slightly adjusted.

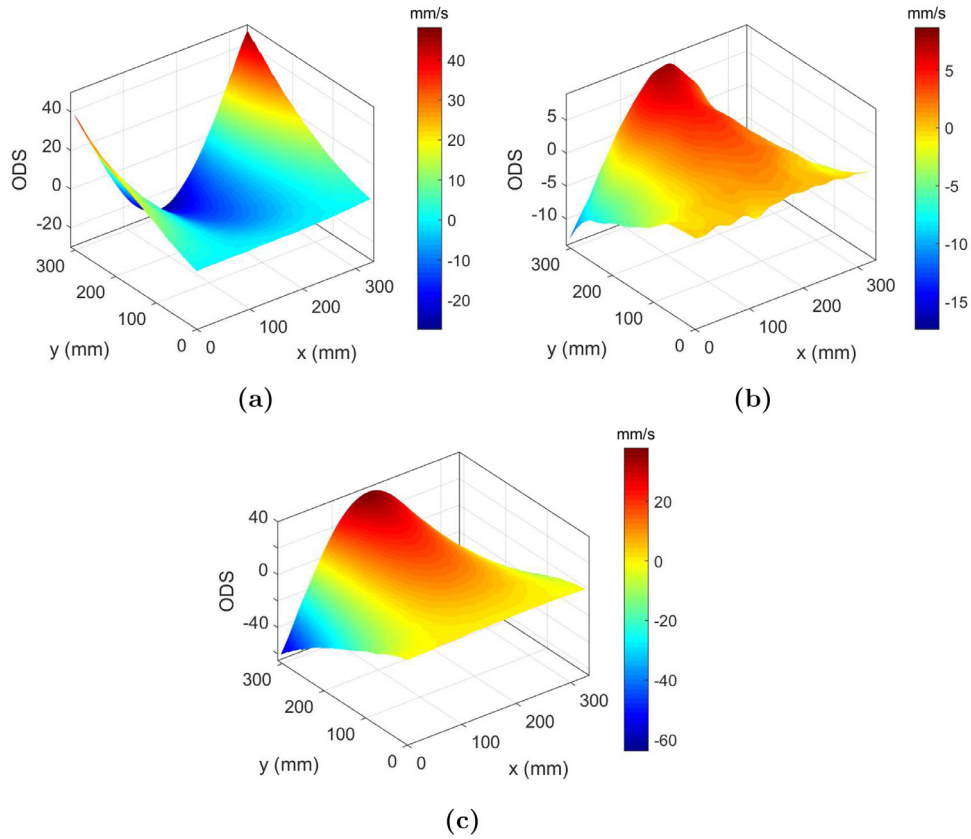


Fig. 12. 3D ODSs of the plate in (a) x , (b) y , and (c) z directions of the MCS obtained by the 3D-CSLDV system with the scan frequency of 1 Hz.

Table 3

MAC values between ODSs from continuous scanning and step scanning in each direction for the four excitation frequencies.

Excitation frequency	x	y	z
98 Hz	99.9%	99.9%	99.9%
317 Hz	98.8%	99.4%	98.4%
640 Hz	99.5%	99.6%	99.9%
1094 Hz	99.4%	99.7%	99.9%

In the beam experiment, the beam was sinusoidally excited by the shaker at four excitation frequencies of 98 Hz, 317 Hz, 640 Hz, and 1094 Hz that were obtained by rounding the first through fourth natural frequencies of the beam in the current experimental setup, respectively. The scan frequency was set to 0.05 Hz, which means that the scan time from one end of the beam to its other end was 10 s. The sampling frequency of the 3D-CSLDV system was set to 10 kHz. In order to examine accuracy of the continuously and synchronously scanning mode developed in this work, vibration of the beam was also measured in the step scanning mode with the Polytec system. ODSs of the beam in x , y , and z directions of the MCS under the four excitation frequencies are shown in Figs. 5–7, respectively. Note that an end-to-end ODS obtained from continuous scanning has $10,000 \times 10 = 100,000$ measurement points, while an ODS obtained from step scanning has 101 measurement points. One can see that continuous scanning can provide denser measurement. It can be seen from Figs. 5–7 that ODSs in each direction obtained by continuous scanning are in good agreement with those obtained by step scanning. Modal assurance criterion (MAC) values [20] between ODSs obtained from continuous scanning and step scanning in each direction were calculated for the four excitation frequencies, as shown in Table 3. All the MAC values are above 98%, which indicates good agreement of ODSs from the two measurement methods. Since the excitation is in the out-of-plane direction and the z direction in the MCS has the smallest angle with the excitation direction, ODSs in the z direction have the largest amplitude for all the four excitation frequencies. Also, out-of-phase relations of ODSs between the z direction and other two directions can be seen. The relative root mean square (RMS) error is used to evaluate amplitude difference between ODSs

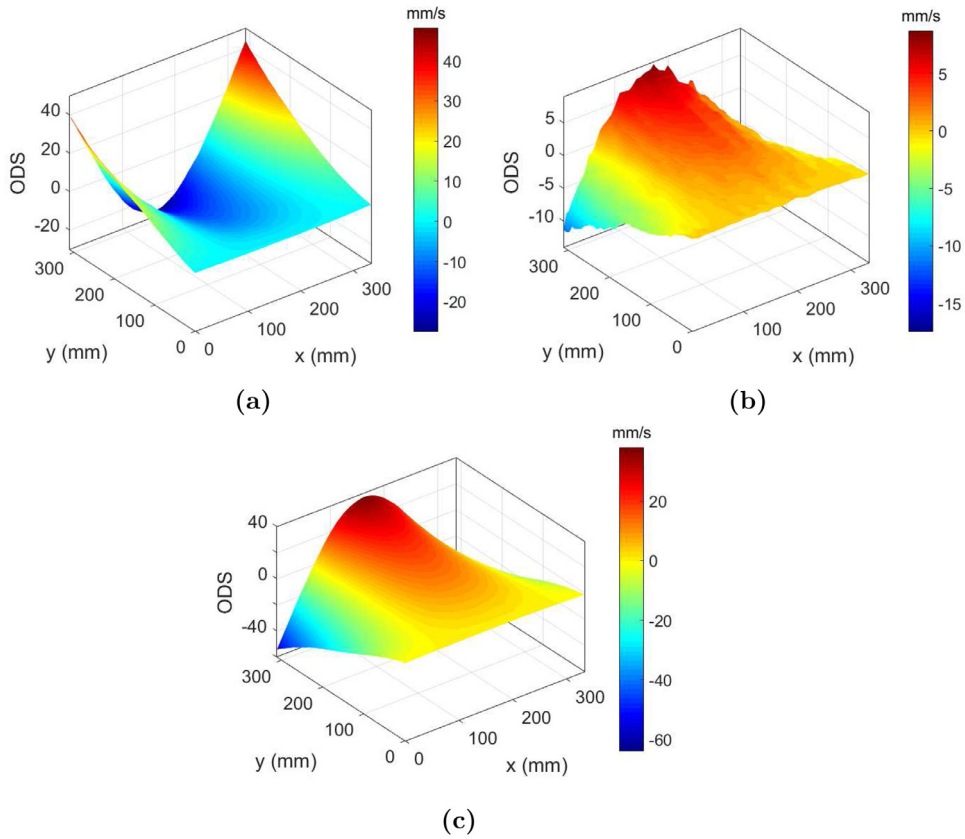


Fig. 13. 3D ODSs of the plate in (a) x, (b) y, and (c) z directions of the MCS obtained by the PSV-500-3D system in the step scanning mode.

Table 4

Relative RMS errors between ODSs from continuous scanning and step scanning in each direction of the MCS for four different excitation frequencies.

Excitation frequency	x	y	z
98 Hz	0.18%	2.26%	1.21%
317 Hz	0.62%	2.18%	1.28%
640 Hz	1.67%	1.22%	2.70%
1094 Hz	1.38%	2.83%	0.96%

obtained from 3D-CSLDV and 3D-SLDV systems, which is expressed by

$$RE\% = \frac{|\text{RMS}(\mathbf{ODS}_{\text{CSLDV}}) - \text{RMS}(\mathbf{ODS}_{\text{SLDV}})|}{\text{RMS}(\mathbf{ODS}_{\text{SLDV}})} \times 100 \quad (18)$$

where $\text{RMS}(\cdot)$ denotes the RMS value of a vector, and $\mathbf{ODS}_{\text{CSLDV}}$ and $\mathbf{ODS}_{\text{SLDV}}$ are ODSs from continuous scanning and step scanning, respectively. Results corresponding to each direction in the MCS for four different excitation frequencies are shown in Table 4. It can be seen that most of the relative RMS errors are less than 2% and the largest one is 2.83%, which shows that there is little amplitude difference between results from the two measurement methods.

3.3. Plate experiment

The second experiment conducted by the 3D-CSLDV system was to measure 3D ODSs of a six-ply $[0^\circ 90^\circ 0^\circ]_s$ IM-7 fiber reinforced composite plate with a thickness of 0.98 mm, as shown in Fig. 8(a). A strip area with the width of 25.4 mm adjacent to the lower boundary of the plate, which was in the vertical plane, was clamped by two aluminum beams with the width of 25.4 mm that were clamped by a bench vice, as shown in Fig. 8(b). A speaker located at the back side of the plate was used to excite it in a non-contact manner. The PSV-A-450 reference object in Fig. 8(b) was mounted on a tripod with an arbitrary angle to define the MCS. A zigzag scan trajectory similar to that in Fig. 2(b) was defined to cover the

Table 5

Comparison between continuous scanning and step scanning for the plate.

	Scan frequency of 0.1 Hz	Scan frequency of 1 Hz	Step scanning
Spatial resolution	$39 \times 50,000$	39×5000	39×65
Scan time	585 s	58.5 s	2516 s

Table 6

MAC values between ODSs from continuous scanning and step scanning in each direction of the MCS for the two scan frequencies.

Scan frequency	x	y	z
0.1 Hz	99.8%	97.9%	99.7%
1 Hz	99.8%	97.4%	99.7%

Table 7

Relative RMS errors between ODSs from continuous scanning and step scanning in each direction of the MCS for the two scan frequencies.

Scan frequency	x	y	z
0.1 Hz	4.07%	4.54%	3.60%
1 Hz	3.95%	4.67%	3.51%

whole surface of the plate. A 3M removable reflection graphic film was attached on the front side of the plate to enhance laser reflection.

In order to investigate the effect of the scan frequency on 3D ODSs obtained by the 3D-CSLDV system, two different scan frequencies, i.e., 0.1 Hz and 1 Hz, were used to scan the same prescribed trajectory. The sampling frequency of the 3D-CSLDV system was also set to 10 kHz. Using steps described in the beam experiment in Section 3.1, rotation angles corresponding to each CSLDV were obtained to continuously and synchronously scan the same trajectory. Input signals to scan mirrors of three CSLDVs corresponding to scan frequencies of 0.1 Hz and 1 Hz are shown in Figs. 9 and 10, respectively. The plate was sinusoidally excited by the speaker at an excitation frequency of 148 Hz, which was close to the first natural frequency of the plate. Results of 3D ODSs in the MCS corresponding to scan frequencies of 0.1 Hz and 1 Hz are shown in Figs. 11 and 12, respectively. One can see that 3D ODSs in the MCS obtained with the two different scan frequencies agree well with each other. Note that while the scan frequency does not affect ODSs, it does introduce more noise in calculation of curvatures of the ODSs when damage detection is conducted [9]. Regarding 3D-CSLDV measurement, since more matrix calculation is involved, it is recommended to use a scan frequency not larger than 1 Hz. A lower scan frequency can also help visually evaluate accuracy of continuous and synchronous scanning.

In order to validate accuracy of 3D ODSs obtained by the 3D-CSLDV system, the PSV-500-3D system was used to measure 3D ODSs of the plate in the same MCS in a step scanning mode at the same excitation frequency of 148 Hz. The surface of the plate was divided into a 39×65 measurement grid. The total time to complete the whole grid in a step scanning mode was 2516 s and results of 3D ODSs are shown in Fig. 13. It can be seen that the 3D ODSs in the MCS obtained in the step scanning mode are in good agreement with those obtained by the 3D-CSLDV system in a synchronously scanning mode. Spatial resolutions and total scan times in these two modes are compared in Table 5. The total scan time is much less and spatial resolution of 3D ODSs is much higher when the 3D-CSLDV system is used. With a lower scan frequency, spatial resolution is higher but more scan time is needed. Note that the sampling frequency of the 3D-CSLDV system can also affect spatial resolution. A higher sampling frequency would yield more sample points during the same scan time and increase spatial resolution. However, in the step scanning mode, the sampling frequency would have no effect on spatial resolution. MAC values between ODSs obtained from continuous scanning and step scanning in each direction of the MCS for the two scan frequencies are shown in Table 6 and they are all above 97%. Relative RMS errors of ODSs corresponding to each direction in the MCS for each scan frequency are shown in Table 7. Most of them are below 4% and the largest one is 4.67%, which shows good correlation between ODSs obtained by the two different scanning modes.

4. Conclusion

An investigation of 3D vibration measurement by a 3D-CSLDV system in a continuously and synchronously scanning mode is presented in this work for the first time. Results from beam and plate experiments show good agreement between the continuous scanning mode and step scanning mode, while 3D ODSs obtained from continuous scanning provide denser measurement. The plate experiment shows that 3D ODSs obtained with different scan frequencies are almost the same, which demonstrates effectiveness of using the 3D-CSLDV system for fast vibration measurement. In these two experiments, the excitation direction was mainly in the out-of-plane direction of the structures and 3D ODSs were obtained in an arbi-

trarily defined MCS. Complicated structures with curved surfaces would be tested using the 3D-CSLDV system in a follow-up work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Da-Ming Chen: Methodology, Software, Validation, Writing - original draft. **W.D. Zhu:** Conceptualization, Methodology, Funding acquisition, Writing - review & editing.

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