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VIEWPOINT

40 years of the quantum Hall effect

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Abstract | The discovery of the quantum Hall effect (QHE) marked a turning point in condensed-matter physics. The measurement of the Hall resistance showed that electronic resistance could be defined precisely in terms of fundamental constants, even in a disordered and irregular sample. Over the past 40 years, the QHE has inspired new theories and led to experimental discoveries in a range of fields going beyond solid-state electronics to photonics and quantum entanglement. In this Viewpoint, physicists reflect on how the QHE has influenced their research.

The beginning

Klaus von Klitzing. Until 1980, nobody had imagined the impact of electrical measurements on a semiconductor with uncontrolled impurities, imprecise geometric dimensions and alloyed contacts — a silicon field-effect transistor. Yet, when we exposed such a device to a strong magnetic field, we found that the Hall resistance was precisely quantized in integer multiples of h/e^2 (where h is Planck's constant and e is the elementary charge; BOX 1 and REF.[1]). This precision triggered the realization of a new international system of units based on fundamental constants, introduced worldwide in 2019. This unexpected direction is an excellent demonstration of how basic research on a practical device can open up a completely new field of research with new theories and unforeseeable applications — a real quantum leap.

The realization of an energy gap in the spectrum of a 2D electron system in strong magnetic fields, which is essential for the quantum Hall effect (QHE), and the strong correlation between electrons in a partially occupied discrete Landau level have led to a multitude of new phenomena and theories with the fractional QHE as the most prominent example. Compound fermions, the quantum spin and quantum anomalous Hall effect (QAHE), exciton condensation with superfluidity and, not least, the QHE as an example of a topological insulator with potential for quantum computation have enriched the scientific activities in this field. Personally, I would like to see in the near future convincing experiments demonstrating the braiding of non-Abelian anyons of certain excitations in the fractional QHE.

Even outside the solid-state community, QHE science became popular in many fields of research, from quantum Hall quarks to black holes and quantum Hall physics in string theory.

Fractional quantum Hall effect

Tapash Chakraborty. The extreme accuracy of the QHE — that the Hall resistance could be quantized to precisely h/ie^2, where $i = 1, 2, 3\ldots$ came as a surprise. This was soon followed by another unexpected discovery: the fractionally quantized Hall effect (FQHE), where the Hall resistance was quantized extremely accurately to h/fe^2, where $f = 1/3$ and $2/3$. This discovery required a complete rethinking of the QHE theory, involving a many-electron correlated system. Within a year, Robert Laughlin proposed a many-body wave function to explain this effect. His radical idea was to predict an incompressible ground state of a quantum fluid with 'fractionally charged' quasiparticles and quasiholes as excitations that could explain the observed effect[2]. This extraordinary proposal inspired many researchers, triggering a new wave of ideas with impact beyond condensed-matter physics. Few other developments in physics can match the impact of these three discoveries made in the space of 3 years.

Many-body theories are notoriously complicated and it is not always easy to find accurate solutions. In addition to making several ground-breaking predictions mentioned above, Laughlin's theory turned out to agree very well with accurate numerical schemes developed in parallel. What is surprising is that even after four decades of intense research, the fundamental problem of understanding the origin of incompressibility in the ground state, the presence of which explains the effect so well, has remained unsolved. None of the existing theories is capable of answering this question, but lately there has been some progress in this direction. I hope that the answer will be found by the time the golden anniversary of the QHE is celebrated.

Quantum Hall effect in graphene

Philip Kim. About 15 years ago, soon after graphene was first isolated, an unusual QHE was discovered in graphene. This was my first personal encounter with a QHE not appearing in conventional semiconductors. The unexpected feature was that the sequence of the plateaus was displaced by a half-integer multiple of 4 for both electrons and holes. The fact that the plateaus are half-integer signifies that the electrons are quasirelativistic and have a Berry phase with their valley and spin degrees of freedom. The quasirelativistic band structure of graphene also provides an unusual Landau level spectrum, the energy of which can be large, allowing the QHE to be observed even at room temperature.

Since its discovery, graphene has provided a unique material platform to study quantum Hall physics. Discoveries include multicomponent (from both spin and valley entanglement) quantum Hall ferromagnetism, FQHE and fractal QHE due to the quasiperiodic moiré pattern formed between graphene and its cousin, a boron nitride substrate. The history of the QHE has been full of wonders and unexpected discoveries. Combining graphene into other 2D materials, provides versatile platforms to discover exotic quantum states of matter in the years to come.

Quantum spin Hall effect

Vidya Madhavan. When graphene was isolated in 2004, I was a young assistant professor with a new lab. I came from a background of studying complex materials that harbour strongly interacting electrons. These strong interactions result in a landscape of rich phenomenology, but are theoretically challenging to model. In contrast, electrons in graphene do not interact strongly and the theory is relatively simple. I was therefore convinced that graphene was a short-lived fad and stubbornly refused to jump into the field. But as time went on, graphene revealed one surprising discovery after another, and I severely lamented my lack of judgement in not entering the field right at the beginning.

When 2D topological insulators (also known as quantum spin Hall insulators; part f of the figure in BOX 1) and their 3D counterparts were first hypothesized and then discovered in 2007 (REF.[3]), I was ready. Owing to the non-trivial topology of the bulk, the boundaries of topological insulators, such as surfaces and edges, carry special states where electrons behave like relativistic massless particles, becoming fermionic analogues of light. This new lens of topology offers a wonderful new view of materials and provides a pathway to realizing a host of new phases and particles such as quantized edge states that can carry currents without dissipation, and long-sought-after Majorana particles that we have spent the past decade studying. The next frontier is to combine topology with interactions where new kinds of particles consisting of magnetic monopoles or chargeless spinons are waiting to be discovered.

Quantum anomalous Hall effect

Xi Dai. The QAHE allows edge states to form in a material without the need for an external magnetic field (part d of the figure in BOX 1). When my colleagues and I were working on the theoretical prediction of the first QAHE material system[4], we viewed it as a quantum spin Hall system, Bi_2Te_3 thin film, with spontaneous magnetization from doping with the magnetic ion, Cr. The quantum spin Hall can often be approximated by two pseudo-spin subspaces connected by time-reversal symmetry with each of them having opposite QAHE. If spontaneous magnetization is introduced to break the exact cancellation between the two subspaces, the QAHE can appear when such a transition has already happened in one subspace, but not yet in another one.

The key component of the mechanism of the QAHE in Cr-doped Bi_2Te_3 thin film is the spin–orbital coupling in Bi_2Te_3. However, last year, the QAHE was

The contributors

Klaus von Klitzing received the Nobel Prize in Physics in 1985 for discovering the quantum Hall effect, 5 years after the unexpected observation at the Grenoble High Magnetic Field Laboratory. His research focuses on electrical and optical measurements on low-dimensional electron systems. In 2018, he retired as director at the Max Planck Institute for Solid State Research in Stuttgart.

Tapash Chakraborty is a retired professor of physics from the University of Manitoba, Canada. He was also the Canada Research Chair in Nanoscale Physics (2003–2017). He has worked on various aspects of the quantum Hall effect since the early days of the discovery. He has also worked on the electronic properties of quantum dots and various other nanoscale systems. In addition to numerous articles, he has authored books and book chapters. He is a fellow of the American Physical Society.

Philip Kim is professor of physics at Harvard University. His group and Andre Geim's group at Manchester University are the first who observed the half-integer shift in the quantum Hall effect in graphene in 2005, experimentally demonstrating the linear Dirac dispersion. The Kim group has actively been pursuing novel quantum transport in graphene and other 2D materials.

Vidya Madhavan obtained her bachelor's and master's degrees from the Indian Institute of Technology in India. After obtaining her Ph.D. from Boston University in 2000, she held a postdoctoral appointment at the University of California, Berkeley, from 1999 to 2002. She joined the physics faculty at Boston College in 2002 and is currently a full professor at the University of Illinois, Urbana-Champaign.

Xi Dai received his Ph.D. in 1999 at the Institute of Theoretical Physics, Chinese Academy of Sciences. He worked there for more than 10 years before he joined Hong Kong University of Science and Technology in 2017 as a chair professor in the physics department.

James McIver received his Ph.D. from Harvard University in 2014. He subsequently did a postdoc at the Max Planck Institute for the Structure and Dynamics of Matter, where he is currently a research group leader. His research focuses on the electrical transport properties of optically driven quantum materials, including transport from topological Floquet states in graphene.

Yoshinori Tokura graduated with a Ph.D. from the University of Tokyo in 1981. He is currently Distinguished University Professor of the University of Tokyo as well as the founding director of the RIKEN Center for Emergent Matter Science. He has been working on correlated and topological electron physics, including high-temperature superconductors, Mott transitions, colossal magnetoresistance oxides, multiferroics, skyrmion science and magnetic topological insulators.

Lucile Savary received her Ph.D. from University of California, Santa Barbara, in 2014. After a postdoctoral position at MIT, she joined Ecole Normale Superior at Lyon as a permanent researcher. Her research focuses on exotic phenomena in real systems, with an emphasis on frustrated magnetism, and includes quantum spin liquids, the anomalous Hall effect and non-centrosymmetric superconductors.

Daria Smirnova received her Ph.D. in Physics in 2016 from the Australian National University, where she currently holds a prestigious Discovery Early Career Research Fellow position supported by the Australian Research Council. Her research interests include topological photonics, nonlinear nanophotonics and multipolar electrodynamics.

Ana Maria Rey received her Ph.D. from the University of Maryland at College Park in 2004. She is currently a JILA fellow, a NIST fellow and an adjoint professor in the physics department at the University of Colorado, Boulder. Rey's research is on how to control and manipulate ultracold atomic systems for use as quantum simulators, precision measurements and quantum computation.

Claudia Felser studied at the University of Cologne, completing her doctorate in physical chemistry in 1994. She is currently a director at the Max Planck Institute for Chemical Physics of Solids in Dresden. She is a fellow of the American Physical Society and the Institute of Physics, London, a member of the Leopoldina, the German National Academy of Sciences, and an International Member of the National Academy of Engineering, USA. In 2019, she received the American Physical Society James C. McGroddy Prize for New Materials together with Andrei Bernevig and Xi Dai.

Johannes Gooth studied physics at the University of Hamburg and Lund University, completing his doctorate in physics in 2014. After two postdocs at IBM Research – Zurich and Harvard University, he became an independent research group leader at the Max Planck Institute for Chemical Physics of Solids in Dresden in 2018.

Xiaoliang Qi received his Ph.D. from Tsinghua University in 2007, then moved to the United States for postdoctoral work. Since 2010, he has been a faculty member at Stanford University. His research interest is the interplay of quantum entanglement, quantum gravity and quantum chaos and he has also worked on topological states and topological phenomena in condensed-matter systems.

discovered in twisted bilayer graphene, where the spin–orbital coupling strength is almost zero[5]. Interestingly, it appears that the underlying physics can still be understood in a similar way by replacing the spin by the valley degree of freedom. In addition, the QAHE here is completely intrinsic and the ferromagnetism is induced by the exchange processes between the Dirac electrons, which is greatly enhanced by the flat bands appearing at the so-called magic twisting angles. Owing to the great tunability and abundance of the Dirac electrons in the twisted graphene systems, fascinating QAHE-related physics can be expected.

Floquet topological insulators

James McIver. In the traditional QHE, a strong magnetic field is required to force the electrons to move in circular trajectories. Another way to drive electrons in circles is to use the strong optical fields in ultrafast pulses of circularly polarized light. This type of coherent light–matter interaction is currently being explored as an alternative way of inducing the QHE, to realize a so-called Floquet topological insulator.

Recently, my colleagues and I observed a Hall effect in graphene induced by a femtosecond pulse of circularly polarized light[6]. By changing the helicity of the light, the direction of the Hall effect could be controlled in a similar manner to changing the magnetic field direction in a conventional Hall effect experiment. Remarkably, we found that the graphene quantum states were modified by the light to resemble those in the QHE.

The results raise the intriguing possibility that light can create and manipulate chiral edge states in zero magnetic field and at ultrafast speeds. Proving the existence of such light-induced edge states will be challenging given the ultrafast timescales involved. However, advances in ultrafast scanning probes enable the direct imaging of these states. This exciting research direction has just started, and it has the discovery of the QHE to thank for its inspiration.

Topological Hall effect

Yoshinori Tokura. In the 1980s, two big discoveries were made: the QHE and first high-temperature superconductor. These led to two major directions of condensed-matter physics: topological quantum matter and strongly correlated electron systems. These two streams of science are now merging to produce new fields. One of the most spectacular discoveries in this merging field is the QAHE realized in magnetic topological insulators.

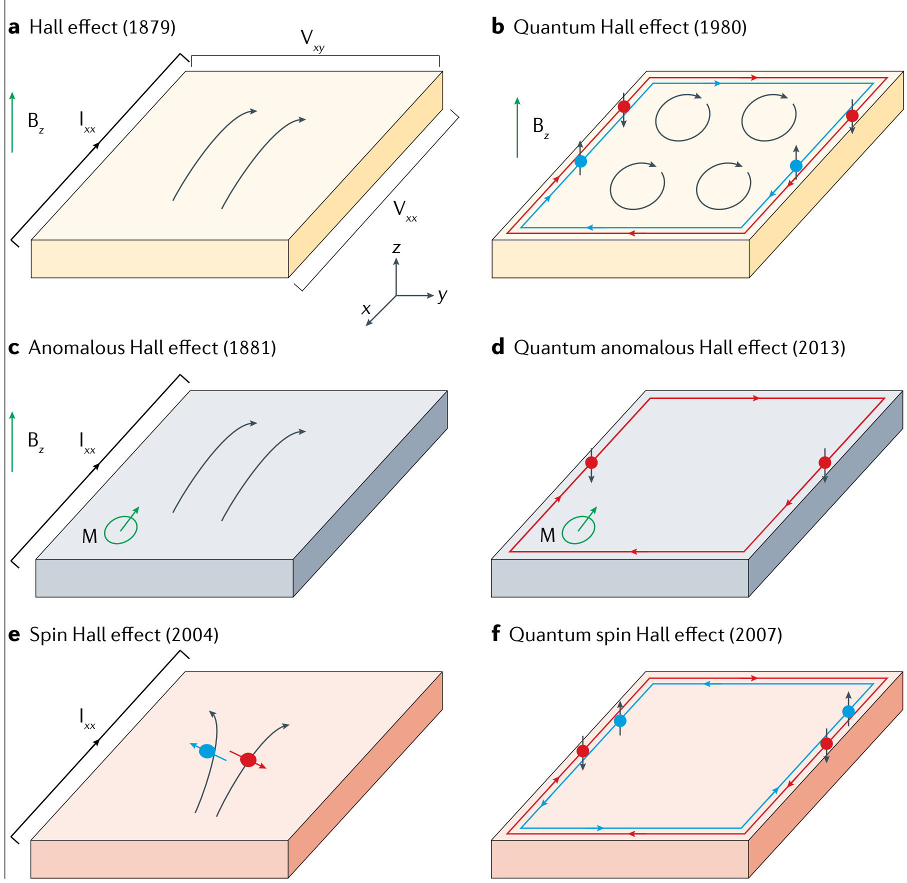
Box 1 | **Classical and quantum Hall effects**

In classical electrodynamics, when a perpendicular magnetic field (B_z) is applied to a sample that has current (I_x) flowing along it, the Lorentz force deflects the electrons, leading to a build-up on one side of the sample. This charge accumulation causes a voltage drop across the sample (V_{xy}) known as the Hall voltage (see the figure, part **a**). This classical model works well for small magnetic fields, for which the Hall resistance is linear with magnetic field.

In the quantum description, the perpendicular magnetic field forces the electrons to move on cyclotron orbits. Whereas the electrons at the centre of the sample form closed orbits, those at the edges form skipping orbits, which bounce off the edges as edge states (see the figure, part **b**). However, it is important to note that these edge states exist even if no current is applied. The magnetic field also leads to the quantization of the allowed energies of the electrons into discrete Landau levels. As the field strength increases, the spacing between these levels also increases, so fewer levels are filled. The number of filled Landau levels determines the Hall resistance, which forms plateaus with the value h/ie^2, where i is an integer depending on the number of filled levels.

In ferromagnetic samples exposed to a perpendicular magnetic field, the Hall resistance is found to be much larger than in non-ferromagnetic samples and this is known as the anomalous Hall effect (see the figure, part **c**). In ferromagnetic samples that also have strong spin-orbit coupling, this effect becomes quantized. In the quantum anomalous Hall effect, the edge states all carry the same spin direction (see the figure, part **d**). These edge states exist regardless of an external bias or magnetic field.

The spin Hall effect does not require an external magnetic field and can be observed in materials that have strong spin–orbit coupling. When an external bias is applied, there is spin accumulation at the edge of the sample (see the figure, part **e**). In the quantum spin Hall effect, the strong spin–orbit coupling leads to the formation of helical edge states, in which the spin orientation of the edge state depends on the direction of travel (see the figure, part **f**). The edge states, which exist regardless of an external bias, are considered to be topologically protected, and these systems are commonly known as topological insulators. Image credit: Ankita Anirban and Charlotte Gurr.



The microscopic origin of the QAHE is spin–orbit coupling. Yet, another intrinsic magnetism-based Hall effect, termed the topological Hall effect, is possible even without spin–orbit coupling[7]. The topological Hall effect can be seen in metallic magnets with non-coplanar spin arrangement or spin chirality, most typically, magnetic skyrmions. This effect stems from the quantum Berry phase of conduction electrons hopping on mutually canted spin sites. The time-dependent motion of the spin chirality can also give rise to an emergent electric field, to create electromagnetic induction of genuinely quantum-mechanical origin. The topological Hall effect occasionally shows a comparable (or larger) magnitude to (than) the AHE, and its quantized version, the quantum topological Hall effect, is also expected to manifest in two dimensions, for example, in magnetic kagome lattices. The realization of a room-temperature, zero-field quantum topological Hall effect in a magnetically ordered state is a big challenge, but a most rewarding functionality enabling dissipationless quantum-conduction circuits.

Quantum thermal Hall effect

Lucile Savary. The QHE relies on the existence of electrically charged and itinerant particles. Moreover, the quantization of the effect necessitates fermionic particles. This a priori evicts the possibility of a similar effect in electrical insulators. However, a quantized thermal Hall effect can be generated by emergent fermionic particles in a special phase of strongly interacting magnetic systems, known as the 'Ising anyon phase'. In the quantized thermal Hall effect, similar to the electrical QHE, the magnetic degrees of freedom produce zero longitudinal thermal conductivity, but an emergent (Majorana) fermion that propagates along the edge of the sample leads to a quantized thermal Hall conductivity[8].

Only a highly quantum-entangled state of the magnetic degrees of freedom (a quantum spin liquid) can support such an emergent fermion. At odds with the discovery of the quantum electrical Hall effect, theory is ahead of experiment in the thermal case, but materials candidates are emerging. Once the Ising anyon phase is found in experiment — it will be! — it will be interesting to see how far it can be taken technologically. One 'obvious' application will be to harness the edge Majorana fermion for non-Abelian-based computations.

Topological states of light

Daria Smirnova. The photonic counterpart of the QHE with scattering-resistant edge propagation was first experimentally realized in 2009 with the use of a gyromagnetic microwave photonic crystal[9]. The design incorporated magnetized ferrite rods to create a periodic spatial modulation of the gyrotropic permeability that breaks time-reversal symmetry, by analogy to the electronic QHE. By this means, bulk bands of the photonic crystal acquire topologically non-trivial properties and the system guides gapless unidirectional electromagnetic waves localized to the boundaries. A grand vision of waveguiding and routing of light in a robust manner with built-in immunity to disorder stimulated rapid progress in bringing topological states to time-reversal-invariant photonic platforms and opened the whole new realm of topological photonics. In many cases, the synthetic fields for photons, which mimic the effects of a magnetic bias or spin–orbit coupling on electrons, are judiciously engineered in specially designed artificial structures called metamaterials, or by employing modulation in time or space.

The physics of topological phases of light goes beyond direct analogies with the condensed-matter systems. The distinctions include the bosonic nature of photons, intrinsic non-Hermiticity due to the presence of absorption/radiation losses or, conversely, an optical gain, and optical nonlinearities. As such, active topological cavities hold special promise for a design of lasers. I anticipate that harnessing topological photonic phases in nonlinear and quantum optics will bring the cutting-edge discoveries towards high-speed data processing and quantum computing with improved reliability.

Synthetic fields for atomic QHE

Ana Maria Rey. Ultracold atomic gases are remarkably versatile and controllable systems that can be used for the quantum simulation of condensed-matter phenomena, including the QHE. Atomic quantum gases are charge neutral and not affected by external electromagnetic fields in the same way as electrons are. However, by using properly designed laser fields, synthetic electromagnetic fields to which an atom responds as if it were a charged particle have been generated in ultracold-atom experiments. These synthetic electromagnetic fields can be ultrastrong, over 1,000 T, which would be impossible to achieve in condensed-matter experiments. This provides the opportunity to reach extreme QHE conditions in the lab. Furthermore, the arrays of atoms could also give rise to new behaviours as some of them have bosonic statistics instead of the fermionic statistics obeyed by electrons. Moreover, the dynamic response of atoms to the applied synthetic fields can be probed and controlled in real time by tuning parameters such as the strength of the interatomic interactions or level of disorder.

The time is ripe for using synthetic ultracold-atom systems to investigate rich and unexplored QHE physics, including new quantum phases of matter such as strongly interacting topological insulators, and FQHE-like liquids with fractal excitations that can live in synthetic spaces with arbitrary dimensionality (even larger than three). An overarching goal is the simulation not only of static electromagnetic fields but also of dynamical gauge fields, which appear in intractable problems in high-energy physics, for example, in lattice gauge theories.

Three-dimensional QHE

Claudia Felser and Johannes Gooth. The past 40 years of experimental research have focused on realizing the QHE in 2D systems. Yet, Bertrand Halperin had proposed in 1987 that it should be possible to realize the QHE in a 3D semimetal or doped semiconductor with a particular instability in the Fermi surface. The additional dimension would enable the possibility of new phenomena such as 1D Majorana chains or 3D fractional quasiparticles. The challenge was to realize a single crystal, which fulfils all necessary conditions for such a 3D QHE: high mobility, an adjusted charge carrier concentration and impurity level with the Fermi level tuned to be in an energy gap to the applied magnetic field. In 2D systems, these properties can be tuned by electrostatic gating, but gating is not possible in 3D materials owing to screening. Last year, strong evidence for the 3D QHE was found in single crystals of $ZrTe_5$ in small magnetic fields of just 2 T. The electron density modulation in the direction of the magnetic field of the $ZrTe_5$ single crystals was accounted for by a charge density wave instability[10]. The results were soon reproduced by other groups and, moreover, evidence for a 3D version of the fractional QHE was seen in single crystals of $HfTe_5$.

However, there are still many open questions. As $ZrTe_5$ is a 'low dimensional' compound with a highly anisotropic Fermi surface, is the 3D QHE the sum of quasi-2D contributions, or can we find other 3D single-crystalline materials with appropriate electronic structures? If we can design a single crystal with the desired Fermi surface, defect and charge carrier concentrations, and position of the Fermi energy, can we realize a QHE at room temperature, without a magnetic field, or in magnetic single crystals? Can we realize a 3D QHE in materials systems with new fermions exhibiting Fermi arcs the size of the Brillouin zone? Forty years after the first discovery, the QHE is still as exciting as on the first day.

Quantum entanglement

Xiaoliang Qi. The discovery of QHE is an important milestone in condensed-matter physics because it uncovered how the same simple building block — mainly electrons — can self-organize into dramatically different interaction and correlation patterns, corresponding to different phases of matter. The study of the properties of these different phases of matter revealed that a lot of properties that used to be considered as fundamental — mass, statistics of particles, symmetry and gauge field — can actually emerge from low-energy dynamics of different states of matter. Understanding quantum Hall states and their relatives, topological states of matter, is thus deeply connected to fundamental physics.

A surprising connection between new states of matter and fundamental physics occurs in the relation between quantum entanglement and quantum gravity, which is based on the theory of holographic duality. Holographic duality tells us that a pair of theories with different spatial dimensions can be equivalent, when the higher-dimensional one has dynamical gravity and the other one does not. There are many indications that gravity — and therefore spacetime itself — is a manifestation of the structure of quantum entanglement. Characterizing the structure of many-body quantum entanglement is an open question, and some of the earliest efforts in this direction came from the theory of topologically ordered states. An entirely new language for understanding the relation between matter, quantum information and geometry may be required, which I consider to be one of the most exciting current directions in theoretical physics.

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1. Klitzing, K. v., Dorda, G. & Pepper, M. New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance. *Phys. Rev. Lett.* **45**, 494–497 (1980).
2. Laughlin, R. B. Anomalous quantum Hall effect: an incompressible quantum fluid with fractionally charged excitations. *Phys. Rev. Lett.* **50**, 1395–1398 (1983).
3. König, M. et al. Quantum spin Hall insulator state in HgTe quantum wells. *Science* **318**, 766–770 (2007).
4. Yu, R. et al. Quantized anomalous Hall effect in magnetic topological insulators. *Science* **329**, 61–64 (2010).
5. Serlin, M. et al. Intrinsic quantized anomalous Hall effect in a moiré heterostructure. *Science* **367**, 900–903 (2020).
6. McIver, J. W. et al. Light-induced anomalous Hall effect in graphene. *Nat. Phys.* **16**, 38–41 (2020).
7. Lee, M., Kang, W., Onose, Y., Tokura, Y. & Ong, N. P. Unusual Hall effect anomaly in MnSi under pressure. *Phys. Rev. Lett.* **102**, 186601 (2009).
8. Kasahara, Y. et al. Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid. *Nature* **559**, 227–231 (2018).
9. Wang, Z., Chong, Y., Joannopoulos, J. D. & Soljačić, M. Observation of unidirectional backscattering-immune topological electromagnetic states. *Nature* **461**, 772–775 (2009).
10. Tang, F. et al. Three-dimensional quantum Hall effect and metal–insulator transition in $ZrTe_5$. *Nature* **569**, 537–541 (2019).

Competing interests

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