

Creation and Characterization of Matter-Wave Breathers

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We report the creation of quasi-1D excited matter-wave solitons, “breathers,” by quenching the strength of the interactions in a Bose-Einstein condensate with attractive interactions. We characterize the resulting breathing dynamics and quantify the effects of the aspect ratio of the confining potential, the strength of the quench, and the proximity of the 1D–3D crossover for the two-soliton breather. Furthermore, we demonstrate the complex dynamics of a three-soliton breather created by a stronger interaction quench. Our experimental results, which compare well with numerical simulations, provide a pathway for utilizing matter-wave breathers to explore quantum effects in large many-body systems.

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The nonlinear Schrödinger equation (NLSE) applies to a wide variety of physical systems, such as small amplitude waves in deep water, light waves propagating in optical fiber, Langmuir waves in plasmas, and matter waves [1,2]. A fundamental solution to the NLSE in one dimension (1D) for a self-focusing nonlinearity is a bright soliton, a localized wave packet that maintains its shape and amplitude while propagating. While the soliton is the ground state, the NLSE also supports excited state solutions that contain an integer number N_s of constituent solitons. These solutions are generally supplemented by radiation that reduces the wave amplitude. In the general case, each constituent soliton is spatially separated from the others, and they propagate with different velocities. A breather is a special class of solutions in which N_s fundamental solitons are overlapped, with zero relative velocity, and without radiation. Unlike the case of the sine-Gordon equation, the constituent solitons of a NLSE breather are not bound to each other. In the absence of any binding energy, the relative motion is in a state of neutral equilibrium [3,4]. The density profile of a breather oscillates quasiperiodically with frequencies determined by the differences in the chemical potentials of the constituent solitons. The interference between the constituent solitons leads to complex spatial patterns, giving the appearance of breathing.

Breathers were first observed in optical fiber [5,6], where optical pulses with discrete intensity levels were found to have a quasiperiodically varying pulse shape matching that of the $N_s = 2, 3$, and 4 breathers. An N_s -soliton breather can be formed from a fundamental soliton by quenching the

strength of the nonlinearity by a factor of N_s^2 [4,7], thus, creating an odd-norm-ratio breather [8] whose fundamental solitons that form the breather have an amplitude ratio of $1:3:\dots:2N_s - 1$. If the quench factor deviates from N_s^2 , the breather becomes the closest N_s -soliton breather with a different norm ratio after shedding radiation to properly reduce the amplitude [4].

In the matter-wave context, bright solitons can be formed in a Bose-Einstein condensate (BEC) confined to a quasi-1D trap by tuning the s -wave scattering length $a_s < 0$, corresponding to an attractive nonlinearity. Matter-wave solitons, and their properties, have been the subject of intense investigation in recent years. These properties include the formation of solitons and soliton trains [9–17], the collision of two solitons [18], interactions of solitons with potential barriers [19–21], and soliton interferometry [22,23]. Solitons formed by a BEC of magnon quasiparticles in ^3He have also been recently observed [24]. Recently, a two-soliton breather was created by quenching a_s by a factor close to 4, in combination with a rapid relaxation of the axial confinement [25]. The soliton dynamics of these experiments are well reproduced by the mean-field Gross-Pitaevskii equation (GPE), which is a NLSE that includes the confining potential of a trap.

Even though the solitons in a breather spatially overlap, their binding energies are zero, leaving the relative motion of the constituent solitons sensitive to perturbations. At the same time, integrability of the NLSE protects the solitons from exchanging matter with each other or losing it to radiation. Within the framework of mean field theory,

dissociation of the breather into constituent solitons may occur due to narrow potential barriers [8,26,27]. Perhaps most interestingly, beyond mean-field effects, due to quantum interference, may result in splitting [28–31], dissociation [32,33], relaxation [34,35], or the complete lack of breathing following the quench [36]. In prior theoretical work, we evaluated the affect of quantum fluctuations on the relative velocity of the two components of a two-soliton breather using both the exact Bethe-ansatz method, appropriate for small number of atoms N [32], and, in the limit of large N , the Bogoliubov approach [33]. We found that quantum fluctuations can produce the macro-effect of breather dissociation over a large range of N , thus providing the motivation of the present study to create and characterize matter-wave breathers.

In this Letter, we report the creation and characterization of a two-soliton breather in a BEC of ^7Li atoms, and for the first time, the experimental creation of a three-soliton breather in a BEC. We systematically study the breathing frequency as a function of deviations from a truly 1D system, the strength of the nonlinearity, and the quench ratio, and compare with 1D GPE simulations. We observe the characteristic dynamics of the three-soliton breather, including density splitting and recombination, using minimally destructive sequential imaging.

Our method for preparing an ultracold ^7Li gas has been described previously [37,38]. The atoms are optically pumped into the $|f = 1, m_F = 1\rangle$ state, where the s -wave scattering length a can be controlled by a broad Feshbach resonance with a zero crossing near 544 G [39]. We describe our method for calibrating $a(B)$ in [40]. The atoms are confined in a cylindrically symmetric, cigar-shaped potential formed by a single-beam optical dipole trap with a $1/e^2$ Gaussian radius of $44\ \mu\text{m}$. In combination with axial magnetic curvature, the overall harmonic frequency along the axial (z) direction, ω_z , is tunable between $(2\pi)1.12$ and $(2\pi)11.50$ Hz. The radial trap frequency is $\omega_r = (2\pi)297$ Hz, corresponding to an aspect ratio, $\lambda = \omega_r/\omega_z$, that is between 26 and 265. First, we create a BEC by direct evaporative cooling in the optical dipole trap with $\omega_z = (2\pi)11.50$ Hz and with a tuned to $140\ a_0$, where a_0 is the Bohr radius. Following evaporation, we ramp a from $140\ a_0$ to $0.1\ a_0$ in 1 s. During this stage, ω_z is kept large in order to limit the axial extent of the repulsive BEC, thus, ensuring that only a single soliton is formed when the interaction is changed from repulsive to attractive. Next, a is ramped from $0.1\ a_0$ to $a_i < 0$ in 1 s, while simultaneously reducing ω_z . This creates a single soliton with approximately $N = 5 \times 10^4$ atoms, with minimal excitations. The scattering length is then quenched from a_i to $a_f = A^2 a_i$ in 1 ms, where $|a_f| > |a_i|$, and A^2 is the quench ratio. We use polarization phase-contrast imaging (PPCI) [38,45] to take *in situ* images of the column density after a variable hold time t_h following the quench.

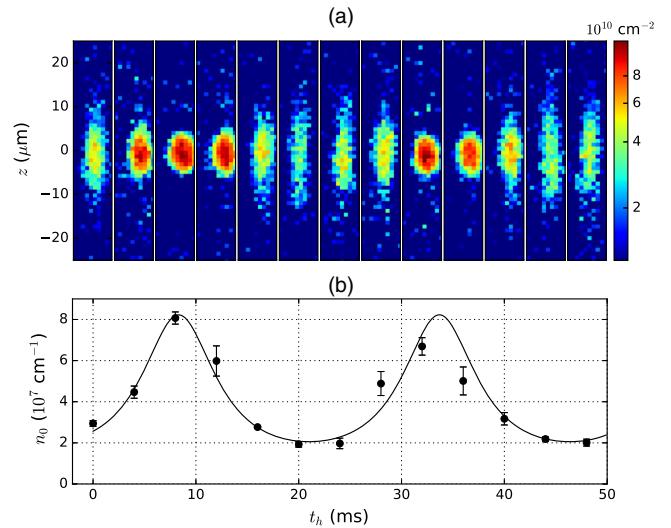


FIG. 1. (a) Experimental images of a two-soliton breather. The values of the parameters are $a_i = -0.15(2)a_0$, $a_f = -0.54(3)a_0$, $N = 5.4(4) \times 10^4$, $N_c = 5.2(3) \times 10^4$, $\omega_r = (2\pi)297(1)$ Hz, and $\omega_z = (2\pi)1.12(2)$ Hz, so that $N/N_c = 1.0(1)$, $\lambda = 265(5)$, and $A^2 = 3.6(6)$. Uncertainties are discussed in Ref. [40]. Each image is a separate realization of the experiment, and the center of the image is adjusted to remove shot-to-shot variation in the center of mass. The color scale represents the column density in this image, as well as in Figs. 2(c) and 3(a). (b) Each data point is the result of fitting the axial density $n(z)$ to find its central density n_0 for each of five images, and averaging the result. The solid line is a fit to Eq. (1), with fitting parameters $\omega_B = (2\pi)39.4(6)$ Hz, and $\phi = (2\pi)0.17(1)$. Error bars in n_0 are the standard error of the mean. The uncertainty in ω_B is the fitting uncertainty.

Figure 1 shows the breathing dynamics of a two-soliton breather. After the quench, the wave function contracts toward the center and forms a large density peak at the half period, followed by expansion back to the initial profile, thus, completing a full breathing period, as shown in Fig. 1(a). The axial density $n(z)$ is obtained by integrating the column density along the remaining radial coordinate perpendicular to the imaging axis. The central density n_0 of the breather is measured by fitting the axial density to a Gaussian function $n(z) = n_0 \exp[-(z/l_z)^2]$, where n_0 and the Gaussian radius l_z are the fitting parameters. Although $n(z)$ is not strictly a Gaussian, the n_0 found in this way is a good approximation of its true value.

To determine the frequency of an N_s -soliton breather, the central density n_0 is measured as a function of t_h , and is fit to the corresponding analytical solution of the NLSE for two-soliton breathers, which for $A^2 = 4$, is [4]

$$n_0(t_h) = \frac{\alpha}{5 + 3 \cos(\omega_B t_h + \phi)}, \quad (1)$$

where the breather frequency ω_B , phase ϕ , and overall amplitude α are fitted parameters. The solid line in Fig. 1(b) shows Eq. (1) using the extracted parameters.

The breather, as described by the NLSE, is a purely 1D object, while the experiment is in quasi-1D due to the fact that the ratio of the chemical potential to the radial trap frequency is nonzero, and as a result, the transverse wave function profile cannot be factored out. The validity of the exact NLSE breather solution also requires the absence of any axial trapping. Both the proximity to 3D and the weak axial confinement break integrability. As a consequence of being in quasi-1D, a BEC with attractive interactions is unstable to collapse once the atom number exceeds a critical value N_c . For an elongated cigar-shaped harmonic confinement, $N_c = 0.67a_r/|a_f|$, where $a_r = \sqrt{\hbar/m\omega_r} = 2.2 \mu\text{m}$ is the radial harmonic oscillator length [46]. The collapse threshold for the breather is predicted to be different from that of the ground state soliton [47]. We explore the 3D and axial confinement effects by measuring the dependence of ω_B on the trap aspect ratio λ and, separately, on N/N_c .

The measured ω_B as a function of λ is plotted in Fig. 2(a). For this data, $N/N_c = 1.0$, $a_i = -0.15a_0$, and $a_f = -0.54a_0$, giving $A^2 = 3.6$. We find that ω_B monotonically decreases as λ increases from 26 to 265. We compare the measured results with the 1D GPE,

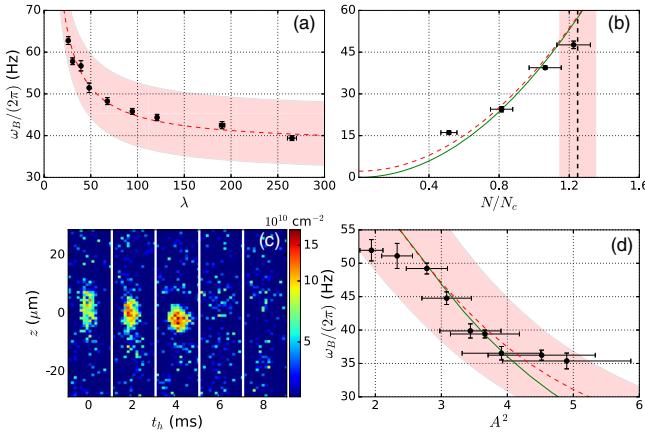


FIG. 2. Two-soliton breather frequency dependence on parameters. The parameters are as shown in the caption to Fig. 1, unless specified otherwise. The red dashed lines in (a), (b), and (d) show the solutions of the 1D GPE simulation, and the red shaded areas show the uncertainty range in ω_B due to the uncertainty in the measured N/N_c . The uncertainties for (a), (b), and (d) are discussed in Ref. [40]. (a) ω_B vs λ . Here, ω_r is fixed while ω_z is varied. The location of the Feshbach resonance zero-crossing field was varied to within its uncertainty (0.2 G) to obtain the best fit of GPE solutions to the data [40]. (b) ω_B vs N/N_c . The solid green line is the solution to the 1D NLSE [Eq. (3)]. The vertical dashed line indicates the value of N/N_c above which collapse is observed. (c) Images showing collapse for t_h between 4 and 6 ms after the quench and for $N/N_c = 1.2(1)$. This sequence of images is taken from a single experimental realization. (d) ω_B vs A^2 . Here, a_f is fixed while a_i is varied. The solid green line is the solution of the 1D NLSE [Eq. (4)].

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_z^2\psi + \frac{1}{2}m\omega_z^2z^2\psi + g_{1D}N|\psi|^2\psi, \quad (2)$$

where $g_{1D} = 2\hbar\omega_r a$ is the nonlinear coupling constant [48]. The ground state at $a = a_i$ is used as the initial wave function, and Eq. (2) is then numerically integrated with $a = a_f$ up to a few breathing periods. The resulting ω_B , using the measured parameters, is shown by the dashed red line in Fig. 2(a). The shaded region in Fig. 2(a) represents the range of solutions of the 1D GPE that includes the measured uncertainty in N/N_c [40]. The measured frequency is consistent with the simulation, to within the measurement uncertainties. We also calculated ω_B using the 3D GPE for several values of the parameters and found excellent agreement with the 1D GPE for $N/N_c \lesssim 0.7$. The 3D and 1D GPE differ at larger N/N_c due to the proximity to the collapse threshold, which signals the breakdown of one dimensionality, and eventually, of the GPE itself. Further consideration of the limits of the applicability of the 1D and 3D mean-field approximations is warranted, particularly in the case of excited states [49,50], such as breathers.

As mentioned above, the breather strictly exists only in 1D on a flat background, thus, requiring $\omega_B/\omega_z \gg 1$. The experiment demonstrates that, for $\lambda = 265$, ω_z is significantly less than ω_B , ensuring that the breather dynamics is, indeed, dominated by the nonlinear interactions, rather than the trap.

Figure 2(b) shows the measurement of ω_B vs N/N_c for $\lambda = 265$ and $A^2 = 3.6$, corresponding to the conditions to excite a two-soliton breather. The analytic result given by the 1D NLSE for $A^2 = 4$ [7],

$$\omega_{B,1D} = \frac{N^2 a_f^2}{4a_r^2} \omega_r = 0.11(N/N_c)^2 \omega_r, \quad (3)$$

is shown by the solid green curve in Fig. 2(b). The results of the 1D GPE simulation is again shown by the dashed red curve. The experimental data follows the quadratic trend given by Eq. (3).

For $N/N_c \geq 1.2(1)$, we observe collapse of the two-soliton breather for $t_h \gtrsim 4$ ms following the quench, at the time when the density grows rapidly. An example is shown in Fig. 2(c). The collapse threshold for the fundamental soliton occurs at $N/N_c = 1.0$, which has been observed in the in-phase collisions of two fundamental solitons [18]. A numerical simulation based on the 3D GPE [47] provides an estimate of the collapse threshold for the two-soliton breather, which is found to be $N/N_c = 1.1$, for the experimental parameters of Fig. 2(b). Additionally, a factorization ansatz in the mean-field limit [51] provides an analytical estimate for the collapse location to be $N/N_c > N_s^2/\sqrt{2N_s^2 - 1}$, which gives 1.5 for $N_s = 2$ [40].

The NLSE can predict the number of atoms in each of the two fundamental solitons when $1.5 < A < 2.5$.

They are found to be $N_1 = (2A - 1)N/A^2$ and $N_2 = (2A - 3)N/A^2$. When $A \neq 2$, the number of atoms in the two solitons, $N_1 + N_2$, is less than the total number of atoms N , with the remaining atoms radiated away [4]. In principle, a measurement of N vs A^2 could reveal the efficiency of the quench, but the radiated loss fraction is predicted to be less than $N/10$ and was not resolved in our experiment.

A change in A^2 modifies the chemical potentials of the constituent solitons and, therefore, the breather frequency. The measured ω_B vs the quench ratio A^2 is shown in Fig. 2(d), where the dashed red line and shaded region again correspond to the 1D GPE simulation, including uncertainties in N/N_c . The dependence of ω_B on A for the two-soliton breather with no axial potential can be evaluated as the soliton chemical potential difference,

$$\omega_{B,1D}(A) = \frac{16(A-1)}{A^4} \omega_{B,1D}(A=2), \quad (4)$$

which is shown by the solid green curve in Fig. 2(d).

We also excited a three-soliton breather by quenching by a factor of $A^2 = 7.1$. The results are given in Fig. 3(a), where a series of sequential images using PPCI are displayed for a single realization of the experiment. The $N_s = 3$ breather displays more complex dynamics than does the $N_s = 2$ breather as it contains more than one frequency component. A superposition of two solitons can exhibit shape oscillations, but it cannot undergo a transition between single- and double-peak shapes, which requires a superposition of no fewer than three solitons. The breather frequencies are the differences between the chemical potentials, μ , of the constituent fundamental solitons. Since $\mu \propto (N/N_c)^2$, and the number ratio of the $N_s = 3$ breather is 1:3:5 [4], the ratio of μ values is 1:9:25, giving frequency ratios of 8:16:24. Identifying the smallest frequency as ω_B , we have the three frequencies: ω_B , $2\omega_B$, and $3\omega_B$, appropriate for $A^2 = 9$.

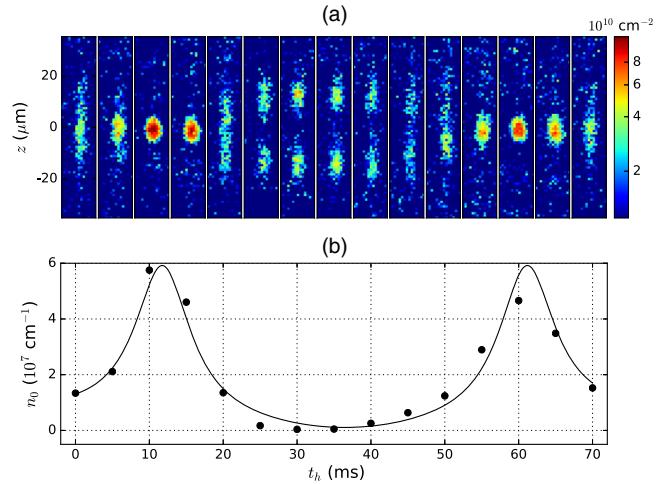


FIG. 3. (a) Experimental images of a three-soliton breather produced by $A^2 = 7(2)$. A series of phase-contrast images were taken at 5 ms intervals after the quench in a single realization of the experiment. The center of each image is adjusted to remove the center-of-mass variation between the images. Parameters for this data are $\lambda = 265(5)$, $a_i = -0.08(2)a_0$, and $a_f = -0.57(3)a_0$, and for the initial image ($t_h = 0$), $N/N_c = 1.0(1)$. Uncertainties are discussed in Ref. [40]. In each subsequent image, N is reduced by 3% due to spontaneous emission by the probe. (b) The closed circles are n_0 extracted from the column density images shown in (a). The solid line is a fit of the data to Eq. (5), giving $\omega_B = (2\pi)10.6(2)$ Hz and $\phi = (2\pi)0.11(1)$.

To analyze the three-soliton breather quantitatively, we fit the integrated 1D density for each t_h to either a single- or double-Gaussian function depending on whether the central density is a local maximum or minimum, respectively. We extracted the central density $n_0(t_h)$ from the fit, and plotted it against t_h , as shown by the discrete points in Fig. 3(b). For three-soliton breathers, $n_0(t_h)$ is fitted to the exact three-soliton breather solution of the NLSE for $A^2 = 9$ obtained from the general theory [4,52],

$$n_0(t_h) = \alpha \left(1 + \frac{32[3 + 5 \cos(\omega_B t_h + \phi)] \sin^2 \frac{1}{2}(\omega_B t_h + \phi)}{55 + 18 \cos(\omega_B t_h + \phi) + 45 \cos 2(\omega_B t_h + \phi) + 10 \cos 3(\omega_B t_h + \phi)} \right), \quad (5)$$

with fitting parameters ω_B , ϕ , and α . The result is $\omega_B = (2\pi)10.6(1)$ Hz and $\phi = (2\pi)0.11(1)$. The solid line in Fig. 3(b) is Eq. (5) using these values. Equation (5) pertains to the specific case of $A^2 = 9$, where the quench produces a pure three-soliton breather with no radiation. We find that Eq. (5) is a good approximation to the central density of a three-soliton breather even when A^2 is close to, but not exactly equal to 9. This result is consistent with exact theory [4] in which a breather composed of three fundamental solitons is created for $6.25 < A^2 < 12.25$.

In conclusion, we have observed the two- and three-soliton breathers in a BEC by quenching the atomic interaction using a zero crossing of a Feshbach resonance in ^7Li . We have shown that, by reducing the axial confinement, the breather frequency approaches the 1D limit and is well described by the 1D NLSE. Like fundamental bright matter-wave solitons, higher-order solitons undergo collapse for a nonlinearity that is too strong. Collapse arises when the soliton is brought close to the 3D boundary, but notably, the collapse threshold for breathers is higher than it

is for fundamental solitons with the same total particle number.

In the strict 1D limit, breathers are exact solutions of the NLSE. Breathers are superficially similar to time crystals [53–56], although breathers are not a consequence of a spontaneously broken symmetry. Breathers are particularly sensitive to beyond-mean-field quantum effects, which, according to Ref. [32], lead to formation of a quantum superposition of two fundamental solitons with different relative velocities and numbers of atoms after the quench. Spontaneous dissociation of the breather is predicted to occur after multiple breathing periods [32,33]. In our experiment, the two-soliton breather survives for at least two breathing periods. An extension of this work is to measure the breathing duration, which determines whether spontaneous dissociation can be observed. Preliminary experiments indicate that noise in the center-of-mass coordinates poses a technical limit to the breather lifetime. Further progress will require better stability of the magnetic field and laser pointing to mitigate center-of-mass fluctuations and drift.

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