



# Boys' advantage on the fractions number line is mediated by visuospatial attention: Evidence for a parietal-spatial contribution to number line learning

David C. Geary | John E. Scofield | Mary K. Hoard | Lara Nugent

University of Missouri, Columbia, MO,  
USA

## Correspondence

David C. Geary, Department of  
Psychological Sciences, Interdisciplinary  
Neuroscience, University of Missouri,  
Columbia, MO 65211-2500, USA.  
Email: GearyD@Missouri.edu

## Funding information

National Science Foundation, Grant/  
Award Number: DRL-1659133; Eunice  
Kennedy Shriver National Institute of  
Child Health and Human Development,  
Grant/Award Number: R01 HD087231

## Abstract

The study tested the hypotheses that boys will have an advantage learning the fractions number line and this advantage will be mediated by spatial abilities. Fractions number line and, as a contrast, fractions arithmetic performance were assessed for 342 adolescents, as was their intelligence, working memory, and various spatial abilities. Boys showed smaller placement errors on the fractions number line ( $d = -0.22$ ) and correctly solved more fractions arithmetic problems ( $d = 0.23$ ) than girls. Working memory and intelligence predicted performance on both fractions measures, and a measure of visuospatial attention uniquely predicted number line performance and fully mediated the sex difference. Visuospatial working memory uniquely predicted fractions arithmetic performance and fully mediated the sex difference. The results help to clarify the nuanced relations between spatial abilities and formal mathematics learning and the sex differences that often emerge in mathematical domains that have a visuospatial component.

## KEY WORDS

mathematics, number line, sex differences, spatial ability, visuospatial attention

## 1 | INTRODUCTION

An emerging understanding of the magnitudes represented by numerals and number words is central to children's mathematical development (Geary, 1994; National Mathematics Advisory Panel, 2008). The insight that numerical magnitudes form a continuum along the mathematical number line is a key aspect of this development (Case & Okamoto, 1996; Siegler & Braithwaite, 2017). Practically, an understanding of how numerical magnitudes can be ordered along a visuospatial number line is an important aspect of students' formal mathematical learning and undergirds more complex mathematics (e.g., plotting functions in coordinate space). Theoretically, the visuospatial representation of the number line sits at the juncture between evolved cognitive systems for representing magnitudes and the evolutionarily novel learning that occurs in modern schools (Feigenson et al., 2004; Geary, 1995; Siegler & Opfer, 2003). In this case, people's intuitive representations of the relations among

magnitudes conflicts with the formal mathematical definition of these relations. The conflict results in well-documented errors during the early learning of how numerals are ordered on the visuospatial number line, as well as an opportunity to explore how cognitive abilities interact with intuitive knowledge during this learning.

The system that supports intuitive representations of numerical magnitudes overlaps to some extent the systems that support representations of space (Bueti & Walsh, 2009; Giles et al., 2018; Hubbard et al., 2005; Walsh, 2003). The overlap provides cognitive and neural links between numerical-spatial abilities and at least some aspects of formal mathematics. The overlap is important here because it might provide a potential explanation for boys' and men's advantage in some areas of mathematics (Bull et al., 2013), and the finding that these sex differences are often related to boys' and men's advantage in spatial abilities (Casey et al., 1997). Adolescence might be a particularly important period for the emergence of sex differences in the spatial-mathematics relation because the magnitude of sex

differences in both spatial abilities and some areas of mathematics increases during this time (Hyde et al., 1990; Lauer et al., 2019). The study of the spatial-mathematics relation more generally is important because it is related to long-term accomplishments in some areas of science, technology, engineering, and mathematics (STEM) and may contribute to sex differences in some of these areas (Kell et al., 2013; Lubinski & Benbow, 2006; Webb et al., 2007).

With respect to the number line, Thompson and Opfer (2008) predicted and confirmed that boys' advantage in spatial abilities should result in a sex difference during the early learning of the ordering of numerals along the visuospatial line. However, they did not explicitly test whether it was spatial abilities that contributed to boys' advantages. We replicated this finding for the fractions number line and extended it by showing that adolescent boys' advantage is mediated by a sex difference in visuospatial attention independent of the domain-general abilities (i.e., intelligence, working memory) that contribute to ease of formal, evolutionarily novel mathematical learning (Geary et al., 2017; Lee & Bull, 2016). Second, we provide unique evidence for the specificity of this relation by showing that visuospatial attention does not contribute to boys' advantage in fractions arithmetic.

## 1.1 | Sex differences

Sex differences in mathematics often favor boys, but mean differences are small and vary from one topic to the next (Hyde et al., 1990). Larger sex differences are found at the high end of performance (Ceci & Williams, 2010; Wai et al., 2018) and oftentimes in mathematical areas that have a spatial component (Geary, 1996; Halpern et al., 2007). For instance, boys and men typically outperform girls and women on mathematical word problems (Casey et al., 1997; Geary et al., 2000). The sex difference here is related, at least in part, to boys' and men's spontaneous use of diagrams that set up the relation between the quantities described in the problem, a strategy that reduces problem-solving errors (Johnson, 1984; Lewis, 1989). Learning the visuospatial representation of the mathematical number line and related competencies associated with processing numerical magnitudes is another area in which boys and men often have an advantage (Bull et al., 2013; Gilligan et al., 2019; Hutchison et al., 2019; Rivers et al., 2020; Thompson & Opfer, 2008). However, it is not known if these advantages are related to the sex difference in visuospatial abilities, but there is reason to suspect that they are related.

The advantage of boys and men in many spatial domains (e.g. Voyer et al., 1995, 2017) is associated with more surface area in the parietal cortex, controlling overall brain size (Koscik et al., 2009; Salinas et al., 2012). As described below, some of these areas are also implicated in spatial-numerical representations of magnitude and the mental number line (Hubbard et al., 2005). One possibility is that boys have more grey matter associated with these representations that could result in better discrimination of associated quantities. However, this is unlikely because there are no consistent sex differences on measures of inherent magnitude representations (Kersey

## Highlights

- Adolescent boys were more accurate than girls in placing fractions on the number line
- Placement accuracy was predicted by visuospatial attention
- Boys' advantage in visuospatial attention mediated their advantage on the number line
- Visuospatial attention was unrelated to performance in fractions arithmetic

et al., 2018). Following the sex difference for word problems, another possibility is that the sex differences in spatial abilities enable boys to use more sophisticated strategies during the dynamic placement of numerals on the number line or to more accurately situate them during this process. If so, then boys should be more accurate in their placement of numerals on the number line or use more sophisticated strategies during these placements, and any such differences should be mediated by spatial abilities.

## 1.2 | Mental number line

Areas of the parietal cortex subserve the formation of magnitude representations of many features of the physical world, such as distance (Summerfield et al., 2020). These magnitudes are typically represented along a single continuous dimension that encodes the relations among them (e.g., closer to farther). The approximate number system (ANS)—situated in the intraparietal sulcus of the parietal lobe—is the associated system for representing quantities and is evident in a wide range of species (Feigenson et al., 2004; Gallistel, 1990; Gallistel & Gelman, 2000; Geary et al., 2015). The representations of quantity magnitudes also appear to be along a single smaller-to-larger continuum that forms a spatially based mental number line (Hubbard et al., 2005; Zorzi et al., 2002). The continuum allows quantity representations to be compared, albeit not precisely. The ease of discriminating one quantity from another depends on the ratio between them and not their absolute difference, such that discriminating 3 from 4 objects (1.33 ratio) is easier than discriminating 13 from 14 objects (1.08 ratio).

One core question is whether people's intuitive sense of quantities and the relations among them supports the learning of symbolic mathematics, including the visuospatial number line. The issue is vigorously debated and remains to be resolved (e.g. Mussolin et al., 2016; Szkudlarek & Brannon, 2017; Szűcs & Myers, 2017). One finding that is often interpreted as evidence for an influence of the ANS is the pattern of whole-numeral placements along the visuospatial number line. During the early phases of learning, these placements show a logarithmic pattern that mirrors the theoretical pattern of the ordered continuum of quantity representations within the ANS (Siegler & Opfer, 2003).



A related and early proposition was that education resulted in the formation of more linear quantity representations within the ANS that in turn resulted in linear placements of whole numbers on the visuospatial number line (Dehaene et al., 2008). The direct formation of extensive linear representations is unlikely, however, as this would involve more precise numeral-to-magnitude mappings than can be supported by the ANS. Moreover, it is unclear how ANS representations could support students' placement of fraction magnitudes on the visuospatial number line. This is because the ANS does not support fractional representations, but an analogous ratio processing system situated near the ANS has been proposed (M. R. Lewis et al., 2016). However, the same difficulty arises whereby a potentially infinite number of precise fraction magnitudes would need to be represented in a system that is sensitive to only approximate ratios.

Moreover, many novice students' do not show the ANS-signature logarithmic pattern for the placement of fractions on the visuospatial number line (Siegler et al., 2011), although Thompson and Opfer (2008) demonstrated that approaches for mapping whole numbers to the visuospatial number line are transferred to the placement of fractions (see also Opfer & DeVries, 2008). Students' placements of fractions improve significantly from fourth to eighth grade and becomes more or less linear, although the relative magnitude of placement errors is larger than those found for whole numbers (Braithwaite et al., 2018; Siegler et al., 2001). The placements are often made by changing proper fractions (e.g.,  $\frac{7}{2}$ ) to mixed numbers (e.g.,  $3\frac{1}{2}$ ) and then situating them by segmenting the line to create an anchor (e.g.,  $2\frac{1}{2}$  for the 0 to 5 line). The mental segmentation or use of anchor points on the visuospatial number line to situate fractions is similar to that found for whole number placements (Barth & Paladino, 2011; Rouder & Geary, 2014; Slusser et al., 2013).

One interpretation of this strategic approach is that students come to understand the smaller-to-larger spatial alignment of numerals and learn the conceptual constraint that the distance between successive whole numbers is invariant across the line. Eventually many students learn that the same smaller-to-larger alignment applies to fractions magnitudes, although this comes later in their education (Siegler et al., 2011). This type of structure mapping maintains the visuospatial representation of the number line and potentially engages the inherent bias to represent features of the world along a magnitude-based continuum but does not require the formation of precise one-to-one mappings of numerals or fraction magnitudes to the ANS or analogous ratio-system representations (Sullivan & Barner, 2014). In other words, the inherent contributions to number line learning are in the bias to represent magnitudes (of any kind) along a single continuous dimension that often has a spatial component, along with school-taught conceptual constraints based on the formal mathematical properties of the number line (e.g., the distance between any two consecutive whole numbers is identical at all locations on the line).

Once students understand the mathematical properties of the number line, linear placements occur during a dynamic process

as people adjust their placements during the act of positioning numerals on the visuospatial line (Dotan & Dehaene, 2016; Kim & Opfer, 2018). Such a dynamic process allows for strategic or contextual influences on number line performance, above and beyond an influence of the ANS (Barth & Paladino, 2011; Cicchini et al., 2014; Cohen & Blanc-Goldhammer, 2011; Rouder & Geary, 2014; Slusser et al., 2013). As noted, whole numbers strategic approaches include the use of anchors, such as the midpoint and endpoints of the line, to help to situate placements. A similar process of breaking the line into segments to facilitate placements occurs for students who are learning the fractions number line (Siegler et al., 2011).

The use of such strategies need not preclude the existence of the ANS, as these representations could still manifest when students are in the early stages of learning the number line or under conditions (e.g., dual task) that disrupt the ability to use top-down strategies (Dotan & Dehaene, 2016; Kim & Opfer, 2018). In any case, a bias to represent quantities along a continuum provides a more direct link to spatial abilities than does the precision of ANS representations per se. This is because the dynamic and strategic construction of the linear pattern during number line tasks should, in theory, be facilitated by visuospatial abilities. For instance, injury-related (e.g., stroke) deficits in visuospatial attention are often associated with difficulties determining the midpoint of two presented numerals (e.g., 17–51; Zorzi et al., 2012). The same bias is found in healthy individuals when visuospatial attention is experimentally disrupted (Longo & Lourenco, 2007). The pattern suggests that one or several components of spatial ability could contribute to individual and sex differences in the accuracy of number line placements and the strategies used in making them.

### 1.3 | Domain-general and spatial abilities

The ease of learning evolutionarily novel academic material, such as the mathematical number line, is consistently related to intelligence and working memory (Bull & Lee, 2014; Geary, 2008; Geary et al., 2017; Lee & Bull, 2016). Intelligence is particularly important in learning novel concepts (Cattell, 1963), which in this case includes the learning that fractions represent magnitudes and that these can be linearly situated on the visuospatial number line (Braithwaite et al., 2019; Siegler et al., 2011). Working memory also contributes to the ease of learning in multiple academic domains (Paas & Ayres, 2014) and likely contributes to performance on measures that involve the execution of multiple steps, as would occur if students dynamically use one strategy or another to make number line placements. In short, we anticipated that intelligence and working memory would emerge as predictors of performance on our fractions measures. Their inclusion as covariates is important because it provides a more rigorous assessment of the relation between visuospatial abilities and fractions performance than would otherwise be the case.

Our study included three measures of visuospatial abilities that show sex differences (Collaer et al., 2007; Voyer et al., 1995, 2017) and engage a distributed brain network, including parietal regions that have been associated with the ANS and spatial-numerical representations. The first is the Judgment of Line Angle and Position Test (JLAP; Collaer et al., 2002) that is sensitive to visuospatial attention deficits that often result from damage to the right parietal cortex (Benton et al., 1978; García-Sánchez et al., 1997; Tranel et al., 2009). The second is the Mental Rotation Test (MRT; Peters et al., 1995) that typically involves the top-down manipulation of images and is associated with widespread bilateral parietal activation (Carpenter et al., 1999). The third is a measure of visuospatial working memory (Corsi Blocks), which is supported in part by parietal regions that are inferior to those typically associated with the ANS (e.g., angular gyrus; Chechlacz et al., 2014), although these areas might contribute to spatial-numerical representations (Göbel et al., 2006).

#### 1.4 | Current study

The current study tested the hypotheses that boys will have an advantage on the fractions number line and this advantage will be at least partially mediated by one or several spatial abilities. The hypotheses follow from the proposal that spatial and numerical magnitudes are supported by similar brain and cognitive systems and from the male advantage in spatial abilities. The hypotheses are consistent with prior studies showing that boys and men often more accurately place whole numbers on the visuospatial number line than do girls and women (Bull et al., 2013; Rivers et al., 2020; Thompson & Opfer, 2008), but any such sex difference might be influenced by level of expertise, that is, largest during the initial learning of the number line (Hutchison et al., 2019). We extend these studies to the fractions number line, assess students who are still learning fractions, and provide a more thorough assessment than previous studies of the hypothesis that any associated sex differences are related to spatial abilities.

Although we anticipated sex differences, favoring boys, on all of the spatial measures included in the study and that these would contribute to a male advantage on the fractions number line (Thompson & Opfer, 2008), we were agnostic as to which spatial measure would emerge as the most relevant to number line performance. Thus, in the context of our general *a priori* predictions, the assessments for specific spatial measures should be considered exploratory.

We also included a measure of fractions arithmetic that spanned the same magnitude range as was used for the number line. Procedural competencies should not be as dependent on spatial-numerical representations as the number line (Hubbard et al., 2005), although they are often predicted by visuospatial working memory (Li & Geary, 2017; Zhang et al., 2019). Still, inclusion of fractions arithmetic provides a means to assess the discriminant validity of the results for the number line. Discriminant validity would be

demonstrated if the spatial predictors of number line performance and any associated sex differences differed from those that emerged for fractions arithmetic.

## 2 | METHODS

### 2.1 | Participants

The sample consisted of 342 (169 boys) adolescents from two cohorts of students enrolled in an ongoing sixth- to ninth-grade longitudinal study in collaboration with the public schools in Columbia MO (USA), and included aspects of their sixth- and seventh-grade assessments. Their mean age at the sixth-grade assessment was 12 years and 3 months ( $SD = 4.51$  months), and 13 years and 1 month ( $SD = 4.41$ ) at the final seventh-grade assessment.

#### 2.1.1 | Demographics of the participants

Demographic information was obtained through a parent survey ( $n = 281$ ). Parents reported on their child's sex/gender (male, female, I choose not to answer) and all chose male or female; for students whose parents did not return the survey, child sex was obtained from the schools that in turn obtained the information from parents. Eighty-eight percent of the students were non-Hispanic, 6% Hispanic or Latino, with the remaining unknown. The racial composition of the sample was 70% White, 14% Black, 3% Asian, 1% Native American, 10% multi-racial, and the remaining unknown. Self-reported annual household income was as follows: \$0–\$24,999 (12%); \$25,000–\$49,999 (18%); \$50,000–\$74,999 (12%); \$75,000–\$99,999 (22%); \$100,000–\$149,999 (19%); and \$150,000+ (17%). Seventy-one percent of the students had at least one parent with a college degree. Sixteen percent of families received food assistance, and six percent housing assistance. The intelligence ( $M = 104.57$ ,  $SD = 13.09$ ) and standardized seventh-grade mathematics ( $M = 99.93$ ,  $SD = 18.92$ ) and word reading ( $M = 104.30$ ,  $SD = 13.22$ ) achievement of the sample were average.

### 2.2 | Fractions measures

The fractions number line and fractions arithmetic measures were from a broader assessment of the mathematical competencies and attitudes of sixth graders. The additional measures and tests that are not reported here can be accessed on the Open Science Framework (OSF; [https://osf.io/fxu4c/?view\\_only=e9ed1203a11b4d72afe5017714f1cbd4](https://osf.io/fxu4c/?view_only=e9ed1203a11b4d72afe5017714f1cbd4)). The fractions addition task was administered using paper and pencil, as was the number line task for the first cohort; for the second cohort, an electronic version of the number line task was developed in-house using Qualtrics (Provo, UT) and administered on iPads using the Qualtrics Offline App (<http://www.qualtrics.com>). There were no cohort differences for mean absolute error on the

## Number Line Estimation

In this section, we would like you to practice placing fractions on number lines.

In each problem, you will see a number line going from 0 to 5 and a fraction above the line. Your job will be to move the circle on the line to about where the fraction belongs on the number line. The closer the fraction's size is to 0, the closer the circle should be to 0. The closer the fraction's size is to 5, the closer the circle should be to 5.

In this example,  $2\frac{1}{2}$  is half of 5, so we can put the circle halfway between 0 and 5 on the number line. Practice moving the circle now.

Move the slider to about where  $2\frac{1}{2}$  belongs on a number line going from 0 to 5:

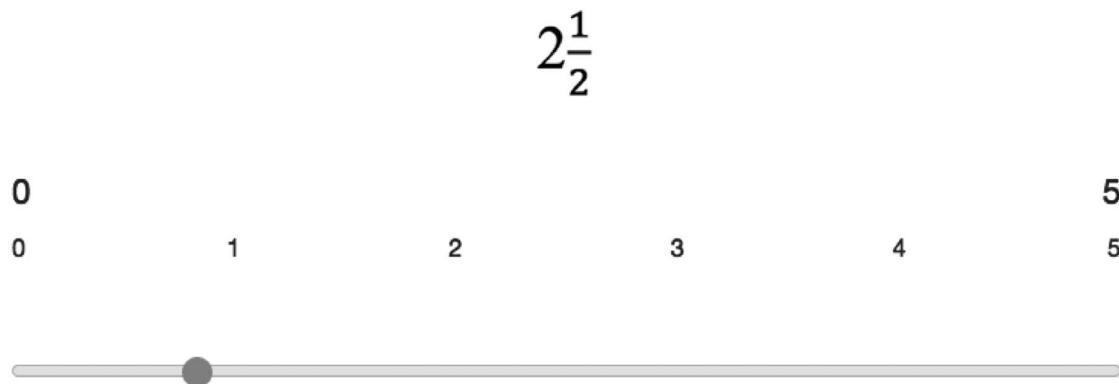


FIGURE 1 Instruction screen for number line task. Note that the 0 1 2 3 4 5 numerals above the line were not present for the test items

fractions number line task ( $p = .193$ ) or number correct for fractions arithmetic ( $p = .531$ ).

### 2.2.1 | Fractions number line

Students were asked to sequentially place, one at a time, 10 fractions on a 0-to-5 number line ( $10/3, 1/19, 7/5, 9/2, 13/9, 4/7, 8/3, 7/2, 17/4$ , and  $11/4$ ). Target fractions were presented in large font and centered above the number line; Figure 1 shows the instruction screen for the iPad administration. There was a 4 min time limit to complete the 10 items. The order of administration

was initially randomized, but the same sequence was used for all students. The students completed 94.4% of the lines in the allotted time and responses to the remaining 5.6% were estimated based on the average of five imputed scores using the multiple imputations procedure in SAS (2014). Items were scored as the absolute error between the correct fraction location and student responses, that is,  $||R - C|| \times 0.2$ , where  $R$  = response and  $C$  = correct placement. As a result, lower values represent more accurate placements. The overall score was the mean of the 10 items ( $M = 0.17, SD = 0.14, \alpha = 0.71$ ). The estimation of students' placement strategies (below) was based on their actual placements, not the absolute error.



## 2.2.2 | Fractions arithmetic

The items were sets of 12 addition (e.g.,  $1/3 + 1/6$ ), 12 multiplication (e.g.,  $1/4 \times 1/8$ ), and 10 division (e.g.,  $2 \div 1/4$ ) problems. All of the numbers included in these problems were  $<5$ , as were 29 of the 34 answers; in other words, the problems largely covered the same magnitudes as in the number line task. Students had 1 min for each operation. The score was the number of correctly solved problems ( $M = 12.50$ ,  $SD = 7.56$ ;  $\alpha = 0.67$ ).

## 2.3 | Cognitive measures

The cognitive tasks were administered on iPads using customized programs developed through Inquisit by Millisecond (<https://www.millisecond.com>); manuals and detailed descriptions are available on OSF (<https://osf.io/qwfk6/>). All of the tasks are standard measures of short-term and working memory, and various aspects of spatial ability.

### 2.3.1 | Spatial ability

As noted, the first spatial ability measure was the JLAP (Collaer et al., 2002, 2007). As shown in Figure 2, the task requires students to match the angle of a single line to 1 of 15-line options in an array below the target line. There are 20 test items presented sequentially, with students using the touch screen of an iPad to select the correct angle. There was a 10 sec time limit on each trial, with a self-paced inter-trial interval. The outcome was the number correct ( $M = 13.34$ ,  $SD = 3.07$ ).

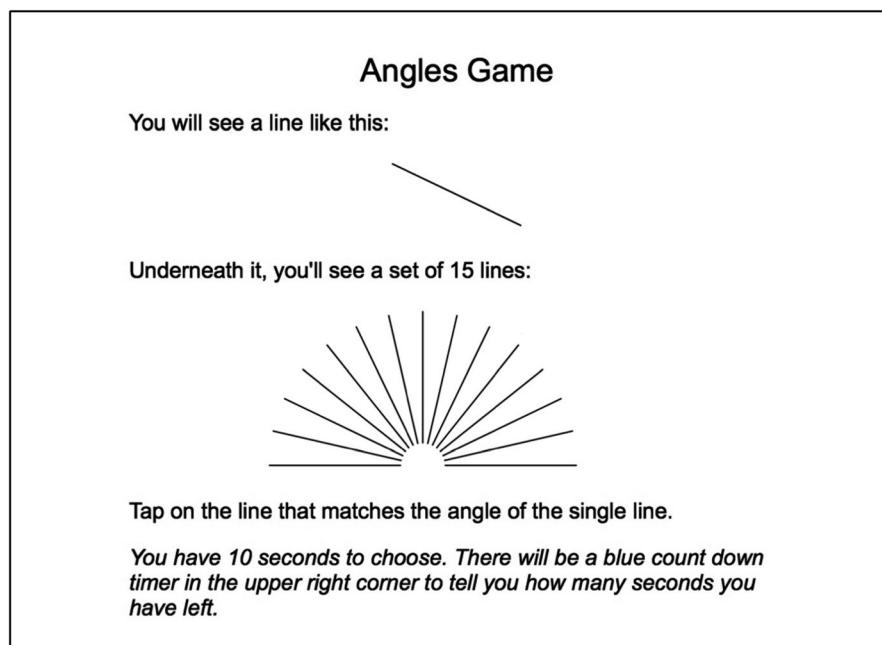
The second measure was the MRT (Peters et al., 1995). On each of 24 trials, students viewed 3D images of 10 connected cubes. For each trial, there was a target image along with four choice options. The task was to select the two options that matched the target image, only rotated to various degrees. Four self-paced practice problems were administered, followed by two blocks of 12 problems each, with a time limit of 3 min per block. The MRT was scored as the number of problems on which the student chose both correct options ( $M = 8.80$ ,  $SD = 4.18$ ).

### 2.3.2 | Spatial span

The Corsi Block Tapping Task was used as a measure of visuospatial working memory. Students were presented with a display of nine squares that appeared to be randomly arranged. The squares "lit up" in a predetermined sequence (constant across participants), and students were asked to tap on the squares in the same order they were lit. The sequence length started at two squares (level = 2) and could increase to up to nine squares. Students had two attempts at each sequence length. If one of the sequences was recalled correctly, the next sequence level began; if both sequences at a level were recalled incorrectly, the task was terminated. The score was the total number of correctly recalled sequences across the whole task ( $M = 8.34$ ,  $SD = 1.87$ ).

### 2.3.3 | Digit span

The tasks included both forward and backward digit spans. The forward assessment started with 3 digits and the backward with 2. For



**FIGURE 2** Instruction screen for Judgment of Line Angle and Position Test (JLAP)



each trial, students heard a sequence of digits at 1 sec intervals. The task was to recall the digit list by tapping on a circle of digits displayed on the iPad screen. If the response was correct (in digits and presentation order), the student advanced to the next level. If the response was incorrect, the same level was presented a second time. If a consecutive error occurred, the student regressed one level. Each direction (forward and then backward) ended after 14 trials. The student's score was the highest digit span correctly recalled before making two consecutive errors at the same span length ( $M = 5.68$ ,  $SD = 1.13$ , 1.20 for forward and backward, respectively).

### 2.3.4 | n-back

The measure was an adaptive version of a single n-back task with letters, following Jaeggi et al. (2010). The student was shown a "target" letter and then a random sequence of 20 consonants (6 are target; 14 are not). The task was to indicate whether the currently presented letter was a target by tapping the screen or withholding a response when the target is not present. The target letter could be the first stimuli presented ( $N = 0$ ) or the one that preceded the currently presented letter ( $N = 1$ ) or one presented two ( $N = 2$ ) or three ( $N = 3$ ) trials that preceded it.

Each trial presented a letter for 500 ms, followed by a blank screen for 2,500 ms, and then by the next letter in the sequence. Students had the entire 3,000 ms to respond if they detected a target. After instructions and three 10-item practice blocks for levels  $N = 0$  to  $N = 2$ , all participants started on level  $N = 0$ . Depending on performance, they moved up, stayed on the current level, or moved down a level for five total blocks (<3 errors—move up; 3–5 errors—repeat level; >5 errors—move down). Performance feedback (% correct) was displayed after each block. Hits (H), Misses, False Alarms (FA), and Correct Rejections were recorded and summarized by block. The score was  $(H - FA) / (\text{total blocks})$ ;  $M = 3.81$ ,  $SD = 0.75$ .

## 2.4 | Standardized measures

### 2.4.1 | Intelligence

Students were administered the Vocabulary and Matrix Reasoning subtests of the *Wechsler Abbreviated Scale of Intelligence* (WASI; Wechsler, 1999). Based on standard procedures, subscale scores were used to generate an estimated full-scale IQ.

### 2.4.2 | Achievement

Mathematics and reading achievement were assessed with the Numerical Operations and Word Reading subtests from the *Wechsler Individual Achievement Test-Third Edition* (Weschler, 2009), respectively. The Numerical Operations items included basic arithmetic and continued through fractions, algebra, geometry, and calculus.

The Word Reading assessment involved a measure of single word reading. Words begin with simple, one-syllable items and progress to more complex vowel, consonant, and morphology types.

## 2.5 | Procedure

Sixth-grade assessments lasted approximately 45 min. Students in groups of 14 to 32 were administered the mathematics tests in their mathematics classroom ( $M$  age = 12.22 and 12.27 years,  $SDs = 4.79$ , 4.20, respectively, for girls and boys,  $p = .194$ ). The remaining measures were administered in sessions in seventh grade; the cognitive measures were administered in the first semester ( $M$  age = 12.70 and 12.75 years,  $SDs = 4.69$ , 4.17, respectively, for girls and boys,  $p = .208$ ) and the intelligence ( $M = 12.95$  and 12.99 years,  $SDs = 4.76$ , 4.15, respectively, for girls and boys,  $p = .331$ ) and achievement ( $M = 13.05$  and 13.09 years,  $SDs = 4.66$ , 4.14, respectively, for girls and boys,  $p = .333$ ) measures in the second semester. Each 45 min session occurred one-on-one in a quiet location in their school. Informed written consent was obtained from students' parents, alongside assent obtained from adolescents for each assessment. This study was approved by the University of Missouri Institutional Review Board (IRB; Approval # 2002634).

## 2.6 | Analyses

There were two key sets of analyses. We first used a Bayesian approach to identify the best cognitive predictors of performance on the fractions number line and fractions arithmetic measures. The critical questions were whether one or several of the JLAP, MRT, or Corsi measures emerged as predictors of number line but not fractions arithmetic performance, and whether one of these measures mediated the anticipated advantage of boys on the number line. The second key set of analyses followed that used in the study number line placements for whole numbers (Barth & Paladino, 2011; Rouder & Geary, 2014; Slusser et al., 2013), that is, we fitted computational models indicative of logarithmic, linear, two-anchor, and three-anchor models to students' number line placements. The approach is not as optimal as having students verbally report their problem-by-problem strategic approaches (Fazio et al., 2016; Siegler et al., 2011). Nevertheless, the analyses allow for an initial assessment of whether boys and girls may have differed in these strategic approaches and an assessment of whether there were sex differences in placement accuracy for students using the same approach.

### 2.6.1 | Bayesian models

The Bayesian regressions were implemented using the *BayesFactor* package in R (v0.9.12-4.2; Morey & Rouder, 2018). Default prior scales for standardized slopes were used,  $r_{\text{scale}} = \frac{1}{2}$ . Bayes factors provide information regarding whether the inclusion of specific

variables improves model fit above and beyond other specified variables and is more robust than standard linear regression with potential multicollinearity among predictors, as is the case for our measures. These analyses allowed us to determine if the set of predictors differed across the fractions measures and to reduce the number of potential mediators of any sex differences. In separate analyses, we selected the best combination of predictors of mean absolute error on the fractions number line task and number correct for the fractions arithmetic test, and subsequently used these in the analyses of sex differences.

The first set of Bayes factors are noted as  $MFNL_m$ , where  $m$  = the specific set of predictors in the model (M) and comparisons as  $BFNL_{mn}$ , with B representing the comparison ratio of Bayes factors between models  $m$  and  $n$ .  $BFNL_{m0}$  represents a contrast of the selected predictors to a null model with no predictors. To illustrate, the full model  $MFNL_1$  included the n-back, JLAP, and IQ measures (below) as predictors of fractions number line (FNL) performance. Each of these predictors was then iteratively dropped one-by-one and change in the odds of the model was evaluated. Dropping JLAP, for instance, resulted in model  $MFNL_3$  and the comparison to the full model as  $BFNL_{31}$ . The latter resulted in a Bayes factor ratio of 0.0596, which means that the model without JLAP was 5.96% as probable as the model with JLAP, or the model including JLAP was preferred 16.8 times to 1 (1/0.0596). As a rule of thumb, models that are less than 33% as probable (preferred 3 to 1 or less) without the variable provide evidence for retaining it, and models that are less than 10% as probable provide strong evidence for retaining it (Jeffreys, 1961; Raftery, 1995). We used the 33% criterion to retain variables (Kass & Raftery, 1995).

### 2.6.2 | Strategy classification modeling

As was described in the introduction, students' placements on the visuospatial number line fit several distinct patterns that can reflect their mental representation of the underlying magnitudes or strategic approaches to making the placements. The four examined patterns are logarithmic (log), linear, and two- and three-anchor. The two-anchor strategy involves using the endpoints as anchors to help situate the placement of the fractions, and the three-anchor strategy involves the same but with inclusion of a midpoint anchor (Barth & Paladino, 2011). Following Rouder and Geary (2014), we fitted student number line placements using mixed-effects modeling (nesting trials within students) with intercepts randomly varying. To facilitate model fitting, we first rescaled the 0-to-5 line to a 0-to-1 line and then fitted four models to each students' pattern of placements; before the transform, 0 values were changed to 0.0001, 2.5 to 2.4999, and 5 to 4.999 to avoid model fit complications (i.e., log of 0). The first model served as a baseline (M1; linear model) with no number line placement transformations. The second model (M2; log model) was fitted with log transformed number line placements and correct locations. The third model (M3; 2-anchor model) was fitted where  $\emptyset(x)$  is a log-odds transformation, that is,  $\log[x/(1-x)]$ . The

fourth and final model (M4; three-anchor model) was fitted where  $\theta(x) = \log(x/0.5-x)$  when  $x < 0.5$ , and  $\log(x-0.5/1-x)$  when  $x \geq 0.5$ . More detailed descriptions of these models can be found elsewhere (Rouder & Geary, 2014), but are formalized as follows:

$$y_{ij} = \alpha_j + \beta x_{ij} + \epsilon_{ij} \quad (M1)$$

$$\log(y_{ij}) = \alpha_j + \beta \log(x_{ij}) + \epsilon_{ij} \quad (M2)$$

$$\emptyset(y_{ij}) = \alpha_j + \beta \emptyset(x_{ij}) + \epsilon_{ij} \quad (M3)$$

$$\theta(y_{ij}) = \alpha_j + \beta \theta(x_{ij}) + \epsilon_{ij} \quad (M4)$$

where for each model:

$$\alpha_j \sim N(\alpha_0, \sigma_\alpha^2)$$

Strategy classifications for each student were contrasted by comparing observed number line placements versus model predictions. As all models shared the same number of free parameters, the model with the smallest root mean squared error (RMSE) was retained as the most probable cognitive strategy used for each particular student.

## 3 | RESULTS

As described below, the strategy classification analyses indicated that the placements of only three students were best fitted by the linear model. These students were dropped, and all subsequent analyses are based on the remaining 339 (168 boys) students; the results do not change with the inclusion of these students. The dropping of these students ensured that all presented results are based on the same sample.

### 3.1 | Sex differences in overall fractions performance

The first section identifies the best predictors of the accuracy of students' placements on the visuospatial number line (i.e., n-back, JLAP, and intelligence) and their performance in fractions arithmetic (i.e., n-back, intelligence, backward digit span, and Corsi blocks). The second section describes sex differences, favoring boys, on the number line and fractions arithmetic, and reveals that these differences were mediated by JLAP and Corsi blocks, respectively.

#### 3.1.1 | Fractions performance

The potential predictors of fractions performance included digit span forward, digit span backward, n-back, intelligence, Corsi, JLAP, and MRT. As shown in the upper section of Table 1, the best set

**TABLE 1** Bayes factor analyses of predictors of fractions number line and fractions arithmetic

Model: Fractions			
Number Line	BFNL <sub>m0</sub>	Excluded	BFNL <sub>m1</sub>
MFNL <sub>1</sub> n-back +JLAP + Intelligence	$8.97 \times 10^{24}$	—	1
MFNL <sub>2</sub> n-back +JLAP	$1.31 \times 10^{13}$	Intelligence	.0000
MFNL <sub>3</sub> n-back +Intelligence	$5.35 \times 10^{23}$	JLAP	.0596
MFNL <sub>4</sub> JLAP +Intelligence	$1.18 \times 10^{23}$	n-back	.0132
Added			
MFNL <sub>5</sub> n-back +JLAP + Intelligence +Corsi	$5.45 \times 10^{24}$	Corsi	.6068
MFNL <sub>5</sub> n-back +JLAP + Intelligence +MRT	$4.10 \times 10^{24}$	MRT	.4570
Model: Fractions Arithmetic			
MFA <sub>1</sub> DSB +n-back +Corsi + Intelligence	$1.96 \times 10^{23}$	—	1
MFA <sub>2</sub> DSB +n-back +Corsi	$4.70 \times 10^{17}$	Intelligence	.0000
MFA <sub>3</sub> DSB +n-back +Intelligence	$5.23 \times 10^{21}$	Corsi	.0267
MFA <sub>4</sub> DSB +Corsi + Intelligence	$2.88 \times 10^{22}$	n-back	.1468
MFA <sub>5</sub> n-back +Corsi + Intelligence	$2.89 \times 10^{21}$	DSB	.0147
Added			
MFA <sub>6</sub> DSB +n-back +Corsi + Intelligence +JLAP	$1.44 \times 10^{23}$	JLAP	.7321
MFA <sub>7</sub> DSB +n-back +Corsi + Intelligence +MRT	$1.84 \times 10^{21}$	MRT	.2481

Abbreviations: DSB, Digit Span Backward; Corsi, Corsi Block Tapping Task; JLAP, Judgment of Line Angle and Position Test; MFNL, Models for fractions number line; MFA, Models for fractions arithmetic.

of predictors of the mean absolute error of students' number line placements was the n-back, JLAP, and intelligence measures (alternative models are in the SOM). Dropping intelligence resulted in a model that was <1% as probable as the model that included it and dropping JLAP and n-back resulted in models that were 5.96% and 1.32% as probable as the models with them. Thus, all measures were retained.

On the basis of the findings for fractions arithmetic (below) and to ensure that different visuospatial abilities are predicting number line and fractions arithmetic performance, we included the Corsi measure with these three variables. The model that included Corsi was only 60.68% as probable as the model without it. The model that included n-back, JLAP, and intelligence was preferred 11.79 to 1 over the model that included n-back, Corsi, and intelligence (see SOM). To further increase our confidence in the JLAP result, we

added MRT to the three core predictors, which produced a model that was 45.7% as probable as the model without it (Table 1). The model that included n-back, JLAP, and intelligence was preferred 18.51 to 1 over the model that included n-back, MRT, and intelligence (SOM). The combination provides strong evidence that JLAP is a better predictor of number line performance than Corsi or MRT.

As shown in the lower section of Table 1, the best set of predictors of fractions arithmetic performance also included n-back and intelligence, as well as the backward digit span and Corsi measures. Dropping each of these measures in succession resulted in models that were <15% as probable as models with them, and thus all were retained.

Adding JLAP resulted in a model that was only 73.21% as probable as the model without it. The model that included n-back, intelligence, backward digit span, and Corsi was preferred 20.86 to 1 over the model that included n-back, intelligence, backward digit span, and JLAP (SOM). Adding MRT resulted in a model that was 24.81% as probable as the model without it, suggesting MRT might contribute to the prediction of fractions arithmetic. However, the model that included n-back, intelligence, backward digit span, and Corsi was preferred 106.9 to 1 over the model that included n-back, intelligence, backward digit span, and MRT (SOM). On the basis of the latter result, we retained the four original predictors (i.e., n-back, intelligence, backward digit span, and Corsi).

Follow-up multi-level analyses using Proc Mixed (SAS, 2014) were used to further assess the specificity of these findings. All multi-level models used students as level 1 units and six schools as level 2 units, allowing intercepts to vary randomly for schools. We used the core variables identified in the Bayesian analyses and included the two other visuospatial measures as predictors. So, number line accuracy was predicted by n-back, intelligence, and JLAP (as identified in the Bayesian analyses), and Corsi and MRT were included to further assess the specificity of the relation between JLAP and fraction number line performance.

Lower mean absolute error on the fractions number line was associated with higher n-back,  $t(328) = -3.19$ ,  $p = .002$ , JLAP,  $t(328) = -2.69$ ,  $p = .008$ , and intelligence,  $t(328) = -6.63$ ,  $p < .001$ , scores, but was not related to performance on the Corsi,  $t(328) = -1.81$ ,  $p = .071$ , or MRT,  $t(328) = -1.40$ ,  $p = .161$ , measures. The same approach (including the variables identified in the Bayesian analyses and the two other spatial measures) revealed that better fractions arithmetic scores were associated with higher backward digit span,  $t(327) = 3.38$ ,  $p = .001$ , n-back,  $t(327) = 2.63$ ,  $p = .009$ , intelligence,  $t(327) = 4.88$ ,  $p < .001$ , and Corsi,  $t(327) = 3.18$ ,  $p = .002$ , scores, but not with performance on the JLAP,  $t(327) = 1.64$ ,  $p = .102$ , or MRT,  $t(327) = 0.86$ ,  $p = .392$ , measures.

One key finding is that independent of intelligence and performance on commonly used measures of working memory (i.e., n-back, backward digit span), performance on the fractions number line and fractions arithmetic measures was related to different aspects of spatial ability. Performance on a measure of visuospatial attention (i.e., JLAP) emerged as a much better predictor of number line performance than did performance on a spatial span task (i.e., Corsi),

Measure	Girls	Boys	t	p	d
	M (SD)	M (SD)			
Fractions number line	0.19 (0.13)	0.16 (0.14)	2.49	.013	-0.22
Fractions arithmetic	11.63 (7.49)	13.34 (7.52)	2.17	.030	0.23
Digit span: Backward	4.59 (1.18)	4.54 (1.22)	0.10	.921	-0.04
n-back	3.77 (0.70)	3.86 (0.80)	1.27	.205	0.12
Corsi	8.10 (1.99)	8.55 (1.71)	2.37	.018	0.24
JLAP	12.66 (3.05)	14.04 (2.96)	4.22	.001	0.45
Intelligence	104.23 (13.65)	104.98 (12.62)	0.99	.324	0.06
7th-Grade Numerical Operations	99.28 (18.80)	100.74 (19.08)	1.15	.252	0.08
7th-Grade Word Reading	103.93 (13.40)	104.74 (12.95)	0.86	.391	0.06

Note: The score for fractions number line is absolute percent error and thus lower scores represent better performance.  $d$  = mean difference/(pooled SD). Positive  $d$  values represent higher scores for boys. The sex differences were tested using multi-level models, with students nested in schools,  $df = 332$ .

Abbreviation: JLAP, Judgment of Line Angle and Position Test.

**TABLE 2** Mean sex differences for core variables

whereas spatial span emerged as a much better predictor of fractions arithmetic than did visuospatial attention. Thus, the JLAP and Corsi measures were assessed as potential mediators of sex differences on the fractions measures (below).

### 3.1.2 | Sex differences

Table 2 shows sex differences for the fractions measures, the cognitive measures that were retained based on the Bayesian analyses and the achievement tests; correlations among these measures are in the Appendix. Sex differences were assessed with multi-level models. The core prediction of a male advantage on the number line was confirmed ( $p = .013$ ,  $d = -0.22$ ). As shown in the density plots in Figure 3, boys had smaller placement errors than did girls. The advantage for boys also extended to fractions arithmetic ( $p = .03$ ,  $d = 0.23$ ), which indicates that a sex difference on the fractions number line in and of itself is not sufficient evidence for a number-spatial foundation for representing the mental number line.

In other words, Thompson and Opfer (2008) predicted (but did not directly assess) that boys would make more accurate placements on the number line than would girls because of boys' advantage in spatial abilities. The results thus far are consistent with this prediction but the finding of a sex difference for fractions arithmetic leaves open the possibility that boys' advantage on the number line is due to a general advantage in fractions knowledge rather than directly related to visuospatial abilities.

The Bayesian analyses, however, suggested that boys' advantage on the number line could be related to their advantage in visuospatial abilities. This is because the Bayesian analyses indicated that JLAP was strongly predictive of fractions number line but, critically, not fractions arithmetic performance, controlling other

identified predictors (e.g., n-back, intelligence). Moreover, boys had an advantage on the JLAP ( $p < .001$ ,  $d = 0.45$ , and it was significantly correlated with number line performance ( $r = -.34$ ,  $p < .001$ ). A mediation analysis using the *lavaan* package in R (Rosseel, 2012) revealed that performance on the JLAP fully mediated boys' advantage on the number line. The associated path estimates and indirect effect of sex on fractions number line performance are shown in Figure 4.

Corsi was the only task to emerge as a predictor of fractions arithmetic performance that showed a sex difference, favoring boys ( $p = .018$ ,  $d = 0.24$ ). A post hoc analysis revealed that performance on the Corsi task was significantly correlated with fractions arithmetic ( $r = .35$ ,  $p < .001$ ), and fully mediated boys' advantage in fractions arithmetic. The associated path estimates and indirect effect of sex on fractions arithmetic performance are shown in Figure 5.

### 3.2 | Sex differences in placement strategies

Students' placement strategies were first restricted to the linear and log models that have been extensively studied in the context of whole number placements on the visuospatial number line. Here, 56% (190, 100 boys) of the students were better fitted by the linear model and 44% (152, 69 boys) by the log model, as is typically found for school-age students (e.g., Thompson & Opfer, 2008). With consideration of all four models, the placements of only three students (2 girls) were best fitted by the linear model and, thus, these students were dropped because their numbers were too small to assess any potential sex differences. Of the remaining students, the placements of 23% (77, 30 boys) of them were best fitted by the log model and 14% (47, 26 boys) and 63% (215, 112 boys) were best fitted by the two- and

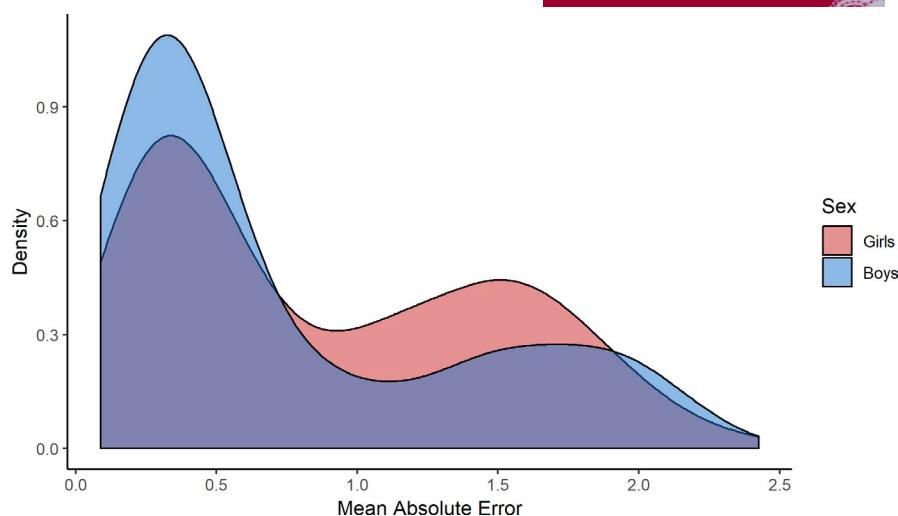


FIGURE 3 Density plots of the distributions of the accuracy of boys' and girls' fractions number line placements

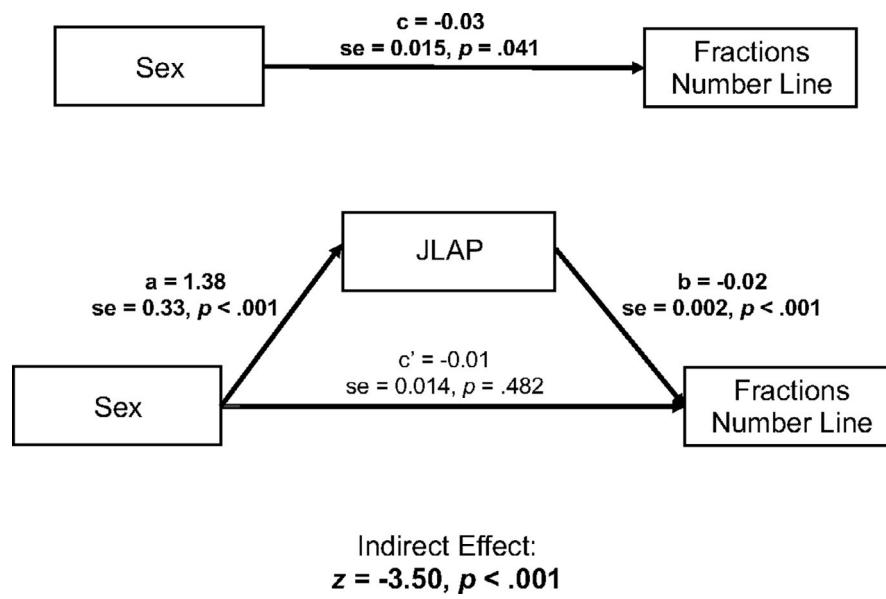
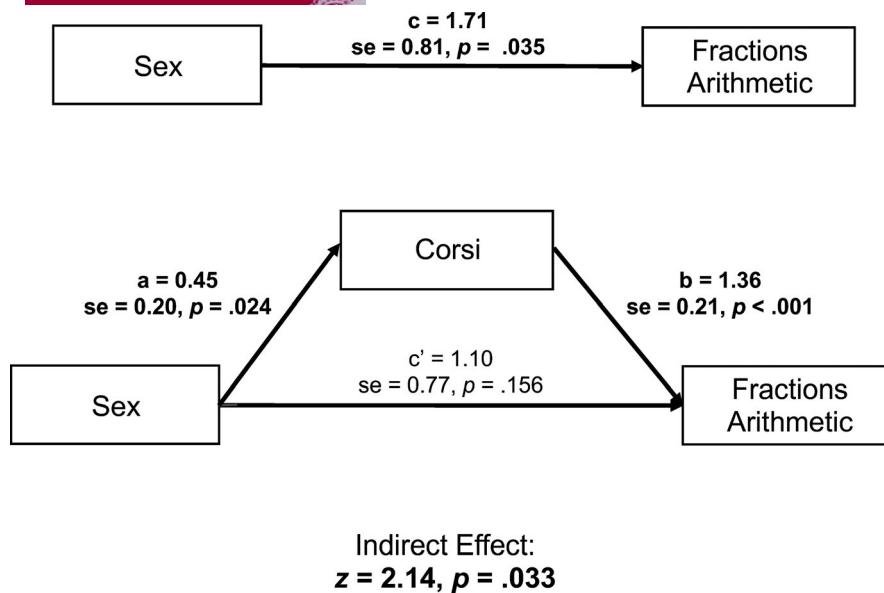


FIGURE 4 Sex differences on JLAP (Judgment of Line Angle and Position) fully mediated the sex difference for accuracy of placements on the fractions number line. Significant effects are in bold

three-anchor models, respectively. Table 3 shows the mean root mean square error (RMSE) or model fits for these students.

Figure 6 shows the pattern of placements for boys and girls in each of these groups. There was no sex difference in the frequency of boys and girls across these strategies,  $\chi^2(2) = 4.64, p = .099$ , suggesting, at least within the limitations of this approach, that boys' advantage was not due to different strategic approaches to number line placements. There were not enough students in the log and two-anchor groups to examine sex differences, but multi-level analyses for the three-anchor group showed that boys had smaller errors than did girls, as shown in Table 4. In fact, the average placement of boys in this group was close to linear, but with enough subtle within-line variation in placements to suggest the use of the three-anchor strategy.

Next, we used a logistic regression to classify students into the log and three-anchor groups using n-back, intelligence, and JLAP as predictors; we excluded the two-anchor group due to the relatively small sample size. These analyses provided insight into whether one or several of the predictors of individual differences in the accuracy of number line placements are also important for predicting broader strategic approaches to the task. The students in the log group ( $M = 3.56, SD = 0.71$ ) had lower n-back scores than those in the three-anchor group ( $M = 3.92, SD = 0.75$ ,  $p < .001, d = -0.47$ ). The same was found for intelligence ( $M = 97.75, SD = 12.42, M = 107.25, SD = 12.34$ , respectively),  $p < .001, d = -0.73$ , and JLAP, ( $M = 11.96, SD = 2.76, M = 13.95, SD = 2.99$ , respectively),  $p < .001, d = -0.65$ .



**FIGURE 5** Sex differences on the Corsi Block Tapping Task fully mediated the sex difference for fractions arithmetic. Significant effects are in bold

**TABLE 3** Mean RMSE for students classified into the strategy groups

Strategy group	Root mean square error (Model fit)			
	Linear	Log	2-Anchor	3-Anchor
Log	0.1890	<b>0.1260</b>	0.3114	0.2006
2-Anchor	0.1381	0.1528	<b>0.0891</b>	0.1133
3-Anchor	0.1481	0.1781	0.1174	<b>0.0702</b>

Bold values represents the smallest number in each column is the preferred model.

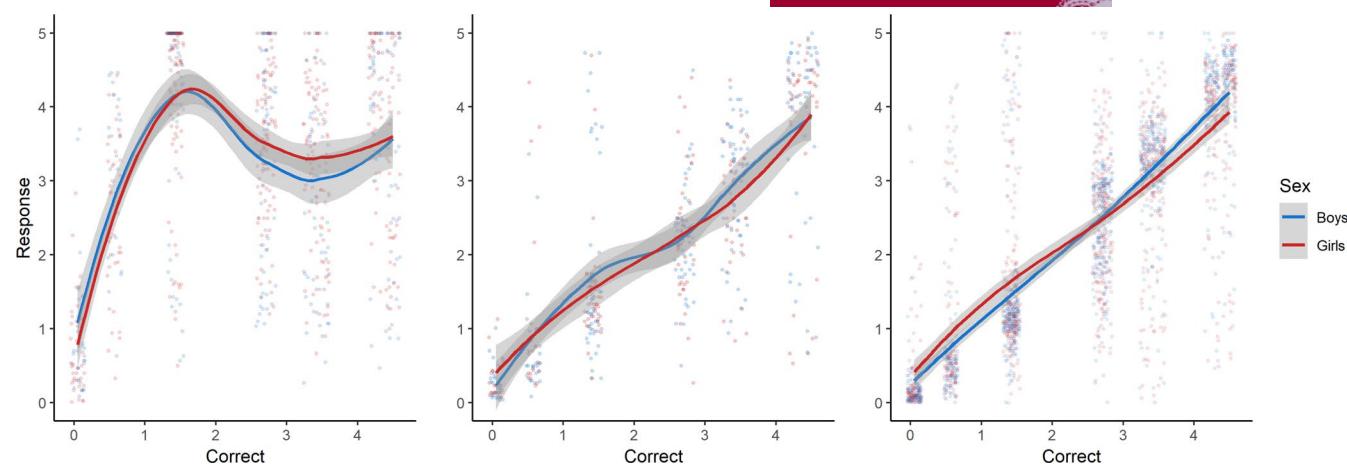
The initial logistic results revealed that n-back was not a significant predictor of group membership ( $p = .265$ ), controlling intelligence and JLAP, and was dropped. Using only intelligence and JLAP, the classification of students as using the log or three-anchor strategy was significant,  $\text{Wald } \chi^2(2) = 35.32, p < .001$ , as were the estimates for intelligence,  $\chi^2(1) = 16.13, p < .001$ , and JLAP,  $\chi^2(1) = 10.76, p = .001$ . A 1 SD increase in JLAP scores resulted in a 18.4% increase in the odds of being in the three-anchor group, controlling intelligence, whereas a 1 SD increase in intelligence resulted in a 4.9% increase in the odds of being in the three-anchor group, controlling JLAP. Overall, 74.2% of students were correctly classified into their strategy group. The Cohen's  $d$  of the log odds of group membership is identical to the multivariate Mahalanobis distance (i.e., multivariate  $d$ ) and was large,  $d = 0.92$  [95% CI = 0.61 to 1.21]. These analyses provide an additional confirmation that, independent of any sex differences, performance on the JLAP is an important differential predictor of performance on the fractions number line, potentially related to competence at mentally segmenting the line and using anchors to facilitate placements on the line.

## 4 | DISCUSSION

Independent of working memory and intelligence, a measure of visuospatial attention predicted individual and sex differences in the accuracy with which fractions magnitudes were placed on the visuospatial number line. Visuospatial attention might have also contributed to the adoption of the sophisticated three-anchor strategy for making dynamic number line placements but did not contribute to individual or sex differences in fractions arithmetic, controlling other factors. We discuss the implications in terms of overall performance on the visuospatial number line and strategic approaches to number line placements.

### 4.1 | Number line performance

The learning of fractions concepts and to correctly use procedures during fractions arithmetic is central to children's mathematical development (National Mathematics Advisory Panel, 2008), but is a prolonged and difficult undertaking (Siegler & Lortie-Forgues, 2017). There are multiple reasons for this, including interference from whole number learning (e.g., whole number multiplication results in larger products, whereas fractions multiplication results in smaller ones). From a broader perspective, the learning is difficult because the content is evolutionarily novel. Intelligence and working memory are important from this perspective because they have been framed as evolved systems for coping with variation and change or novelty within and across lifetimes (Geary, 2005). Much of the academic learning that occurs in schools is novel and thus intelligence and working memory are expected to contribute to individual differences in the rate of academic learning (Geary, 2008; Paas & Ayres, 2014; Sweller et al., 2019), and this was the case for



**FIGURE 6** Plots of boys' and girls' placements on the 0-to-5 fractions number line for students characterized as using log, two-anchor, and three-anchor strategies, respectively

**TABLE 4** Sex differences in the three-anchor group

Measure	Girls		Boys		
	M (SD)	M (SD)	t	p	d
Fractions number line	0.14 (0.11)	0.11 (0.11)	2.10	.037	-0.27
n-back	3.87 (0.70)	3.97 (0.80)	1.06	.290	0.13
JLAP	13.34 (2.83)	14.51 (3.03)	2.93	.004	0.39
Intelligence	107.34 (13.41)	107.16 (11.33)	0.40	.686	-0.01

Note: The score for fractions number line is absolute percent error and thus lower scores represent better performance.  $d$  = mean difference/(pooled SD). Positive  $d$  values represent higher scores for boys.

Abbreviation: JLAP, Judgment of Line Angle and Position Test.

the fractions competencies assessed here and for mathematics generally (Geary et al., 2017; Lee & Bull, 2016).

Whether or not we take an evolutionary perspective, the empirical control of intelligence and working memory strengthens the evidence for unique contributions of visuospatial abilities to fractions learning. The critical result was the finding that performance on the JLAP—a measure that is sensitive to the visuospatial attention systems supported (in part) by parts of the right parietal cortex (Benton et al., 1978; García-Sánchez et al., 1997; Tranel et al., 2009)—was uniquely predictive of accuracy of number line placements and was more important than was the Mental Rotation Test or visuospatial working memory. The combination suggests that visuospatial attention is more important than other spatial abilities for the accurate placement of fractions on the visuospatial number line. This finding is consistent with prior neuropsychological and experimental studies of the functioning of the underlying parietal brain regions and accuracy of positioning whole numbers along a continuum (Longo & Lourenco, 2007; Zorzi et al., 2002, 2012).

The sex differences on the fractions number line are consistent with previous studies and relatively small in magnitude (Bull et al., 2013; Hutchison et al., 2019; Rivers et al., 2020; Thompson & Opfer, 2008). For sixth graders' placements of whole numbers on

the visuospatial number line, Hutchinson et al. found that boys had an advantage on both the 0–100 ( $d = -.27$ ) and 0–1000 ( $d = -.23$ ) line. The magnitude of these sex differences is consistent with our finding for the fractions number line (i.e.,  $d = -.22$ ). Our finding that boys had an advantage on the JLAP is also consistent with previous results (Collaer et al., 2002; Gur et al., 2000), and the combined findings are consistent with the predicted relation between spatial abilities and number line performance (Gilligan et al., 2019; Thompson & Opfer, 2008). The sex difference on the number line emerged because boys were able to make more precise fraction magnitude to visuospatial number line mappings that, in turn, were mediated by their advantage on the JLAP.

Collaer and Nelson (2002) suggested the sex difference on the JLAP may be due to boys' and men's greater allocation of attention to Euclidean features of space as related to navigation, which would result in a heightened sensitivity to directional orientation (Goyette et al., 2012); for instance, implicitly understanding that moving from the current location to the desired one requires orienting 30 degrees westward and maintaining that orientation during travel. Even if the JLAP is sensitive to an evolved bias in the systems that support relative orientation during navigation, the development of these competencies and any associated sex differences is almost certainly also

influenced by spatial experiences during development (Geary, 2021; Levine et al., 2005).

Whatever the contributing factors, accurate placements on the number line require precise angular orientation as students move their hands to position the numeral on the line (Dotan & Dehaene, 2016; Kim & Opfer, 2018). Boys' heightened ability to attentionally focus on subtle deviation in the angular orientation of environmental cues appears to provide them with an advantage in precisely situating numerals on visuospatial representations of the mathematical number line. This conclusion must be considered tentative, however, and in need of replication. This is because our finding for the JLAP and number line performance was not based on an *a priori* prediction, that is, we anticipated a spatial contribution to boys' advantage on the number line but did not know which spatial ability would emerge as the most critical to number line performance.

The results for fractions arithmetic were unexpected because sex differences in computational arithmetic typically favor girls (Hyde et al., 1990), although most of these studies have focused on whole number computations (Marshall & Smith, 1987). Even with the unexpected sex difference, the critical finding is that boys' advantage was not related to the sex difference on the JLAP that in turn provides discriminant validity for its relation to fractions number line performance. Individual differences in fractions arithmetic were predicted by a combination of intelligence and working memory measures, including visuospatial working memory. The latter is often associated with general mathematics achievement (Li & Geary, 2017), but the specific relation between this form of working memory and fractions arithmetic has not been well studied (Peng et al., 2016). The specificity of the relation between visuospatial working memory and fractions arithmetic performance, as well as the associated sex differences would benefit from replication.

The overall results also point to specificity in the relations between visuospatial abilities and mathematics learning and achievement, and any associated sex differences. The specificity is shown in this study with the finding that performance on the number line was predicted by the JLAP and not by visuospatial working memory, whereas fractions arithmetic was predicted by the latter and not the former. In the broader literature, performance on the Mental Rotation Test is often correlated with performance in more complex mathematical domains (e.g., word problems, the mathematics section of the SAT), and appears to contribute to sex differences in these domains (Casey et al., 1997; Geary et al., 2000; Halpern et al., 2007). Most of these previous studies only used a single measure of spatial ability and often general measures of mathematics achievement, which would obscure any more specific relations between different spatial abilities and performance in different mathematical domains. Placed in the context of the broader literature, our results suggest more nuance in the relation between sex differences in mathematics and spatial abilities than is commonly appreciated and calls for more fine-grain assessments of these relations in future studies.

## 4.2 | Strategic approaches

Recent studies suggest that the process of situating whole numbers and fractions on the visuospatial number line is dynamic, whereby any bias to separate smaller values and compress larger ones (i.e., the logarithmic pattern) is inhibited in favor of some type of strategic approach to make the placements more linear (Dotan & Dehaene, 2016; Kim & Opfer, 2018; Siegler et al., 2011). A dynamic approach allows for multiple influences on the process of placing numerals on the number line, including room for inherent biases and use of strategies to enable more linear placements. The current study does not address the details of these dynamics but provides evidence for something similar to a logarithmic pattern of placements among the lower-achieving students in our sample and the use of endpoint and midpoint anchors for higher-achieving ones.

The number line placements of students in the log group (Figure 6) are consistent with the often-found logarithmic pattern up through 2, but the placements for higher values are near the midpoint of the line. We cannot be certain, but such a pattern could emerge if many of these students were unsure about the magnitudes represented by our larger-valued fractions (e.g., 17/4) and thus made an informed guess as to where they should be situated on the line. Any such guesses were informed and not random because most of the placements were beyond the midpoint of the line, which indicates they knew the values were relatively large but were not yet able to place them accurately. Verbal reports of the specific strategies used in making these placements will be needed to more fully understand how these students were mapping larger-valued fractions (e.g., based on the size of the numerator or denominator) to the number line (Siegler et al., 2011). Whatever strategy they were using, the accuracy of placements around and larger than the midpoint would have been improved had they used an anchor strategy (Barth & Paladino, 2011; Rouder & Geary, 2014; Slusser et al., 2013), assuming they understood the magnitude of larger-valued fractions (Siegler et al., 2011).

It is not clear why they had not adopted this approach when the majority of their peers had, but the logistic regression suggested that their relatively lower (but still average) intelligence and visuospatial attention (JLAP) might have been contributing factors. The finding that working memory (i.e., n-back) emerged as a predictor of individual but not group differences in number line performance suggests it might be more important during the act of making placements than for adopting an anchor strategy. Intelligence is likely to be related to how quickly students realize that the smaller-to-larger structure of the mental number line can be analogically mapped onto visuospatial representations of the mathematical number line (Sullivan & Barner, 2014), and most of these students appear to have made this mapping (e.g., smaller-valued fractions are placed at the left of the line). Intelligence might also be related to the ease with which students discover the usefulness anchors in making number line placements. The effect for the JLAP was even stronger than that for intelligence. The implication is that the visuospatial attentional abilities that support people's ability to represent and bisect



the mental number line might also, in combination with intelligence, contribute to the discovery of strategic approaches to dealing with the mathematical number line.

On this view, we might have expected boys to use more sophisticated placement strategies than girls, but this is not what we found. However, our computational models should be interpreted as relatively coarse measures of strategic approaches and verbal reports of the varied ways in which people process fractions magnitudes may reveal more subtle sex differences (Fazio et al., 2016; Siegler et al., 2011). Whatever the strategic approaches, the sex difference for number line performance is found during the process of moving from novice to expert and becomes smaller as most students learn the line and make accurate placements (Hutchison et al., 2019; Thompson & Opfer, 2008). The common use of the three-anchor strategy and relatively low placement errors for the majority of our students indicates that the overall sample was well along the path to expertise, at least with respect to understanding fractions along the 0-to-5 number line. Longitudinal studies will be needed to determine if boys adopt the two- and three-anchor strategies—or other approaches to segmenting the line—earlier than girls, and if the developmental sex differences in fractions number line learning follows that found for whole numbers (Hutchison et al., 2019); specifically, whether the sex differences on the fractions number line will eventually disappear.

### 4.3 | Limitations and conclusions

The correlational nature of the data precludes causal statements. Although we assessed a much broader array of spatial and cognitive abilities as potential predictors of number line performance than is typical in this literature, there may be other factors that we did not include. Based on findings for whole numbers, we anticipated that boys would have an advantage on the fractions number line and that this advantage would be mediated by spatial abilities but were agnostic as to which spatial measure would emerge as the most critical. Thus, as noted, the findings for the JLAP are consistent with prior studies (Longo & Lourenco, 2007; Zorzi et al., 2002), but should be considered exploratory and in need of replication. Despite these limitations and caveats, the study provided a more thorough assessment of the relation between spatial ability, number line learning, and associated sex differences than has heretofore been conducted. The results add nuance to our understanding of the specific visuospatial abilities that contribute to learning the evolutionarily novel number line and provide directions for follow-up studies.

### ACKNOWLEDGEMENTS

The study was supported by grants R01 HD087231 from the Eunice Kennedy Shriver National Institute of Child Health and Human Development and DRL-1659133 from the National Science Foundation. We thank Dana Hibbard of Columbia Public Schools, as well as the staff, faculty, students, and families of Columbia Public Schools for their assistance with the logistics of the study. We thank Kristin Balentine, Mandar Bhojar, Amanda Campbell, Maria Ceriotti,

Felicia Chu, Anastasia Compton, Alexis Currie, Kaitlynn Dutzy, Amanda Evans, James Farley, Amy Jordan, Joseph LaMendola, Bradley Lance, Kate Leach, Joshua McEwen, Kelly Mebruer, Heather Miller, Sarah Peterson, Nicole Reimer, Laura Roider, Brandon Ryffe, Logan Schmidt, and Melissa Willoughby for their assistance with data collection and processing. We also thank Edward Merkle for advice on the computational modeling.

### DATA AVAILABILITY STATEMENT

The data that support the findings of this study and associated R code are available in the Open Science Framework, <https://osf.io/2kjt6/>.

### REFERENCES

- Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science*, 14, 125–135. <https://doi.org/10.1111/j.1467-7687.2010.00962.x>
- Benton, A. L., Varney, N. R., & Hamsher, K. D. (1978). Visuospatial judgment: A clinical test. *Archives of Neurology*, 35, 364–367. <https://doi.org/10.1001/archneur.1978.00500300038006>
- Braithwaite, D. W., Leib, E. R., Siegler, R. S., & McMullen, J. (2019). Individual differences in fraction arithmetic learning. *Cognitive Psychology*, 112, 81–98. <https://doi.org/10.1016/j.cogpsych.2019.04.002>
- Braithwaite, D. W., Tian, J., & Siegler, R. S. (2018). Do children understand fraction addition? *Developmental Science*, 21, e12601. <https://doi.org/10.1111/desc.12601>
- Bueti, D., & Walsh, V. (2009). The parietal cortex and the representation of time, space, number and other magnitudes. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364, 1831–1840. <https://doi.org/10.1098/rstb.2009.0028>
- Bull, R., Cleland, A. A., & Mitchell, T. (2013). Sex differences in the spatial representation of number. *Journal of Experimental Psychology: General*, 142, 181–192. <https://doi.org/10.1037/a0028387>
- Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, 8, 36–41. <https://doi.org/10.1111/cdep.12059>
- Carpenter, P. A., Just, M. A., Keller, T. A., Eddy, W., & Thulborn, K. (1999). Graded functional activation in the visuospatial system with the amount of task demand. *Journal of Cognitive Neuroscience*, 11, 9–24. <https://doi.org/10.1162/089892999563210>
- Case, R., & Okamoto, Y. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development*, 61(1/2), 1–2. <https://doi.org/10.2307/1166077>
- Casey, M. B., Nuttal, R. L., & Pezaris, E. (1997). Mediators of gender differences in mathematics college entrance test scores: A comparison of spatial skills with internalized beliefs and anxieties. *Developmental Psychology*, 33, 669–680. <https://doi.org/10.1037/0012-1649.33.4.669>
- Cattell, R. B. (1963). Theory of fluid and crystallized intelligence: A critical experiment. *Journal of Educational Psychology*, 54, 1–22. <https://doi.org/10.1037/h0046743>
- Ceci, S. J., & Williams, W. M. (2010). *The mathematics of sex: How biology and society conspire to limit talented women and girls*. Oxford University Press.
- Chechlacz, M., Rotstein, P., & Humphreys, G. W. (2014). Neuronal substrates of Corsi Block span: Lesion symptom mapping analyses in relation to attentional competition and spatial bias. *Neuropsychologia*, 64, 240–251. <https://doi.org/10.1016/j.neuropsychologia.2014.09.038>

- Cicchini, G. M., Anobile, G., & Burr, D. C. (2014). Compressive mapping of number to space reflects dynamic encoding mechanisms, not static logarithmic transform. *Proceedings of the National Academy of Sciences of the United States of America*, 111, 7867–7872. <https://doi.org/10.1073/pnas.1402785111>
- Cohen, D. J., & Blanc-Goldhammer, D. (2011). Numerical bias in bounded and unbounded number line tasks. *Psychonomic Bulletin & Review*, 18, 331–338. <https://doi.org/10.3758/s13423-011-0059-z>
- Collaer, M. L., & Nelson, J. D. (2002). Large visuospatial sex difference in line judgment: Possible role of attentional factors. *Brain and Cognition*, 49, 1–12. <https://doi.org/10.1006/brcg.2001.1321>
- Collaer, M., Reimers, L., & Manning, S. (2007). Visuospatial performance on an internet line judgment task and potential hormonal markers: Sex, sexual orientation, and 2D:4D. *Archives of Sexual Behavior*, 36, 177–192. <https://doi.org/10.1007/s10508-006-9152-1>
- Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science*, 320, 1217–1220. <https://doi.org/10.1126/science.1156540>
- Dotan, D., & Dehaene, S. (2016). On the origins of logarithmic number-to-position mapping. *Psychological Review*, 123, 637–666. <https://doi.org/10.1037/rev0000038>
- Fazio, L. K., DeWolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 42, 1–16. <https://doi.org/10.1037/xlm0000153>
- Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8, 307–314. <https://doi.org/10.1016/j.tics.2004.05.002>
- Gallistel, C. R. (1990). *The organization of learning*. Cambridge, MA: The MIT Press.
- Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, 4, 59–65. [https://doi.org/10.1016/s1364-6613\(99\)01424-2](https://doi.org/10.1016/s1364-6613(99)01424-2)
- García-Sánchez, C., Estévez-González, A., Suárez-Romero, E., & Junqué, C. (1997). Right hemisphere dysfunction in subjects with attention-deficit disorder with and without hyperactivity. *Journal of Child Neurology*, 12, 107–115. <https://doi.org/10.1177/088307389701200207>
- Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.
- Geary, D. C. (1995). Reflections of evolution and culture in children's cognition: Implications for mathematical development and instruction. *American Psychologist*, 50, 24–37.
- Geary, D. C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences*, 19, 229–284. <https://doi.org/10.1017/S0140525X00042400>
- Geary, D. C. (2005). *The origin of mind: Evolution of brain, cognition, and general intelligence*. Washington, DC: American Psychological Association.
- Geary, D. C. (2008). An evolutionarily informed education science. *Educational Psychologist*, 43, 279–295. <https://doi.org/10.1080/00461520802392133>
- Geary, D. C. (2021). *Male, female: The evolution of human sex differences* (3rd ed.). Washington, DC: American Psychological Association.
- Geary, D. C., Berch, D. B., & Mann Koepke, K. (Eds.) (2015). *Evolutionary origins and early development of number processing* (Vol. 1, *Mathematical Cognition and Learning*). San Diego, CA: Elsevier Academic Press.
- Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study. *Journal of Educational Psychology*, 109, 680–693. <https://doi.org/10.1037/edu0000159>
- Geary, D. C., Saults, S. J., Liu, F., & Hoard, M. K. (2000). Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. *Journal of Experimental Child Psychology*, 77, 337–353. <https://doi.org/10.1006/jecp.2000.2594>
- Giles, O. T., Shire, K. A., Hill, L. J. B., Mushtaq, F., Waterman, A., Holt, R. J., Culmer, P. R., Williams, J. H. G., Wilkie, R. M., & Mon-Williams, M. (2018). Hitting the target: Mathematical attainment in children is related to interceptive-timing ability. *Psychological Science*, 29, 1334–1345. <https://doi.org/10.1177/0956797618772502>
- Gilligan, K. A., Hodgkiss, A., Thomas, M. S., & Farran, E. K. (2019). The developmental relations between spatial cognition and mathematics in primary school children. *Developmental Science*, 22, e12786. <https://doi.org/10.1111/desc.12786>
- Göbel, S. M., Calabria, M., Farne, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: simulating 'spatial' neglect in healthy subjects. *Neuropsychologia*, 44(6), 860–868. <https://doi.org/10.1016/j.neuropsychologia.2005.09.007>
- Goyette, S. R., McCoy, J. G., Kennedy, A., & Sullivan, M. (2012). Sex differences on the judgment of line orientation task: A function of landmark presence and hormonal status. *Physiology & Behavior*, 105, 1045–1051. <https://doi.org/10.1016/j.physbeh.2011.11.018>
- Gur, R. C., Alsop, D., Glahn, D., Petty, R., Swanson, C. L., Maldjian, J. A., Turetsky, B. I., Detre, J. A., Gee, J., & Gur, R. E. (2000). An fMRI study of sex differences in regional activation to a verbal and a spatial task. *Brain and Language*, 74, 157–170. <https://doi.org/10.1006/brln.2000.2325>
- Halpern, D., Benbow, C., Geary, D. C., Gur, R., Hyde, J., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological Science in the Public Interest*, 8(1), 1–51. <https://doi.org/10.1111/j.1529-1006.2007.00032.x>
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, 6, 435–448. <https://doi.org/10.1038/nrn1684>
- Hutchison, J. E., Lyons, I. M., & Ansari, D. (2019). More similar than different: Gender differences in children's basic numerical skills are the exception not the rule. *Child Development*, 90, e66–e79. <https://doi.org/10.1111/cdev.13044>
- Hyde, J. S., Fennema, E., & Lamon, S. (1990). Gender differences in mathematics performance: A meta-analysis. *Psychological Bulletin*, 107, 139–155. <https://doi.org/10.1037/0033-2909.107.2.139>
- Jaeggi, S. M., Studer-Luethi, B., Buschkuhl, M., Su, Y.-F., Jonides, J., & Perrig, W. J. (2010). The relationship between n-back performance and matrix reasoning—Implications for training and transfer. *Intelligence*, 38(6), 625–635. <https://doi.org/10.1016/j.intell.2010.09.001>
- Jeffreys, H. (1961). *Theory of probability* (3rd ed.). New York, NY: Oxford University Press.
- Johnson, E. S. (1984). Sex differences in problem solving. *Journal of Educational Psychology*, 76, 1359–1371. <https://doi.org/10.1037/0022-0663.76.6.1359>
- Kass, R. E., & Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association*, 90, 773–795. <https://doi.org/10.1080/01624595.10476572>
- Kell, H. J., Lubinski, D., Benbow, C. P., & Steiger, J. H. (2013). Creativity and technical innovation: Spatial ability's unique role. *Psychological Science*, 24, 1831–1836. <https://doi.org/10.1177/0956797613478615>
- Kersey, A. J., Braham, E. J., Csumitta, K. D., Libertus, M. E., & Cantlon, J. F. (2018). No intrinsic gender differences in children's earliest numerical abilities. *NPJ Science of Learning*, 3, 12. <https://doi.org/10.1038/s41539-018-0028-7>
- Kim, D., & Opfer, J. E. (2018). Dynamics and development in number-to-space mapping. *Cognitive Psychology*, 107, 44–66. <https://doi.org/10.1016/j.cogpsych.2018.10.001>
- Koscik, T., O'Leary, D., Moser, D. J., Andreasen, N. C., & Nopoulos, P. (2009). Sex differences in parietal lobe morphology: Relationship

- to mental rotation performance. *Brain and Cognition*, 69, 451–459. <https://doi.org/10.1016/j.bandc.2008.09.004>
- Lauer, J. E., Yhang, E., & Lourenco, S. F. (2019). The development of gender differences in spatial reasoning: A meta-analytic review. *Psychological Bulletin*, 145, 537–565. <https://doi.org/10.1037/bul0000191>
- Lee, K., & Bull, R. (2016). Developmental changes in working memory, updating, and math achievement. *Journal of Educational Psychology*, <https://doi.org/10.1037/edu0000090>
- Levine, S. C., Vasilyeva, M., Lourenco, S. F., Newcombe, N., & Huttenlocher, J. (2005). Socioeconomic status modifies the sex differences in spatial skill. *Psychological Science*, 16, 841–845. <https://doi.org/10.1111/j.1467-9280.2005.01623.x>
- Lewis, A. B. (1989). Training students to represent arithmetic word problems. *Journal of Educational Psychology*, 81, 521–531. <https://doi.org/10.1037/0022-0663.81.4.521>
- Lewis, M. R., Matthews, P. G., & Hubbard, E. M. (2016). Neurocognitive architectures and the nonsymbolic foundations of fractions understanding. In D. B. Berch, D. C. Geary & K. Mann Koepke (Eds.), *Development of mathematical cognition* (pp. 141–164). Elsevier/Academic Press.
- Li, Y., & Geary, D. C. (2017). Children's visuospatial memory predicts mathematics achievement through early adolescence. *PLoS One*, 12(2), e0172046. <https://doi.org/10.1371/journal.pone.0172046>
- Longo, M. R., & Lourenco, S. F. (2007). Spatial attention and the mental number line: Evidence for characteristic biases and compression. *Neuropsychologia*, 45, 1400–1407. <https://doi.org/10.1016/j.neuropsychologia.2006.11.002>
- Lubinski, D., & Benbow, C. P. (2006). Study of mathematically precocious youth after 35 years: Uncovering antecedents for the development of math-science expertise. *Perspectives on Psychological Science*, 1, 316–345. <https://doi.org/10.1111/j.1745-6916.2006.00019.x>
- Marshall, S. P., & Smith, J. D. (1987). Sex differences in learning mathematics: A longitudinal study with item and error analyses. *Journal of Educational Psychology*, 79, 372–383. <https://doi.org/10.1037/0022-0663.79.4.372>
- Morey, R. D., & Rouder, J. N. (2018). BayesFactor: Computation of Bayes Factors for common designs. R package version 0.9.12-4.2. <https://CRAN.R-project.org/package=BayesFactor>
- Mussolin, C., Nys, J., Leybaert, J., & Content, A. (2016). How approximate and exact number skills are related to each other across development: A review. *Developmental Review*, 39, 1–15. <https://doi.org/10.1016/j.dr.2014.11.001>
- National Mathematics Advisory Panel (2008). *Foundations for Success: Final Report of the National Mathematics Advisory Panel*. Washington, DC: United States Department of Education. <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>
- Opfer, J. E., & DeVries, J. M. (2008). Representational change and magnitude estimation: Why young children can make more accurate salary comparisons than adults. *Cognition*, 108, 843–849. <https://doi.org/10.1016/j.cognition.2008.05.003>
- Paas, F., & Ayres, P. (2014). Cognitive load theory: A broader view on the role of memory in learning and education. *Educational Psychology Review*, 26, 191–195. <https://doi.org/10.1007/s10648-014-9263-5>
- Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, 108, 455. <https://doi.org/10.1037/edu0000079>
- Peters, M., Laeng, B., Latham, K., Jackson, M., Zaiyouna, R., & Richardson, C. (1995). A redrawn Vandenberg and Kuse mental rotations test: Different versions and factors that affect performance. *Brain and Cognition*, 28, 39–58. <https://doi.org/10.1006/brcg.1995.1032>
- Raftery, A. E. (1995). Bayesian model selection in social research. *Sociological Methodology*, 25, 111–163.
- Rivers, M. L., Fitzsimmons, C. J., Fisk, S. R., Dunlosky, J., & Thompson, C. A. (2020). Gender differences in confidence during number-line estimation. *Metacognition and Learning*, <https://doi.org/10.1007/s11409-020-09243-7>
- Rosseel, Y. (2012). Lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, 48, 1–36.
- Rouder, J. N., & Geary, D. C. (2014). Children's cognitive representation of the mathematical number line. *Developmental Science*, 17, 525–536. <https://doi.org/10.1111/desc.12166>
- Salinas, J., Mills, E. D., Conrad, A. L., Koscik, T., Andreasen, N. C., & Nopoulos, P. (2012). Sex differences in parietal lobe structure and development. *Gender Medicine*, 9, 44–55. <https://doi.org/10.1016/j.genm.2012.01.003>
- SAS Institute (2014). *Statistical analysis system* 9.2. Cary, NC: Author.
- Siegler, R. S., & Braithwaite, D. W. (2017). Numerical development. *Annual Review of Psychology*, 68, 187–213. <https://doi.org/10.1146/annurev-psych-010416-044101>
- Siegler, R. S., & Lortie-Forgues, H. (2017). Hard lessons: Why rational number arithmetic is so difficult for so many people. *Current Directions in Psychological Science*, 26, 346–351. <https://doi.org/10.1177/0963721417700129>
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14, 237–250. <https://doi.org/10.1111/1467-9280.02438>
- Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, 62, 273–296. <https://doi.org/10.1016/j.cogpsych.2011.03.001>
- Slusser, E. B., Santiago, R. T., & Barth, H. C. (2013). Developmental change in numerical estimation. *Journal of Experimental Psychology: General*, 142, 193–208. <https://doi.org/10.1037/a0028560>
- Sullivan, J., & Barner, D. (2014). Inference and association in children's early numerical estimation. *Child Development*, 85, 1740–1755. <https://doi.org/10.1111/cdev.12211>
- Summerfield, C., Luyckx, F., & Sheahan, H. (2020). Structure learning and the posterior parietal cortex. *Progress in Neurobiology*, 184, 101717. <https://doi.org/10.1016/j.pneurobio.2019.101717>
- Sweller, J., van Merriënboer, J. J., & Paas, F. (2019). Cognitive architecture and instructional design: 20 years later. *Educational Psychology Review*, 31, 261–292. <https://doi.org/10.1007/s10648-019-09465-5>
- Szkudlarek, E., & Brannon, E. M. (2017). Does the approximate number system serve as a foundation for symbolic mathematics? *Language Learning and Development*, 13, 171–190. <https://doi.org/10.1080/15475441.2016.1263573>
- Szűcs, D., & Myers, T. (2017). A critical analysis of design, facts, bias and inference in the approximate number system training literature: A systematic review. *Trends in Neuroscience and Education*, 6, 187–203. <https://doi.org/10.1016/j.tine.2016.11.002>
- Thompson, C. A., & Opfer, J. E. (2008). Costs and benefits of representational change: Effects of context on age and sex differences in symbolic magnitude estimation. *Journal of Experimental Child Psychology*, 101, 20–51. <https://doi.org/10.1016/j.jecp.2008.02.003>
- Tranel, D., Vianna, E., Manzel, K., Damasio, H., & Grabowski, T. (2009). Neuroanatomical correlates of the Benton facial recognition test and judgment of line orientation test. *Journal of Clinical and Experimental Neuropsychology*, 31, 219–233. <https://doi.org/10.1080/13803390802317542>
- Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: A meta-analysis and consideration and consideration of critical variables. *Psychological Bulletin*, 117, 250–270.
- Voyer, D., Voyer, S. D., & Saint-Aubin, J. (2017). Sex differences in visual-spatial working memory: A meta-analysis. *Psychonomic*

- Bulletin & Review, 24, 307-334. <https://doi.org/10.3758/s13423-016-1085-7>
- Wai, J., Hodges, J., & Makel, M. C. (2018). Sex differences in ability tilt in the right tail of cognitive abilities: A 35-year examination. *Intelligence*, 67, 76-83. <https://doi.org/10.1016/j.intell.2018.02.003>
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7, 483-488. <https://doi.org/10.1016/j.tics.2003.09.002>
- Webb, R. M., Lubinski, D., & Benbow, C. P. (2007). Spatial ability: A neglected dimension in talent searches for intellectually precocious youth. *Journal of Educational Psychology*, 99, 397-420. <https://doi.org/10.1037/0022-0663.99.2.397>
- Wechsler, D. (1999). *Wechsler Abbreviated Scale of Intelligence*. San Antonio, TX: Psychological Corporation.
- Wechsler, D. (2009). *Wechsler Individual Achievement Test* (3rd ed.). San Antonio, TX: Psychological Corporation.
- Zhang, Y., Liu, T., Chen, C., & Zhou, X. (2019). Visual form perception supports approximate number system acuity and arithmetic fluency. *Learning and Individual Differences*, 71, 1-12. <https://doi.org/10.1016/j.lindif.2019.02.008>
- Zorzi, M., Bonato, M., Treccani, B., Scalambri, G., Marenzi, R., & Priftis, K. (2012). Neglect impairs explicit processing of the mental number line. *Frontiers in Human Neuroscience*, 6, 125. <https://doi.org/10.3389/fnhum.2012.00125>
- Zorzi, M., Priftis, K., & Umiltà, C. (2002). Neglect disrupts the mental number line. *Nature*, 417, 138-139. <https://doi.org/10.1038/417138a>

## SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section.

**How to cite this article:** Geary DC, Scofield JE, Hoard MK, Nugent L. Boys' advantage on the fractions number line is mediated by visuospatial attention: Evidence for a parietal-spatial contribution to number line learning. *Dev Sci*. 2021;24:e13063. <https://doi.org/10.1111/desc.13063>



## APPENDIX

TABLE A1 Grand means and correlations among the core variables.

Measure	M (SD)	1	2	3	4	5	6	7	8	9	10
1. Fractions number line	0.17 (0.14)	1.00									
2. Fractions arithmetic	12.47 (7.55)	-.58	1.00								
3. Digit span: Backward	4.56 (1.20)	-.30	.38	1.00							
4. n-back	3.81 (0.75)	-.36	.34	.25	1.00						
5. Corsi	8.32 (1.87)	-.29	.35	.25	.31	1.00					
6. JLAP	13.34 (3.08)	-.34	.30	.24	.28	.28	1.00				
7. Intelligence	104.60 (13.14)	-.51	.47	.41	.34	.30	.33	1.00			
8. 7th-Grade Numerical Operations	100.00 (18.92)	-.59	.70	.38	.33	.33	.33	.55	1.00		
9. 7th-Grade Word Reading	104.33 (13.17)	-.42	.41	.42	.27	.17	.26	.56	.50	1.00	
10. Gender	—	-.11	.11	-.03	.06	.12	.22	.03	.04	.03	1.00

Note: Correlations  $> .10$  are significant ( $p < .05$ ).