Radiative transitions of charmoniumlike exotics in the dynamical diquark model

Justin M. Gens,* Jesse F. Giron,† and Richard F. Lebed‡
Department of Physics, Arizona State University, Tempe, Arizona 85287, USA

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Using the dynamical diquark model, we calculate the electric-dipole radiative decay widths to $X(3872)$ of the lightest negative-parity exotic candidates, including the four $I = 0$, $J^{PC} = 1^{--}$ ("Y") states. The $O(100–1000\ \text{keV})$ values obtained test the hypothesis of a common substructure shared by all of these states. We also calculate the magnetic-dipole radiative decay width for $Z_c(4020)^0 \to \gamma X(3872)$, and find it to be rather smaller (<10 keV) than its predicted value in molecular models.

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I. INTRODUCTION

The number of new heavy-quark exotic-hadron candidates, presumptive tetraquark and pentaquark states, increases every year. In the past 18 years, over 40 candidates have been observed at multiple facilities and their hosted experiments. However, no single theoretical picture to describe the structure of these states has emerged as an undisputed favorite. Both the broad scope of experimental results and competing theoretical interpretations have been reviewed by many in recent years [1–11].

Among these competing physical approaches, the dynamical diquark picture [12] was developed to provide a mechanism through which diquark (δ)-antidiquark ($\bar{\delta}$) states could persist long enough to be identified as such experimentally. Diquarks are formed through the attractive channels $\mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}}$ $[\delta \equiv (Qq)_{\bar{\mathbf{3}}}]$ and $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \to \mathbf{3}$ $[\bar{\delta} \equiv (\bar{Q}\bar{q}')_{\mathbf{3}}]$ between color-triplet quarks. In this physical picture, the heavy quark Q must first be created in closer spatial proximity to a light quark q than to a light antiquark $\bar{q}'$ (and vice versa for $\bar{Q}$). This initial configuration provides an opportunity for the formation of fairly compact δ and $\bar{\delta}$ quasiparticles, in distinction to an initial state in which the strongly attractive $\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1}$ coupling immediately leads to $(Q\bar{q}')(\bar{Q}q)$ meson pairs. Second, the large energy release of the production process (from a heavy-hadron decay or in a collider event) drives apart the δ-$\bar{\delta}$ pair before immediate recombination into a meson pair can occur, creating an observable resonance. A similar mechanism extends the picture to pentaquark formation [13], by means of using color-triplet "antitriquarks" $\bar{\theta} \equiv [\bar{Q}_{\bar{\mathbf{3}}}(q_1 q_2)_{\bar{\mathbf{3}}}]_{\mathbf{3}}$.

This physical picture was subsequently developed into the dynamical diquark *model* [14]: The separated δ-$\bar{\delta}$ pair is connected by a color flux tube, whose quantized states are best described in terms of the potentials computed using the Born-Oppenheimer (BO) approximation. These are the same potentials as appear in QCD lattice gauge-theory simulations that predict the spectrum of heavy-quarkonium hybrid mesons [15–19]. The BO potentials are introduced into coupled Schrödinger equations that are solved numerically in order to produce predictions for the δ-$\bar{\delta}$ spectrum, as shown in Ref. [20]. As one of the primary results of that work, all the observed exotic candidates are shown to be accommodated within the ground-state BO potential Σ_g^+, with the specific multiplets in order of increasing average mass being $1S$, $1P$, $2S$, $1D$, and $2P$. A full summary of the BO potential notation is presented in Ref. [14].

The mass spectrum and preferred decay modes (organized by eigenstates of heavy-quark spin) of the 6 isosinglets and 6 isotriplets comprising the $c\bar{c}q\bar{q}'$ positive-parity $\Sigma_g^+(1S)$ multiplet (where $q, q' \in \{u, d\}$) were studied in Ref. [21]. This was the first work to differentiate $I = 0$ and $I = 1$ states in a diquark model. The specific model of Ref. [21] naturally produces scenarios in which $X(3872)$ is the lightest $\Sigma_g^+(1S)$ state, and moreover predicts that the lighter of the two $I = 1$, $J^{PC} = 1^{+-}$ states in $\Sigma_g^+(1S)$ $[Z_c(3900)]$ naturally decays almost exclusively to J/ψ and the heavier one $[Z_c(4020)]$ to h_c, as is observed. The model of Ref. [21] uses a 3-parameter Hamiltonian consisting of a common multiplet mass, an internal diquark-spin coupling, and a long-distance isospin- and spin-dependent coupling (analogous to π exchange) between the light quark q in δ and light antiquark $\bar{q}'$ in $\bar{\delta}$. Similar conclusions using QCD sum rules have been obtained in Ref. [22].

*jgens@asu.edu
†jfgiron@asu.edu
‡Richard.Lebed@asu.edu

The dynamical diquark model was developed further through the corresponding analysis [23] of the negative-parity $c\bar{c}q\bar{q}'$ $\Sigma_g^+(1P)$ multiplet and its 28 constituent isomultiplets (14 isosinglets and 14 isotriplets), which includes precisely four Y ($I = 0$, $J^{PC} = 1^{--}$) states. In this case, the simplest model has 5 parameters: the 3 listed above, plus spin-orbit and tensor terms. An earlier diquark analysis using a similar Hamiltonian, but not including isospin dependence, appears in Ref. [24].

The success of Ref. [20] in predicting the correct mass splittings between the observed bands ($1S, 1P, 2S$) of exotic hadrons, and Ref. [21] in effectively representing the fine structure within the lowest multiplets [especially $\Sigma_g^+(1S)$] provides strong *a posteriori* support for the applicability of the dynamical diquark model. In particular, one may certainly question whether treating the exotics as quasi-two-body states within a BO approximation, rather than including full 4- (or 5-) body interactions to represent the internal evolution of the quasiparticles, is sensible. However, while such effects are undoubtedly present at some level, the current experimental evidence appears to support the presence of a scale separation that allows the quasiparticles to be treated identifiable subunits within the hadrons. As an example, Ref. [20] showed in numerical simulations that the diquarks need not be pointlike particles, but could have substantial spatial extent (characteristic radii as large as 0.4 fm) before the full hadron mass spectrum changes significantly.

An analysis within this model of the 12 isomultiplets comprising the $b\bar{b}q\bar{q}'$ $\Sigma_g^+(1S)$ multiplet and the 6 states of the $c\bar{c}s\bar{s}$ $\Sigma_g^+(1S)$ multiplet appears in Ref. [25]. By using only experimental inputs for the states $Z_b(10610)$ and $Z_b(10650)$, which includes their masses and relative probability of decay into h_b versus Υ states, the entire $b\bar{b}q\bar{q}'$ mass spectrum is predicted. In particular, the mass of the bottom analogue to $X(3872)$ is highly constrained (≈10600 MeV), and the lightest $b\bar{b}q\bar{q}'$ state ($I = 0$, $J^{PC} = 0^{++}$) lies only a few MeV above the $B\bar{B}$ threshold. Furthermore, starting with the assumption that $X(3915)$ is the lowest lying $c\bar{c}s\bar{s}$ state [26] and $Y(4140)$ is the sole $J^{PC} = 1^{++}$ $c\bar{c}s\bar{s}$ state in $\Sigma_g^+(1S)$, the remaining 4 masses in the multiplet are predicted. Emerging naturally in the spectrum is $X(4350)$, a $J/\psi - \phi$ resonance seen by Belle [27], while $Y(4626)$ and $X(4700)$ are found to fit well within the $\Sigma_g^+(1P)$ and $\Sigma_g^+(2S)$ $c\bar{c}s\bar{s}$ multiplets, respectively.

The dynamical diquark model has also recently been extended to the case in which the light quarks q are replaced with heavy quarks Q to produce fully heavy tetraquark states $Q_1\bar{Q}_2Q_3\bar{Q}_4$, where $Q_i = c$ or b. Sparked by the recent LHCb report of at least one di-J/ψ resonance near 6900 MeV [28], Ref. [29] determined the spectrum of $c\bar{c}c\bar{c}$ states in the dynamical diquark model. In this system, the minimal model predicts each S-wave multiplet to consist of 3 degenerate states ($J^{PC} = 0^{++}, 1^{+-}, 2^{++}$) and 7 P-wave states. $X(6900)$ was found to fit most naturally as a $\Sigma_g^+(2S)$ state, with other structures in the measured di-J/ψ spectrum appearing to match $C = +$ members of the $\Sigma_g^+(1P)$ multiplet.

In this paper we use the dynamical diquark model to predict radiative transitions between exotic states. So far, very few theoretical papers have investigated exotic-to-exotic transitions (and of these papers, only diquark models have been considered [30,31]). One of the distinctive features of the P-wave study in Ref. [23] is the direct calculation of decay probabilities to eigenstates of heavy-quark spin. Indeed, Ref. [23] uses the heavy quark-spin content of states as the main criterion for associating observed resonances with particular states in the $\Sigma_g^+(1P)$ multiplet, and identifies using likelihood fits two particularly plausible assignments for the states. Using the same decay probabilities, we calculate here the transition amplitudes for $\Sigma_g^+(1P) \to \gamma\Sigma_g^+(1S)$. We directly adapt the well-known expression for electric dipole (E1) radiative transitions used to great effect for conventional quarkonium. Since the E1 transition formula depends sensitively upon the initial and final wave functions, a comparison between our predictions and data provides an important test of the hypothesis that the purported $\Sigma_g^+(1P)$ and $\Sigma_g^+(1S)$ states, such as in $Y(4220) \to \gamma X(3872)$, truly share a common structure. The corresponding magnetic dipole (M1) expression within this model is also presented, in anticipation of the observation of relevant transitions such as $\Sigma_g^+(2S) \to \gamma\Sigma_g^+(1S)$, or even between two $\Sigma_g^+(1S)$ states such as $Z_c(4020)^0 \to \gamma X(3872)$.

This paper is organized as follows: In Sec. II we review the current experimental data on transitions between $c\bar{c}q\bar{q}'$ states. Section III reprises the relevant phenomenological aspects of Ref. [23]. In Sec. IV we calculate the decay widths and decay probabilities for exotic-to-exotic radiative transitions and focus upon two of the more probable P-wave state assignments in Ref. [23]. We conclude in Sec. V.

II. EXPERIMENTAL REVIEW OF EXOTIC-TO-EXOTIC TRANSITIONS

Although the number of exotic-candidate discoveries continues to increase at a remarkable pace, only a handful of exotic-to-exotic decays have been observed to date, through radiative [32,33] and pionic [34–36] transitions.

Considering first the radiative decays that form the topic of this work, thus far only E1 transitions (as indicated by changing parity $\Delta P = -$) have been observed in two states at BESIII, the $J^{PC} = 1^{--}$ $Y(4260)$ [32] and $Y(4220)$ [33], both seen to decay to a photon and the $J^{PC} = 1^{++}$ $X(3872)$. Indeed, an increasing amount of evidence from BESIII (e.g., in Ref. [37]) suggests that the well-known $Y(4260)$ is actually a collection of resonances, of which $Y(4220)$ is just one component. Observed exotic-to-conventional radiative transitions are also rather few in number, due to the

TABLE I. $J^{PC} = 1^{--}$ charmoniumlike exotic-meson candidates catalogued by the Particle Data Group (PDG) [41], which are identified with specific states within the $\Sigma_g^+(1P)$ multiplet of the dynamical diquark model, as summarized by the cases presented in Ref. [23] and repeated in Sec. III. Both the particle name most commonly used in the literature and its label as given in the PDG are listed.

Particle	PDG label	$I^G J^{PC}$	Mass [MeV]	Width [MeV]	Production and decay
$Y(4220)$	$\psi(4230)$	0^-1^{--}	4218^{+5}_{-4}	59^{+12}_{-10}	$e^+e^- \to Y;\ Y \to \begin{cases} \omega\chi_{c0} \\ \eta J/\psi \\ \pi^+\pi^- h_c \\ \pi^+\pi^-\psi(2S) \\ \pi^+ D^0 D^{*-} \\ \pi^0 Z_c^0(3900) \\ \gamma X(3872) \end{cases}$
$Y(4260)$	$\psi(4260)$	0^-1^{--}	4230 ± 8	55 ± 19	$e^+e^- \to \gamma Y \text{ or } Y;\ Y \to \begin{cases} \pi^+\pi^- J/\psi \\ f_0(980) J/\psi \\ \pi^\mp Z_c^\pm(3900) \\ K^+K^- J/\psi \\ \gamma X(3872) \end{cases}$
$Y(4360)$	$\psi(4360)$	0^-1^{--}	4368 ± 13	96 ± 7	$e^+e^- \to \gamma Y \text{ or } Y;\ Y \to \begin{cases} \pi^+\pi^-\psi(2S) \\ \pi^0\pi^0\psi(2S) \end{cases}$
$Y(4390)$	$\psi(4390)$	0^-1^{--}	4392 ± 7	140^{+16}_{-21}	$e^+e^- \to Y;\ Y \to \begin{cases} \eta J/\psi \\ \pi^+\pi^- h_c \end{cases}$
$Y(4660)$	$\psi(4660)$	0^-1^{--}	4643 ± 9	72 ± 11	$e^+e^- \to \begin{cases} \gamma Y; Y \to \pi^+\pi^-\psi(2S) \\ Y; Y \to \Lambda_c^+\Lambda_c^- \end{cases}$

large decay widths of exotics that follows from the dominance of their strong decay modes. To date, only $X(3872) \to \gamma J/\psi$ and $\gamma\psi(2S)$, also both E1 transitions, have definitely been seen (e.g., in Ref. [38]). BESIII has also recently announced an interesting negative result [39], an upper limit for $Z_c(4020)^0(J^{PC} = 1^{+-}) \to \gamma X(3872)$. Indeed, to date no M1 radiative decay ($\Delta P = +$) of any exotic candidate has yet been seen at any experiment.

As for pionic transitions, both BESIII [34] and Belle [35] have observed (indeed, discovered) $Z_c(3900)^\pm$ through $Y(4260) \to \pi^+\pi^- J/\psi$, and BESIII recently observed $Z_c(3900)^0$ via $Y(4220) \to \pi^0\pi^0 J/\psi$ [36]. Assuming just a similarity of hadronic structure between various exotic candidates, one may expect several more exotic-to-exotic pionic (or other light-meson) transitions to be observed in the future. An essential criterion for how such transitions may best be studied relies on the size of the pion momentum p_π in such processes; for example, in the decays listed above, $p_\pi \approx 300$ MeV. Processes with smaller p_π values may be reliably studied using conventional chiral perturbation theory, while studies of processes with larger p_π values require modifications to the perturbative calculation to improve their convergence. Since the methods associated with radiative transitions (particularly E1 transitions) present fewer computational ambiguities, we defer a study of exotic-to-exotic pionic transitions for future work.

The expressions for E1 and M1 transition widths used below [Eqs. (9) and (12), respectively] are almost identical to the forms derived in standard quantum mechanics textbooks. As such, they are manifestly nonrelativistic, and furthermore are developed using the photon long-wavelength approximation, $\exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \to 1$. Nevertheless, the expressions can also be derived directly from the fundamental Lagrangian $j_\mu A^\mu$ couplings of the electromagnetic current j^μ of charged quarks to the photon field A^μ (see, e.g., Ref. [40]). In Sec. III we discuss the effect of including certain corrections to the textbook expressions.

In the dynamical diquark model, all states in the multiplet $\Sigma_g^+(1S)[(1P)]$ have $P = +[-]$ [14]. The current observed properties of the $J^{PC} = 1^{--}$ (Y) states identified with the multiplet $\Sigma_g^+(1P)$, whose spectroscopy is analyzed extensively in Ref. [23], are summarized in Table I.

III. THEORETICAL REVIEW OF *P*-WAVE EXOTIC STATES

The full spectroscopy of diquark-antidiquark (δ-$\bar{\delta}$) tetraquarks and diquark-antitriquark (δ-$\bar{\theta}$) pentaquarks connected by a gluonic field of arbitrary excitation quantum numbers, and including arbitrary orbital excitations between the δ-$\bar{\delta}$ or δ-$\bar{\theta}$ pair, is presented in Ref. [14]. As discussed in that work, the gluonic-field excitations combined with the quasiparticle sources $\delta, \bar{\delta}, \bar{\theta}$ produce states analogous to ordinary quarkonium hybrids; therefore, these states may likewise be classified according to the quantum numbers provided by BO-approximation static gluonic-field potentials. The numerical studies of Ref. [20] show that the exotic analogues to hybrid quarkonium states lie above the exotic states within the corresponding BO ground-state potential Σ_g^+ by at least 1 GeV (just as for conventional quarkonium). Since the entire range of observed hidden-charm exotic candidates [not counting $c\bar{c}c\bar{c}$ candidates such as $X(6900)$] spans only about 800 MeV [1], it is very likely that all known hidden-charm exotic states occupy energy levels within the Σ_g^+ BO potential. All known $c\bar{c}q\bar{q}'$ candidates can be accommodated by the lowest Σ_g^+ levels: $1S$, $1P$, $2S$, $1D$, and $2P$, in order of increasing mass [20].

A detailed enumeration of the possible $Q\bar{Q}q\bar{q}'$ states, in which the light quarks q,$\bar{q}'$ do not necessarily carry the same flavor, is straightforward for the S wave. Assuming zero relative orbital angular momenta between the quarks, any two naming conventions for the states differ only by the order in which the 4 quark spins are coupled. In the diquark basis, defined by coupling in the order $(qQ) + (\bar{q}\,\bar{Q})$, the 6 possible states are denoted by [30]:

$$\begin{aligned}
J^{PC} &= 0^{++}:\; X_0 \equiv |0_\delta, 0_{\bar{\delta}}\rangle_0, \qquad X_0' \equiv |1_\delta, 1_{\bar{\delta}}\rangle_0,\\
J^{PC} &= 1^{++}:\; X_1 \equiv \frac{1}{\sqrt{2}}(|1_\delta, 0_{\bar{\delta}}\rangle_1 + |0_\delta, 1_{\bar{\delta}}\rangle_1),\\
J^{PC} &= 1^{+-}:\; Z \equiv \frac{1}{\sqrt{2}}(|1_\delta, 0_{\bar{\delta}}\rangle_1 - |0_\delta, 1_{\bar{\delta}}\rangle_1),\\
&\qquad\qquad Z' \equiv |1_\delta, 1_{\bar{\delta}}\rangle_1,\\
J^{PC} &= 2^{++}:\; X_2 \equiv |1_\delta, 1_{\bar{\delta}}\rangle_2, \qquad (1)
\end{aligned}$$

where outer subscripts indicate total quark spin S. The same states may be expressed in any other basis by using angular momentum recoupling coefficients in the form of the relevant $9j$ symbol. For the purposes of this work, the most useful alternate basis is that of definite heavy-quark (and light-quark) spin, $(Q\bar{Q}) + (q\bar{q})$:

$$\begin{aligned}
&\langle (s_q s_{\bar{q}}) s_{q\bar{q}}, (s_Q s_{\bar{Q}}) s_{Q\bar{Q}}, S | (s_q s_Q) s_\delta, (s_{\bar{q}} s_{\bar{Q}}) s_{\bar{\delta}}, S\rangle\\
&\quad = ([s_{q\bar{q}}][s_{Q\bar{Q}}][s_\delta][s_{\bar{\delta}}])^{1/2} \begin{Bmatrix} s_q & s_{\bar{q}} & s_{q\bar{q}} \\ s_Q & s_{\bar{Q}} & s_{Q\bar{Q}} \\ s_\delta & s_{\bar{\delta}} & S \end{Bmatrix}, \qquad (2)
\end{aligned}$$

where $[s] \equiv 2s + 1$ denotes the multiplicity of a spin-s state. Using Eqs. (1) and (2), one then obtains

$$\begin{aligned}
J^{PC} &= 0^{++}:\; X_0 = \frac{1}{2}|0_{q\bar{q}}, 0_{Q\bar{Q}}\rangle_0 + \frac{\sqrt{3}}{2}|1_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_0,\\
&\qquad\quad X_0' = \frac{\sqrt{3}}{2}|0_{q\bar{q}}, 0_{Q\bar{Q}}\rangle_0 - \frac{1}{2}|1_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_0,\\
J^{PC} &= 1^{++}:\; X_1 = |1_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_1,\\
J^{PC} &= 1^{+-}:\; Z = \frac{1}{\sqrt{2}}(|1_{q\bar{q}}, 0_{Q\bar{Q}}\rangle_1 - |0_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_1),\\
&\qquad\quad Z' = \frac{1}{\sqrt{2}}(|1_{q\bar{q}}, 0_{Q\bar{Q}}\rangle_1 + |0_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_1),\\
J^{PC} &= 2^{++}:\; X_2 = |1_{q\bar{q}}, 1_{Q\bar{Q}}\rangle_2. \qquad (3)
\end{aligned}$$

Once light-quark flavor is included, one obtains 12 states: 6 each with $I = 0$ and $I = 1$, and spin structures in the form of Eqs. (1) or (3).[1] Using these states and the most minimal 3-parameter Hamiltonian [the M_0, κ_{qQ}, and V_0 terms of Eq. (4) below], Refs. [21,25] calculate the masses of all 12 S-wave states in the hidden-charm and hidden-bottom sectors using known masses of $X(3872)$, $Z_c(3900)$, and $Z_c(4020)$ for the former; and the known masses of $Z_b(10610)$, $Z_b(10650)$, and their relative h_b to Υ branching fractions for the latter. These results incorporate isospin dependence (the V_0 term), a feature not explicitly integrated into other diquark models.

Reference [23] extends this analysis by examining the P-wave multiplet, whose mass spectrum is dictated by the most minimal 5-parameter Hamiltonian:

$$\begin{aligned}
H &= M_0 + 2\kappa_{qQ}(\mathbf{s}_q \cdot \mathbf{s}_Q + \mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{Q}}) + V_{LS}\mathbf{L}\cdot\mathbf{S}\\
&\quad + V_0 \boldsymbol{\tau}_q \cdot \boldsymbol{\tau}_{\bar{q}} \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}} + V_T \boldsymbol{\tau}_q \cdot \boldsymbol{\tau}_{\bar{q}} S_{12}^{(q\bar{q})}, \qquad (4)
\end{aligned}$$

where M_0 is the common mass of the multiplet, κ_{qQ} represents the strength of the spin-spin coupling within each diquark, V_{LS} is the spin-orbit coupling strength, V_0 is the isospin-dependent coupling,[2] V_T represents the tensor coupling, and $S_{12}^{(q\bar{q})}$ is the tensor operator defined as

$$S_{12}^{(q\bar{q})} \equiv 3\boldsymbol{\sigma}_q \cdot \boldsymbol{r}\, \boldsymbol{\sigma}_{\bar{q}} \cdot \boldsymbol{r}/r^2 - \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}}. \qquad (5)$$

The well-known tabulated expressions for matrix elements of $S_{12}^{(q\bar{q})}$ (e.g., in Ref. [42]) directly apply neither in the basis of $s_{q\bar{q}}, s_{Q\bar{Q}}$ spins nor $s_\delta, s_{\bar{\delta}}$ spins, but rather refer to the basis of total light-quark angular momentum $J_{q\bar{q}}$:

$$\mathbf{J}_{q\bar{q}} \equiv \mathbf{L}_{q\bar{q}} + \mathbf{s}_{q\bar{q}}. \qquad (6)$$

[1] If strange quarks are included, one obtains 6 $SU(3)_{\rm flavor}$ octets and 6 singlets.

[2] V_0 in Eq. (4) is analogous to the axial coupling in $NN\pi$ interactions.

Assuming that δ and $\bar{\delta}$ have no internal orbital excitation so that $L_{q\bar{q}} = L$, the matrix elements of $S_{12}^{(q\bar{q})}$ are most easily computed in the $J_{q\bar{q}}$ basis, with results that are then related back to the $s_{q\bar{q}}, s_{Q\bar{Q}}$ basis by means of recoupling using $6j$ symbols:

$$\mathcal{M}_{J_{q\bar{q}}} \equiv \langle (L, s_{q\bar{q}}), J_{q\bar{q}}, s_{Q\bar{Q}}, J | L, (s_{q\bar{q}}, s_{Q\bar{Q}}), S, J \rangle$$
$$= (-1)^{L+s_{q\bar{q}}+s_{Q\bar{Q}}+J} \sqrt{[J_{q\bar{q}}][S]} \begin{Bmatrix} L & s_{q\bar{q}} & J_{q\bar{q}} \\ s_{Q\bar{Q}} & J & S \end{Bmatrix}. \quad (7)$$

Using this expression, $S_{12}^{(q\bar{q})}$ matrix elements for all relevant states are tabulated in Ref. [23].

The experimental status of the P-wave $J^{PC} = 1^{--}$ exotic candidates remains in flux, with BESIII providing the majority of the most recent data. With reference to the information presented in Table I, we have already noted that the analysis of the BESIII Collaboration [37] favors the interpretation of $Y(4260)$ as a superposition of states, the lowest component of which is $Y(4220)$. They identify the higher component with $Y(4360)$, although the previous mass measurements of this state given in Table I are rather higher, and one of several scenarios considered in Ref. [23] proposes that $Y(4360)$ and $Y(4390)$ are the same state, while the higher-mass component in Ref. [37] can be interpreted as a distinct "$Y(4320)$". Alternately, if the only lower states are $Y(4220)$, $Y(4360)$, and $Y(4390)$, then $Y(4660)$ becomes the fourth $I = 0$, 1^{--} candidate state in $\Sigma_g^+(1P)$.

With the mass spectrum of these charmoniumlike states not yet entirely settled, Ref. [23] also employs information on their preferred charmonium decay modes as classified by heavy-quark spin: ψ ($s_{Q\bar{Q}} = 1$) or h_c ($s_{Q\bar{Q}} = 0$). Assuming heavy-quark spin symmetry as expressed by the conservation of $s_{Q\bar{Q}}$ in the decays, the heavy-quark spin content $P_{s_{Q\bar{Q}}}$ of each state becomes an invaluable diagnostic in disentangling the $J^{PC} = 1^{--}$ spectrum. For example, from Table I one sees that $Y(4220)$ decays to both ψ states and h_c, while if $Y(4360)$ and $Y(4390)$ are in fact one state, the same can be said for them as well. Reference [23] also introduces a parameter ϵ designed to enforce the goodness-of-fit to a particular value f of $P_{s_{Q\bar{Q}}}$, which in the case of $s_{Q\bar{Q}} = 0$ reads

$$\Delta\chi^2 = \left(\frac{\ln P_{s_{Q\bar{Q}}=0} - \ln f}{\epsilon} \right)^2. \quad (8)$$

In terms of the parameters $P_{s_{Q\bar{Q}}}$, f, and ϵ, the 5 cases discussed in Ref. [23] designed to represent a variety of interpretations of the current data are:

(1) $Y(4220)$, $Y(4260)$, $Y(4360)$, $Y(4390)$ masses are as given in the PDG (Table I). No constraint is placed upon $P_{s_{Q\bar{Q}}=0}^{Y(4220)}$ or $P_{s_{Q\bar{Q}}=0}^{Y(4390)}$.

(2) $Y(4220)$, $Y(4260)$, $Y(4360)$, $Y(4390)$ masses are as given in the PDG $P_{s_{Q\bar{Q}}=0}^{Y(4220)}$ is fit to $f = \frac{1}{3}$ with $\epsilon = 0.1$, and $P_{s_{Q\bar{Q}}=0}^{Y(4390)}$ is unconstrained.

(3) $Y(4220)$, $Y(4360)$, and $Y(4390)$ masses are as given in the PDG, while $m_{Y(4260)} = 4251 \pm 6$ MeV, which is the weighted average of the 3 PDG values not including the low BESIII value [37]. $P_{s_{Q\bar{Q}}=0}^{Y(4220)}$ is fit to $f = \frac{1}{3}$ with $\epsilon = 0.2$, and $P_{s_{Q\bar{Q}}=0}^{Y(4390)}$ is fit to $f = \frac{2}{3}$ with $\epsilon = 0.05$.

(4) $Y(4360)$, $Y(4390)$, and $Y(4660)$ masses are as given in the PDG, but $Y(4260)$ is assumed not to exist, and $m_{Y(4220)} = 4220.1 \pm 2.9$ MeV is the weighted average of the PDG values combined with the newer BESIII measurements [43,44]. $P_{s_{Q\bar{Q}}=0}$ values are as given in Case 3.

(5) $m_{Y(4220)}$ is as given in Case 4; $m_{Y(4260)}$ is as given in Case 3; $m_{\text{"}Y(4320)\text{"}} = 4320 \pm 13$ MeV is the lower BESIII $Y(4360)$ mass measurement from [37]; $m_{Y(4390)} = 4386 \pm 4$ MeV is the weighted average of the PDG value and the upper BESIII $Y(4360)$ mass measurement from [45]. $P_{s_{Q\bar{Q}}=0}$ values are as given in Case 3.

We previously suggested the importance of heavy-quark spin-symmetry ($s_{Q\bar{Q}}$) conservation in the decays of exotics, particularly for $Z_c(3900)$ and $Z_c(4020)$, but also for several other exotic candidates that to date have only been observed to decay to charmonium states carrying one specific value of $s_{Q\bar{Q}}$ (e.g., to ψ or to h_c). We assume that a state like $Y(4220)$ is able to decay to channels with either value of $s_{Q\bar{Q}}$ due to the initial state being a mixture of $s_{Q\bar{Q}}$ eigenstates, rather than to the value of $s_{Q\bar{Q}}$ changing in the decay process through a heavy-quark spin-symmetry violation. In addition, in this analysis we take the well-known radiative transition selection rules to apply to the light degrees of freedom, which carry the total angular momentum $J_{q\bar{q}}$ defined in Eq. (6). As usual, the operators defining E1 and M1 transitions transform as $J^P = 1^-$ and $J^P = 1^+$, respectively.

Explicit expressions for radiative transitions between quarkonium states (themselves transcribed from textbook atomic-physics formulas) appear in the literature (e.g., Ref. [46]), and may readily be adapted to the present case. In particular, the quarkonium orbital angular momentum L is replaced with $J_{q\bar{q}}$, and the heavy quark mass m_Q is replaced with the diquark mass m_δ. For E1 partial widths, one has

$$\Gamma_{\mathrm{E1}}(n^{2s_{Q\bar{Q}}+1}(J_{q\bar{q}})_J \to n'^{2s'_{Q\bar{Q}}+1}(J'_{q\bar{q}})_{J'} + \gamma)$$
$$= \frac{4}{3} C_{fi} \delta_{s_{Q\bar{Q}} s'_{Q\bar{Q}}} \alpha Q_\delta^2 |\langle \psi_f | r | \psi_i \rangle|^2 E_\gamma^3 \frac{E_f^{(Q\bar{Q}q\bar{q}')}}{M_i^{(Q\bar{Q}q\bar{q}')}}, \quad (9)$$

where

$$C_{fi} \equiv \max\,(J_{q\bar{q}}, J'_{q\bar{q}})(2J'+1)\begin{Bmatrix} J'_{q\bar{q}} & J' & s_{Q\bar{Q}} \\ J & J_{q\bar{q}} & 1 \end{Bmatrix}^2. \quad (10)$$

The labels i and f refer to initial and final states, respectively. The initial exotic state $Q\bar{Q}q\bar{q}'$, of mass $M_i^{(Q\bar{Q}q\bar{q}')}$, decays in its rest frame into a final exotic state with the same flavor content and energy $E_f^{(Q\bar{Q}q\bar{q}')}$, and a photon of energy E_γ. α is the fine-structure constant. ψ denotes radial wave functions of the exotic hadrons, and r is the spatial separation between the δ-$\bar{\delta}$ pair centers. Q_δ is the total electric charge (in units of proton charge) to which the photon couples; in Ref. [31], the diquarks are treated as pointlike, in which case one simply takes $Q_\delta = Q_Q + Q_q$. Alternately, one may argue that the diquarks δ are of sufficient spatial extent that the photon couplings to the distinct quarks in δ should add through incoherent diagrams, in which case one takes $Q_\delta^2 = Q_Q^2 + Q_q^2$. In our calculation we use the first option, but note in addition that a Y state, being an isosinglet, contains an equal superposition of u and d quarks. We thus take

$$Q_\delta^2 \to \frac{1}{2}[(Q_c+Q_u)^2 + (Q_c+Q_d)^2] = \frac{17}{18}. \quad (11)$$

Other schemes give rise to coefficients that differ from this value only at $O(1)$. Corrections that arise from treating the distinct quarks within each diquark as separated entities, for example through electromagnetic form factors of the $\delta, \bar{\delta}$ composite quasiparticles, would be incorporated in this model through the factor Q_δ^2.

The corresponding expression for M1 partial widths, involving no change in parity but a flip of the heavy-quark spin $s_{Q\bar{Q}}$ (hence breaking heavy-quark spin symmetry), reads

$$\Gamma_{\rm M1}(n^{2s_{Q\bar{Q}}+1}(J_{q\bar{q}})_J \to n'^{2s'_{Q\bar{Q}}+1}(J'_{q\bar{q}})_{J'} + \gamma)$$
$$= \frac{4}{3}\frac{2J'+1}{2J_{q\bar{q}}+1}\delta_{J_{q\bar{q}}J'_{q\bar{q}}}\delta_{s_{Q\bar{Q}},s'_{Q\bar{Q}}\pm 1} Q_\delta^2 \frac{\alpha}{m_\delta^2}|\langle\psi_f|\psi_i\rangle|^2 E_\gamma^3 \frac{E_f^{(Q\bar{Q}q\bar{q}')}}{M_i^{(Q\bar{Q}q\bar{q}')}}. \quad (12)$$

This expression is presented here for completeness, in light of the current lack of experimental evidence for such transitions. However, in Sec. IV we use it to calculate the expected radiative width for the yet-unobserved [39] transition $Z_c(4020)^0 \to \gamma X(3872)$.

As noted above, Eqs. (9) and (12) are almost identical to textbook nonrelativistic results. The only exception in each case is the inclusion of a factor E_f/M_i to represent relativistic phase space associated with recoil of the final-state hadron. In fact, Ref. [40] discusses several distinct relativistic corrections that could be included in a more complete study. Since this work represents the first attempt to calculate the radiative widths for a spectrum of states whose experimental interpretation remains ambiguous, we include only a minimal set of physical effects in the analysis.

Lastly, corrections to the long-wavelength approximation discussed in Sec. III that are derived by retaining the full photon plane-wave factor $\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$ have also been computed (e.g., Ref. [40]). Explicitly, Eqs. (9) and (12) are modified through the substitutions

$$\langle\psi_f|r|\psi_i\rangle \to \langle\psi_f|\frac{3}{k}\left[\frac{kr}{2}j_0\left(\frac{kr}{2}\right) - j_1\left(\frac{kr}{2}\right)\right]|\psi_i\rangle, \quad (13)$$

and

$$\langle\psi_f|\psi_i\rangle \to \langle\psi_f|j_0\left(\frac{kr}{2}\right)|\psi_i\rangle, \quad (14)$$

respectively,where j_0 and j_1 are spherical Bessel functions. The corresponding series expansions of these functions read

$$r - \frac{1}{20}k^2r^3 + O(k^4r^5), \quad (15)$$

and

$$1 - \frac{1}{24}k^2r^2 + O(k^4r^4). \quad (16)$$

Note especially the small subleading-term numerical coefficient in each case, suggesting that the long-wavelength approximation holds relatively well even for substantial values of kr. We examine specific examples in Sec. IV.

IV. ANALYSIS AND RESULTS

Possible assignments of observed Y states to members of the $\Sigma_g^+(1P)$ multiplet in this model are described by the 5 cases discussed extensively in Ref. [23] and summarized in Sec. III. Of these cases, all have excellent goodness-of-fit values $\chi^2_{\rm min}$/d.o.f. except Case 3; however, we argue this case and Case 5 to be the most phenomenologically relevant ones, since they enforce the important physical constraint that both $Y(4220)$ and $Y(4390)$ are observed (see Table I) to have substantial couplings to h_c ($s_{Q\bar{Q}} = 0$). Since $\Sigma_g^+(1S)$ contains only one $I=0$, $J^{PC} = 1^{--}$ state with $s_{Q\bar{Q}} = 0$, the requirement of providing a substantial component of this state to both of the well-separated $Y(4220)$ and $Y(4390)$ mass eigenstates is one of the primary obstacles to achieving a good fit.

Case 5 relieves the tension of Case 3 by identifying, as discussed in Sec. II, a new state "$Y(4320)$" from the data of Ref. [37]. In addition, Cases 1, 2, 3, and 5 all predict the

TABLE II. Decomposition of Y ($I=0$, $J^{PC}=1^{--}$) charmoniumlike exotic candidates into a basis of good light-quark spin $s_{q\bar{q}}$, heavy-quark spin $s_{Q\bar{Q}}$, and total light-quark angular momentum $J_{q\bar{q}}$, performed for the 4 experimentally observed candidate states as described in Case 3 above and in Ref. [23]. A minus sign on the probability $(-|P|)$ means that the corresponding amplitude is $-|P|^{1/2}$, the same convention as is used for Clebsch-Gordan coefficients by the PDG [41].

Particle	$s_{q\bar{q}}$	$s_{c\bar{c}}$	$J_{q\bar{q}}$	Probability
$Y(4220)$	0	0	0	+0.231
	1	1	0	+0.012
			1	−0.577
			2	+0.181
$Y(4260)$	0	0	0	+0.061
	1	1	0	+0.004
			1	+0.352
			2	+0.583
$Y(4360)$	0	0	0	+0.069
	1	1	0	+0.835
			1	+0.020
			2	−0.075
$Y(4390)$	0	0	0	+0.638
	1	1	0	−0.149
			1	+0.051
			2	−0.161

sole $I=1$, $J^{PC}=0^{--}$ state in $\Sigma_g^+(1P)$ to lie in the range 4220–4235 MeV, which agrees well with the unconfirmed state $Z_c(4240)$ carrying these quantum numbers that is observed in the LHCb paper [47] confirming the existence of $Z_c(4430)$.

Case 4 also satisfies the $Y(4220)/Y(4390) s_{Q\bar{Q}}=0$ criterion, but additionally assigns the rather high-mass $Y(4660)$ to the $\Sigma_g^+(1P)$ multiplet; the cost is a much higher prediction (≈4440 MeV) for the mass of the $\Sigma_g^+(1P)$ $I=1$, $J^{PC}=0^{--}$ state, in conflict with the value of $m_{Z_c(4240)}$.

TABLE III. Decomposition of Y ($I=0$, $J^{PC}=1^{--}$) charmoniumlike exotic candidates as in Table II, except now performed for the 4 experimentally observed candidate states as described in Case 5 above and in Ref. [23].

Particle	$s_{q\bar{q}}$	$s_{c\bar{c}}$	$J_{q\bar{q}}$	Probability
$Y(4220)$	0	0	0	−0.264
	1	1	0	−0.007
			1	+0.543
			2	−0.186
$Y(4260)$	0	0	0	+0.060
	1	1	0	+0.036
			1	+0.380
			2	+0.523
"$Y(4320)$"	0	0	0	+0.025
	1	1	0	+0.870
			1	$+8\times10^{-4}$
			2	−0.105
$Y(4390)$	0	0	0	−0.651
	1	1	0	+0.086
			1	−0.076
			2	+0.187

We therefore single out the fits of Cases 3 and 5 for the decomposition of Y states with respect to the total light-quark angular momentum $J_{q\bar{q}}$ in Tables II and III, respectively. For completeness, we also provide the corresponding information for Cases 1, 2, and 4 in Table IV.

Using the mass eigenvalues for the Y states in Table I, the state decompositions according to $J_{q\bar{q}}$ in Tables II, III, and IV, the coefficient factors in Eq. (10), and the effective squared-charge Q_δ^2 from Eq. (11), one may calculate the E1 radiative partial decay widths for $\Sigma_g^+(1P)\to\gamma\Sigma_g^+(1S)$ transitions from Eq. (9). The only nontrivial new input to the calculation is that of the transition matrix element $\langle\psi_f|r|\psi_i\rangle$. Using the numerical methods for solving Schrödinger equations developed in Ref. [20], and particularly the fits performed in Ref. [25] to obtain the fine structure of the $\Sigma_g^+(1S)$ multiplet, the optimal diquark mass is found to be

$$m_\delta = m_{\bar{\delta}} = 1.933 \pm 0.005 \text{ GeV}, \tag{17}$$

as one varies over the static gluonic-field potentials Σ_g^+ obtained in the lattice calculations of Refs. [15–19]. We then compute the relevant matrix element to be

$$\langle\psi_f(1S)|r|\psi_i(1P)\rangle = 0.402 \pm 0.001 \text{ fm}. \tag{18}$$

Note in particular that this numerical input appears in all $\Sigma_g^+(1P)\to\gamma\Sigma_g^+(1S)$ transitions, not simply those of $Y\to\gamma X(3872)$ that are compiled according to the 5 cases in Table V. Moreover, Table V and additional calculated width values presented subsequently in this work exhibit only central values for Γ; the small uncertainties in Eqs. (17) and (18) only refer to variation over different lattice simulations, and do not take into account other much more significant potential sources of uncertainty, such as effects due to finite diquark size. Nevertheless, such effects were shown [25] to change expectation values like $\langle r\rangle$ no more than 10%, a value that we adopt as a benchmark uncertainty for all Γ values computed here.

Also noteworthy is the magnitude of $k\langle\psi_f(1S)|r|\psi_i(1P)\rangle$ for each case, which provides an indication of the reliability of the long-wavelength approximation. Indeed, for $Y(4220)$, $k=334$ MeV, and using Eq. (18) gives $kr\to0.680$, while the corresponding value for $Y(4660)$ ($k=699$ MeV) is 1.424. However, the same simulations as in Eq. (18) also produce

$$\langle\psi_f(1S)|r^3|\psi_i(1P)\rangle = 0.135 \pm 0.001 \text{ fm}^3, \tag{19}$$

from which one computes the relative magnitude of the first correction term in Eq. (15) to be only 0.033 for $Y(4220)$ and, surprisingly, only 0.300 for $Y(4660)$.

TABLE IV. Decomposition of Y ($I = 0$, $J^{PC} = 1^{--}$) charmoniumlike exotic candidates as in Tables II–III, except now performed for the 4 experimentally observed candidate states as described in Cases 1, 2, and 4 above and in Ref. [23].

Case 1					Case 2					Case 4				
Particle	$s_{q\bar{q}}$	$s_{c\bar{c}}$	$J_{q\bar{q}}$	Probability	Particle	$s_{q\bar{q}}$	$s_{c\bar{c}}$	$J_{q\bar{q}}$	Probability	Particle	$s_{q\bar{q}}$	$s_{c\bar{c}}$	$J_{q\bar{q}}$	Probability
$Y(4220)$	0	0	0	+0.771	$Y(4220)$	0	0	0	−0.336	$Y(4220)$	0	0	0	−0.233
	1	1	0	−0.019		1	1	0	+0.032		1	1	0	−0.048
			1	−0.211				1	+0.631				1	+0.376
			2	-4×10^{-7}				2	$+8 \times 10^{-4}$				2	−0.343
$Y(4260)$	0	0	0	+0.212	$Y(4260)$	0	0	0	+0.588	$Y(4360)$	0	0	0	−0.119
	1	1	0	+0.130		1	1	0	+0.056		1	1	0	+0.101
			1	+0.597				1	+0.246				1	−0.473
			2	+0.062				2	+0.109				2	−0.308
$Y(4360)$	0	0	0	+0.006	$Y(4360)$	0	0	0	−0.046	$Y(4390)$	0	0	0	+0.647
	1	1	0	+0.252		1	1	0	−0.117		1	1	0	+0.003
			1	-5×10^{-6}				1	−0.012				1	+0.009
			2	−0.742				2	+0.824				2	−0.342
$Y(4390)$	0	0	0	−0.012	$Y(4390)$	0	0	0	−0.029	$Y(4660)$	0	0	0	−0.002
	1	1	0	+0.599		1	1	0	+0.795		1	1	0	+0.848
			1	−0.193				1	−0.111				1	+0.143
			2	+0.196				2	+0.066				2	+0.007

One observes from Table V that the widths $\Gamma_{Y(4220)\to\gamma X(3872)}$ and $\Gamma_{Y(4260)\to\gamma X(3872)}$ assume almost the same values in Cases 3 and 5 (102–105 keV and 211–216 keV, respectively). $\Gamma_{Y(4390)\to\gamma X(3872)}$ also exhibits fairly modest variation, from 254–319 keV. Indeed, some of the large radiative width values in Table V, such as 3.4 MeV for $Y(4660) \to \gamma X(3872)$ in Case 4, can serve as vital criteria for eliminating possible assignments of Y states to the $1P$ multiplet: Glancing at the measured total $\Gamma_{Y(4660)}$ in Table I, one sees that were $Y(4660)$ truly a $1P$ state, then its large phase space for radiative decay to $X(3872)$ [evident from the E_γ^3 factor of Eq. (9)] would generate a radiative branching fraction of at least several percent.

The transition matrix element of Eq. (18) has already been noted to apply to all $\Sigma_g^+(1P) \to \gamma\Sigma_g^+(1S)$ transitions. The only observed hidden-charm tetraquark candidates with $P = -$ apart from the Y states are $Z_c(4240)$ and $Y(4626)$; the latter has thus far been observed to decay only to various D_s meson pairs [48,49], and therefore is very likely a $c\bar{c}s\bar{s}$ state [25]. As for $Z_c(4240)$, only its charged

TABLE V. Radiative E1 partial widths (in keV) to $\gamma X(3872)$ calculated using Eq. (9), for the 5 cases of possible Y state assignments defined above and in Ref. [23]. For each case, note that two of the Y states (indicated by dots) are assumed either not to exist or not to belong to the $\Sigma_g^+(1P)$ multiplet.

Case	$Y(4220)$	$Y(4260)$	"$Y(4320)$"	$Y(4360)$	$Y(4390)$	$Y(4660)$
1	30.4	145.6	⋯	721.3	981.8	⋯
2	81.1	80.6	⋯	616.0	1127.0	⋯
3	105.1	211.2	⋯	1016.1	319.2	⋯
4	136.0	⋯	⋯	432.1	231.6	3363.9
5	102.4	216.2	807.6	⋯	253.8	⋯

isobar has yet been observed, but assuming the existence of a degenerate $Z_c(4240)^0$, one may input its quantum numbers $s_{Q\bar{Q}} = 1$, $J_{q\bar{q}} = 1$, $J = 0$ [23] into Eq. (9) to obtain

$$\Gamma[Z_c(4240)^0 \to \gamma X(3872)] = 503 \text{ keV}. \qquad (20)$$

Lastly, we noted with Eq. (12) that M1 transitions occur only with a flip of the heavy-quark spin. Such is the case for the $\Sigma_g^+(1S) \to \gamma\Sigma_g^+(1S)$ transition $Z_c(4020)^0 \to \gamma X(3872)$ ($s_{Q\bar{Q}} = 0 \to s_{Q\bar{Q}} = 1$). Using Eq. (17), we calculate

$$\Gamma[Z_c(4020)^0 \to \gamma X(3872)] = 7.91 \text{ keV}, \qquad (21)$$

noting from Eq. (12) that the underlying matrix element $\langle\psi_f|\psi_i\rangle = 1$ since both states share the same radial wave function. In comparison, the molecular model, in which $X(3872)$ and $Z_c(4020)$ are $D^0\bar{D}^{*0} + \bar{D}^0 D^{*0}$ and $D^*\bar{D}^*$ bound states, respectively, and the decay $Z_c(4020)^0 \to \gamma X(3872)$ proceeds via $D^{*0} \to \gamma D^0$, produces a rather larger radiative width: The calculation of Ref. [50] predicts a branching fraction of about 5×10^{-3}, which for $\Gamma_{Z_c(4020)} = 13 \pm 5$ MeV [41] amounts to at least 40 keV.

In light of our investigations for $\Sigma_g^+(1P) \to \Sigma_g^+(1S)$ E1 transitions, the long-wavelength approximation for M1 transitions within the single multiplet $\Sigma_g^+(1S)$ is undoubtedly satisfactory [for example, in $Z_c(4020)^0 \to \gamma X(3872)$, k is only 150 MeV]. Indeed, one may press the approximation of Eq. (16) to consider a transition that is forbidden in the long-wavelength limit of Eq. (12) due to the orthogonality of wave functions, such as $\Sigma_g^+(2S) \to \gamma\Sigma_g^+(1S)$. Assuming that $Z_c^0(4430)$ is the $2S$ partner to $Z_c^0(4020)$, then $Z_c^0(4430) \to \gamma X(3872)$ has $k \approx 565$ MeV, while we compute

$$\langle\psi_f(1S)|r^2|\psi_i(2S)\rangle = 0.152 \pm 0.001\ \mathrm{fm}^2, \quad (22)$$

and the first nontrivial term of Eq. (16) evaluates to −0.052. Using this correction in Eq. (12) leads to a radiative width $\Gamma_\gamma \simeq 1$ keV, to be compared with $\Gamma_{Z_c^+(4430)} \approx 180$ MeV [41]. The observation a radiative transition with such a small branching fraction is not impossible, but likely will not occur in the near future.

V. CONCLUSIONS

In this paper we have calculated exotic-to-exotic hadronic radiative transitions using the dynamical diquark model. The most phenomenologically relevant final state is $X(3872)$, which is a member of the model's hidden-charm ground-state multiplet $\Sigma_g^+(1S)$. We use the results from a recent study [23] of this model for the lowest P-wave multiplet $[\Sigma_g^+(1P)]$ of hidden-charm tetraquark states, in which the $\Sigma_g^+(1P)$ states are identified with the observed $I=0$, $J^{PC}=1^{--}$ (Y) states according to a variety of scenarios, based upon both their mass spectra and preferred decay modes to eigenstates of heavy-quark spin (e.g., J/ψ vs h_c). We calculate E1 and M1 transition amplitudes for $\Sigma_g^+(1P) \to \gamma\Sigma_g^+(1S)$ and $\Sigma_g^+(1S) \to \gamma\Sigma_g^+(1S)$ processes, respectively, and present corresponding values for the radiative decay widths of a number of particular exclusive channels.

This analysis shows that if $Y(4220)$ and $X(3872)$ have a similar underlying diquark structure, then one expects $\Gamma_{Y(4220)\to\gamma X(3872)} \approx 100$ keV. Moreover, similar values (albeit somewhat larger due to increased γ phase space) are expected for the heavier Y states in $\Sigma_g^+(1P)$. The extreme possibility of $Y(4660)$ belonging to the $1P$ multiplet would lead to a $\gamma X(3872)$ branching fraction of several percent, and so the absence of such a remarkably large signal would appear to relegate $Y(4660)$ instead to the $\Sigma_g^+(2P)$ multiplet.

Furthermore, we found that the observed but unconfirmed $Z_c(4240)$, a candidate for the sole $I=1$, $J^{PC}=0^{--}$ state in $\Sigma_g^+(1P)$, should have a substantial (≈500 keV) radiative decay width to $X(3872)$ through its neutral isobar, and therefore this decay is a good candidate for future experimental investigation. Indeed, many of the $\Sigma_g^+(1P)$ states have not yet been observed, offering multiple potential future tests of the model.

M1 transitions within a single multiplet, such as $Z_c(4020)^0 \to \gamma X(3872)$, produce much narrower widths (<10 keV in this model), and can provide sensitive tests of substructure (e.g., diquarks *vs* meson molecules).

One may also study exotic-to-exotic radiative transitions in other heavy-quark sectors (e.g., hidden-bottom or $c\bar{c}s\bar{s}$ exotics). Indeed, Eqs. (9) and (12) are general for any tetraquark state in the diquark-antidiquark configuration. For example, Ref. [25] calculates the mass of X_b [the hidden-bottom analogue to $X(3872)$] to lie in a rather narrow range $m_{X_b} \in [10598, 10607]$ MeV, only slightly below the observed $Z_b(10610)^0$. The M1 transition $Z_b(10610)^0 \to \gamma X_b$ is thus expected from Eq. (12) to produce a tiny $[O(\mathrm{eV})$ or less] radiative width, owing to not only the small phase space, but also the larger (b-containing) diquark mass. We conclude that even very coarse experimental results in other sectors can be decisive in verifying or falsifying particular models.

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