



Iterative dominance in young children: Experimental evidence in simple two-person games[☆]



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ABSTRACT

We investigate iterative reasoning in children from pre-kindergarten to first grade (4–7 years old). We consider two-person games that have a unique Trembling Hand Perfect Equilibrium but vary in three key features: iterative complexity (the number of iterations required to reach the equilibrium), perspective requirement (the identity of the player with whom the iteration should start) and action symmetry. The beliefs of participants regarding the decision of their partners are elicited either before or after their own choices are made. We obtain the following findings. (1) Iterative complexity is not necessarily a cause of equilibrium failure. (2) Games where the equilibrium action is the same for both players are correctly solved more often. (3) Starting a recursion by solving the problem of a different player is non-intuitive in all our age groups but it can be overcome by asking participants to think about their partner's decision before making their own choices. (4) Such ex-ante belief elicitation can, however, be detrimental in complex asymmetric games. We discuss these results in the context of the development of logical abilities and theory of mind abilities.

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1. Introduction

Strategic reasoning is a cornerstone of equilibrium play in game theoretic paradigms. From a conceptual point of view, strategic reasoning requires two fundamental cognitive abilities: the ability to take the perspective of others and the ability to reason logically. The ability to take the perspective of others, known as Theory of Mind (ToM), develops gradually. Children move from a situation in which they neither infer nor care about what others think to a situation in which they attribute beliefs to others and empathize with them (Apperly and Butterfill, 2009; Perner, 1991; Premack and Woodruff, 1978; Wellman et al., 2001; Wellman and Liu, 2004). Evidence suggests that critical development occurs between 4 and 8 years of age (Perner and Wimmer, 1985; Wimmer and Perner, 1983), even though instances of ToM are already present in infants (Baron-Cohen, 1991; Meltzoff, 2002) and the ability develops throughout childhood and into late adolescence (Valle et al., 2015). Logical thinking develops in stages. Children first learn to draw logical conclusions from their observa-

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tions, and later learn to reason abstractly (Feeney and Heit, 2007; Rafetseder et al., 2013; Shultz and Cloghesy, 1981; Tecwyn et al., 2014).

In principle, the development of these cognitive abilities should support the development of strategic reasoning and result in a gradual improved performance in game theoretic experimental paradigms. Recent research makes two important contributions in that respect. First, it confirms the existence of a link between strategic reasoning and these underlying cognitive abilities (Blake et al., 2015; Brocas and Carrillo, 2017; 2018; Brocas et al., 2017; Sher et al., 2014) in specific game theoretic contexts. This indicates that, as we grow, we acquire cognitive skills that gradually transfer into our ability to find equilibrium strategies. Second, it demonstrates that the age at which children are capable of playing at equilibrium depends on the paradigm and the complexity of the task. Hence strategic reasoning is multi-faceted and the abilities underlying it are acquired and/or transferred differentially.

Strategic reasoning requires the integration of ToM and logical abilities. In particular, it relies on the ability to utilize a belief about what a partner will do as input in own decisions. ToM supports belief formation while logical abilities support its logical utilization to formulate a strategy, or best response. Importantly, a player should also realize that partners are making the same reasoning, which should be taken into account to iteratively formulate a strategy. The objective of this study is to examine iterative reasoning in the context of simple games that young children (4–7 years old) can play. More precisely, we restrict to games that are intuitive, feature a unique Trembling Hand Perfect Equilibrium (THPE) and can be solved using a few steps of iterated elimination of (weakly or strictly) dominated strategies. This class of games is interesting because they are simple to play, yet contain most features pertaining to strategic reasoning. The age range 4–7 is particularly relevant because children are in the process of developing ToM abilities: first-level beliefs of the form ‘I think that you think’ appear around 4 years of age (Wimmer and Perner, 1983) while second-level beliefs of the form ‘I think that you think that he thinks’ develop at around 8 years of age (Perner and Wimmer, 1985). We argue that beliefs of the form ‘I think that you think that I think’ should be developing in our window of observation.

We focus on two important aspects. First, we want to measure how best response behavior relates to beliefs. To this purpose, we elicit the beliefs of participants about the other player's choice either *after* or *before* they have made their decisions. This allows us to assess ToM in our participants and to determine how prompting them to think about others changes their beliefs and actions. It also allows us to verify the consistency between their beliefs and their decisions. We conjecture that prompting children to think about the decisions of their partners before making a decision will help the selection of equilibrium strategies. Second, we want to tie performance to specific features of the game. We design simple games that require the same iterative reasoning abilities but vary in iterative complexity, perspective requirements, and action symmetry. Iterative complexity refers to the number of iterations that may be required to reach an equilibrium. Perspective requirement refers to the identity of the player one should consider to start the iterative reasoning. Action symmetry refers to whether the equilibrium action is the same for both players or not. We conjecture that games that are symmetric, feature low levels of iterative complexity and can be started from any player's viewpoint should exhibit higher equilibrium rates.

We find that the perception of difficulty by children often but not always coincides with our conjectures. In particular, iterative complexity is not necessarily a cause of failure to play at equilibrium, which indicates that children are able to act as if they are iterating towards the fixed point of the game. Perspective requirements however have a strong effect on behavior. The presence of a strictly dominant strategy in the rival's choice set – which requires to start the recursion by identifying that strategy – is complex to find, even when iterative thinking is simple. Last, games where both players have the same equilibrium action are more often solved, independently of iterative complexity. Interestingly, we find that eliciting beliefs before their action can affect performance positively but also negatively. Indeed, prompting children to think about the decision of the partner allow subjects in all ages to find more frequently the dominant strategy of the rival. However, it negatively impacts the ability to reach the fixed point in iteratively complex games where the equilibrium actions of players are not identical. This suggests that thinking about the decision of the partner may crystalize initial non-equilibrium beliefs rather than help eliminating them. Last, the results are strongly modulated by age. Younger children are less likely to reach the equilibrium action and also believe that their partners are less sophisticated.¹ These differences suggest differences in ToM abilities. Younger children have trouble forming second-level beliefs of the kind ‘I think that you think that I think’, which is consistent with the ToM literature. We describe the experiment and theoretical background in Section 2, report our findings in Section 3, and provide some concluding comments in Section 4.

2. Experiment

2.1. Design and procedures

The experiment was reviewed and approved by the IRB of the University of Southern California (USC). We recruited 122 children in three grades –pre-kindergarten (PK, ages 4–5), kindergarten (K, ages 5–6) and first (1, ages 6–7)– at the Los Feliz campus of the Lycée International de Los Angeles (LILA). LILA is a french-english bilingual private school in Los

¹ Aggregate beliefs about sophistication of the partner are correct in equilibrium, implying that results on different beliefs at different ages may come from correct anticipations or differences in own sophistication. One way to disentangle between these two possibilities would be to match subjects of different ages.

Table 1
Participants by grade and gender.

Grade	PK	K	1
Male	18	21	21
Female	20	18	24
Total	38	39	45

Angeles. Families are predominantly of caucasian ethnicity from north America and western Europe, with a mix of french and english native speakers. Some children speak a third language at home. Parents are predominantly from upper-middle socio-economic status, with an over-representation of professionals from the entertainment industry. It is therefore not representative of the US population. The distribution of participants is reported in Table 1.

For comparative purposes, we ran 4 sessions with 52 USC students. These were conducted at the Los Angeles Behavioral Economics Laboratory (LABEL) in the department of Economics at the University of Southern California.²

2.1.1. Games

We focus on games that are easy to grasp, require simple logic and a few steps of reasoning to be solved but, at the same time, some amount of strategic thinking. There are numerous studies in psychology on reasoning in young children. While the majority of the literature focuses on individual decision making (Asato et al., 2006; Eliot et al., 1979; Moffett et al., 2017; Tecwyn et al., 2014) some fall in the category of multi-person games of strategy (Shultz and Cloghesy, 1981; Warneken et al., 2014). However, to make them attractive to children, they are typically embedded in a physical environment such as a box with levers (Tecwyn et al., 2014; Warneken et al., 2014). This has its own advantages. In particular, it helps maintaining the attention especially for the younger children. However, it also makes it difficult to isolate the fundamental game theoretic aspects of the game. More importantly, studies typically focus on one single game, that children may or may not successfully solve. Instead, we are interested in understanding which game theoretic elements of strategic situations are more or less conducive to equilibrium behavior.

We selected games with a unique Trembling Hand Perfect Equilibrium (THPE).³ We designed four novel games, all of which featured an intuitive solution and could be solved using a few steps of iterated elimination of (weakly or strictly) dominated strategies. The games are presented in a simple, graphical way and explained in an accessible manner. Each participant (from now on, he) completed all four games in a counterbalanced order. Participants were paired randomly within each grade and allocated a role as player 1 or player 2 (except for one pair that had one child from K and one from 1). The games are depicted in Fig. 1.⁴

- In the *Matching* game, each player is allocated a deck with 5 colored cards (Fig. 1(a)). Each player simultaneously selects a card. They both win if the colors of their cards match, and they both lose otherwise.
- The *Fighting* game has the opposite logic. Each player is allocated a deck with 4 colored cards (Fig. 1(b)). Each player simultaneously selects a card. They both win if the colors of their cards do not match, and they both lose otherwise.
- In the *Tower* game, players simultaneously place their character (the red or the green person) in one floor of the building (Fig. 1(c)). A player wins if he select the same or a higher floor than their partner.⁵ These three games are always played with the same partner.
- The *Shape* game takes place between a child and the experimenter. The experimenter has 4 shapes (triangle, star, circle and moon) with different colors and two puzzles (star and rhombus) while the player receives a deck of 4 cards, each with a color that matches one of the shapes (Fig. 1(d)). The experimenter wins if she selects a shape that fits in one of the puzzles and the player wins if he selects a card that matches the color of the shape selected by the experimenter. The player is explicitly instructed that the experimenter will try to win.

At the end of each game, players learned the choice of their partner and the payoff of both.

2.1.2. Treatments

Half of the participants in each age group completed each game first and then answered the question “what do you think your partner did?”. The other half first answered the question “what do you think your partner will do?” and then made their choices. We will refer to these two treatments as the *After* and *Before* (belief elicitation of the partner’s choice). Overall, the experiment has a 3×2 between-subject design with 3 age groups (PK, K, 1) and 2 belief elicitation timings (*After*, *Before*).

² We thank an anonymous referee for suggesting us to run the adult population.

³ THPE is an intuitive refinement of Nash equilibrium introduced by Selten (1975). In two-person games, it amounts to ruling out Nash equilibria in which players use weakly dominated strategies.

⁴ To help the reader, who may be viewing Fig. 1 in Black and White, we indicate the color of each card in words. The children saw each card in color during the experiment. Therefore words were not necessary.

⁵ This is a simplified version of the two-person beauty contest game that has been studied in adults using analytical (Grosskopf and Nagel, 2008) and graphical (Chou et al., 2009) interfaces. Brocas and Carrillo (2017) tested a two-dimensional graphical version with children and adolescents.

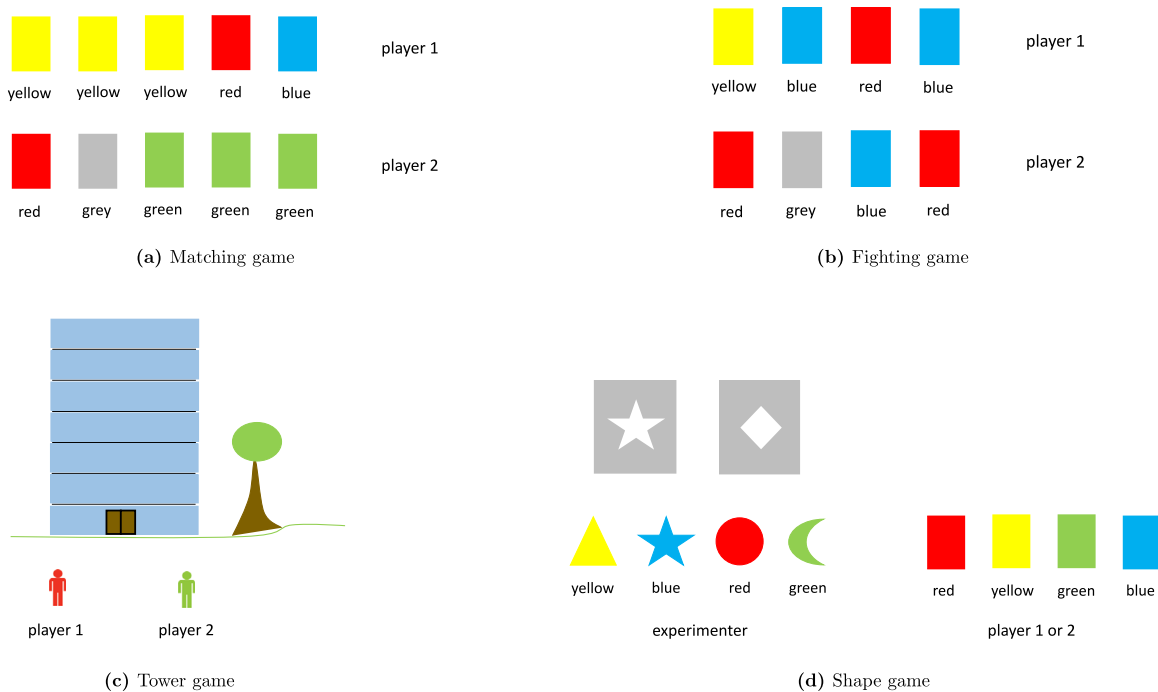


Fig. 1. Experimental design.

2.1.3. Implementation

For each game, we first read the instructions and then raised a cardboard wall between each pair of players to ensure privacy in decision-making. In the Matching, Fighting and Shape games, each child had a set of physical cards as well as a laminated mat depicting both his deck and the deck of the partner (or the experimenter). This procedure ensured that children knew at all times the options of all players. It also facilitated the belief elicitation, as they could simply point in the mat to the card they believed the partner (or experimenter) had chosen. For the Tower game, children had a mat with the building and were given the outline of one character. They physically placed their character in the mat and pointed to the floor they believed the partner had chosen.

2.1.4. Demographic information

To control for age-related differences within grade, we collected information regarding the age (in months) of our participants. We also recorded their gender for analysis.

Games were administered in a classroom at LILA reserved for the experimental sessions. Games were incentivized. Children earned 40 tokens for each game they won and 20 tokens for each game they did not.⁶ They used the tokens to purchase toys in our ‘toy shop’ located in an adjacent space.⁷ It took about 15 min to administer the games in PK and 10 min in K and 1. A copy of the instructions can be found in [Appendix B](#).

2.2. Theory

Despite the simplicity of the games, they vary in terms of equilibrium features. All games have a unique THPE, which requires some strategic reasoning but, at the same time, can be played instinctively. With a slight abuse of language, we will refer to the THPE as the “equilibrium” of the game. In the Matching game, the equilibrium is for both players to choose the only matching card, namely the red one. In the Fighting game, the equilibrium is for players to choose the only card whose color is not present in the other deck (yellow for player 1 and grey for player 2). In the Tower game, the equilibrium is for

⁶ We provided tokens in case of losing, thereby increasing the mean and decreasing the variance in final payoffs, to make sure the experience was enjoyable.

⁷ The shop had 20–25 pre-screened, age-appropriate toys. They included gel pens, friendship bracelets, erasers, figurines, die-cast cars, trading cards, bouncy balls and fidget spinners. Each child got between 3 and 8 toys depending on the tokens accumulated and the token price of the options selected. For the youngest children, we helped them determine the toys they could buy with their budget.

Table 2
Game features.

	Iterative complexity	Perspective requirements	Action symmetry
Matching	simple	own or rival	yes
Tower	complex	own or rival	yes
Fighting	complex	own or rival	no
Shape	simple	rival	no

both players to place their character in the top row. Finally, in the Shape game the experimenter's dominant strategy is to choose the blue star, so by iterative dominance the player's equilibrium strategy is to choose the blue card.⁸

Overall, our games differ mainly in three dimensions. First, the iterative complexity. In the Matching game, once a player understands that colors much match, it takes one step to find the only common color in the two decks. In the Shape game, the player needs to complete exactly two steps: find the dominant strategy of the rival and best respond to it. The Fighting and Tower games however may require more iterations to determine the color that is unique in each deck and the top floor, respectively. Second, games differ in the role that should be examined to start the recursion. In the Shape game, the equilibrium cannot be reached if the dominant strategy of the rival is not pinpointed. By contrast, in all other games, it does not matter a priori how the recursion is started. Last, in the Matching and Tower games the equilibrium action is identical for both players so, once the action of one player is found, the action of the other can be trivially replicated. By contrast, in the Fighting and Shape games players have different equilibrium actions.⁹ We summarize these features in Table 2.

Notice also that the Matching, Fighting and Tower games have other Nash equilibria, where subjects play weakly dominated strategies (for example, if my partner does not choose red in the Matching game, it is in my best response correspondence to not choose red either). By contrast, the Nash equilibrium in the Shape game is unique.¹⁰

Since we explicitly elicit beliefs, we can assess the degree of sophistication of our participants in two different ways. First, we can determine whether they play the THPE (equilibrium actions) and whether they expect their partner to play the THPE (equilibrium beliefs). Second, among participants who do not play the THPE, we can determine if, at least, they best respond to their beliefs about their partner's choice.¹¹

2.3. Predictions

Consistent with the literatures on the development of logical thinking and ToM, we hypothesize that age will be a strong predictor of both equilibrium behavior and equilibrium beliefs about other's behavior.

We also hypothesize different qualitative results across games. We expect the Matching game to be easiest for all participants, resulting in frequent equilibrium play and equilibrium beliefs at all ages. Indeed, the alignment of interests and symmetry of equilibrium actions makes the red card a natural choice even for the youngest participants. Thinking that the rival thinks (first-level beliefs) and simple logic should be enough to reach the equilibrium.

We expect lower levels of equilibrium play and equilibrium beliefs in the Fighting and Tower games, as they necessitate multiple iterations between a subject's choice and the rival's best response to reach the equilibrium (iterative complexity). Not only children need to iterate logically several times but they also need to use different levels of beliefs (about what the other thinks, and about what the other thinks I think) as starting point of each iteration.

Last, we expect lowest levels of equilibrium play and equilibrium beliefs in the Shape game. Indeed, we anticipate that children will have problems realizing the importance of identifying the unique course of action of the other player before making their choices (perspective requirements). This task requires only first-level beliefs of the form 'I think that you think' and a short recursion. However, to solve the game it is essential to hypothesize a starting point for the other player and deduce the solution from there. This form of logic develops slowly and is apparent in specific experimental settings (Dias and Harris, 1988; Richards and Sanderson, 1999). Furthermore, and contrary to the other games, the Shape game does not have a weakly dominant strategy that can be heuristically deduced and safely played when the subject is uncertain about the other player's behavior.

⁸ We decided against having two experimental subjects playing the Shape game with each other for two reasons. First, because we are mostly interested in the behavior of the player who needs to perform one step of elimination of dominated strategies. Second, because we want to minimize the likelihood that a sophisticated subject does not choose the blue card only because he believes that his partner is not sophisticated enough to select the star.

⁹ In the Fighting game, both players have identical equilibrium strategies (choose the only color not present in the other deck). However, they translate into different equilibrium actions (grey card vs. yellow card).

¹⁰ In the matching game, the other Nash equilibria involve a lower payoff than the THPE (e.g., yellow-green) whereas in the Fighting and Tower games, they involve the same payoff as the THPE (e.g., blue-red in Fighting and both in the same non-top floor in Tower).

¹¹ Notice that we asked for point predictions of beliefs and we did not incentivize the belief elicitation procedure. While this could be suboptimal in many settings, we believe it is the right procedure in an experiment with 4–7 years-old participants, who would not understand probabilistic predictions and would be puzzled if we gave them tokens for the accuracy of their beliefs. A simple, direct question like "what do you think your partner did / will do?" seems to us the most appropriate procedure.

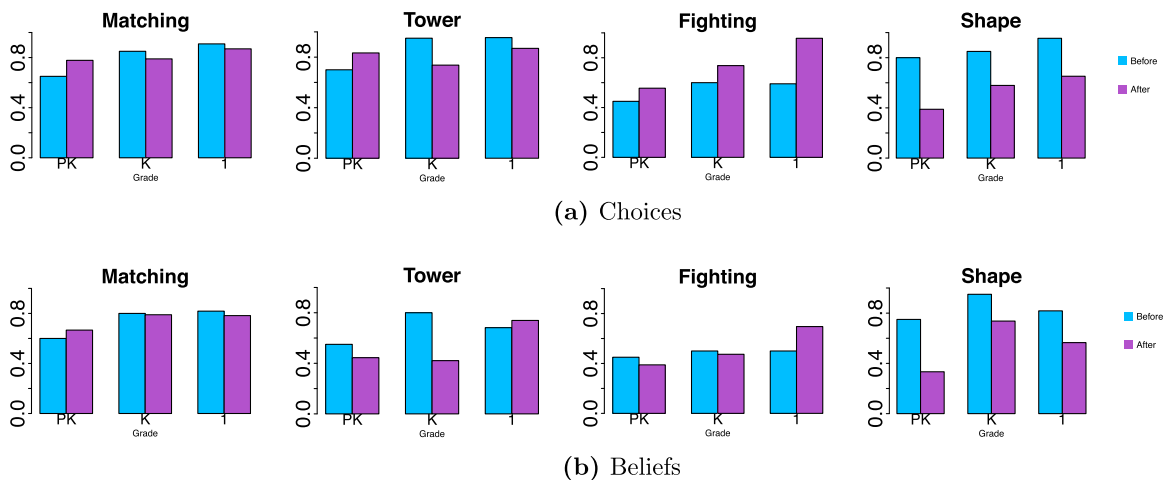


Fig. 2. Equilibrium choices and beliefs by age and treatment.

Finally, we conjecture that strategic reasoning does not come naturally to some children. Therefore, we have a strong expectation that prompting children to think about their partner's choice before making their own decisions will significantly increase the proportion of equilibrium beliefs and equilibrium choices.

3. Results

We present below the results related to the children population. The results of the adult control group can be found in [Appendix A](#).

3.1. Equilibrium actions and equilibrium beliefs

We start with an aggregate measure of the percentage of equilibrium actions and equilibrium beliefs by game, grade, and treatment. The results are reported in [Fig. 2](#). For statistical analysis, we perform the Pearson's chi-square test of comparison of proportions and use a 5%-level as the benchmark threshold for significance.

As predicted, the rate of equilibrium play in the Matching game is high in all ages and all treatments (between 0.65 and 0.91). There is also no significant treatment or age group effect. The same is true in the Tower game which, contrary to our hypothesis, children also find intuitive and reasonably “easy.” Indeed, equilibrium play in the Tower game is even higher than in the Matching game (between 0.70 and 0.96). There are also few differences in beliefs across age in these games, with first graders holding equilibrium beliefs more often than kindergartners and pre-kindergartners in the *After* treatment of the Tower game ($p = .038$ and $p = .055$, respectively). We found no statistically significant differences in equilibrium beliefs for the Matching game. Overall, the results indicate that the two games are simple enough for the children of all ages to play the equilibrium action. If anything, younger children do not necessarily believe that their partner is as sophisticated as they are.

Results are sharper in the Fighting game. In the *After* treatment, subjects in grade 1 choose the equilibrium action at a higher rate (0.96) than K (0.74) and PK (0.55), $p = .057$ and $p = .004$. These subjects also believe more often that their partner has picked the equilibrium solution (0.70 against 0.47 and 0.38), although the difference is only significant between 1 and PK ($p = 0.050$). The result suggests that the game is simple for the oldest children but quite difficult for the younger ones, both in terms of choice and beliefs. Most surprisingly and in contrast to our prediction, there is a *negative* treatment effect in grade 1: participants who were prompted to think about the strategy of their partner play significantly less often the equilibrium action (0.59 vs. 0.96 $p = .010$). The treatment does not affect play or beliefs in the other two grades.

Last, children reveal the greatest disparity in the Shape game, with equilibrium actions ranging from 0.39 to 0.95. This game appears to be the most challenging to our participants, who exhibit lowest equilibrium play of all games in the *After* treatment. However, when they are prompted to think about the behavior of the other player (*Before* treatment), the equilibrium action is selected more often in all age groups: PK (from 0.39 to 0.80, $p = 0.012$), K (from 0.58 to 0.85, $p = .064$) and 1 (from 0.65 to 0.95, $p = .015$). Prompting also increases (significantly or marginally) the belief that the other player will choose the equilibrium action: PK (from 0.33 to 0.75, $p = .012$), K (from 0.74 to 0.95, $p = .081$) and 1 (from 0.57 to 0.82, $p = .065$). Overall, this game conforms to our expectations, featuring a rate of equilibrium behavior increasing with age and a positive effect of prompting the player to think strategically.

While comparisons of proportions are instructive, they can also be misleading since all participants born in a calendar year are pooled in the same age group. To study our data further, we run a series of Probit regressions for each game separately. The dependent binary variables are equilibrium beliefs (*Belief*) and equilibrium action (*Action*). As for the dependent

Table 3

Probit regressions of equilibrium beliefs and actions in each game.

	Matching		Tower		Fighting		Shape	
	Belief	Action	Belief	Action	Belief	Action	Belief	Action
<i>Constant</i>	−0.546 (0.882)	−0.666 (1.140)	−1.256 (0.839)	−1.491 (1.034)	−0.799 (0.816)	−1.214 (0.864)	−0.639 (0.890)	−2.520* (1.197)
<i>Age</i>	0.225 (0.146)	0.160 (0.190)	0.253+ (0.137)	0.349* (0.176)	0.162 (0.133)	0.323* (0.143)	0.159 (0.146)	0.322+ (0.195)
<i>Before</i>	−0.058 (0.250)	−0.124 (0.328)	0.303 (0.236)	0.191 (0.297)	−0.150 (0.230)	−0.670** (0.248)	0.845*** (0.254)	0.716* (0.348)
<i>Belief</i>	—	1.823*** (0.330)	—	0.687* (0.298)	—	0.238 (0.244)	—	2.188*** (0.341)
<i>Male</i>	−0.183 (0.250)	−0.750* (0.344)	−0.209 (0.236)	0.057 (0.295)	−0.135 (0.230)	−0.091 (0.245)	−0.317 (0.251)	−0.702* (0.350)
Obs.	122	122	122	122	122	122	122	122
AIC	143.26	85.21	164.19	103.02	174.94	153.82	142.59	80.82

Standard errors in parenthesis. + $p < .1$; * $p < .05$; ** $p < .01$; *** $p < .001$.

variables, we include the age in months of the participant (*Age*), a treatment dummy (1 = *Before*), and a gender dummy (1 = *Male*). For the *Action* regression, we also include the *Belief* dummy as a dependent variable.¹² The results are compiled in Table 3.

The regression results support and complement the previous analysis. In the Matching game, equilibrium beliefs and actions are high in both treatments and all ages, so our dependent variables have no explanatory power. There is a strong correlation between beliefs and actions. This is natural in this game since for an agent to pinpoint the optimal strategy, it is necessary to also find the optimal strategy of the other player (with the caveat mentioned above that some subjects may know what the equilibrium is and still believe their partner will not play it). In the Tower game, age is a predictor of both equilibrium belief and equilibrium action. As in the Matching game, equilibrium beliefs predict equilibrium actions, although less strongly. This is partly because the equilibrium strategy can be determined independently of the other player's choice and, as we saw in Fig. 2, subjects tend to underestimate the rationality of their partners. The Fighting game is more puzzling. Beyond the natural positive effect of age on equilibrium actions, there is a strong negative treatment effect. This effect is not channeled through beliefs which, contrary to all other games, have also no effect on choice. The Shape game neatly conforms to our predictions. Prompting subjects to think about the other player's choice increases very significantly the likelihood of an equilibrium belief which, in turn, highly affects the likelihood of an equilibrium choice. Although this is the main channel towards equilibrium behavior, age also has an independent effect on it. Finally, we also find that, depending on the game, our female participants play the equilibrium action either at the same or at higher rates than their male counterparts.

Overall and despite their apparent similarity, we notice substantial differences across games. Because of the symmetry in the players' equilibrium actions, subjects in the Matching and Tower games have a reasonably easy time figuring out the optimal course of action. By contrast, the other two games are significantly more complex. Interestingly, while the Fighting game has a (weakly) dominant strategy that ensures the high payoff, thinking first about the other player's choice can be detrimental. Indeed, depending on the belief about the other person's choice, some (or even all) of the choices can be best response. We conjecture that this is the reason why prompting subjects to think first about the other player is typically counterproductive in this game. The Shape game, on the other hand, must be solved through iterated elimination of dominated strategies, starting from the other player. This is a natural game-theoretic process but non-intuitive for children. As a result, participants who are not induced to think first about the other player's choice get confused and do not choose often the equilibrium action. By contrast, those who are induced to look at the problem first through the other player's lenses, systematically succeed in identifying the partner's optimal strategy and invariably best respond to it. This is true even (or rather, especially) in the youngest population.

We finally conduct Probit regressions pooling all games together. We use as the binary dependent variables whether the subject played the equilibrium action (*Action* - columns (1) and (2)) and whether the subject both played the equilibrium action and reported the equilibrium belief (*Action&Belief* - columns (3) and (4)). We include the same dependent variables as in Table 3, and interaction terms between treatment and game. The results are reported in Table 4, with robust standard errors clustered at the individual level.

Once again, the regressions reinforce our previous findings. Age and belief about the other player's choice have a highly significant effect on equilibrium play. Male subjects perform worse than female. Equilibrium choices are also slightly lower in the Fighting and Shape games. From regression (2), we notice that in the Fighting game, lower performance is entirely

¹² One approach to study actions and beliefs would be to set a simultaneous equation model where beliefs affect actions and actions affect beliefs, as implied by a standard game theoretic equilibrium reasoning. Note, however, that we ask our subjects which action they believe their partner will play / has played, and not which action their partner should play / should have played. So, while actions should be a best response to beliefs, beliefs are not always affected by actions, which is why we decided against the simultaneous equation approach.

Table 4
Probit regressions pooling all games.

	Action		Action&Belief	
	(1)	(2)	(3)	(4)
Constant	−1.188** (0.451)	−1.245** (0.476)	−0.681 (0.427)	−0.625 (0.447)
Tower	0.284 (0.214)	0.199 (0.334)	−0.383* (0.175)	−0.531* (0.249)
Fighting	−0.326+ (0.192)	0.039 (0.286)	−0.921*** (0.165)	−0.665** (0.222)
Shape	−0.313+ (0.163)	−0.681** (0.238)	−0.157 (0.166)	−0.534* (0.240)
Age	0.255*** (0.073)	0.277*** (0.076)	0.207** (0.069)	0.211** (0.071)
Before	−0.008 (0.138)	−0.046 (0.257)	0.229+ (0.120)	0.083 (0.246)
Belief	1.121*** (0.151)	1.075*** (0.152)	—	—
Male	−0.239+ (0.137)	−0.275+ (0.143)	−0.236* (0.120)	−0.245* (0.124)
Tower × Before	—	0.171 (0.411)	—	0.298 (0.350)
Fighting × Before	—	−0.673+ (0.372)	—	−0.505 (0.327)
Shape × Before	—	0.858* (0.339)	—	0.812* (0.339)
Obs.	488	488	488	488
AIC	451.42	440.43	628.80	618.89

Clustered st. errors in parenthesis. + $p < .1$, * $p < .05$; ** $p < .01$; *** $p < .001$.

Table 5
Probability of best response to own beliefs.

	PK	K	1
Matching	0.97	0.97	0.93
Tower	1.00	1.00	0.93
Fighting	0.97	1.00	1.00
Shape Before	0.90	0.90	0.86
Shape After	0.88	0.63	0.65

driven by the *Before* treatment whereas in the Shape game it is entirely driven by the *After* treatment. Given the importance of holding equilibrium beliefs for playing equilibrium actions, it is not surprising that similar results are obtained when we jointly consider equilibrium action and belief as the dependent variable. Age, and to a lesser extent gender, remain major determinants. Deviations from equilibrium are stronger in the Fighting game and the *After* treatment of the Shape game. The only significant novelty is the lower performance in the Tower game mostly due to the lower rate of equilibrium beliefs relative to the Matching game, as previously documented in Fig. 2(b).

Some final comments are in order. First, it would be interesting to determine whether the (weakly) closer to equilibrium performance of females over males is a trait that persists in adulthood (as documented for example by Cadsby and Maynes, 1998 in the context of a public good game with multiple equilibria) or an indication of a superior intellectual maturity at this specific age. Second, a key policy question is to determine whether early schooling and a nurturing environment affect cognitive decision-making in young children. It would be interesting to compare the behavior of our population with that of children in a poor inner-city school as well as with home-educated children of similar socio-economic status as ours.¹³

3.2. Best response strategies

Even though children do not always play the equilibrium strategy, their actions may be consistent with their beliefs about their partner's behavior. In this section, we compute the best response of each participant in each game to their own elicited beliefs. The results are reported in Table 5.

¹³ See Cappelen et al. (2016) for a comparison of the evolution of social preferences between children who attend preschool, children in a parenting program and children in no program.

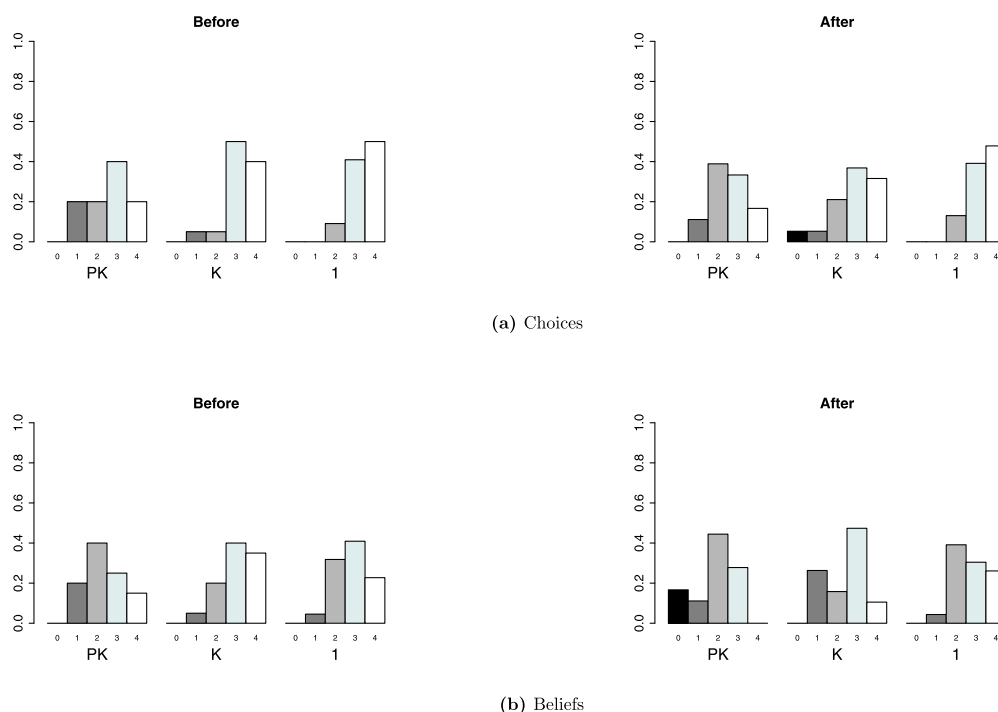


Fig. 3. Number of equilibrium choices and equilibrium beliefs by age and treatment.

Children in all ages almost always best respond to their beliefs in the Matching, Tower and Fighting games. This result should not be overemphasized as best response is almost guaranteed by construction.¹⁴ It confirms, however, that children are not confused by the games (a possibility worth keeping in mind with subjects of such young age) and do not treat the Fighting game as a Matching game, for example.

The most interesting result in Table 5 relates to the treatment effect in the Shape game. In the *After* treatment, pre-kindergartners have the highest rates of best response, despite their lowest rate of equilibrium choice. It means that, conditional on their (often incorrect) belief about the experimenter's behavior, they are able to pick the correct color. It is also possible that the reasoning is reversed: they first choose one color and, when asked, they simply assign to their partner the shape that matches their color. Kindergartners and first graders do not best respond as well, and often do not choose the equilibrium action while revealing that the experimenter chose the correct shape. It is likely that those children realize the strategy they should have played after being asked about the strategy of their partner. In the *Before* treatment, these children best respond at a higher rate, although only marginally significantly in the K population ($p = .054$) and not significantly in the other two. All in all, in the context of the Shape game, prompting participants to think about the behavior of the other player has a marginally positive effect on best response reasoning.

3.3. Individual analysis

In previous sections we have studied the behavior of children in each game separately, thereby abstracting from individual differences. We now study individual behavior across games, to determine whether participants have consistent reasoning abilities. Fig. 3 reports the fraction of individuals with 0–4 equilibrium choices (Fig. 3 - top row) and 0–4 equilibrium beliefs (Fig. 3 - bottom row) in each age group and treatment.

While the percentage of participants who always choose the equilibrium action is higher among the older population, the overwhelming majority of children in all ages play the equilibrium strategy at least twice. On average, pre-kindergartners solve 2.56 games, kindergartners solve 2.84 games and first graders solve 3.34 games in the *After* treatment. Differences are significant only between PK and 1 (t -test, $p = .025$). In the *Before* treatment, pre-kindergartners solve on average 2.60 games, kindergartners solve 3.25 games and first graders solve 3.4 games. Differences are significant between PK and K (t -test, $p = .035$) as well as between PK and 1 (t -test, $p = .009$). Table 6 presents an OLS regression of the number of games solved by children in each treatment on their age. It shows a significant effect of age.

¹⁴ The requirements to be classified as not best responding are very stringent. In the Matching game, the subject must not select the red card but believe that the partner selected the red card. In the Tower game, the subject must choose a floor strictly below the floor believed to be selected by the partner. In the Fighting game, the subject must believe that the partner did not choose the equilibrium card and then select that same color card.

Table 6
OLS regression of the number of games solved.

	number of equilibrium choices	
	Before	After
Constant	0.881 (0.777)	1.091 (0.792)
Age	0.380** (0.126)	0.367** (0.131)
Male	−0.125 (0.218)	−0.581* (0.228)
Obs.	60	62
Adj. R ²	0.111	0.168

St. errors in parenthesis. +, $p < .1$, *, $p < .05$; **, $p < .01$; ***, $p < .001$.

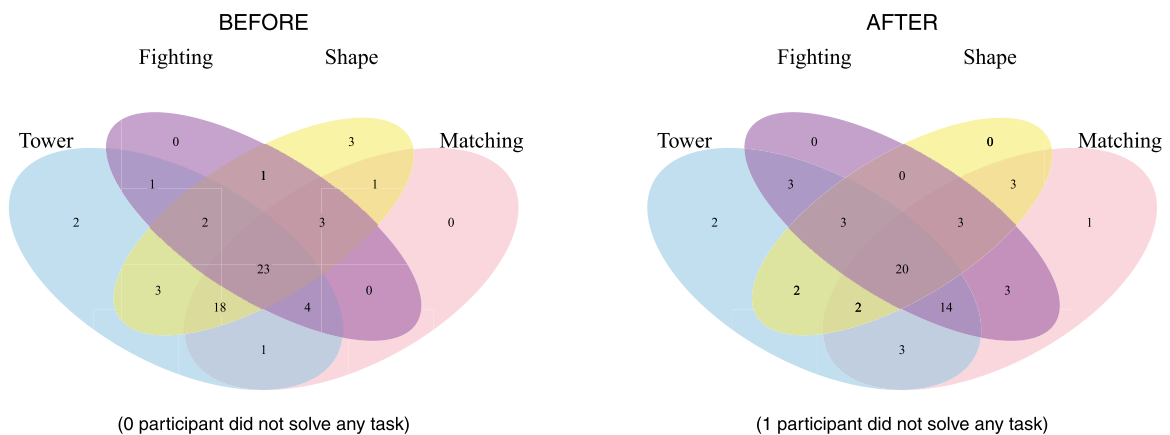


Fig. 4. Distribution of equilibrium play across games.

On average, pre-kindergartners have equilibrium beliefs in 1.83 games, kindergartners in 2.42 games and first graders in 2.78 games in the After treatment. Differences are significant only between PK and 1 (t -test, $p = .010$). In the Before treatment, pre-kindergartners have equilibrium beliefs in 2.35 games, kindergartners in 3.05 games and first graders in 2.82 games. Differences are only marginally significant between PK and K (t -test, $p = .054$).

Fig. 4 presents a Venn diagram of equilibrium play in each treatment pooling all ages together. The exercise reveals which games are solved in combination. Interestingly, few children solve zero, one or two games. In fact, the majority of them (74% in Before and 65% in After) play the equilibrium in the Matching game, the Tower game and at least one of the two other (more difficult) games. The most noticeable result is the shift in the game that is solved in conjunction with Matching and Tower. In treatment Before, 18 out of 46 subjects solve only the Shape game and 4 out of 46 solve only the Fighting game whereas in treatment After, 2 out of 39 subjects solve only the Shape game and 14 out of 39 solve only the Fighting game. This indicates once again that the Shape game is the most difficult one when participants are not prompted to think first about their partner's action whereas the Fighting game is the most difficult one when they are. It is also interesting to note that the proportion of children solving all games is comparable across treatments (37% in Before and 33% in After).

These points are further investigated using regression analysis. Table 7 presents Probit regressions in three scenarios. The left column reports the probability that an individual plays the equilibrium action in all four games (M+T+F+S). The center and right columns consider only the participants who did not play correctly all four games, and report the probability that they played the equilibrium action in Matching, Tower and Fighting (M+T+F) and in Matching, Tower and Shape (M+T+S), respectively.

The regressions show that completing all four games is associated with age but not with treatment while solving exactly the simple games (Matching and Tower) and one of the difficult games depends on the treatment (with opposite signs in Fighting and Shape) but not the age of the participant. These results indicate that the overall ability to reason in games of strategy improves with age and the fraction of children capable of solving all games increases over time. However, solving one complex game in addition to the simple ones depends crucially on how the game is formulated. This further indicates that children who know how to solve simple games but are still learning to solve complex ones do respond to reasoning cues, such as prompting them to think about the other player's choice first. However, those cues may be either beneficial or detrimental.

Table 7

Probit regressions of solving all games or exactly three games, including Matching and Tower.

	M + T + F + S	M + T + F	M + T + S
<i>Constant</i>	−2.642** (0.882)	−1.198 (1.131)	−2.432* (1.240)
<i>Age</i>	0.393** (0.142)	0.127 (0.190)	0.197 (0.201)
<i>Before</i>	0.061 (0.241)	−0.874* (0.347)	1.449*** (0.395)
<i>Male</i>	−0.272 (0.241)	0.121 (0.334)	−0.621 (0.356)
<i>Obs.</i>	122	79	79
<i>AIC</i>	157.0	85.0	73.6

Clustered st. errors in parenthesis. + $p < .1$, * $p < .05$; ** $p < .01$; *** $p < .001$.

4. Conclusion

We have investigated iterative reasoning among children from pre-kindergarten to first grade in the context of four games exhibiting a unique Trembling Hand Perfect Equilibrium but varying in three key features: iterative complexity, perspective requirements and action symmetry. We have also elicited the beliefs of participants regarding the decisions of their partners either before or after choices were made to study how beliefs contribute to decisions.

We have found that performance is different across games and treatments. First, iterative complexity is not necessarily a cause of equilibrium failure, indicating that children are able to act as if they iterate towards the fixed point of the game.

Second, perspective requirements have a strong effect on behavior. The presence of a strictly dominant strategy but only in the rival's choice set requires the participants to start the recursion by identifying that strategy. This is difficult to find, even when iterative thinking is simple. It indicates that children in our age bracket have problems formulating a hypothesis regarding the starting point of a reasoning. This may be due to a limited ToM ability, which makes them simulate first their own decision and only then find a consistent decision for their partner, rather than simulate first the decision of the partner and then find their consistent own decision. Prompting children to think first about the choice of the partner reverts this difficulty in all ages. This suggests that children can make logical deductions once the starting point of the reasoning is clear. Overall, it seems that children in our window of observation are in the process of acquiring deductive logic skills but have still trouble formulating (abstract) hypotheses.

Third, games where the equilibrium specifies the same action for both players are solved more often, independently of iterative complexity. In these games, attributing the same reasoning to others, as a shortcut, is enough and requires limited ToM. Games where players have different equilibrium actions require to find the best-response of the rival in each iteration. The combination of asymmetric features and iterative complexity revealed developmental differences. It suggests that iterative complexity is an issue when no shortcut is available.

The most unexpected result of the paper is that prompting participants to think about the rival first is detrimental in our complex asymmetric game for our first graders. A possible explanation is that, when prompted to think about the rival, beliefs used to start the recursion crystalize and behavior corresponds to the best response to this belief. By contrast, when not prompted, beliefs are updated through the recursion and behavior converges to equilibrium. This effect might also been caused or reinforced by the presence of other non-THP equilibria.

Overall, our results suggest that performance in simple games results from the application of cognitive abilities (logical thinking and ToM) to the specific strategic context. Games that require a limited ability to model rivals are easily solved. By contrast, games that require to unveil the rival's thoughts in more detail, either to start the iterative process or to revise beliefs in each iteration, are more difficult for young children.

Appendix A. Choice and belief of the control (adult) population

We conducted 2 sessions of the *After* treatment with 10 and 14 subjects and two sessions of the *Before* treatment with 14 subjects each, for a total of 52 adults drawn from the LABEL subject pool at USC. The experiment lasted 20–25 min (including payment). Subjects were paid a \$5 show-up fee and \$2.5 for each game they won. For consistency with the experiment with children, we did not incentivize the belief elicitation portion. Fig. A.5 summarizes the choices and beliefs by game and treatment.

The purpose of running a control adult population is to confirm that, after a certain (young) age, subjects can perform those simple games without difficulty. Behavior in the *After* treatment conforms to that prediction. Indeed, equilibrium compliance levels range between 92% and 100%, depending on the game. These proportions are not statistically different from perfect compliance to equilibrium. At the same time, only for the Shape game is the proportion of equilibrium choice significantly greater than that of first graders ($p = .005$).

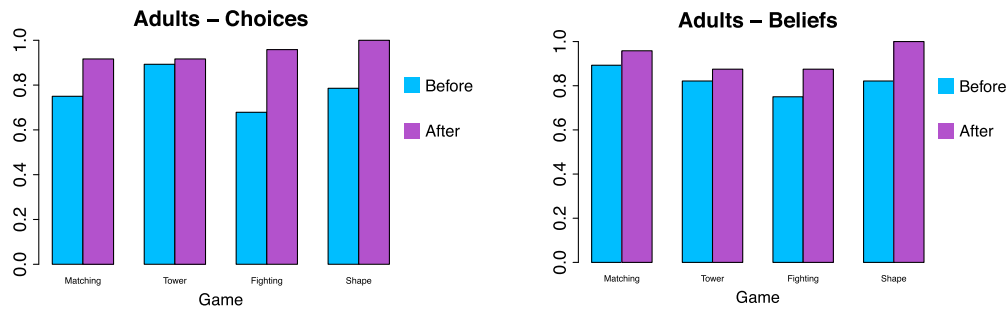


Fig. A.5. Equilibrium choices and equilibrium beliefs of adults.

By contrast and to our astonishment, equilibrium compliance in the *Before* treatment is between 68% and 89%, which is significantly lower than predicted by theory in the Matching, Fighting and Shape games ($p = .015$, $p = .004$, and $p = .031$, respectively). In all but the Fighting game, the equilibrium proportions are also lower for adults than for first graders, although no difference is statistically significant at the 5% level. More generally, the equilibrium choices of adults in the Fighting and Shape games are significantly lower in *Before* than in *After* ($p = .028$ and $p = .048$, respectively).

Overall, behavior in the *Before* treatment significantly departs from equilibrium. One conjecture is that adults understand easily the intuitive solution behind the simple games presented in this experiment. However, when they are asked to think first about the behavior of the other player, some participants “over-think” and end up deviating from the equilibrium solution (Dijksterhuis et al., 2006; Mikels et al., 2011). While our experiment is not designed to formally test this conjecture, it is an interesting possibility worth exploring in future research. In any case, it would seem that either our games are slightly more complex than we anticipated or the non-analytical presentation confused some adult subjects. Either way, it was strongly against our prediction that prompting individuals to think strategically about the behavior of their partner could be detrimental for equilibrium behavior.

Finally, the analysis of beliefs reveals weakly higher levels of equilibrium beliefs in adults than in first graders, although the difference is statistically significant only for the *After* treatment of the Shape game. Within the adult population, there are no statistically significant differences across treatments. Overall and despite the lack of incentives, subjects were closer to equilibrium in beliefs than in choices.

Appendix B. Instructions

B1. Instructions for the experiment with children

The main instructions reported here relate to the *After* treatment. We briefly describe the changes required by the *Before* treatment at the end of this section.

[Seat two children at a table facing each other. The experimenter positions him/herself so that each can see the experimenter]

Hello, my name is [Experimenter's name]. What is your name? [ask each child]
I brought you some games to play. Do you want to play?

B1.1. Tower Game

[Place material for the “The Tower Game” on the table]

Our first game is called the “Tower Game.” In this game, [name of child] has a red boy and [name of other child] has a green boy. Both of you have to decide in which floor to put your boy without telling the other. It is important that you do not tell each other. Now, do you want to know how to win this game? You win if you put your boy in the same floor or in a higher floor than your partner. I am going to put a wall between you so that you can decide where to put your boy without your partner knowing.

[Place divider]

Are you ready? Ok, now you can choose where to put your boy.

[Go to child #1, note the position] - [Go to child #2, note position]

[Go back to child #1, ask:]

Can you point at the floor you think [child2's name] put his boy?

[Note answer Child #1]

[Go back to child #2, ask:]

Can you point at the floor you think [child1's name] put his boy?

[Note answer Child #2]

OK, now, let's find out [Take the divider out]

[Disclose what they each did, determine winners, explain results, record the points]

B1.2. Matching Colors

[Remove material for “Tower Game” and place material for “Matching Colors”]

This game is called “Matching Colors.” In this game each of you has 5 cards. These are [child’s name]’s cards and these are [other child’s name]’s cards. You each need to pick one card without telling the other. How do we win in this game? You both win if you pick cards that have the same color and you both lose if you pick cards that have different colors. Now, I will put a wall between you. You can take as much time as you want to look at all the cards. When you are ready, pick a card and raise your hand.

[Place divider]

[Go to child #1, note card] - [Go to child #2, note card]

[Go back to child #1, ask:]

Can you point me the card you think [child2’s name] chose?

[Note answer Child #1]

[Go back to child #2, ask:]

Can you point me the card you think [child1’s name] chose?

[Note answer Child #2]

Let’s find out [Take the divider out]

[Disclose what they each did, determine winners, explain results, record the points]

B1.3. Fighting Colors

[Remove material for “Matching Colors” and place material for “Fighting Colors”]

This game is called “Fighting Colors.” In this game each of you has 4 cards. These are [child’s name]’s cards and these are [other child’s name]’s cards. As before you need to pick one card without telling the other. Now, you both win if you pick cards that have different colors and you both lose if you pick cards that have the same color. I will again put a wall between you. You can take as much time as you want to look at the cards. When you are ready, pick a card and raise your hand.

[Place divider] - [Go to child #1, note card] - [Go to child #2, note card]

[Go back to child #1, ask:]

Can you point me the card you think [child2’s name] chose?

[Note answer Child #1]

[Go back to child #2, ask:]

Can you point me the card you think [child1’s name] chose?

[Note answer Child #2]

Let’s find out. [Take the divider out]

[Disclose what they each did, determine winners, explain results, record the points]

B1.4. Lucky Shape

[Remove material for “Fighting colors” and place material for “Lucky Shape” game]

This game is called “Lucky shape.” In this game, you will not play with each other. You will each play with me. We will all make our choices without telling the others.

I have 4 different shapes. Each of you has 4 cards.

In this game, I win if I pick the shape that fits in the puzzle. [Child’s name], you win if you pick the same color as the color of the shape I pick. This is the same for you, [Other child’s name]: you win if you pick the same color as the color of the shape I pick.

Let’s bring the dividers and we will all make our choices.

[Place dividers so that children cannot see the experimenter’s choosing]

[Go to child #1, note card] - [Go to child #2, note card]

[Go back to child #1, ask:]

Can you point me the shape you think I chose?

[Note answer Child #1]

[Go back to child #2, ask:]

Can you point me the shape you think I chose?

[Note answer Child #2]

Let’s find out. [Take the divider out]

[Disclose what they each did, determine winners, explain results, record the points]

B1.5. Before Treatment

The order of questions after placing the divider is altered the following way in all games.

[Place divider]

[Go back to child #1, ask:]

Before you make your choice, can you point me the card you think [child 2’s name] will choose?

[Note answer Child #1]

[Go back to child #2, ask:]

Before you make your choice, can you point me the card you think [child 1's name] will choose?

[Note answer Child #2]

Ok, now you can make your choice.

[Go to child #1, note the decision] - [Go to child #2, note the decision]

OK, now, let's find out. [Take the divider out]

[Disclose what they each did, determine winners, explain results, record the points]

B2. Instructions for the experiment with adults

Below we present the slides used with the adult population.

INSTRUCTIONS

- You will be matched with someone in this room.
- You will have a role: **player 1** or **player 2** clearly stated in your paper
- You will perform 4 tasks
- You will be asked two questions for each task
- You will be paid:
 - \$5 show up fee
 - \$2.5 for each task you "win" and \$0 for each task you "lose"
- Use a pen to circle your answer in each task
- Please do not talk or communicate in any way**
- Wait until instructed to move to the next page**

Note: these are tasks to be performed in young children

TASK 1

You each have to choose one card

You both win \$2.5 if you choose the **same** color

cards of player 1

cards of player 2

TASK 2

You each have to choose one card

You both win \$2.5 if you choose **different** colors

cards of player 1

cards of player 2

TASK 3

I (the experimenter) choose a shape.
You (player 1 or player 2) choose a card.

I win if I choose the shape that fits one of the puzzles (I want to win)

You win \$2.5 if you pick the card with the same color as the color of the shape I pick

shapes for the experimenter

shapes for the experimenter

cards for player 1 or player 2

TASK 4

You each have to put your character in one floor of the building

You win \$2.5 if you put your character in the same floor or in a higher floor than your partner.

■

 player 1

■

 player 2

- We will collect your answers.
- Please wait in silence.
- We will call to pay you in the next room
- Thank you for your participation

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