Learning Lyapunov Functions for Hybrid Systems

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***Abstract*—We propose a sampling-based approach to learn Lyapunov functions for a class of discrete-time autonomous hybrid systems that admit a mixed-integer representation. Such systems include autonomous piecewise affine systems, closed-loop dynamics of linear systems with model predictive controllers, piecewise affine/linear complementarity/mixed-logical dynamical system in feedback with a ReLU neural network controller, etc. The proposed method comprises an alternation between a learner and a verifier to find a valid Lyapunov function inside a convex set of Lyapunov function candidates. In each iteration, the learner uses a collection of state samples to select a Lyapunov function candidate through a convex program in the parameter space. The verifier then solves a mixed-integer quadratic program in the state space to either validate the proposed Lyapunov function candidate or reject it with a counterexample, i.e., a state where the Lyapunov condition fails. This counterexample is then added to the sample set of the learner to refine the set of Lyapunov function candidates. By designing the learner and the verifier according to the analytic center cutting-plane method from convex optimization, we show that when the set of valid Lyapunov functions is full-dimensional in the parameter space, our method finds a valid Lyapunov function in a finite number of steps. We demonstrate our stability analysis method on closed loop MPC dynamical systems and a ReLU neural network controlled PWA system.**

***Index Terms*—Hybrid systems, Lyapunov functions, Analytic center cutting-plane, Mixed-Integer quadratic program**

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