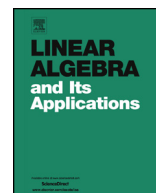




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Corrigendum

Corrigendum to “Subspace controllability of bipartite symmetric spin networks” [Linear Algebra Appl. 585 (2020) 1–23]



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ABSTRACT

We correct a statement in the paper [1] concerning the subspace controllability of a bipartite spin network. Contrary to what stated in [1], subspace controllability is not always verified in this case, and we specify here when it is verified and when it is not.

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The paper [1] studied the subspace controllability of networks of spin $\frac{1}{2}$ particles in the bipartite configuration. One of the two sets of spins called the set of *central spins* is assumed to have $n_c = 1$ or $n_c = 2$ spins. The result is based on a direct calculation of the

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dynamical Lie algebra of the system, which, as it is known from quantum control theory (see, e.g., [3]), gives information about quantum controllability in finite dimensions. While such a calculation is correct, the conclusion drawn in Theorem 6 of [1] is only true for the case $n_c = 1$ but, in general, not true in the case $n_c = 2$. The gap in the proof concerns the matrix $\mathbf{1} + \frac{1}{3}J$ in the discussion leading to the statement of Theorem 6 of [1]. Such a matrix is a scalar matrix on each invariant subspace, but it was overlooked that it can be zero on certain subspaces. In fact, according to formula (24) of [1] the matrix $\mathbf{1} + \frac{2}{3n_p}J$ (which coincides with $\mathbf{1} + \frac{1}{3}J$ for $n_p = 2$) is the Casimir operator (see, e.g., [4]) which is zero on each one dimensional representation of $su(2)$.

The correct statement for Theorem 6 of [1] is as follows:

Theorem 6. *For the system (1) with one or two central spins ($n_c = 1$ or $n_c = 2$) and with any number $n_p \geq n_c$ of surrounding spins we have:*

1. *If $n_c = 1$ then each invariant subspace is controllable.*
2. *If $n_c = 2$, then a subspace of the type $\mathcal{H}_1 \otimes \mathcal{H}_2$ is controllable in all cases except when $\dim(\mathcal{H}_1) = 1$ and $\dim(\mathcal{H}_2) > 2$ or $\dim(\mathcal{H}_2) = 1$ and $\dim(\mathcal{H}_1) > 2$.*

In the case $n_c = 1$ most of the proof goes as in the discussion leading to Theorem 6 in [1]. Consider an invariant subspace $\mathcal{H}_1 \otimes \mathcal{H}_2$. One only has to notice that $\mathbf{1} + \frac{2}{3n_p}J$ equal to zero (when $\dim(\mathcal{H}_2) = 1$) does not compromise subspace controllability in this case, since on $\mathcal{H}_1 \otimes \mathcal{H}_2$ which has dimension 2, the dynamical Lie algebra acts as the two dimensional (standard) representation of $su(2)$. Therefore controllability is verified.

For the case $n_c = 2$, if $\dim(\mathcal{H}_1) \geq 2$ and $\dim(\mathcal{H}_2) \geq 2$ the proof of subspace controllability can be carried out as in the paper. This is due to the fact that, in this case, the restriction of $\mathbf{1} + \frac{2}{3n_c}J$ or $\mathbf{1} + \frac{2}{3n_p}J$ to the invariant subspaces $\mathcal{H}_1$ or $\mathcal{H}_2$, which coincides (according to formula (24) in [1]) with the Casimir operator on a representation of dimension ≥ 2 , is different from zero. It remains to investigate what happens when $\dim(\mathcal{H}_1) = 1$ and-or $\dim(\mathcal{H}_2) = 1$. In order to do that, we directly analyze the action of the Hamiltonian of the system (formula (10) of [1]) on $\mathcal{H}_1 \otimes \mathcal{H}_2$. If $\dim(\mathcal{H}_1) = \dim(\mathcal{H}_2) = 1$, then both $S_{x,y,z}^C$ and $S_{x,y,z}^P$ on $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, are zero. So H is zero on the space $\mathcal{H}_1 \otimes \mathcal{H}_2$, which is one dimensional. Subspace controllability is verified trivially on this space. If $\dim(\mathcal{H}_1) = 1$, all the $S_{x,y,z}^C$ in formula (10) are zero on this space. So the action of H coincides with the irreducible representation of $su(2)$ given by $\{iS_{x,y,z}^P\}$ on $\mathcal{H}_2$. In the case $\dim(\mathcal{H}_2) = 2$ this is the standard representation of $su(2)$ and therefore controllability is verified. However controllability is not verified in the other cases ($\dim(\mathcal{H}_2) > 2$). The same argument holds by reversing the role of C and P if $\dim(\mathcal{H}_2) = 1$.

The discussion in section 5 of [1] remains valid in that it shows that, for the considered generalizations, the dynamical Lie algebra is always greater than or equal to the dynamical Lie algebra calculated for the main case of the paper. However, this does not necessarily mean subspace controllability since subspace controllability is not always verified as we have seen above.

We have recently given in [2] a generalization of the results of [1] to the multipartite case.

Declaration of competing interest

The authors declare no competing interests.

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