Accelerating the Adoption of Automated Vehicles by Subsidies: A Dynamic Games Approach

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Abstract

Early deployment of automated vehicles (AVs) may likely cause a loss of efficiency in the transportation system. However, after there are a sufficient number of such vehicles in the traffic stream, many benefits can be realized. It thus appears sensible to provide subsidies to promote the early adoption of AVs and shorten the transition period. This paper investigates an optimal subsidy policy that accelerates the deployment of AVs from lower to higher market penetration rates. The policy can maximize the government agency's expected total payoff associated with the AV deployment. The main contribution is a dynamic games approach that considers the uncertainty in the market forecast and the information asymmetry between the government agency and the subsidized entities.

Keywords: Automated vehicles, subsidy policy, dynamic Stackelberg games, diffusion of innovations

1. Introduction

Automated Vehicles (AVs) are anticipated to tremendously enhance the efficiency, safety, and convenience of the existing transportation system, with new businesses springing up around them. Pioneering companies such as Waymo and GM Cruise have been working to develop, test, and pilot commercial services. A stream of research has been conducted to quantify the profound and far-reaching implications of AVs on the transportation system, society, and the economy (Greenblatt and Shaheen, 2015; Fagnant and Kockelman, 2015; Wadud et al., 2016; Clements and Kockelman, 2017; Milakis et al., 2017; Chen et al., 2017b,c). In general, the efficiency gains from AVs are from the following three sources: (a) Vehicle platooning can improve substantially the throughput of highway facilities; (b) Advanced traffic management schemes (e.g., adaptive speed control and harmonization) that leverage vehicle connectivity can further increase the throughput and improve the stability

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of the traffic stream; (c) Automation can make on-demand shared mobility services more cost-effective. Low-cost shared mobility services have the potential to yield higher vehicle occupancy and reduce overall vehicular traffic demand.

All these efficiency gains hinge on the market penetration level of AVs being sufficiently high. At low market shares, AVs exert little impact on enhancing transportation system efficiency (Mahmassani, 2016). Worse yet, early deployment of AVs will likely compromise efficiency. At early stages of the deployment, car manufacturers (original equipment manufacturers or OEMs) will likely configure their AVs with a lower operation speed and excessive safety clearance, i.e., longer time gap or headway, to ensure safety and avoid liability. Undoubtedly, the presence of these types of AVs in the traffic stream will slow other vehicles down (imposing so-called congestion externalities) and thus compromise the efficiency of transportation systems (Seo and Asakura, 2017; Chen et al., 2017a; Ghiasi et al., 2017; Calvert et al., 2017). Such an efficiency degradation could last for a very long time until the market penetration of AVs reaches a certain threshold (Seo and Asakura, 2017). In addition, the benefits promised by AVs can be offset by the increase in vehicle miles traveled (VMT) generated by empty trips of AVs and induced travel demand (Childress et al., 2015; Fagnant and Kockelman, 2018; Maciejewski and Bischoff, 2017; Loeb and Kockelman, 2019), particularly at the early stage of AV deployment when necessary transportation policies are not in place and the shared mobility market is not large enough to facilitate ride-sharing.

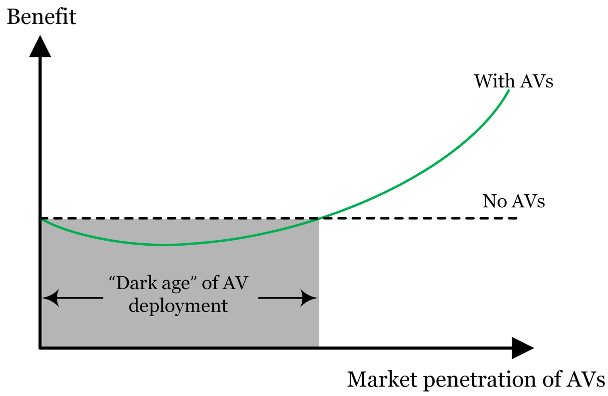


Figure 1: Efficiency benefit of AV deployment.

With all these considerations, we envision that the overall efficiency benefit offered by AVs with various market shares likely follow a trend depicted in Figure 1. The benefit initially drops and then rises with the market penetration of AVs. For lack of a better word, we call the initial stage of performance drop the "dark age" of the AV deployment. *AV market failure* may occur if the market share stagnates in this "dark age" for a long time. Such an initial drop is pretty unusual and does not arise in the deployment of other new technologies such as electric or connected vehicles.

A powerful policy to address the aforementioned negative impacts of AVs is to price their utilization in a way that internalizes their external costs incurred by longer time headway or empty trips, for example. However, the timing of implementing this policy is critical, since such a congestion charge or Pigouvian tax may increase the cost of using AVs, thereby discouraging their early adoption. It appears plausible that we need to endure short-term pains for long-term benefits.

A suite of policies needs to be in place to fully correct the possible AV market failure. However, the objective of this paper is modest. Given the social benefit curve as in Figure 1, we investigate a subsidy policy to accelerate the deployment of AVs adaptively from lower to higher market penetration rates. The objective of AV subsidies is to maximize the total *expected* efficiency benefits from the AV deployment over the planning horizon. The questions we are interested in examining are who should be incentivized, and how much subsidies should be enacted. While subsidizing customers can nudge them to adopt the AV technology, subsidizing manufacturers can directly motivate them to innovate the technology. The latter strategy may effectively mitigate the negative externalities of prototype AVs. It thus remains an open question, which side should be directly incentivized. With a time-varying market share, we also need to consider when and how long subsidies should be implemented. Note that previous research treating it as a static problem (Chen et al., 2016; Bonnefon et al., 2016) failed to answer the optimal timing for enacting AV subsidies. In contrast, we investigate how to implement AV subsidy policies that are adaptive to the AV market share. Different from previous work (Shabanpour et al., 2018a; Kalish and Lilien, 1983; Wadud, 2017; Nieuwenhuijsen et al., 2018), our model can also capture the uncertainty in forecasting the diffusion of AV technology.

The contribution of this paper is to develop a new approach to find the optimal AV subsidy policy that shortens the "dark age" of AV deployment. Our approach is able to solve for *non-myopic* subsidy policies with the *information asymmetry* between the government agency and AV manufacturer. Subsidizing new technologies is never a one-way command-and-control process (Armstrong and Sappington, 2007; Decker, 2014), while full information is widely implied in the previous literature. For instance, Janssens and Zaccour (2014) recently explored optimal subsidy paths to make a technology competitive at a given future period. Langer and Lemoine (2017) established a subsidy scheme to induce customers to adopt new technology over time. Results showed that, if customers are myopic, an increasing subsidy scheme is preferred; otherwise, subsidies should decrease over time to induce the adoption in early periods. The effectiveness of those suggested AV subsidy policies may be reduced because of the information asymmetry. This paper considers two essential problems: (a) How to model the non-myopic decision makings of the government agency and the manufacturer; (b) How to avoid the information asymmetry that may lead to an unintended AV market failure. The asymmetric information is caused by government agency's unawareness of the subsidized entity's actual effort in promoting AVs. The new approach integrates two modeling techniques cohesively: a *diffusion of innovations* (DOI) model that describes the evolution of the AV market share and captures how the AVs' efficiency benefit varies with the market share, and *dynamic Stackelberg games* (DSG) in continuous time.

The remainder of the paper is organized as follows. In §2, we formulate the AV subsidy problem by integrating a DOI model and DSG. The optimal subsidy policy is computed by dynamic programming with a set of implementability constraints. In §3, we prove the structure of optimal AV subsidy policies and demonstrate their robustness by numerical experiments. In §4, the model is extended to a changing AV market potential with price incentives. Finally, we conclude the paper in §5.

2. Accelerating AV Market Penetration by Subsidies

Our AV subsidy design framework integrates a DSG model with a DOI process. The main idea is as follows. In the near future, the evolution of AV's market share follows a DOI model with uncertainty. The government agency is the leader (denoted by "G") who establishes statutory subsidy policies with the intention of accelerating the adoption of AVs. AV manufacturers are the followers (denoted by "A") who respond to the subsidy policies by enhancing the AV innovations. Both are non-myopic decision makers because they consider each other's decisions with regard to the time-varying AV market dynamics. The information asymmetry occurs when the government agency expects to advance the AV technology by subsidizing the manufacturers. However, the manufacturer's exact effort in AV innovations is not observable. Such asymmetric information on AV innovations compromises the cost-effectiveness of the AV subsidy policies. On the other hand, given that more advanced AV innovations will accelerate the AV market penetration process, the government agency can use the time-varying AV market share as an imperfect indicator of the manufacturer's effort. The government agency's objective is to find an optimal AV subsidy policy that maximizes its total expected payoff (i.e., the efficiency benefit of AVs) over the planning horizon. In what follows, we first introduce how to model the AV market penetration process with uncertainty, then we integrate this process into a DSG setting.

2.1. AV Market Penetration with Uncertainty

The DOI model depicts the process by which the AV technology spreads in the transportation system. The AV market size, i.e., the cumulative number of AVs sold by time t, is denoted by $N(t)$. The AV market potential, i.e., the population of potential AV consumers, is denoted by $M(t)$. Both AV market dynamics variables are observable to the government agency. The DOI model presumes that AV consumers consist of two groups: innovators who are early users of AVs and imitators whose tendency to purchase AVs depends on the size of innovators. Therefore, the market penetration rate is determined by two diffusion parameters: (a) the coefficient of external influence a (the "power of innovation") that represents the manufacturer's effort in AV innovations, and (b) the coefficient of internal influence b (" the power of contagion") that represents the word-of-mouth effect within the consumers' social network. Although many extensions of the model have been proposed, we simplify the following analysis by using a (generalized) Bass diffusion model (Bass and Bultez, 1982). The Bass diffusion model is widely recognized as a seminal work that initiated a stream of DOI models. Nevertheless, our subsidy design paradigm can easily adopt more complex market penetration models (Nieuwenhuijsen et al., 2018; Talebian and Mishra, 2018; Shabanpour et al., 2018b). At time t, the AV market penetration rate $dN(t)/dt$ follows the dynamics below,

$$\frac{dN(t)}{dt} = a\Big(M(t) - N(t)\Big) + bN(t) \cdot \frac{M(t) - N(t)}{M(t)},$$

which is a combination of the innovation effect (i.e., the rate attributes to innovators as the first term) and the imitation effect (i.e., the rate attributes to imitators as the second term).

The AV market forecast may overestimate or underestimate the AV market growth in the future. It is natural to assume that the uncertain AV market dynamics follows a Gaussian process. Each AV market penetration sample path is generated by an extraneous Brownian motion $B(t)$ (Figure 2), and the AV market dynamics is described by the following stochastic differential equation:

$$dN(t) = \left(a + b\frac{N(t)}{M(t)}\right)(M(t) - N(t))dt + \sigma N(t)dB(t), \tag{1}$$

where the drift term follows the DOI model above, and the diffusion term has a constant volatility σ. Note that the uncertainty aggravates as the time horizon expands in Figure 2.

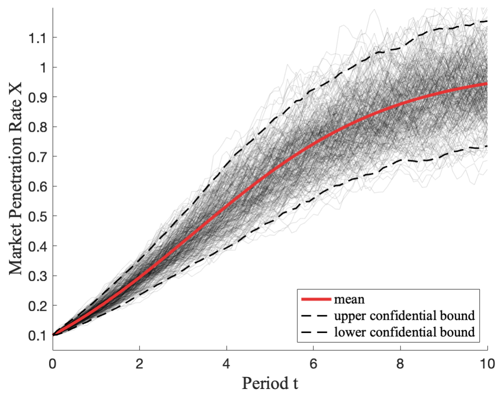


Figure 2: Sample paths, mean and 95% confidence interval of the DOI model for AV market penetration; the AV market penetration rate is $X(t) = N(t)/M(t)$.

The uncertainty in the AV market penetration process implies the possible inefficiency of AV subsidy policies because of information asymmetry. Over the planning horizon $t > 0$, the government agency continuously receives the efficiency benefit of AVs represented by a function of market size $g(N(t))$. As shown in Figure 1, $g(N(t)) < 0$ in the "dark age" of AV deployment. To accelerate the early adoption of AVs, the government agency would like to incentivize the manufacturer to realize a large $a(t)$ during the "dark age" by a sequence of AV subsidies.

2.2. AV Subsidy Policies for Dynamic Stackelberg Games

In what follows, we formulate the AV subsidy problem as a DSG in the simplest setting (Figure 3). An extension that includes varying market size and price discount mechanisms for consumers are discussed in §4. When the manufacturers' responses are additive, we can use a single agent to present a group of agents with linearly aggregated responses. We discuss more complicated interactions between agents at the end of this paper.

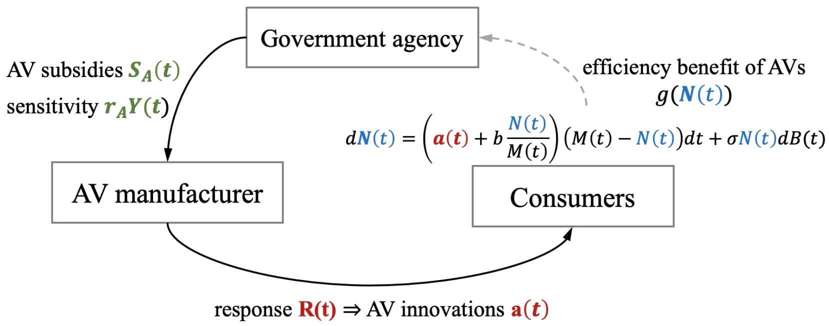


Figure 3: DSG model for the AV subsidy problem at time t. The highlighted variables are decision and state variables.

After implementing the AV subsidies, the efficiency benefit $g(N(t))$ is what the government agency gains, and the total subsidies are what it pays to the manufacturer. As in Kalish and Lilien (1983), we suppose that the government agency's instantaneous payoff is a linear combination of the benefits and costs. The risk-neutral government agency's goal is to initiate a sequence of per-unit AV subsidies $\{S_A(t)\}_{t\geq 0}$ to maximize the total expected discounted payoff:

$$\mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\cdot\frac{dN(t)}{dt}\right]dt\right],$$

where we use a continuously compounded discount factor $e^{-r_G t}$. Note that r_G in front of the integral normalizes the government agency's total payoff to annuity payments.

After receiving the subsides, the AV manufacturer reacts with a response $R(t)$. The response represents a target action that affects the DOI process, and the exact value of $R(t)$ is not observable by the government agency. In practice, the government usually lacks the access to AV manufacturer's private information on innovations, or monitoring the effort is too costly. Thus, the government agency who only observes the AV market size $N(t)$ (i.e., an imperfect indicator of $R(t)$) needs to specify the amount of subsidies S_A and the *sensitivity level* $r_A Y$ in the policy (Figure 3).

In the simple setting, we assume that $R(t)$ directly controls the power of innovation as $R(t) \equiv a(t)$. While most of the DOI literature focuses on controlling the imitation effect represented by b, promoting the AV innovation represented by a is the objective of AV subsidies. This is because the innovation effect is the main driving force in the early adoption of AVs. For a similar setting, the reader can refer to Becker et al. (2009). In the extension, we also discuss how $R(t)$ can include other factors like price mechanisms. The manufacturer's instantaneous utility function is $h(S_A(t), R(t))$. With a given AV subsidy policy, the AV manufacturer's objective is to maximize the expected total discounted utility:

$$\mathbb{E}\left[r_A \int_0^\infty e^{-r_A t} h(S_A(t), R(t))dt\right].$$

To guarantee the existence of the best response, the AV manufacturer is assumed to be risk-averse, i.e., the instantaneous utility h is concave and satisfies $\partial h/\partial S_A > 0$ and $\partial h/\partial R < 0$. Risk aversion expressed by the concavity of the utility function h is a realistic assumption. Non-myopic AV manufacturers who are exposed to the AV market uncertainty prefer not to overexert their innovations now for future market growth. We will give a more rigorous proof of this in Proposition 1.

The *optimal* AV subsidy policy characterizes a sequence $\{S_A(t)\}_{t\geq 0}$ that maximizes the government agency's expected total discounted payoff if only the AV manufacturer cooperates with the best responses. Following the literature of mechanism design, we call the conditions that specify the manufacturer's best responses the IC-constraint (incentive-compatible) and the IR-constraint (individual-rationality), respectively. The IC-constraint guarantees that the AV manufacturer's best response solves the utility maximization problem above.
The IR-constraint guarantees that the AV manufacturer stays in the market as long as the cumulative expected utility over the horizon exceeds some pre-defined quantity W_0. In summary, the optimal AV subsidy policy solves the following optimization problem:

$$\max_{S_A(t):t\geq 0} \mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\frac{dN(t)}{dt}\right] dt\right] \tag{2}$$

$$s.t. \quad a(t) \in \arg\max_a \mathbb{E}\left[r_A \int_t^\infty e^{-r_A u} h(S_A(u), a(u))du\right], \forall t \geq 0 \;\; \text{(IC-constraint)},$$

$$\mathbb{E}\left[r_A \int_0^\infty e^{-r_A t} h(S_A(t), a(t))dt\right] \geq W_0 \;\; \text{(IR-constraint)}.$$

Plugging the AV market penetration process (1) into the objective function of (2) yields $\mathbb{E}[\int_0^\infty \sigma N(t)dB(t)] = 0$. This does not mean that the asymmetric information is eliminated because $a(t)$ is still not directly controlled by the government agency. The government agency's objective function can be rewritten as:

$$\max_{S_A(t):t\geq 0} \mathbb{E}\left[r_G \int_0^\infty e^{-r_G t}\left[g(N(t)) - S_A(t)\left(a(t) + b\frac{N(t)}{M}\right)(M - N(t))\right] dt\right].$$

Before investigating how to find the optimal AV subsidy policies, we first introduce a hierarchy of feasible policies that are in the government agency's interest. Suppose that the joint domain of the government agency's decision $S_A(t)$ and the manufacturer's response $R(t)$ is a known compact set $\mathcal{U} = \mathcal{U}_{S_A} \times \mathcal{U}_R \in \mathbb{R}^2$. The support $\mathcal{U}_{S_A}$ and $\mathcal{U}_R$ are known a priori due to the limited budgets of both sides at time t. In what follows, we define the *admissible*, *implementable*, and *optimal* subsidy policy, respectively.

Definition 1. $\{S_A(t), R(t)\}_{t\geq 0}$ *are admissible if* $(S_A(t), R(t)) \in \mathcal{U}$ *for all* $t \geq 0$ *along any path generated by the stochastic process* $B(t)$.

The implementable policies guarantee that the AV manufacturer will realize the target responses after receiving the subsidies. With the presence of information asymmetry, the attainable implementable policies are second best.

Definition 2. *$\{S_A(t), R^*(t)\}_{t\geq 0}$ is implementable if:*

1. *$\{S_A(t), R^*(t)\}_{t\geq 0}$ is admissible,*
2. *For a given $S_A(t)$, $R^*(t)$ are optimal for the AV manufacturer's utility maximization problem (i.e., the policy is incentive-compatible).*

Finally, our goal is to find the optimal subsidy policy defined below.

Definition 3. *$\{S_A^*(t), R^*(t)\}_{t\geq 0}$ is optimal if it is implementable and maximizes the government's expected total payoff.*

The optimal subsidy policy is dependent on the AV market dynamics and the expected total payoff. Hence, the decisions are adaptive and non-myopic over the planning horizon. Since the bilateral decisions $S_A(t)$ and $R(t)$ are coupled in (2), and there are infinite number of IC-constraints in continuous time, below we shall introduce a tractable scheme to decompose the optimal AV subsidy problem.

2.3. Solving for Optimal AV Subsidies

The optimization for the AV subsidy problem (2) can be transformed into a dynamic program. This tractable scheme for solving the DSG is inspired by the celebrated results in differential games (Sannikov, 2008; Cvitanić et al., 2018). The road map for the dynamic program transformation includes three steps. First, we can find an equivalent representation of the IC-constraint (Theorem 1). Second, the representation can be parameterized and incorporated into the objective function with mild smoothness assumptions (Proposition 1). Finally, the optimal AV subsidy policies can be computed by dynamic programming in continuous time, i.e., a Hamilton-Jacobi-Bellman (HJB) equation (Theorem 2).

We denote the manufacturer's response as a generic variable $R(t)$. One of the central ideas in the DSG is the *continuation value*, which is the expected discounted total payoff/utility at time t with the optimal subsidy policy followed to the end of the horizon. This function is equivalent to the value function in dynamic programming or reinforcement learning. More specifically, the AV manufacturer's continuation value at time $t \geq 0$ is given by

$$W(t) = \mathbb{E}\left[r_A \int_t^\infty e^{-r_A(s-t)} h(S_A^*(s), R^*(s)) ds | \mathcal{F}_t\right],$$

where the filtration $\mathcal{F}_t$ represents the information collected by time t (including the decisions S_A and Y, and the market dynamics X). $W(t)$ is a state variable in the dynamic program. Applying the Martingale representation theorem, we can derive the dynamics of $W(t)$ as follows:

$$dW(t) = r_A \left(W(t) - h(S_A^*(t), R^*(t))\right) dt + \sigma r_A Y(t) dB(t),$$

where $r_A Y(t)$ is the sensitivity level of the manufacturer's continuation value $W(t)$ with respect to the AV market size $N(t)$. $Y(t)$ measures the marginal utility gained by increasing the response $R(t)$. It is a well-defined variable because the dynamics of $W(t)$ and $N(t)$ are adapted to the filtration generated by the process $B(t)$, i.e., the extraneous noise in the AV market forecast.

Since $\mathcal{U}$ is compact, we denote the set $\mathcal{U}_R = [\underline{R}, \bar{R}]$. For instance, $R(t) \equiv a(t) \in [0.01, 0.04]$ in this setting (Mahajan et al., 1995). Over the infinite planning horizon, we assume a transversality condition $\lim_{s\to\infty} \mathbb{E}[e^{-r_A s} W(t+s)] = 0$. Then we can derive the following theorem that converts the subsidy policies respecting $R(t)$ to $Y(t)$ by the comparison principle (see Appendix A.1).

Theorem 1 (Sannikov (2008))**.** *For any given subsidy policy $\{S_A(t), Y(t)\}$ at any $t \geq 0$, the IC-constraint that derives the manufacturer's optimal decision in* (2) *is equivalent to:*

$$h(S_A(t), R(t)) + Y(t)R(t) \geq h(S_A(t), \hat{R}(t)) + Y(t)\hat{R}(t) \text{ for all } \hat{R}(t) \in [\underline{R}, \bar{R}]$$

By imposing mild smoothness and concavity conditions on the AV manufacturer's utility function, we can reduce the countably infinite IC-constraints into a parametric function of S_A and $r_A Y(t)$, as seen in the following proposition.

Proposition 1. *R^* is the best response from the AV manufacturer if*

1. *For any given subsidy policy $\{S_A(t), Y(t)\}$, if the AV manufacturer is risk neutral (i.e., h is linear in R), then $R^*(t)$ satisfies the following optimality condition*

$$R^*(t) = \begin{cases} \bar{R} & \text{if } Y(t) \geq 0 \\ \underline{R} & \text{if } Y(t) < 0. \end{cases}$$

2. *For any given subsidy policy $\{S_A(t), Y(t)\}$, if h is continuously differentiable, concave, and nonlinear in R, then $R^*(t)$ solves*

$$\frac{\partial h(S_A(t), R(t))}{\partial R(t)} + Y(t) = 0$$

for all $t \geq 0$.

The intuition of recommending a sensitivity level $r_A Y(t)$ in the AV subsidy policy is related to the dual roles of $Y(t)$ above. The first role is to ensure no violation of the IC-constraint. In Proposition 1, $Y(t)$ is the AV manufacturer's marginal utility for enhancing AV innovations. Besides the fixed amount of subsidies $S_A(t)$ received over the planning horizon, the manufacturer is incentivized to increase the response $R(t)$ to gain higher utility with the presence of $Y(t)$. $S_A(t)$ are still necessary to guarantee that the IR-constraint is satisfied. The second role is to penalize the information asymmetry. In the dynamics of the AV manufacturer's continuation value $W(t)$ and Theorem 1, a higher sensitivity level $r_A Y(t)$ will potentially increase the stochasticity of its expectation on the future benefit. Therefore, the manufacturer is forced to cooperate with accelerating the early adoption of AVs to reduce the risk of increasing stochasticity. Proofs of Theorem 1 and Proposition 1 can be found in Appendix A.1.

In summary, the subsidy policy including $Y(t)$ and $S_A(t)$ is sufficient to break down the information asymmetry. Proposition 1 demonstrates how to find a mapping from a subsidy policy $\{S_A(t), Y(t)\}$ to the AV manufacturer's optimal response $R^*(t)$. Such a sensitivity level could be easily included in practice, for example, a suggested internal rate of return relevant to the advancements of AV technology. Different from the previous literature (Sannikov, 2008; Cvitanic and Zhang, 2012), the best response R^* is basically a function of S_A and Y because of the DOI model.

To have a more intuitive state variable representing the AV market share, we normalize (1) by a constant market potential M to obtain the dynamics of the *market penetration rate* $X(t) = N(t)/M$,

$$dX(t) = (a(t) + bX(t))\,(1 - X(t))dt + \sigma X(t)dB(t).$$

Let $F(X, W)$, denote a $C^{2,2}$ function with two state variables – AV market share X and the manufacturer's continuation value W. With all these considerations, we can solve the optimal subsidy policies in (2) by dynamic programming in the following theorem.

Theorem 2 (Optimal Subsidies)**.** *The government agency's continuation value is equal to $F(X, W)$ that solves the following HJB equation:*

$$r_G F = \max_{S_A, Y}\Big\{ r_G[g(N) - S_A \cdot M(a^* + bX)(1 - X)] + (a^* + bX)(1 - X)F_X + \\ r_A\Big(W - h(S_A, a^*)\Big)F_W + \frac{1}{2}\sigma^2(X^2F_{XX} + 2r_AXYF_{XW} + r_A^2Y^2F_{WW})\Big\}.$$

The proof of Theorem 2 in Appendix A.2 shows that it is sufficient to solve the above HJB equation to solve for the optimal subsidies in (2). The proof also shows that W is finite if the manufacturer's utility function h is finite. Let $\bar{W}$ denote the upper bound of W. It is clear that $\{S_A^*(t)\}_{t\geq 0}$ are adaptive to the filtration generated the market dynamics for all $t \geq 0$. We can characterize the property of F in the following proposition.

Proposition 2. *For a given X, there exists $W^* \in [0, \bar{W}]$ such that:*

1. $F(X, W^*) = \max_W F(X, W)$.
2. *For any $W \in [0, W^*]$, $F_W \geq 0$.*
3. *For any $W \in [W^*, \bar{W}]$, $F_W \leq 0$.*
4. *For any $W \in [0, \bar{W}]$, $F_{WW} \leq 0$.*

Proposition 2 shows that the government agency's continuation value F is partially concave with regard to the AV manufacturer's continuation value W. A brief proof is as follows. Note that F is assumed to be a twice differentiable function in W with uniformly bounded derivatives over S_A and Y. By definition, $F(0) = 0$ and $F_{WW}(0) < 0$. With the assumptions that h is a concave function (i.e., a risk-averse manufacturer), and that there is a $w \in [0, \bar{W}]$ such that $F_{WW}(w) = 0$, it follows that the entire solution is a linear function, which is a contradiction. So F_{WW} is nonpositive everywhere and F is a concave function in W. With $F_W(0) > 0$, the properties above are proved.

3. Main Results

The main analytical results are (a) comparing the cost-effectiveness of the AV subsidy policies using the DSG model with a standard welfare maximization approach, and (b) characterizing the special structure of optimal AV subsidies. We use numerical methods to verify those results.

3.1. Optimal Subsidy v.s. Welfare-Maximization Subsidies

Social welfare maximization seems to be a more natural approach for designing subsidies. However, the presence of information asymmetry can cause inefficiency and friction in implementing those policies. Since Pareto efficiency is a necessary condition for welfare maximization (Deardorff, 2014), we want to compare our results with the subsidy policies that obtain the most efficient Pareto outcomes. The government agency who has the flexibility to choose a different approach will prefer the one that gives a higher payoff.

The comparison also shows how, in practice, to implement the optimal subsidy policy. The policies derived from Theorem 2 are implementable because: (a) The mapping from the subsidy policy to the manufacturer's best response in Proposition 1 holds; (b) The manufacturer's continuation value $W(t)$ and the AV market share $X(t)$ are observable as state variables. To convey the AV subsidy policies with the state $W(t)$, we propose a simple mechanism to evaluate the IR-constraint below.

The mechanism uses a Pareto optimality argument in Figure 4. We divide the state space of $W \in [0, \bar{W}]$ into three regions, $\mathcal{W}_1, \mathcal{W}_2$, and $\mathcal{W}_3$ as follows:

1. $\mathcal{W}_1 = [0, W^*]$,
2. $\mathcal{W}_3 = \{W \in [0, \bar{W}] : F(W) < 0\}$,
3. $\mathcal{W}_2 = [0, \bar{W}] \setminus \{\mathcal{W}_1, \mathcal{W}_3\}$.

With the computed subsidy policies $\{S_A^*, Y^*\}_{t \geq 0}$, the government agency can decide the continuous value $F(W)$ regarding the entry threshold W_0. The optimal subsidy policies over the three regions are:

1. If $W_0 \in \mathcal{W}_1$, the optimal subsidy policies automatically drive the AV manufacturer to obtain W^* approximately. The government agency can enforce a higher $W \approx W^*$ such that both the government agency and the manufacturer obtain higher payoff/utility.
2. If $W_0 \in \mathcal{W}_2$, the government can obtain the exact continuation value $F(W_0)$ by implementing the optimal subsidies.
3. If $W_0 \in \mathcal{W}_3$, then $F(W) < 0$ so the subsidy policy is rescinded. In other words, no feasible AV subsidy policy exists in this region.

Theorem 3. *For the government agency, the DSG optimal subsidy policy dominates a welfare-maximization subsidy policy by providing a higher expected discounted total payoff for any given market share $X \in [0, 1]$.*

Proof. In Proposition 2, we know that F is a partially concave function in W (Figure 4). Then we have the following observations:

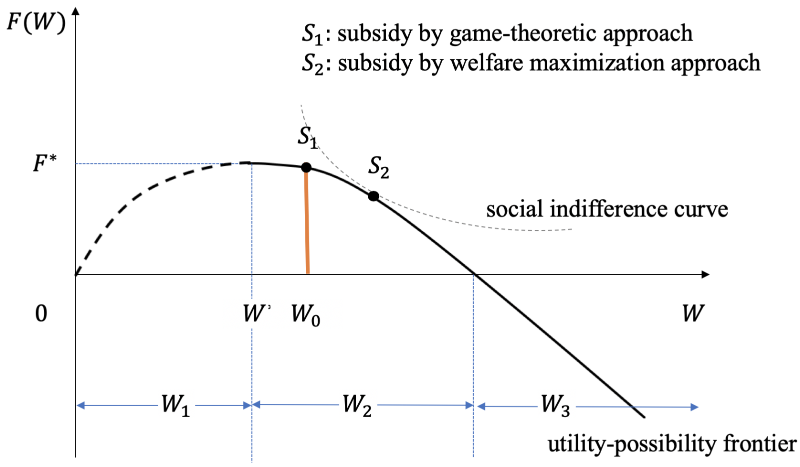


Figure 4: Relationship between the government agency's continuation value F and the manufacturer's continuation value W. The bold curve is the government agency's payoff possibility frontier. For a given entry threshold W_0, S_1 represents the government agency's payoff using the DSG approach; S_2 represents the payoff using a welfare-maximization approach.

1. The solid curve in Figure 4 is the payoff possibility frontier.
2. The welfare-maximization subsidy policy is obtained within $\mathcal{W}_2$.
3. Because the government agency has the flexibility of choosing which policy on the frontier to deploy, and both contracts are implementable, it will prefer to implement the DSG subsidy policy.

Hence, if the IR-constraint threshold W_0 is not large (which is chosen to be 0 by default), the continuation value with DSG subsidy policy dominates that of the welfare-maximization subsidy policy. □

3.2. Structure of Optimal AV Subsidy Policy

Previous literature found the optimal subsidy policies to be monotonic (Kalish and Lilien, 1983; Janssens and Zaccour, 2014; Langer and Lemoine, 2017). This paper discovers a unique structure of the optimal AV subsidies $\{S(t)\}_{t\geq 0}$ in Proposition 3 below. The presence of this two-threshold structure is unprecedented because of the existence of the "dark age" of AV deployment. The structure is also relevant to the weight of innovation and imitation effects in the DOI model (see Figure 5).

Proposition 3. *The optimal subsidy policy has a two-threshold structure during the product life cycle – an early subsidy and a late subsidy.*

1. *The early subsidy is implemented when the AV market share is low, and decreases with the ascending AV market penetration rate X.*
2. *The late subsidy is implemented when the AV market share is high, and increases with the ascending AV market penetration rate X.*

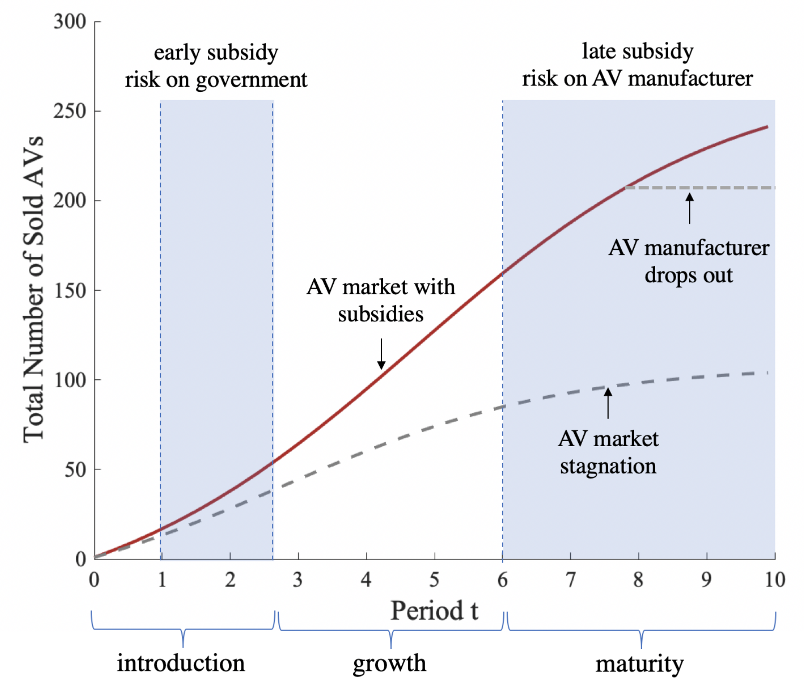


Figure 5: Optimal AV subsidy policy over the AV market penetration process.

Proof. Applying the first-order optimality condition with regard to S_A on the right-hand side of the HJB equation in Theorem 2,

$$S_A^* = (h')^{-1}\left(\frac{M(a^*+bX)(1-X)}{\frac{1}{2}\sigma^2 r_G F_W}\right).$$

The inverse function of an increasing continuous function is also increasing, thus h^{-1} is increasing in its argument. Let the domain of S_A be $\mathcal{U}_{S_A} = [\underline{S}, \bar{S}]$. Because of the S-shape of the DOI model, S_A^* converges to the lower bound $\underline{S}$ when the numerator is large and $F_W < 0$ (W_0 is small). When $F_W = 0$, it is easy to see that $S_A^* = 0$ in Theorem 2. The monotonicity of the h^{-1} also implies a monotonic subsidy policy in early subsidy and late subsidy respectively. Therefore, the optimal subsidies has the two-threshold structure described above. □

The transition of risk-sharing is another dimension of the subsidy policy that is not mentioned in previous literature. Such a transition over the planning horizon is because of the information asymmetry associated with the DOI process. At a low market share, the government agency shall undertake a high risk when it implements the early subsidy policies and creates the early demand for AVs. Such a relationship reverses when the AV market penetration reaches the near-saturation stage. At a high market share, the declining market growth stimulates the AV manufacturer to reduce or even terminate the production of AVs. In response, the late subsidy policy incentivizes the manufacturer to take more risk so to gain efficiency benefit in the future.

3.3. Iterative Algorithm for Optimal AV Subsidy Policy

Since the optimal AV subsidies in Theorem 2 can only be computed numerically, we propose an iterative algorithm that provably converges to the optimal subsidies. We can compute a menu of subsidy policies as a look-up table for different state variables, so the government agency can easily implement them in practice. For notational convenience, let the decision variables be $u = [S_A, Y]$ and $\Lambda(u)$ be the left-hand side of the HJB equation in Theorem 2. The gradient can be evaluated because $\Lambda(u)$ is assumed to be differentiable with regard to the control variables.

- Input data: DOI coefficients b and M, the initial AV market share $X(0)$, parameters r_G, r_A, σ, the stopping criteria $\epsilon > 0$, and functions g and h.
- Step 0: Initialization. For given $u^0 = (\underline{S_A}, \underline{Y}) \in \mathcal{U}$ at iteration $n = 0$, we solve $\Lambda(u^0) = r_G F$ by an implicit finite difference method. This gives the value of F^0 over discretized state space.
- Step 1: Search direction – compute the approximate value of gradient $\nabla_u \Lambda$ by:

$$\nabla_u \Lambda^n = \left[\frac{\Lambda(S_A + \triangle S_A, Y) - \Lambda(S_A, Y)}{\triangle S_A}, \quad \frac{\Lambda(S_A, Y + \triangle Y) - \Lambda(S_A, Y)}{\triangle Y} \right]$$

  If $\|\nabla_u \Lambda^n\| < \epsilon$, stop; else go to Step 2.
- Step 2: Line search – find the step size γ^n

$$\Lambda(u^n + \gamma^n \nabla_u \Lambda^n) = \max_{\gamma} \Lambda(u^n + \gamma \nabla_u \Lambda^n).$$

- Step 3: Update the AV subsidy policies S_A and Y by the gradient ascent $u^{n+1} = u^n + \gamma^n \nabla_u \Lambda^n$. Go to Step 1.

The concavity of F ensures that the algorithm converges to the global optimal solution. We prove this convergence result in Appendix A.3. If changing the subsidies in real time is not possible, we can approximate the subsidy policies with a sequence of subsidies in discrete time. Shan (2017) showed that the gap of optimality also converges in implementation. Note that, with an infinite horizon, we do not need to resolve the menu of AV policies using the renewal theory (Cvitanic and Zhang, 2012).

3.4. Numerical Results

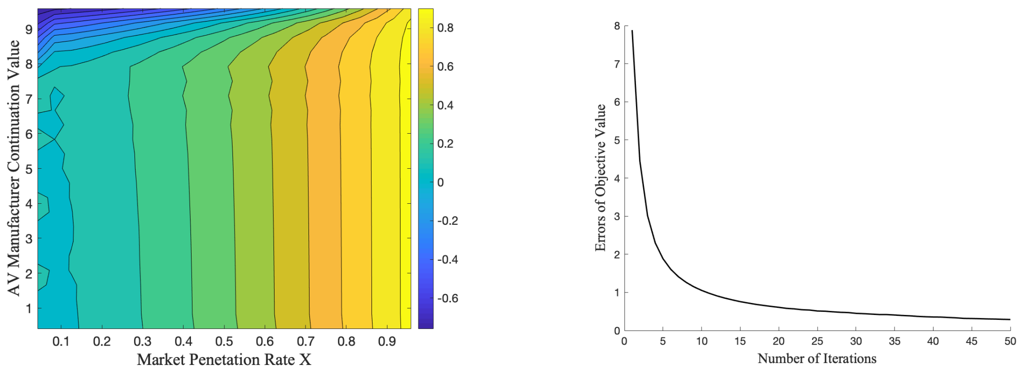
We shall validate the analytical results by numerical experiments and gain some managerial insights using the AV market forecast data. By running sensitivity tests on the optimal subsidies regarding these input data, we can identify the critical research directions that may effect the AV subsidy policies.

The following instances are common to all numerical experiments.

1. The AV market penetration process refers to the DOI model in Shabanpour et al. (2018b). They used a survey-based approach to estimate the individuals' propensities to adopt AVs. The estimated power of innovation was $a = 0.108$, and the estimated power of contagion was $b = 0.957$. The AV market saturation was expected to be 71.3% of the current automotive market in the United States, and thus the AV market potential is $M = 200$ million cars (OECD, 2018).
2. The aggregate efficiency benefit of AVs that is measured by the unit reduced value of driving time refers to Seo and Asakura (2017). The benefit function is quadratic: $g(X) \approx -650 + 6000 \cdot (X - 0.33)^2$. The positive efficiency benefit is not obtained until the AV market surpasses 67% market share. We term the market share that gives most negative efficiency benefit ($X = 33.5\%$) the *worst-case market share*, and the constant 6000 the *social benefit multiplier*.
3. The AV manufacturer's continuation value W is bounded if subsidies S are bounded. Hence, we use a trivial upper bound for the government's continuation value $\bar{F} = g(M)$ (see Appendix A.3).

3.4.1. Solving Optimal AV Subsidies

With these considerations, we computed the optimal subsidies with varying states X and W by the above iterative algorithm. The government's continuation value $F(X, W)$ is shown in Figure 6a. The convergence of the algorithm is validated by tracking the error of the government agency's continuation value F between two subsequent iterations in Figure 6b.



(a) Government agency's relative continuation value

(b) Convergence of the iterative algorithm

Figure 6: Solving the optimal AV subsidy policy by the iterative algorithm. (a) The normalized government agency's continuation value $F/F_{\max}$; (b) The error of F converges to near zero.

Our observations include: (a) When the government's continuation value $F(X, W) > 0$, there exists an optimal subsidy policy (S_A^*, Y^*) that accelerates the early adoption of AVs. In the case that $F < 0$, the AV market failure is unavoidable; (b) Figure 7 verifies the two-threshold structure of optimal subsidies during the market penetration process.

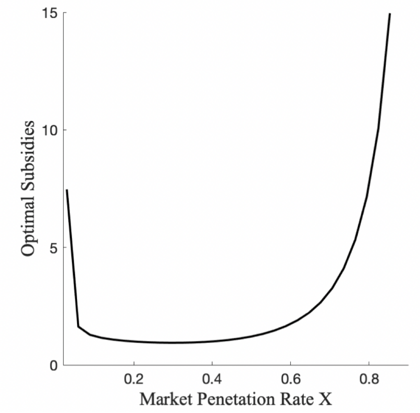
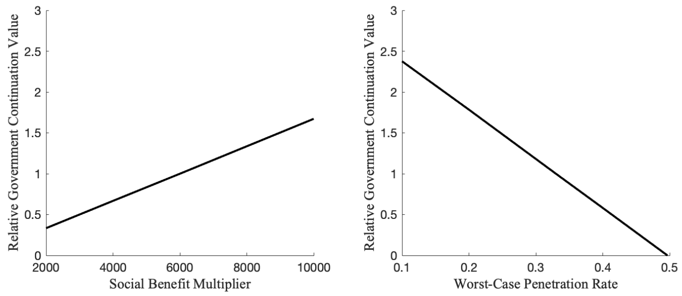


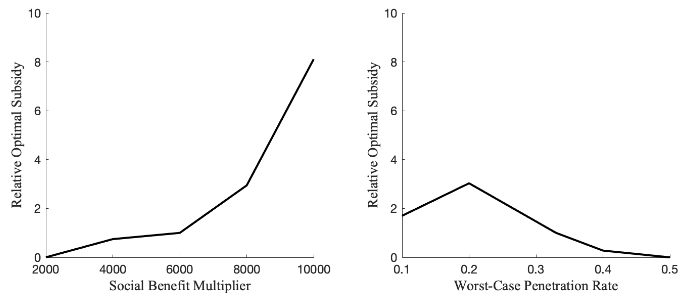
Figure 7: Optimal subsidies (in unit of $1,000) per AV regarding the market penetration rate X.

3.4.2. Sensitivity Analysis of Optimal Subsidies

The government agency may be inquisitive about the robustness of optimal AV subsidy policies with regard to the input data. We considered various uncertainties in the data, among others: (a) the AV efficiency benefit function $g(N)$, (b) the parameters in the DOI model, and (c) the noise of AV market forecast. For convenience, we ran a sensitivity test on a fixed state ($W = 4.2, X = 0.8$), and used the measures of relative objective values and subsidies in respect to the original instance.

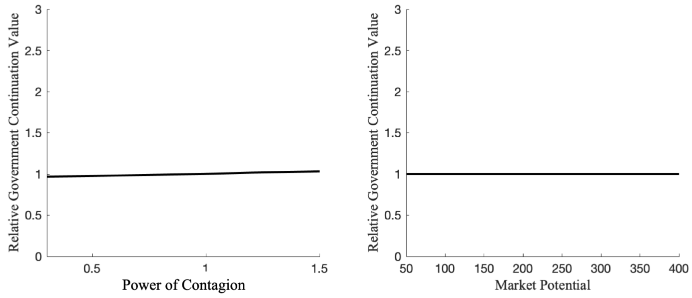


(a) Sensitivity of the government agency's continuation value F.

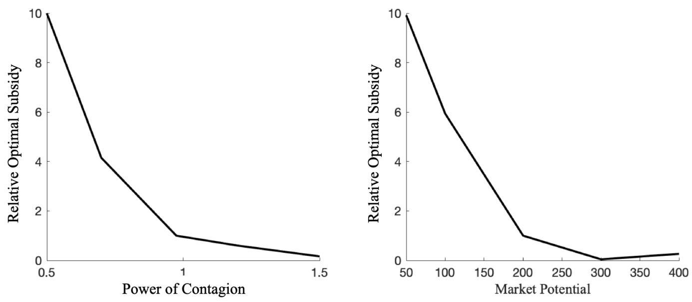


(b) Sensitivity of the optimal AV subsidy policy S_A.

Figure 8: Sensitivity of the optimal AV subsidy policy to the efficiency benefit function.



(a) Sensitivity of the government agency's continuation value F.



(b) Sensitivity of the optimal AV subsidy policy S_A.

Figure 9: Sensitivity of the optimal AV subsidy policy to the market penetration DOI model.

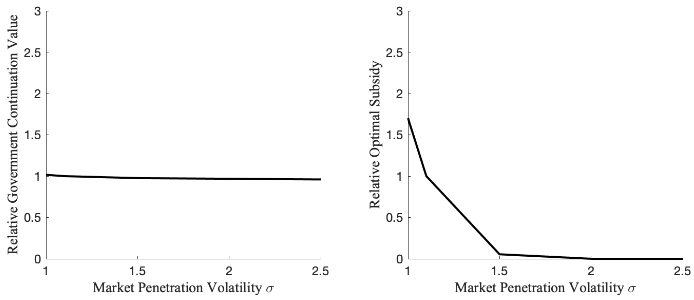


Figure 10: Sensitivity of the government agency's continuation value F and the optimal AV subsidy policy S_A to the market penetration volatility.

We have the following observations from the sensitivity analysis:

1. The government agency's continuation value and optimal subsidy policies are *sensitive* to the AV efficiency benefit function $g(N)$.
   (a) With an increasing social benefit multiplier (i.e., the magnitude of AV's efficiency benefit), the government's expected total payoff increases (Figure 8a).
   (b) With an increasing worst-case penetration rate (i.e., the market share with most negative efficiency benefit), the government agency's total payoff decreases (Figure 8a).

(c) With an increasing efficiency benefit, the government agency is willing to pay higher subsidies (Figure 8b).

Hence, it is noticeably valuable to improve the operational rules of AVs, e.g., the time headway, in order to improve the efficiency benefit of AVs.

2. The government agency's objective value is *insensitive* to the parameters of the market penetration process (Figure 9a).
3. The optimal subsidy policies are *sensitive* to the parameters of the market penetration process. If the initial adoption rate of AVs during the launching period is fast, the required amount of AV subsidies can be reduced significantly (Figure 9b).
4. The optimal subsidy policies are *sensitive* to the AV market forecast uncertainty. Greater uncertainty (represented by the market penetration volatility σ) may increase the gap of information asymmetry (Figure 10). Therefore, with large noise in the AV market forecast, the government should reduce the subsidies to penalize the hidden information of $R(t)$.

4. Extension: Changing AV Market Potential

4.1. AV Subsidy Policies with Concerns of Car Ownership and Incentivizing Consumers

We aim to relax the assumption that the AV market potential M (i.e., the population of potential AV consumers) is fixed. With the rising of shared mobility, previous literature has predicted the possible demise of private car ownership because of the widespread of AVs (Wadud, 2017; Lavieri et al., 2017; Zhang et al., 2018). This implies an extrinsic declining number of the private-car consumers. On the other hand, AVs may be highly priced at the beginning so that the initial AV market potential is restricted, which causes an intrinsic ascending or declining trend of car ownership. In response, the government agency can directly compensate consumers by offering a per-vehicle subsidy $S_C(t)$ to consumers. We model the market potential with a simple exponential function of the extrinsic car ownership trend $\beta(t)$ and the retail price multiplier $P(t)$ (Bass and Bultez, 1982)

$$M(t) = M_0 e^{-\beta(t)-P(t)+S_C(t)}.$$

$\beta(t)$ is a known process that represents the decreasing car ownership because of the widespread use of AVs (Shabanpour et al., 2018b).

Now the AV manufacturer can control the AV market potential $M(t)$ by modifying the price multiplier $p(t) = -dP(t)/dt$. $p(t)$ is termed as the "pricing incentives" for AVs. In the extension, price multiplier is the AV manufacturer's response, i.e., $R(t) \equiv p(t)$. The extension to the DSG model is depicted in Figure 11. It is easy to see that:

$$dM(t) = -M(t)dP(t) = M(t)p(t)dt, \qquad M(0) = M_0.$$

Applying Ito's lemma to (1), we can characterize $X(t)$ that is controlled by $p(t)$ as:

$$dX(t) = [a + (b - a - p(t))X(t) - bX(t)^2]dt + \sigma X(t)dB(t).$$

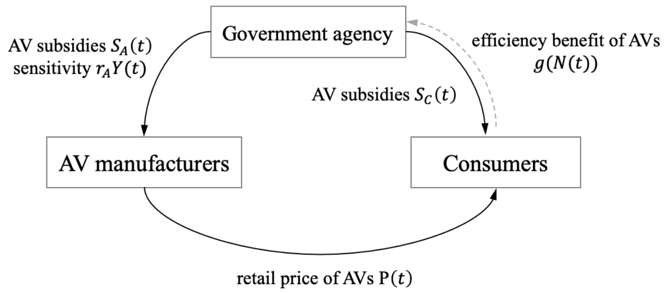
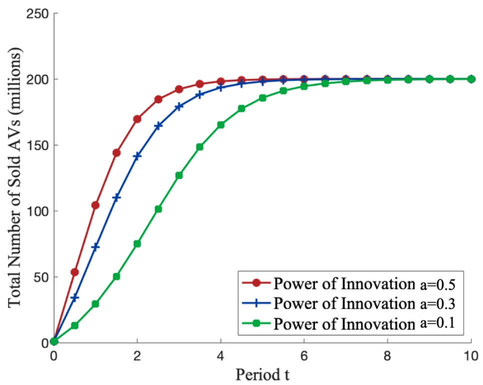
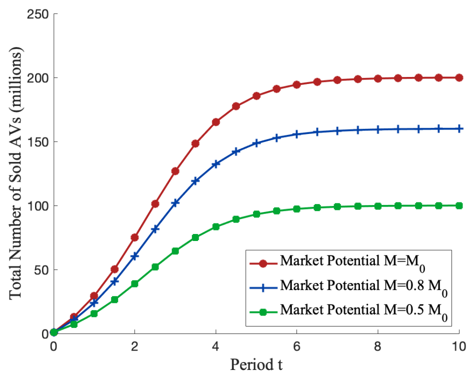


Figure 11: Extension to the DSG model for AV subsidy problem.

Compared to the simpler case above, the impacts of subsidies on the AV market penetration processes are different (Figure 12). When the manufacturer's response $R(t)$ controls the AV innovations (i.e. the value of $a(t)$), the AV market growth rate is tempered at the early adoption stage but ultimately the market size will reach the saturated market share. In contrast, when the manufacturer provides pricing incentives for AVs (the value of $p(t)$), the market potential $M(t)$ is changed. It is worth mentioning that the market failure with inadequate pricing incentives (i.e., $p(t)$ is lower than expected) has more serious consequences than that with inadequate AV innovations (i.e., $a(t)$ is lower than the target). With inadequate AV innovations, the slow market penetration only generates finite negative externalities during the "dark age"; with inadequate pricing incentives, the AV market's long stagnation in low market size may continuously generate negative externalities.



(a) AV manufacturer changes the power of innovations $a(t)$.



(b) AV manufacturer changes the market potential $M(t)$ by pricing incentives.

Figure 12: The mean value of the AV market penetration process under the manufacturer's controls.

Similar to (2), the government agency solves the following an optimization problem with with an IC-constraint for the optimal pricing incentives $p(t)$. It is trivial to include other responses $R(t)$, e.g., a mixture of $a(t)$ and $p(t)$. With all these considerations, we can formulate the optimal subsidy problem as:

$$\max_{S_A(t),S_C(t)} \mathbb{E}\left[r_G\int_0^\infty e^{-r_Gt}\left[g(N(t))-(S_A(t)+S_C(t))\left(a+b\frac{N(t)}{M(t)}\right)(M(t)-N(t))\right]dt\right]$$
$$s.t.\, p(t)\in \arg\max_p \mathbb{E}\left[r_A\int_t^\infty e^{-r_Au}h(S_A(u),S_C(u),p(u))du\right], \forall t\geq 0 \text{ (IC-constraint)} \quad (3)$$
$$\mathbb{E}\left[r_A\int_0^\infty e^{-r_At}h(S_A(t),S_C(t),p(t))dt\right]\geq W_0 \text{ (IR-constraint)}$$

With the extension of decision variables $S_C(t)$ and $p(t)$, the state variables of the dynamic programming should also be expanded. Let $F(P, X, W)$ be a $C^{1,2,2}$ function. Besides the AV manufacturer's continuation value $W(t)$ and AV market penetration rate $X(t)$, government agency also needs to observe the retail price of AVs $P(t)$. With Proposition 1, the best response p^* can be found as a function of S_A, S_C and Y. The following theorem solves the optimal AV subsidy policies.

Theorem 4. *The government agency's continuation value is equal to $F(P, X, W)$ that solves the following HJB equation:*

$$\begin{aligned} r_GF = \max_{S_A,S_C,Y}\Big\{ & r_G[g(N)-(S_A+S_C)M_0e^{-\beta-P+S_C}(a+bX)(1-X)]+p^*F_P+\\ & (a+(b-a-p^*)X-bX^2)F_X+r_A(W-h(S_A,S_C,p^*))F_W\\ & +\frac{1}{2}\sigma^2(X^2F_{XX}+2r_GXYF_{XW}+r_A^2Y^2F_{WW})\Big\}. \end{aligned}$$

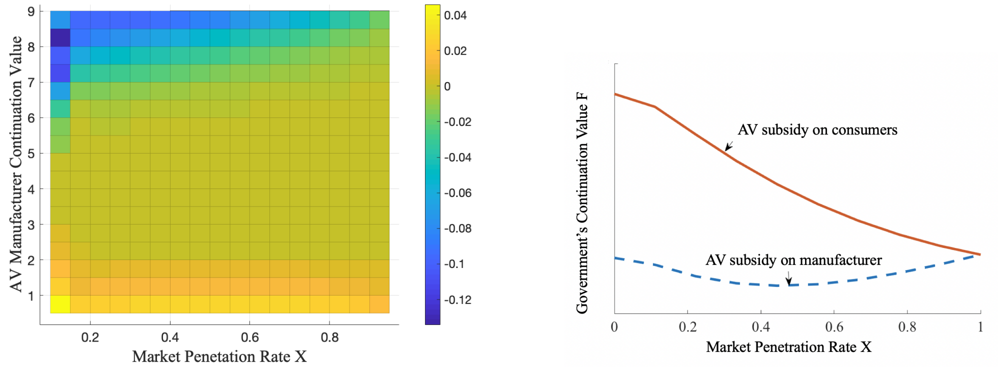
4.2. Result and Discussion

We can use the same iterative method to solve for the optimal AV subsidy policies in Theorem 4. The arguments for two-threshold structure of the AV subsides $S_A(t)$ also hold. However, $S_C(t)$ does not retain this special structure.

Now we briefly discuss the new insights about whom should be subsidized in the AV market. Because of the complexity of the state variables, we find that there is no seemingly simple answer to this question. In numerical experiments, we compared the relative continuation values of giving a fixed amount of subsidies to consumers (F_{consumer}) and to manufacturers ($F_{\text{manufacturer}}$). In Figure 13, we can observe that there are regions where subsidizing AV manufacturers promises a larger expected total payoff, and vice versa. In general, the government agency should give a higher weight on the consumers' subsidies when the market penetration rate is small, or the AV manufacturer's continuation value is low. Otherwise, it is more beneficial to subsidizing AV manufacturers.

In sum, our main results provide the following insights for designing AV subsidy policies:

1. The DSG optimal subsidy has a two-threshold structure because of the presence of a "dark age" in AV deployment. The government agency needs to subsidize AVs in their early deployment as well as the near-saturation stage.



(a) Relative difference $(F_{\text{consumer}} - F_{\text{manufacturer}})/F_{\text{manufacturer}}$.

(b) Government agency's continuation value over the market penetration process.

Figure 13: Comparison of subsidizing AV manufacturers and consumers.

2. The most crucial factor for AV subsidies is the efficiency benefit related to AV deployment. To mitigate the negative externalities of AVs, the government agency needs to consider developing and implementing operational strategies that improve the efficiency of AV technology. Otherwise, the government agency has to pay high subsidies to avoid the potential AV market failure.
3. The optimal AV subsidies also depend on the parameters in the market penetration process. Hence, conducting more research on forecasting the AV market is necessary.
4. The iterative algorithm can compute a menu of optimal subsidy policies for the government agency with a different data instance.

5. Conclusion

The potential efficiency loss during the "dark age" of AV deployment calls for policies and strategies to accelerate the deployment of AVs. As it is extremely difficult, if not impossible, to establish a single modeling framework to prescribe these policies and strategies, this paper investigates an optimal subsidy policy that maximizes the benefits of AV deployment throughout a planning horizon. The possible AV market failure is because of the information asymmetry between the government agency and the AV manufacturers. To prevent this failure due to the static and monopolistic subsidy design practice, we develop a new dynamic games approach. This approach can compute the adaptive subsidies based on the state of the AV market penetration process with uncertainty. Given the optimal subsidies, the AV manufacturer is incentivized to enhance the AV innovations, and provide pricing incentives to potential consumers.

This approach opens the door to many promising research directions and interesting applications in transportation policy. For example, the AV subsidies can be imposed to evolving generations of technology with different efficiency benefit functions. We can model it by replacing the DOI model with a system of diffusion processes. This model can also capture more complex interactions between multiple manufacturers with different utility functions, e.g., a competition for the future AV market. We refer the interested audience to the extension to a multiagent system (Luo and Saigal, 2017). Since DSG is a general framework of prescribing optimal transportation policies, we can generalize our approach to other policy-making processes to include the regulated entity's response into the consideration.

With this new approach, we gain new insights about how to accelerate the adoption of AVs through adaptive subsidies. First, the DOI model and information asymmetry induce a two-threshold structure in the optimal subsidies, i.e., decreasing subsidies at low market share, and increasing subsidies at near-saturation market share. Second, the cost efficiency of the optimal subsidies obtained by this dynamic games approach dominates other approaches. Third, the sensitivity analysis of the optimal subsidies addresses the necessity for more AV market studies. The government agency should consider policies that enhance the AV operations and improve the accuracy of the AV market forecast models. With all considerations, the policy makers can strategically mitigate the negative externalities of AVs while embracing the advances of the technology.

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Appendix A. Technical Proofs and Numerical Method

Appendix A.1. Proof of Theorem 1 and Proposition 1

Given the dynamics of $N(t)$ in equation 1, we can compute the dynamics of the market penetration rate $X(t)$ where the AV manufacturers control the pricing incentives $p(t)$:

$$dN(t) = M(t)\left(a + b\frac{N(t)}{M(t)}\right)(1 - \frac{N(t)}{M(t)})dt + \sigma N(t)dW(t),$$
$$dM(t) = p(t)M(t)dt.$$

Defining $X(t) = (\frac{N(t)}{M(t)})$ and applying Ito's lemma yields

$$\begin{aligned} dX(t) &= (a + b\frac{N(t)}{M(t)})(1 - \frac{N(t)}{M(t)})dt + p(t)\frac{N(t)}{M(t)}dt + \sigma(\frac{N(t)}{M(t)})dW(t), \\ &= (a + (b - a + p(t))X(t) - bX(t)^2)dt + \sigma X(t)dW(t). \end{aligned}$$

Proof. Assume that at time $t > 0$, the AV manufacturer follows the target responses that are obtained by solving the optimization problem (2). For simplicity of notation, we use $r_A = r_G = r$ and define

$$V_A(t) = \int_0^t re^{-rs}h(s)ds + e^{-rt}W(t)$$

where $W(t)$ is the manufacturer's continuation value.

It is easy to see that $V_A(t)$ is a martingale, i.e.,

$$\mathbb{E}\left[V_A(t)|\mathcal{F}_s\right] = V_A(s), \forall s < t.$$

From the Martingale representation theorem, there exists an adopted process $Y(t)$ such that

$$dV_A(t) = re^{-rt}\sigma Y(t)dB(t), \tag{A.1}$$

where $Y(t)$ is adapted to the filtration of $B(t)$, and $rY(t)$ can be interpreted as the sensitivity of $W(t)$ to $X(t)$. Applying Ito's lemma to V_A :

$$dV_A(t) = re^{-rt}h(t)dt - re^{-rt}W(t)dt + e^{-rt}dW(t).$$

Plugging into (A.1) we have the dynamics of $W(t)$:

$$dW(t) = r\left[\sigma Y(t)dB(t) + (W(t) - h(t))dt\right].$$

We use the standard deviation argument. For given time t, let the optimal control be $p^*(t) : t \geq 0$. Define a non-optimal control, where $\hat{p}(u)$, $u \leq t$ is arbitrary:

$$\hat{p}(u) = \begin{cases} \hat{p}(u) & 0 \leq u \leq t \\ p^*(u) & u \geq t \end{cases}$$

and

$$\hat{h}(t) = \hat{h}(\hat{P}(t), S(t), \hat{C}(t), X(t)).$$

Now with

$$\begin{aligned}
\hat{V}_A(t) &= r\int_0^t e^{-ru}\hat{h}(u)du + e^{-rt}W(t), \\
d\hat{V}_A(t) &= e^{-rt}\{r\hat{h}(t)dt + d(t) - rB(t)dt\} \\
&= e^{-rt}\{(r(\hat{h}(t) - h(t))dt + rY(t)\sigma dB(t)\}.
\end{aligned}$$

Define the Girsanov's kernel

$$\phi(t) = \hat{p}(t) - p(t),$$

and the measure change:

$$L(t) = \exp\{-\int_0^t \phi(u)B(u) - \frac{1}{2}\int_0^t \phi^2(u)du\}$$

and $d\hat{P}(t) = L(t)dP(t)$. It follows from the Girsanov's theorem that:

$$\sigma dB(t) = \sigma d\hat{B}(t) + \phi(t)dt.$$

Thus, under measure $\hat{P}$, the $X(t)$ dynamics becomes:

$$dX(t) = \Big(a + (b - a + \hat{p}(t))X(t) - bX(t)^2\Big)dt + \sigma X(t)d\hat{B}(t),$$

and the dynamics of $\hat{V}_A(t)$ becomes:

$$d\hat{V}_A(t) = e^{-rt}\left\{\Big[r(\hat{h}(t) - h(t)) + rY(t)(\hat{p}(t) - p(t))\Big]dt + r\sigma Y(t)d\hat{B}(t)\right\}.$$

By comparison theorem, under p^*, $\hat{V}_A$ must have a negative drift, giving the optimality condition:

$$h(t) + Y(t)p(t) \geq \hat{h}(t) + Y(t)\hat{p}(t) \text{ for all } \hat{p}(t).$$

Similar proof can be applied to the case the manufacturer determines the power of innovation a. We show the result for the case in which the manufacturer chooses the optimal response R. Assuming that function h is differentiable to control R so for any given $Y(t)$, the first-order optimality condition suggests that at optimality,

$$\frac{dh(R(t), S(t), Y(t))}{dR(t)} + Y(t) = 0.$$

In the risk-neutral manufacturer case, i.e., h is linear in R, and since a linear (more generally convex) function achieves it maximum on the boundary so that the optimal pricing policy becomes a bang-bang control:

$$R^*(t) = \begin{cases} \bar{R} & \text{if } Y(t) \geq 0 \\ \underline{R} & \text{if } Y(t) < 0, \end{cases}$$

which means that the government agency should penalize those who frequently manipulate the prices to the extremes.

For a risk-averse agent, if h is continuously differentiable, and its first-order derivative is invertible (this is the case when h is strictly concave in R), then the optimal control is

$$R^*(t) = (h')^{-1}(-Y(t)),$$

where $(h')^{-1}$ is an inverse function of the first derivative of h.

Appendix A.2. Proof of Theorem 2.

Proof. For notational convenience, we denote the government's instantaneous payoff function f. The government's continuation value $C_g(t)$ at time $t > 0$ and the optimal $S(u)$ is adopted in $u \in [t, \infty)$:

$$C_G(t) = \mathbb{E}^{R(y)}\{r_G \int_t^\infty e^{r_G(u-t)} f(u)du|\mathcal{F}_t\}. \tag{A.2}$$

At time $t > 0$, we can rewrite the government's optimal value as:

$$V_G(t) = r_G \int_0^t e^{-r_G u} f(u)du + e^{-r_G t} C_G(t).$$

In case an optimal strategy is adapted, $V_G(t)$ is a $\mathcal{F}_t$ martingale, and thus has a zero drift. By assumption, there exists a $C^{2,2}$ function when the manufacturers choose $a(t)$, or a $C^{1,2,2}$ function F when the manufacturers choose $p(t)$. For convenience, we suppose that $C_G(t) = F(P(t), X(t), W(t))$.

Under the assumption the continuation value of the government can be written as:

$$V_G(t) = r_G \int_0^t e^{-r_G u} f(u)du + e^{-r_G t} F(P(t), C(t), X(t), W(t)).$$

Using Ito's lemma the dynamics of $V_G(t)$ can be derived as:

$$dV_G(t) = r_G e^{-r_G t} f(t)dt - r_G e^{-r_G t} F(P, X, W) + e^{-r_G t} dF(P, X, W),$$

where the drift term is $[pF_P + (a + (b - a + p)X - bX^2)F_X + r_G(W - h)F_W + \frac{1}{2}\sigma^2(X^2 F_{XX} + r_G^2 Y^2 F_{WW} + 2r_G XY F_{XW})]dt$.

By setting the drift term to zero, we have the HJB equations in Theorem 2.

Appendix A.3. Supplementary material for the iterative algorithm

We use the following boundary conditions for the case that the manufacturer chooses the pricing incentives p:

$$\begin{cases} F(\underline{P}, X, W) = 0, & F(\bar{P}, X, W) = 0, \\ F(P, 0, W) = 0, & F(P, 1, W) = \bar{F}(P), \\ F(P, X, 0) = 0, & F(P, X, \bar{W}) = -\bar{F}(P). \end{cases}$$

We briefly explain why we set the boundary conditions as follows:

1. When the selling prices P is less or equal to $\underline{P}$, there is no motivation to produce AV. Hence, the manufacturer's continuation values $W = 0$. This enforces $F = 0$. When the selling prices hits $\bar{P}$, we assume that the market potential M reaches the lower bound so the government's continuation value is also 0.
2. When the manufacturer's continuation value hits $\bar{W}$, the government's continuation value is most negative.
3. When the penetration rate X hits 1, the government agency's continuation value reaches the maximum. Note that the market potential is a function of the selling price so the values of F is monotonically decreasing in P.

The remaining question is whether or not W is bounded (we have assumed that other state variables are in a compact set). It is easy to verify the following upper bound for the government's continuation value:

$$\bar{F}(P) = F(P, C, 1, W) = \sup \mathbb{E}\{r \int_t^\infty e^{r(u-t)} f(P, C, 1) du | \mathcal{F}_t\} = r\frac{1}{r}\bar{g}(N) = g(e^{-P} M_0).$$

Proof. We have shown that function F derived from HJB equation has a numerical solution that converges to an unique weak solution. Next, we show that the optimization also converges to (S_A^*, Y^*).

We show the convergence of sequences as follows. Set $\Gamma = \{u : \Lambda(u) \leq \Lambda(u^0)\}$ be a compact set and there exists a subsequence N such that $\{u^n\}_{n\in N}$ converges to $\bar{u}$ and $\triangledown\Lambda(\bar{u}) = 0$.

Step 2 $\rightarrow$ Step 4 satisfies that $\Lambda(u^{n+1}) > \Lambda(u^n)$ and $\triangledown_u\Lambda$ is the steepest ascent direction. Thus the sequence $\{u^n\} \subset \Gamma$. Since Γ is a compact set, there exists the required subsequence and limit point $\bar{u}$.

Now assume that $\Lambda(\bar{u}) \neq 0$ and $\triangledown\Lambda(\bar{u}) = \bar{d}$. There exists a step size $\bar{\gamma}$ such that

$$\Lambda(\bar{u} + \bar{\gamma}\bar{d}) = \max_\gamma \Lambda(\bar{u} + \gamma\bar{d}),$$

and for some $\delta > 0$, Λ is a $C^{1,1}$ function

$$\Lambda(\bar{u}) = \Lambda(\bar{u} + \bar{\gamma}\bar{d}) - \delta.$$

Now define

$$\begin{cases} z^n = u^n + \bar{\gamma} d^n \\ \bar{z} = \bar{u} + \bar{\gamma}\bar{d} \end{cases}.$$

Note that $z^n \rightarrow \bar{z}$ thus the difference between the current point and the limit point, $v^n = z^n - \bar{z} \rightarrow 0$.

Let $n \in N$, and use the first-order approximation by the Taylor's formula:

$$\Lambda(u^n + \bar{\gamma} d^n) = \Lambda(z^n) = \Lambda(\bar{z}) - \nabla\Lambda(\bar{z})^T v^n.$$

As the sequences converge $z^n \to \bar{z}$ and $u^n \to \bar{u}$, the gradient $\nabla\Lambda(u^n) \to \nabla\Lambda(\bar{z})$. As $v^n \to 0, \nabla\Lambda(\bar{z})^T v^n \to 0$. Hence, for sufficiently large $n \in N$, $\|\nabla\Lambda(\bar{z})^T v^n\| < \frac{\delta}{2}$. So

$$\Lambda(u^n + \bar{\gamma} d^n) > \Lambda(\bar{z}) - \frac{\delta}{2} = \Lambda(\bar{u}) + \frac{\delta}{2}.$$

Since γ^n is a maximizer, thus

$$\Lambda(u^{n+1}) = \Lambda(u^n + \gamma^n d^n) \geq \Lambda(u^n + \bar{\gamma} d^n) > \Lambda(\bar{u}) + \frac{\delta}{2}.$$

This contradicts the fact that $\Lambda(\bar{u}) > \Lambda(u^{n+1})$. Thus such a sequence of solutions in the iteration set exists within such a compact set. This completes the proof of convergence. □