Parking Management of Automated Vehicles in Downtown Areas

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Abstract

Automated vehicles (AVs) eliminate the burden of finding a parking spot upon arrival to the destination, because they can park at a strategic location or cruise until summoned by their users. In this study, we investigate the parking choices of privately-owned AVs in a downtown area. Since each user's choice has impacts on another via cruising-incurred traffic congestion and parking competition, we model the parking choice problem of AVs as a Wardrop equilibrium in which each user cannot further reduce their parking cost by unilaterally changing their choice. The model considers all possible options for parking a private AV, and finds that these parking choices may yield multiple equilibria under which the congestion and social welfare are very different. We further develop an efficient solution algorithm to find all equilibrium solutions. Our analysis shows that even if AVs act in a non-cooperative manner, one possible outcome would involve many AVs choosing to cruise, which creates severe congestion to decrease the cost of cruising. However, this outcome can be avoided by a time-based congestion toll, which discourages AVs from cruising for a long period and increases the social welfare. Our analysis also shows parking pricing and provision may not reduce congestion induced by cruising AVs without the help of a congestion toll.

Keywords: Automated vehicles; parking choice, parking policy; time-based toll; congestion

1 Introduction

Automated vehicles (AVs) are fast approaching and they are going to change the way we travel, and the form and land use of cities, consequently. For instance, AVs could be stacked behind each other inside parking facilities to increase parking supply and decrease the land needed for parking (Nourinejad et al., 2018). This would free up land for commercial and residential land uses, thereby changing cities and their sprawl pattern (Zakharenko, 2016b; Gelauff et al., 2019). Also, AVs can accelerate the growth of shared use of a vehicle via car sharing or on-demand ride-hailing due to greater convenience and cost savings arising from their self-driving capability

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(Stocker and Shaheen, 2017; Gurumurthy et al., 2019). However, surveys show that many users would prefer to own and use their private AVs even if shared mobility services become extremely cheap (Haboucha et al., 2017; Gkartzonikas and Gkritza, 2019).

Private AV users would experience hassle-free parking due to the self-parking capability of AVs. This feature enables users to exit from their AVs at their final destinations, and send their occupant-free AVs to search for parking spots on their own. Hence, it removes the necessities of providing parking for AVs in the proximity of destination, and offers new parking options to them. AVs would still have the option to drive to a nearby parking spot and park there until they are summoned by their users. In addition, AVs may drive all the way back to the users' home or any farther but cheaper parking spots. Furthermore, in a more extreme scenario, AVs may cruise around the destination while their users are running errands. Therefore, it is important to investigate the parking choices of these privately owned AVs and evaluate the impact of various policies on congestion and social welfare.

Parking studies and impacts of parking policies have received much more attentions after the pioneering work by Shoup (Shoup, 2005), which highlights the impact of vehicle cruising for parking on congestion. Since then numerous research has attempted to model the impact of cruising for parking, users' preference in choosing a parking spot, and the effects of different parking policies and management schemes on congestion (e.g., Arnott and Inci, 2006; Van Ommeren et al., 2011; Benenson et al., 2008; Geroliminis, 2015; Arnott et al., 2015; Chen et al., 2016). Understandably, these models consider human-driven vehicles (for recent reviews, see, e.g., Brooke et al., 2014, and Inci, 2015).

Recently, researchers have started to study the impact of AVs on parking. Liu (2018) and Zhang et al. (2019) presented equilibrium models of the choice of parking locations for commuter trips by AVs over a linear city. Zakharenko (2016a) also investigated the parking location of commuting trips in a mono-centric two-dimensional city of a half-circular shape. He considered three options of parking, including parking near the workplace, parking at a farther special parking zone, and parking at residential lots. The analysis showed that 97% of commuter AVs will be parked in the dedicated parking zones, and the optimal location of such zones is just outside of the commuter work zone. Zakharenko (2016a) suggested that a congestion toll for zero-occupant AVs make outskirt parking more expensive than central ones. This leads to a competition between AVs and regular vehicles over central parking spots, thereby decreasing social welfare. More recently, Su and Wang (2020) investigated the parking location choice of commuting AV trips. They considered three parking locations in their model: parking in the downtown area, parking at home, and parking outside downtown area, and showed that congestion can be minimized without a toll with proper parking pricing and supply. All these prior models only consider commuter trips with long dwell times, which enables them to ignore the option of cruising as a substitution for parking. However, if the network becomes congested and the speed drops considerably, cruising can be a viable option even for the long dwell times of commuting trips. By means of an agent-based simulation model, Bahrami (2019) investigated the parking choices of AVs including the cruising option, and tested different policies for a real case study of the City of Toronto. He concluded that zero-occupant toll can make a balance between the parking cost and distance and decrease the congestion. Millard-Ball (2019) also considered cruising as a substitution for parking and investigated its impact on downtown San Francisco. He suggested that AVs would collaborate with each other and choose the most congested cruising path to make a gridlock in order to decrease their parking cost.

In this paper, we model the parking choices of privately-owned AVs in a downtown area and investigate how these choices impact congestion and social welfare. We also explore the effectiveness of different policies on reducing congestion and increasing social welfare. More specifically, we consider a downtown area where users of private AVs come to engage in activities of varying duration; AVs then choose different parking options depending on their users' activity time, with the objective of minimizing their total parking cost, which leads to a Wardrop equilibrium. Under the equilibrium conditions, no AV users can decrease their parking cost by unilaterally changing their choices. With the parking equilibrium model, we then investigate policies such as congestion pricing, parking pricing and provision on internalizing the externalities of parking to increase social welfare. The primary contribution of this work is a parking choice equilibrium model. Developed in a stylized setting, the model provides a simple yet powerful framework for representing all parking considerations or preferences of AVs, capturing their interactions, and revealing various consequences of AV parking choices. It provides a foundation for investigating and optimizing parking policies in the era of AVs. The framework is flexible and can be extended to model more complicated scenarios such as ingress/egress congestion, elastic parking demand, heterogeneous AV fleet, and multiple areas or regions.

The remainder of this paper is organized as follows. Section 2 presents an equilibrium model for AV parking choices. Section 3 presents the proposed solution algorithm. Section 4 investigates the impact of different policies on congestion and proposes optimal ones in terms of social welfare. Section 5 presents numerical examples, and Section 6 concludes the paper.

2 An equilibrium model for AV parking choices

2.1 Model setting

We consider a downtown area where a continuum of users of AVs come to participate in activities whose duration vary from user to user. We assume that users activity time t is continuously distributed between $[0, T]$. Each AV user makes a parking decision upon arrival and chooses to send their AV back home (presented with "h"), travel to and park in an outskirt parking (presented with "o"), search and park in the downtown area (presented with "p"), or cruise within the area as a substitution for parking (presented with "c") as shown in Figure 1. We will then model the interactions and outcome of their parking choices. Below we first present the cost of each parking option. It is assumed that the average running speed in the downtown area is related to the traffic accumulation and that AVs travel with a free-flow speed outside this downtown area; they also experience no delay during the ingress or egress of the area.

We start with the cost of sending the AVs to park and wait at home. We assume that the average round-trip distance to home from the downtown area is l_h. If a user whose activity time is equal to t decides to send the AV back home to park, two situations might happen: if the user activity time is long enough for the AV to complete round-trip travel between the destination and home, the cost of parking at home is equal to $\rho_d l_h$, where ρ_d is the driving cost per unit of distance. Unlike human-driven vehicles, users are not present in the AVs while they are searching

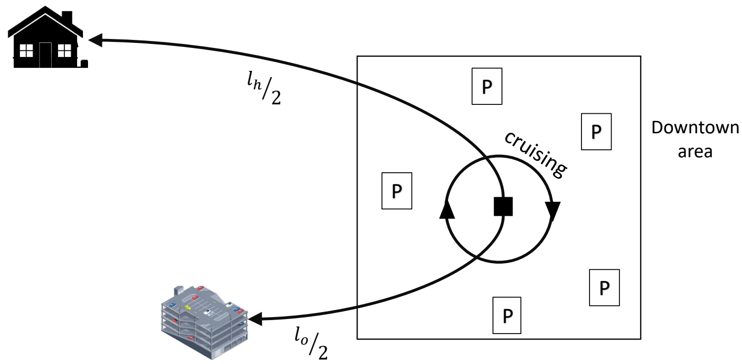


Figure 1: Schematic of parking choices of AV users.

for parking. Therefore, the cost of driving is vehicle operating cost, which is typically measured in distance in the literature (e.g., Berthelot et al., 1996). Otherwise, if the activity time of the user is not long enough, the AV must return in the middle of its route to pick up the user. Note that it is possible that AVs will find the option of home parking cheaper, e.g., in the presence of congestion pricing in downtown, even if the activity time is shorter than the round-trip time. Hence, the cost of sending the AV back home would be equal to $\rho_d v_f t$, where v_f is the free-flow speed, and $v_f t$ gives the total distance traveled by the AV during user activity time t. Therefore, we can write the cost of sending AVs back home to park as

$$\gamma_h(t) = \rho_d \cdot \min(l_h, t v_f). \tag{1}$$

Another option for AV users is to send the AVs to an outskirt parking facility that is located $l_o/2$ from the downtown area and charges τ_o per unit of time. Similar to parking at home, two situations might occur in this scenario. If the user's activity time t is long enough, the AV reaches the parking facility and parks there for $t - \frac{l_o}{v_f}$, where $\frac{l_o}{v_f}$ is the round-trip travel time to parking. Hence, the total cost of using the parking facility is equal to $\rho_d l_o + \tau_o(t - \frac{l_o}{v_f})$, where the first term is the round-trip travel cost to the parking facility, and the second term is the cost charged by the parking facility. If the activity time of the user is not long enough for the AV to reach the outskirts parking, it must return in the middle of trip and the parking cost is the same as the home option and equal to $\rho_d v_f t$. Hence, we can write the cost of sending AVs to park at an outskirt parking facility as

$$\gamma_o(t) = \rho_d \cdot \min(l_o, t v_f) + \tau_o \cdot \max(t - \frac{l_o}{v_f}, 0). \tag{2}$$

The AVs also have the option to stay in the downtown area and search for a parking spot. The parking search time, denoted by t_p, depends on the parking vacancy rate and average speed in the area. Similar to previous choices, two options might happen to an AV user with activity time t who chooses to park in the area. First, if the activity time is longer than the average time needed to find a parking spot ($t \geq t_p$), the AV parks and pays the parking charge τ_p for the parking period $t - t_p$. Such a scenario also incurs a driving cost up until finding a parking spot, which is $\rho_d v(n) t_p$, where $v(n)$ is the moving speed in the downtown area, which depends on traffic accumulation in the network, n. Also, there might be a time-based congestion pricing toll applied to AVs in the downtown area, denoted by τ_c, to disincentivize the zero-occupant movements of AVs. Hence, the AVs must also pay a toll equal to $\tau_c t_p$. If the activity time is shorter than the average search time to find a parking spot, the AV is summoned before finding a parking spot. In other words, the AV searches for a vacant parking spot for the whole activity time of its user. In this case, the parking cost is equal to the summation of the driving cost and tolling cost, which is $\rho_d v(n)t + \tau_c t$. Hence, the cost of parking in the downtown area can be summarized as

$$\gamma_p(t) = \rho_d \cdot v(n) \cdot \min(t, t_p) + \tau_c \cdot \min(t, t_p) + \tau_p \cdot \max(t - t_p, 0). \tag{3}$$

The parking search time t_p will be the parking search distance divided by the cruising speed. The former is assumed to be a function of parking occupancy that is the ratio of the number of parked vehicles divided by the parking supply. The function can take various forms, depending on, e.g., the availability of parking information, the possibility of parking reservation or the presence of a service that matches searching vehicles with available parking.

Finally, AVs also have the option to stay in the downtown area and cruise for the entire activity time of their users. The cost of this option is

$$\gamma_c(t) = \rho_d \cdot t \cdot v(n) + \tau_c \cdot t, \tag{4}$$

where the first term is the driving cost and the second term is the toll cost.

2.2 Equilibrium conditions and formulation

We are now ready to model the parking choice equilibrium problem. It is assumed that users are rational and would compete against each other to minimize their parking costs, which leads to the equilibrium conditions, where no user can further decrease their parking cost by unilaterally changing the parking choice. Given the number of users or AVs is infinite, the equilibrium is a Wardrop equilibrium or a mean-field equilibrium. Let's assume that the total mass of AV users is q and the activity time $t \in [0, T]$ follows a continuous probability density function f. Let k_j be the jam density, and A_u be the total utilizable road area. Also, P is the total number of parking spots in the downtown area. Below we present the equilibrium conditions conceptualized above:

$$x_r(t) \cdot (\gamma_r(t) - \mu(t)) = 0 \qquad \forall t, \forall r \in \{h, o, p, c\} \tag{5a}$$

$$\gamma_r(t) - \mu(t) \geq 0 \qquad \forall t, \forall r \in \{h, o, p, c\} \tag{5b}$$

$$x_r(t) \geq 0 \qquad \forall t, \forall r \in \{h, o, p, c\} \tag{5c}$$

$$\sum_r x_r(t) = q \cdot f(t) \qquad \forall t \tag{5d}$$

$$v(n) = V\left(\frac{n}{k_j A_u}\right) \tag{5e}$$

$$n = \int_0^T x_c(t) \cdot t \cdot dt + \int_0^T x_p(t) \cdot t_p \cdot dt + q_b \cdot \frac{l_b}{v(n)} \tag{5f}$$

$$q_b = Q_b(\frac{l_b}{v(n)}, \tau_c) \tag{5g}$$

$$t_p = \frac{L_p\left(\frac{\int_0^T x_p(t)\cdot\max(t-t_p,0)dt}{P}\right)}{v(n)} \tag{5h}$$

where $\mu(t)$ is the minimum parking cost incurred by each user whose activity time is t. γ_r is the cost of parking option $r \in \{h, o, p, c\}$, and $x_r(t)$ is the AV flow density with activity time t choosing option r for parking. Also, q_b is the background traffic, and l_b is the average travel distance of background traffic, which is assumed to be given.

Constraints (5a)-(5c) indicate that all parking options used by a group have the same cost, which is equal to or smaller than the cost of all unused parking options for that group. Constraints (5d) assure that the total AV flow density in each group is equal to the sum of flow density choosing different parking options. Equation (5e) states that the speed in the downtown area is a function of network accumulation, the jam density, and the utilizable road area, as described by a network macroscopic fundamental diagram (e.g., Geroliminis and Daganzo, 2008). Equation (5f) states the traffic accumulation in the downtown area, including those who are cruising, searching for parking spots, and from the background traffic. Each cruising AV remains in traffic for the whole activity time of its user, while the AVs that park in the downtown area are only present for the parking search time in traffic. Equation (5g) captures the background traffic, which is given by a demand function of their travel time in the downtown area and the toll. Equation (5h) states the parking search time as a function of parking search distance divided by cruising speed. The former is a function of parking occupancy that is the ratio of the number of parked vehicles divided by the parking supply.

Define $\Lambda = \{Y | x_r(t) \geq 0, \sum_r x_r(t) = q \cdot f(t), \forall t, r; 0 \leq n \leq k_j A_u, 0 \leq q_b \leq Q_b^0, 0 \leq t_p \leq T\}$, where Q_b^0 is the potential background demand. The above conditions yield an equivalent fixed-point problem of finding an $Y \in \Lambda$ such that

$$F(Y) = Y, \tag{6}$$

where

$$Y = \begin{pmatrix} \mathbf{x} \\ n \\ q_b \\ t_p \end{pmatrix} \tag{7}$$

$$F(Y) = \begin{pmatrix} H_{\gamma_r}(x_r(t), n) \\ \int_0^T x_c(t)\cdot t \cdot dt + \int_0^T x_p(t)\cdot t_p \cdot dt + q_b \cdot \frac{l_b}{v(n)} \\ Q_b(\frac{l_b}{v(n)}, \tau_c) \\ \frac{L_p\left(\frac{\int_0^T x_p(t)\cdot\max(t-t_p,0)\cdot dt}{P}\right)}{v(n)} \end{pmatrix}, \tag{8}$$

in which $H_{\gamma_r}(x_r(t), n) = \max\left(0, x_r(t) - (\gamma_r(t) - \mu(t))\right)$.

Since the users activity time is a continuous random variable, the fixed-point problem (6) is of infinite dimension, similar to traffic assignment models with continuous value of time previously investigated in the literature (Marcotte and Zhu, 2009; Zhu et al., 2015). Note that Λ is a compact convex set in a Banach space and F is continuous if the parking search distance function L_p is continuous. Based on the Schauder fixed-point theorem, which generalizes the Brouwer fixed-point theorem (Franklin, 2002), there exists a solution to the fixed-point problem (6). However, there may be multiple solutions as F is not necessarily monotone, e.g., a fall in speed can decrease cost of cruising and induce an increase in number of users choosing to cruise. Moreover, solutions may arise either in the non-congested or congested regime. It is thus critical to find these equilibra for the purpose of proposing parking policies, as the congestion and parking externalities are much different under these solutions. We further note that these equilibrium solutions may not necessarily be stable. An equilibrium solution is considered stable if the system returns to it after a small disruption.

Below we present a special case for the sake of demonstrating the properties of the equilibrium solutions and motivating a solution algorithm to find all equilibrium solutions of the general case.

2.3 A special case

We now assume that there are no parking spots in the downtown area and AVs must choose between cruising, outskirts parking, or returning home. We also assume that the background traffic accumulation, n_b, in the downtown area is fixed. In this special case, finding a solution to the nonlinear complementarity system (5) is equivalent to solving the following mathematical program:

$$\min \mathcal{Z} = \int_0^T \left(\gamma_h(t) \cdot x_h(t) + \gamma_o(t) \cdot x_o(t) + \tau_c \cdot t \cdot x_c(t) + \int_0^{\int_0^T x_c(t) \cdot t \cdot dt} \rho_d \cdot v(\omega) \cdot d\omega \right) \cdot dt \tag{9a}$$

$$\text{s.t. } x_h(t) + x_o(t) + x_c(t) = q \cdot f(t) \qquad \forall t \tag{9b}$$

$$x_h(t),\, x_o(t),\, x_c(t) \geq 0 \qquad \forall t, \tag{9c}$$

$$\text{in which: } v = V\left(\frac{\int_0^T x_c(t) \cdot t \cdot dt + n_b}{k_j A_u} \right). \tag{9d}$$

The equivalence of the above formulation can be established by writing out the optimality conditions and comparing them with the reduced system (5) for the special case. We note that the formulation (9) appears very similar to the equilibrium traffic assignment formulation (Beckmann et al., 1956). In the parking choice problem, each parking choice can be treated as a route in a parallel network with one origin-destination pair. However, a closer inspection of the objective function will find that, as the average network speed and density are negatively correlated, the objective function is concave, and thus the problem is a concave minimization problem with linear constraints. This suggests that the problem may have a number of locally optimal solutions and there exists a globally optimal solution that is an extreme point of the feasible region.

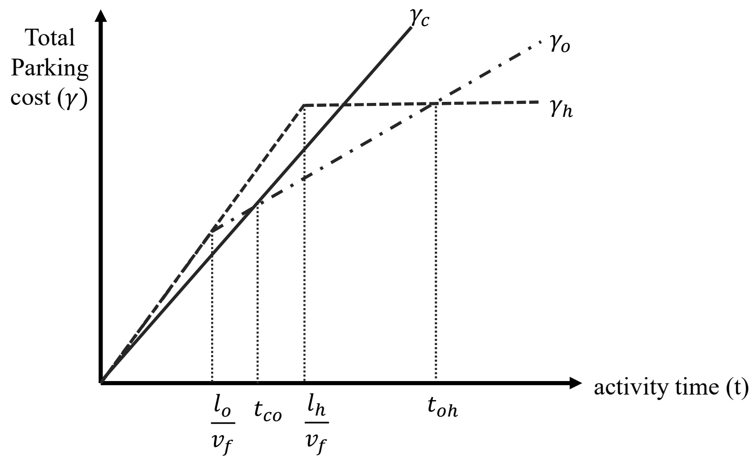


Figure 2: An example of the cost function of different parking choices with respect to the activity time of a user.

3 Equilibrium solution by finding indifference activity times

3.1 Special case

Note that the formulation (9) is NP-hard. In this section, we develop an efficient solution algorithm to solve it. We first show that each class of users can only select one parking option based on their activity time. Figure 2 shows an example of the cost function of different parking choices for different users' activity times in the special case in which we assume that we know the speed in the downtown area. We can see in this example that the cruising option has the lowest cost for any activity time up to t_{co}, and users with a shorter activity time than t_{co} choose this option based on the equilibrium conditions, consequently. From t_{co} to t_{oh}, the outskirts parking has the lowest cost; users whose activity times are in this range send their AVs to outskirts parking. Finally, the home option has the lowest cost for any activity time longer than t_{oh}. Hence, we can solve the AV parking problem (Eqs. (9a)-(9c)), if we know the boundary points in activity time (t_{co} and t_{oh}), where the option with the lowest cost changes.

To find all the boundary points of the AV parking problem (Eqs. (9a)-(9c)), we define the indifference points in the activity time of users. The indifference point, denoted by $t_{rr'}$, is activity time in which the cost of two choices r and r' are equal and users are indifferent in choosing between the two. Also, the indifference point $t_{rr'}$ indicates that the cost of option r is less than r' for shorter activity times, while the option r' has the lower cost for longer activity times. Figure 3 shows all the possible scenarios for the cost function of different parking choices in the special case. The slopes of home and the outskirts parking choices are fixed based on Equations (1) and (2). However, for the cruising option, the slope depends on the speed in the downtown area and three scenarios might occur, as shown in Figure 3. We can see that the type and location of the indifference points vary between different scenarios. For instance, t_{co} is the activity time in which the cost of cruising and outskirts parking are equal in scenario *II*, while there is no such indifference point under scenarios *I* and *III*.

We now discuss the three scenarios and find the indifference points in the activity times,

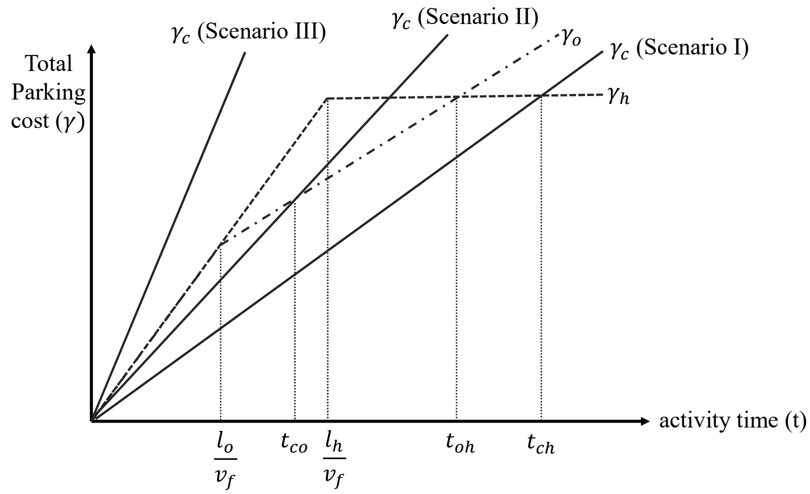


Figure 3: Possible scenarios for the cost function of different parking choices with respect to the activity time of a user.

which can give us the mass of users choosing different parking choices under the equilibrium conditions.

3.1.1 Scenario I

If we have $v \leq \frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *I* happens. In this scenario, as shown in Figure 3, cruising is the cheapest option from the minimum activity time up until t_{ch}. The cost of cruising and parking at home are equal at t_{ch}, and parking at home is cheaper for any activity time longer than t_{ch}. Under this scenario, the outskirts parking is dominated and no user would use it, consequently. The indifference point of activity time, t_{ch}, is

$$t_{ch} = \frac{\rho_d l_h}{\rho_d v + \tau_c}. \tag{10}$$

3.1.2 Scenario II

Scenario *II* happens if we have $\frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq v_f - \frac{\tau_c}{\rho_d}$ under equilibrium conditions. In this scenario, cruising is the cheapest option for the minimum activity time up to the activity time t_{co}. The cost of cruising and outskirts parking is equal at t_{co}. Then, outskirts parking is the cheapest option up to the activity time t_{oh}, in which the cost of outskirts parking is equal to the home option. Parking at home becomes the cheapest option for any activity time longer than t_{oh}. Under this scenario, all three parking options are used by the AVs and the indifference points in the activity times are

$$t_{co} = \frac{l_o(\rho_d v_f - \tau_o)}{v_f(\rho_d v + \tau_c - \tau_o)} \tag{11}$$

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{12}$$

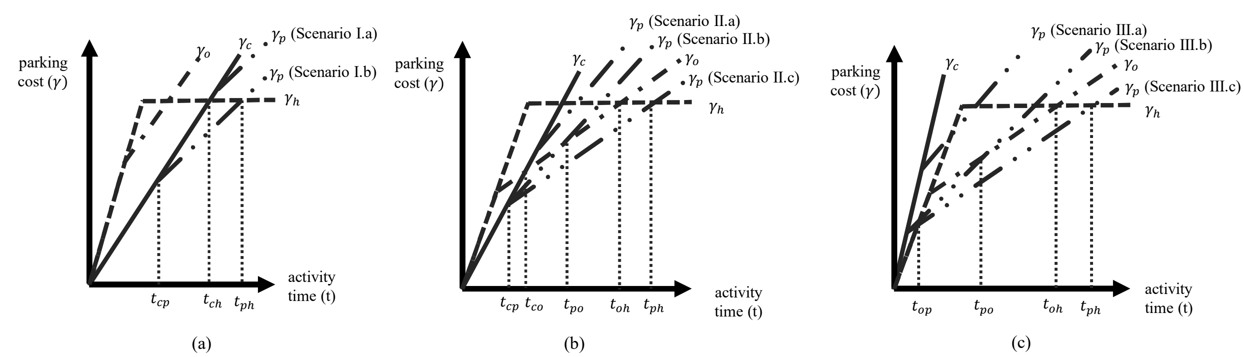


Figure 4: The cost function of different parking choices with respect to the activity time of a user.

3.1.3 Scenario III

Finally, if $v \geq v_f - \frac{\tau_c}{\rho_d}$ under equilibrium conditions, scenario *III* occurs. In this scenario, cruising is always more expensive than the other two options for any activity time duration. Outskirts parking is cheaper than park at home for minimum activity time up to t_{oh}, in which the cost of two options are equal. Then, park at home is cheaper for any activity time greater than t_{oh}. No AVs cruise instead of parking and the indifference point, t_{oh}, is

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{13}$$

In scenarios *I* and *II*, the indifference points in the activity times are a function of speed in the downtown area, which itself is a function of the number of cruising vehicles and their cruising time, yielding a fixed-point problem. There may be multiple solutions that satisfy all the conditions depending on the type of function assumed for speed and the distribution of users based on their activity times. Also, all three scenarios might be possible for the same set of input parameters, implying that any of them can arise in reality. We discuss these phenomena in more detail in Section 5 with the help of a numerical example.

3.2 General case

Although the general case is modeled as a fixed-point problem, we can still use the indifference point approach to find all the solutions. In the general case, there are eight different scenarios, as shown in Figure 4. Table 1 presents the indifference points in activity times for general case and the conditions on parking search time and travel speed in the downtown area under the equilibrium conditions for these scenarios to happen. We discuss these scenarios in further detail in Appendix A.

Similar to the special case, multiple solutions may exist in each scenario depending on the type of speed function, the parking search time function, and the distribution of users based on their activity times. Also, multiple scenarios might be feasible for the same set of input parameters and any of the equilibrium solutions might happen in reality. Figure 5 shows a flowchart with a step-by-step approach to find all of the solutions.

Scenario	Conditions	Indifference points in activity times
I.a	$v \leq \frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d}$ $t_p \geq \frac{\rho_d l_h}{\rho_d v + \tau_c}$	$t_{ch} = \frac{\rho_d l_h}{\rho_d v + \tau_c}$
I.b	$v \leq \frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d}$ $t_p \leq \frac{\rho_d l_h}{\rho_d v + \tau_c}$	$t_{cp} = t_p$ $t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}$
II.a	$\frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq v_f - \frac{\tau_c}{\rho_d}$ $t_p \geq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$	$t_{co} = \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$ $t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d (l_h - l_o)}{\tau_o}$
II.b	$\frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}\left(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f}\right) \leq v \leq v_f - \frac{\tau_c}{\rho_d}$ $t_p \leq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$	$t_{cp} = t_p$ $t_{po} = \frac{\rho_d l_o - \frac{\tau_o l_o}{v_f}}{\tau_p - \tau_o} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p - \tau_o}$ $t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d (l_h - l_o)}{\tau_o}$
II.c	$\frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}\left(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f}\right)$ $t_p \leq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$	$t_{cp} = t_p$ $t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}$
III.a	$v \geq v_f - \frac{\tau_c}{\rho_d}$ $t_p \geq \frac{l_o}{v_f}$	$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d (l_h - l_o)}{\tau_o}$
III.b	$v \geq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}\left(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f}\right)$ $t_p \leq \frac{l_o}{v_f}$	$t_{op} = \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\rho_d v_f - \tau_p}$ $t_{po} = \frac{\rho_d l_o - \frac{\tau_o l_o}{v_f}}{\tau_p - \tau_o} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p - \tau_o}$ $t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d (l_h - l_o)}{\tau_o}$
III.c	$v_f - \frac{\tau_c}{\rho_d} \leq v \leq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}\left(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f}\right)$ $t_p \leq \frac{l_o}{v_f}$	$t_{op} = \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\rho_d v_f - \tau_p}$ $t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}$

Table 1: The indifference points in activity times for general case.

4 Parking management policies

With the proposed framework of equilibrium analysis of the AVs parking decision, we are now ready to formulate an optimal policy design problem to prescribe parking management policies to improve social welfare. Below we demonstrate our formulation by considering policies including pricing of downtown and outskirt parking, downtown parking provision, and congestion pricing, which can be used by policymakers to manage parking in the age of AVs.

More specifically, a congestion toll can increase the cost of driving in the downtown area, thereby increasing the costs of the options of cruising and parking in the downtown area. Therefore, it can be used as a leverage to incentivize AV users to go outside the downtown area for parking and reduce congestion. The pricing of outskirt parking can further regulate the choices between home and parking outskirt. In contrast, the pricing and provision of downtown parking can change the cost of parking in the downtown area. However, their impact on congestion in the downtown area is not as clear as the congestion toll. For instance, a low parking provision can increase the parking search time and the cost of parking in the downtown area. This can lead more AVs to go and park outside the area or choose to cruise for a longer period. The former case can decrease the congestion in the downtown while the later one increases it. From this

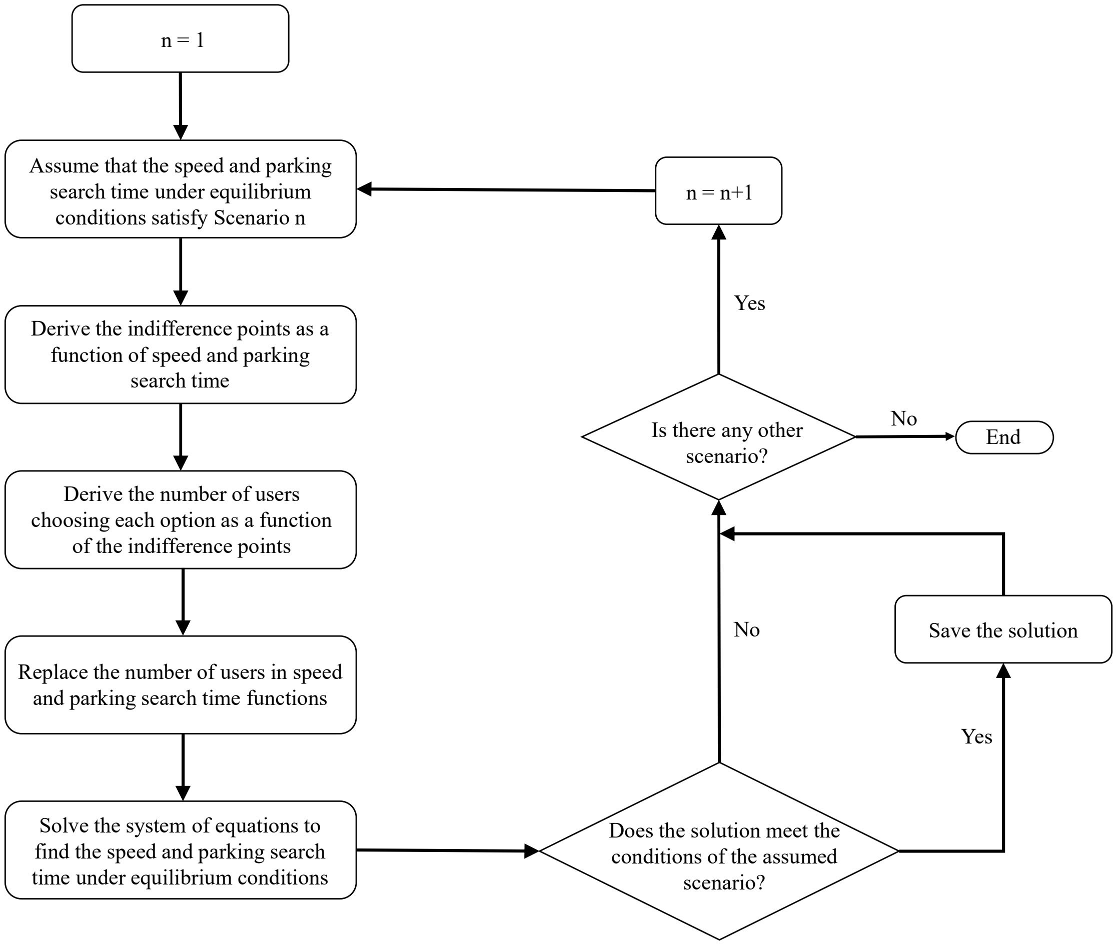


Figure 5: Flowchart of the proposed algorithm to find all equilibrium solutions.

perspective, therefore, the pricing and provision of downtown parking is a double-edged sword.

The nonuniqueness of solutions to the parking equilibrium model presents a challenge when we prescribe an optimal parking management policy, because its outcome will be uncertain. Below we develop an optimization model to determine a policy whose worst-case social welfare is maximized. In the literature, the resulting policy is considered to be robust (Lou et al., 2010; Li et al., 2020).

$$\min_{(\tau_p,\tau_o,\tau_c,P)} \max_{x,q_b} \int_0^T \Big(x_h(t)\cdot\rho_d\cdot\min(l_h, tv_f) + x_o(t)\cdot\rho_d\cdot\min(l_o, tv_f) + x_p(t)\cdot\rho_d\cdot v(n)\cdot\min(t, t_p) + x_c(t)\cdot\rho_d\cdot v(n)\cdot t\Big)dt + \alpha\left(q_b\cdot\frac{l_b}{v(n)} - \int_0^{q_b} Q_b^{-1}(\vartheta)d\vartheta\right) + \beta P \tag{14}$$

s.t. (5),

where α is the value of time of background traffic users and Q_b^{-1} is the inverse function of background traffic. Also, β is the cost of providing a parking spot in the downtown area. In the above equation, the objective is to minimize the maximum social cost (negative social welfare).

As formulated, the optimal parking policy design problem is a mathematical program with equilibrium constraints, difficult to solve in general. However, as the decision variables of the outer minimizing layer are of a low dimension and we have developed an efficient inference point approach to find all equilibrium solutions, a derivative-free method such as the Nelder–Mead method can be applied to solve the problem effectively.

We also note that when the background traffic is fixed in the special case, social welfare defined above can increase with a drop in speed in the downtown area. Such a "paradox" may not arise with the presence of elastic background traffic in the general case. A speed drop or higher congestion will increase the cost of the background traffic and discourage them from coming to the downtown area, which yields a loss of consumers' surplus.

5 Numerical studies

In this section, we assume that the speed function follows a Greenshields-type function (Greenshields et al., 1935) as

$$v = v_f \left[1 - \frac{n}{k_j A_u} \right]. \tag{15}$$

In addition, we assume that the parking search time in the downtown area can be estimated using the following function

$$t_p = \frac{-1}{\ln\left(\frac{\int_0^T x_p(t) \cdot \max(t - t_p, 0) \cdot dt}{P} + 0.01\right) v}. \tag{16}$$

The demand function of background traffic is assumed to be as follows:

$$q_b = Q_b^0 \left(\left(\frac{\kappa_b l_b}{v}\right)^{-2} - \psi_b \tau_c \right), \tag{17}$$

where κ_b and ψ_b represent the sensitivity of the background traffic demand to travel time and toll, respectively.

In the numerical examples, we consider a Manhattan-like downtown area and assume that users' activity times follow a uniform distribution between 0 and 10 hours. This function can be replaced by any other distribution. Table 2 presents the default value of the used parameters in this section.

We first consider that there is no on-street parking ($P = 0$) and the background traffic accumulation is fixed ($n_b = 15,000$) to explore the special case presented in Section 2.3. The purpose is to demonstrate the use of the proposed indifference point method and the existence of multiple solutions. First, we assume that scenario I happens and we have $v \leq \frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d}$. Then, as users' activity times follow a uniform distribution, we can drive

$$x_c = \frac{D}{10} t_{ch} = \frac{D \rho_d l_h}{10(\rho_d v + \tau_c)}. \tag{18}$$

Replacing Eq. (18) into the speed function, we have a fixed-point equation as

$$v = v_f \left[1 - \frac{\frac{D}{20}\left(\frac{\rho_d l_h}{\rho_d v + \tau_c}\right)^2 + n_b}{k_j A_u} \right]. \tag{19}$$

Table 2: Parameters and their default values in the numerical example.

Notation	Interpretation	Default value
A_u	Total utilizable area for roads [$mi \cdot lane$]	300
D	Total AV demand for parking [veh]	40,000
Q_b^0	Potential background traffic [veh]	20,000
ρ_d	Driving cost per unit of distance[$\frac{\$}{mi}$]	0.2
τ_o	Outskirt parking hourly cost [$\frac{\$}{hr}$]	1
τ_p	On-street parking hourly cost [$\frac{\$}{hr}$]	3
τ_c	Congestion price cost [$\frac{\$}{hr}$]	1
κ_b	Sensitivity of background traffic demand to travel time [$\frac{1}{hr}$]	4
ψ_b	Sensitivity of background traffic demand to congestion toll [$\frac{1}{\$}$]	0.001
k_j	Jam density [$\frac{veh}{mi \cdot lane}$]	300
l_b	Average travel distance of background traffic [mi]	5
l_h	Average round-trip distance to home [mi]	20
l_o	Average round-trip distance to off-street parking [mi]	10
P	Number of provided parking spots in the downtown area [veh]	5,000
v_f	Free-flow travel speed [mph]	30
α	Background traffic value of time [$\frac{\$}{hr}$]	10
β	Cost of providing an on-street parking spot in downtown area [$]	100

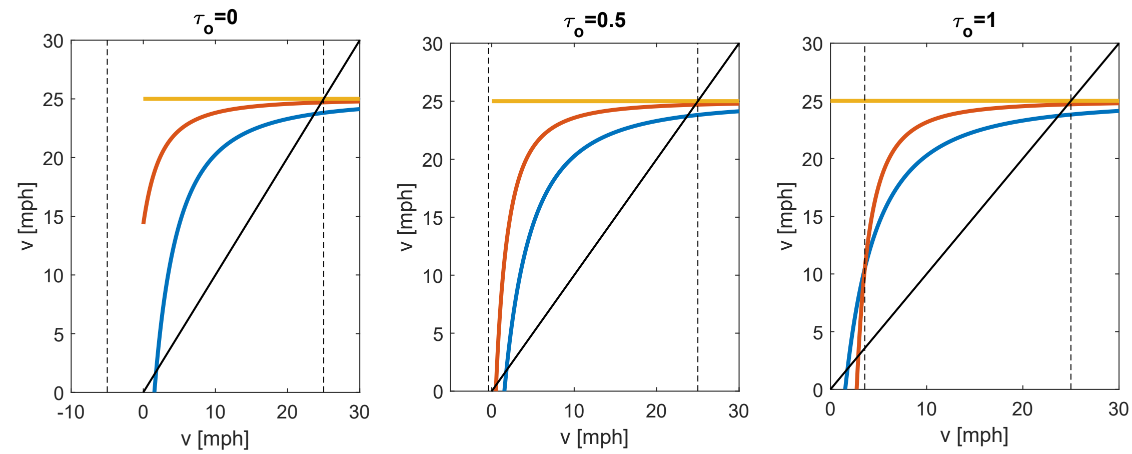
If scenario *II* happens and we have $\frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq v_f - \frac{\tau_c}{\rho_d}$, then again as users' activity times follow a uniform distribution, following similar steps, we have another fixed-point equation as

$$v = v_f \left[1 - \frac{\frac{D}{20}\left(\frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}\right)^2 + n_b}{k_j A_u} \right]. \tag{20}$$

Finally, if scenario (III) happens and we have $v \geq v_f - \frac{\tau_c}{\rho_d}$, then $x_c = 0$, and the speed function simplifies to

$$v = v_f \left[1 - \frac{n_b}{k_j A_u} \right]. \tag{21}$$

Figure 6 and Table 3 present the properties of different solutions for different outskirts parking costs. In Figure 6, the curves show the right-hand functions of Equations (19), (20), and (21) in blue, red, and yellow, respectively. The two dotted horizontal lines are the limits on speed and the curves intersection with the line $y = x$ will indicate the solutions to the fixed-point problems of (19) and (20), if the intersections lie in the assumed range of speed. More specifically, the scenario *I* solutions are only feasible if the blue line and $y = x$ intersect each other before the left-hand side of the dash line. Similarly, scenario *II* solutions are only feasible if the red line and $y = x$ intersect between the two horizontal lines. Finally, the scenario *III* solution is only feasible if it intersects with $y = x$ beyond the right-hand side of the horizontal line. As shown, the equilibrium problem does not necessarily have a unique solution. Scenario *III* always provides a feasible solution in which $v = 25$. We can also see that when $\tau_o = 0.5$ scenario *II* provides two solutions itself that are related to the congested and un-congested regimes of traffic, respectively, arising from the assumed relation between speed and density. This shows that multiple scenarios may be feasible for the same input parameters. More importantly, as shown in Table 3, the solutions have very different implications. While 30% of AVs cruise under the first equilibrium solution, which makes the downtown area congested with the running speed of $v = 0.5[mph]$, only 3% of AVs cruise under the second one, and the running speed in the downtown area and the total travel cost increases, consequently.



(a) (b) (c)

Figure 6: Speed fixed-point functions for different outskirt parking costs.

Table 3: The properties of different solutions for different outskirt parking costs.

τ_o	v	x_c	x_o	x_h	Scenario
0	24.7	1,347	38,653	0	II
	25	0	40,000	0	III
0.5	0.5	12,118	5,215	22,667	II
	24.7	1,348	15,985	22,667	II
	25	0	17,320	22,680	III
1	1.78	7,120	0	32,880	I
	24.7	1,350	7,984	30,666	II
	25	0	9,334	30,666	III

We now consider the general case. The problem has one solution for the input parameters presented in Table 2 ($\tau_c = 1[\frac{\$}{hr}]$), which happens under scenario *III.b*. Under the equilibrium conditions, the speed in the downtown area is $v = 28.93[mph]$, and the indifference points in the activity times are $t_{op} = 0.03[hr]$, $t_{po} = 0.79[hr]$, and $t_{oh} = 2.33[hr]$. No AVs cruise in the downtown area due to the high speed and cost associated with it, and consequently most of potential background traffic comes to the downtown area (18,578 [veh]). 3,039 AVs park in the downtown area, and their parking search time is 0.02 hours due to the high running speed in the area and their short parking time. Also, 6,294 AVs go to outskirts parking, from which 117 return in the middle of their route. Finally, the remaining 30,667 AV users send their vehicles home. If we remove the toll, i.e., $\tau_c = 0[\frac{\$}{hr}]$, we would see four equilibrium solutions under scenarios

I.a, *I.b*, *II.b*, and *III.b*. The speed in the downtown are is $v = 3.16[mph]$ and $v = 0.34[mph]$ under scenarios *I.a* and *I.b*, respectively, due to the high number of cruising AVs. The low running speed in the downtown area results in the background traffic avoiding the area and the background traffic demand is only 222 [veh] and 3 [veh], respectively, under scenarios *I.a* and *I.b*. In contrast, under the equilibrium conditions in scenarios *II.b* and *III.b*, the speed in the downtown area increases to $v = 28.9[mph]$ and $v = 28.93[mph]$, respectively, and we will observe few cruising AVs in the downtown area. In the literature, Millard-Ball (2019) speculated that AVs would collaborate with each other to slow down and jam downtown to decrease their parking cost. This example shows this can happen even if AVs behave in a non-cooperative manner. This motivates the need for parking policies to avoid such situations.

It is worth noting that the equilibrium solutions under scenarios *I.a* and *I.b* are not stable, while the other solutions are stable. Under scenario *I.a* equilibrium, there are 25,325 cruising AVs, and the rest 14,675 AVs go to park at home. The indifference point in activity time of users between these two choices is 6.33 [hrs] and the cost of cruising and parking at home for $t = 6.33$ is equal to 4 [$]. If we make a small perturbation to the system that decreases the number of cruising AVs to 25,000 and the maximum cruising time to 6.25 [hrs], then speed will increase to 3.85 [mph]. As a result the cost of cruising for the new indifference point ($t = 6.25$) increases to 4.8 [$], which is higher than 4 [$] cost of parking at home, and thus more AVs will shift from cruising to park at home and the system further deviates from this equilibrium solution. In this example, the worse equilibrium solutions in terms of congestion and social welfare are not stable, implying that even if the system happens to achieve such a state, it may not stay there for long. However, it is inconclusive whether this observation is generally applicable.

The previous examples highlight the need of policies and setting the prices of parking and tolls optimally. Hence, we seek for the optimal value of parking fees, congestion pricing, and parking provision by solving model (14) using the derivative-free method of Nelder-Mead. The optimum value of tolls, and parking fees are $\tau_c = 0.9[\frac{\$}{hr}]$, $\tau_p = 5[\frac{\$}{hr}]$, and $\tau_o = 0[\frac{\$}{hr}]$. Also, the optimal parking provision is 1,500 spots. We only observe one equilibrium solution under scenario *III.b* for these parameter values, in which the speed in the downtown area is $v = 28.93[mph]$ and most AVs park in outskirt parking (38,555 [veh]). Figure 7 illustrates the contours of the social welfare with respect to on-street parking fees and tolls for free outskirt parking. We can see that the total travel cost of users and social welfare do not change considerably with respect to on-street parking fees, while tolls have considerable impact on them. However, its impact would be eliminated after passing a threshold. Figure 8 similarly shows the changes in social welfare with respect to outskirts and downtown parking fees for no tolling. We can see that the parking fee in the downtown area cannot push AVs outside the area to increase the speed and let more background traffic travel through this region. Also, even though the outskirts parking fee has some impact on social welfare, it is marginal. Previously, without considering the option of cruising, Zakharenko (2016a) concluded that a toll would cause AVs to park closer to their destination, thereby decreasing social welfare by increasing parking competition in the downtown areas. Similarly, Su and Wang (2020) showed that parking pricing and parking provision policies are sufficient to reduce congestion without the help of the toll. However, our analysis shows that congestion pricing is the key policy factor in the age of AVs. Without a time-based toll, more AVs choose to cruise to slow down the speed and decrease their parking cost. Yet, the toll is not

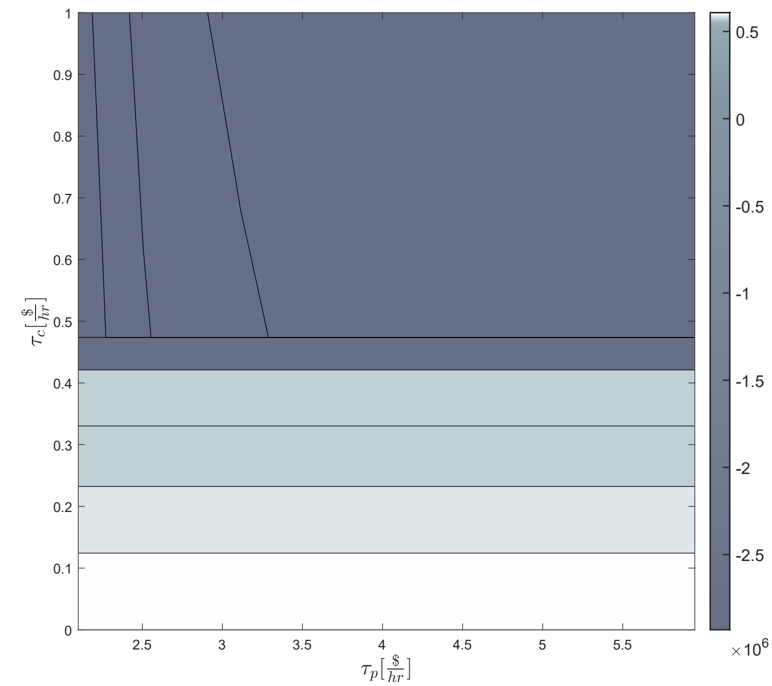


Figure 7: Social welfare changes with respect to parking fee in the downtown area (τ_p) and toll (τ_c) for free outskirts parking ($\tau_o = 0$).

sufficient and needs to be supplemented by parking pricing or parking supply.

6 Conclusion

AVs will change users' sensitivity to parking policies by means of their self-parking capability. This feature allows AV users to send their vehicles to cruise around the block or travel back home instead of parking. In this paper, the AV users' parking choice in a downtown area is investigated with the use of Wardrop equilibrium. We model the AVs parking choice problem in downtown areas as a fixed-point problem and propose an efficient method to solve it. We show that the equilibrium problem might have multiple solutions in which congestion and externalities are considerably different. We also provide a robust parking policy design formulation to recommend parking management policies to improve social welfare. The model can be used to properly set parking fees in and outside the downtown area, congestion toll, and parking provision in the downtown area. Prior studies suggested that AVs may collaborate and intentionally slow down to substitute cruising for parking, which creates severe congestion. Our work shows that it can happen, even if AVs do not collaborate with each other and act in a non-cooperative manner. Our analysis also highlights that time-based congestion pricing is the key factor to hinder AVs from cruising that exacerbates congestion. Without such a toll, the number of AVs that cruise in downtown areas increases substantially to decrease the running speed and their parking cost dramatically. The resulting congestion from cruising AVs discourages the background traffic from coming to the area due to high travel time, which decreases social welfare.

We also note that the other types of congestion toll schemes, such as cordon-based or distance-

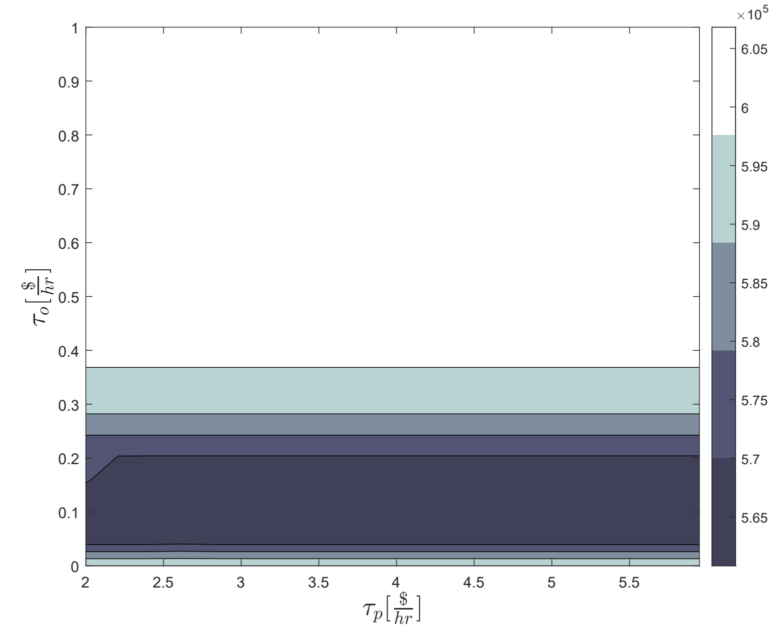


Figure 8: Social welfare changes with respect to parking fees in the downtown area (τ_p) and outskirts (τ_o) for not toll ($\tau_c = 0$).

based toll, are not effective in reducing congestion caused by AVs cruising instead of parking. The distance-based toll can be interpreted as a higher travel cost in the downtown area. However, as AVs' travel distance decrease with more congestion, such type of toll cannot discourage AVs from cruising and choosing other parking options, and even their parking cost would decrease further with an increase in the number of cruising AVs. Also, if there is a cordon-based toll, all AVs have to pay it to egress to the downtown area regardless of what option they choose afterward for their parking. Therefore, it also cannot push cruising AVs to choose other parking options available to them.

In this paper, we have developed a static framework to investigate the parking choice of privately-owned AVs. In future research, it would be valuable to develop a dynamic framework in which the cost of different options evolves over time. In such a dynamic setting, the departure-time decision of users can be also considered to investigate the impact of parking choice on ingress/egress time of those who leave later. In addition, shared AVs are not considered in this paper. Shared AVs may yield a significant number of empty miles between two occupied trips, which can be partially addressed by providing dedicated layover or parking spaces to shared AVs (Xu et al., 2017). An interesting future study would be to model the parking competition between shared and privately-owned AVs. Also, as AVs can sense their surroundings when traveling, they can detect whether on-street parking spots are empty or full, and share parking availability information with the rest of traffic. It would be interesting to study the design and implications of such a crowd-sourced parking information system.

Acknowledgement

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Appendix A

We now discuss the different scenarios of equilibrium in general case in more detail.

A.0.1 Scenarios I.a and I.b

Scenarios *I.a* and *I.b* take place when the speed in the downtown area is low enough that the outskirts parking is exceeded by either cruising or parking in the downtown area options. Figure 4a shows the cost functions for these scenarios. If we have $t_p \geq \frac{\rho_d l_h}{\rho_d v+\tau_c}$ and $v \leq \frac{v_f l_h \tau_o}{l_o \tau_o+\rho_d v_f (l_h-l_o)} - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *I.a* happens. This scenario is similar to scenario *I.a* in the special case. In this case, cruising is chosen by AV users whose activity times are up to t_{ch}, while others with longer activity times send their AVs home. In this scenario, outskirts parking and parking in the downtown area are not used by any user. The indifference point of activity time, t_{ch}, is

$$t_{ch} = \frac{\rho_d l_h}{\rho_d v + \tau_c}. \tag{22}$$

If we have $t_p \leq \frac{\rho_d l_h}{\rho_d v+\tau_c}$ and $v \leq \frac{v_f l_h \tau_o}{l_o \tau_o+\rho_d v_f (l_h-l_o)} - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *I.b* happens. In this scenario, as shown in Figure 4a the cruising is the cheapest option from the minimum activity time up until t_{cp}. The cost of cruising and parking are equal at t_{cp}. Then parking becomes the most viable option up until t_{ph}, where its cost is equal to the home option. Parking at home is cheaper for any activity time longer than t_{ph}. In this scenario, outskirts parking is not chosen by any users again. The indifference points in activity times are

$$t_{cp} = t_p \tag{23}$$

$$t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}. \tag{24}$$

A.0.2 Scenarios II.a to II.c

Scenarios *II.a* to *II.c* happen when the speed in the downtown area is neither very low nor very high. Figure 4b shows the cost functions for these scenarios. If we have $t_p \geq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v+\tau_c-\tau_o\right)}$ and $\frac{v_f l_h \tau_o}{l_o \tau_o+\rho_d v_f (l_h-l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq v_f - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *II.a* occurs. In this scenario, the parking in the downtown area is always dominated by other options, and there are two indifference points. One such point is between cruising and outskirts parking, t_{co}, and the other is between outskirts parking and home, t_{oh}, which are

$$t_{co} = \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)} \tag{25}$$

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{26}$$

If we have $t_p \leq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$ and $\frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f}) \leq v \leq v_f - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *II.b* happens. In this scenario, cruising, parking, outskirt parking, and home are the most viable options, respectively. Thus, all options are used by AVs and the indifference points in activity times are

$$t_{cp} = t_p \tag{27}$$

$$t_{po} = \frac{\rho_d l_o - \frac{\tau_o l_o}{v_f}}{\tau_p - \tau_o} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p - \tau_o} \tag{28}$$

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{29}$$

If we have $t_p \leq \frac{l_o(\rho_d v_f - \tau_o)}{v_f\left(\rho_d v + \tau_c - \tau_o\right)}$ and $\frac{v_f l_h \tau_o}{l_o \tau_o + \rho_d v_f (l_h - l_o)} - \frac{\tau_c}{\rho_d} \leq v \leq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f})$ under the equilibrium conditions, then scenario *II.c* happens. In this scenario, as shown in Figure 4b cruising is the cheapest option from the minimum activity time up until t_{cp}. The cost of cruising and parking are equal at t_{cp}. Then parking becomes the most viable option up to t_{ph} where its cost is equal to the home option. Parking at home is cheaper for any activity time longer than t_{ph}. In this scenario, outskirts parking is not chosen by any users, and the indifference points in activity times are

$$t_{cp} = t_p \tag{30}$$

$$t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}. \tag{31}$$

A.0.3 Scenarios III.a to III.c

Scenarios *III.a* to *III.c* occur when the speed in the downtown area is such that the cruising option is always dominated by other options. Figure 4c shows the cost functions for these scenarios. If we have $t_p \geq \frac{l_o}{v_f}$ and $v \geq v_f - \frac{\tau_c}{\rho_d}$ under the equilibrium conditions, then scenario *III.a* happens. In this scenario, both options of parking in the downtown area and cruising are always dominated by other options. Thus, there is only one indifference point between outskirts parking and home, t_{oh}, which is

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{32}$$

If we have $t_p \leq \frac{l_o}{v_f}$ and $v \geq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f})$ under the equilibrium conditions, then scenario *III.b* happens. In this scenario, the outskirts parking option is the cheapest one from the minimum activity time up until t_{op}, where its cost is equal to parking in the downtown area. The outskirt parking option becomes the cheapest option again at activity time t_{po}. The parking in the downtown area is the cheapest for any activity time between t_{op} and t_{po}. Actually, the AVs with activity time less than t_{op} would not reach outskirts parking and would return mid-route, while those with activity time longer than t_{po} would arrive at outskirts parking and park there. Finally, the cost of outskirts parking becomes equal to the home option at t_{oh}, with the latter being the cheapest option for any activity time longer than that. The indifference points in the activity times are

$$t_{op} = \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\rho_d v_f - \tau_p} \tag{33}$$

$$t_{po} = \frac{\rho_d l_o - \frac{\tau_o l_o}{v_f}}{\tau_p - \tau_o} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p - \tau_o} \tag{34}$$

$$t_{oh} = \frac{l_o}{v_f} + \frac{\rho_d(l_h - l_o)}{\tau_o}. \tag{35}$$

If we have $t_p \leq \frac{l_o}{v_f}$ and $v_f - \frac{\tau_c}{\rho_d} \leq v \leq \frac{\tau_p - \tau_c}{\rho_d} + \frac{l_h}{t_p} - \frac{\tau_p}{t_p}(\frac{l_h - l_o}{\tau_o} + \frac{l_o}{\rho_d v_f})$ under the equilibrium conditions, then scenario *III.c* happens. In this scenario, as shown in Figure 4c the outskirts parking option is the cheapest from the minimum activity time up until t_{op}. The cost of outskirt parking and parking in the area are equal at t_{op}. Then parking becomes the most viable option up until t_{ph}, where its cost is equal to the home option. Parking at home is cheaper for any activity time longer than t_{ph}. In this scenario, cruising is not chosen by any users. The indifference points in the activity times are

$$t_{op} = \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\rho_d v_f - \tau_p} \tag{36}$$

$$t_{ph} = \frac{\rho_d l_h}{\tau_p} - \frac{t_p(\rho_d v + \tau_c - \tau_p)}{\tau_p}. \tag{37}$$

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