The QCD transition line from lattice simulations

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The QCD transition line from lattice simulations

Claudia Ratti[a], Rene Bellwied[a], Szabolcs Borsanyi[b], Zoltan Fodor[b,c,d,e], Jana N. Guenther [b,f], Ruben Kara[b], Sandor Katz[c], Paolo Parotto[b], Attila Pasztor[c], Kalman Szabo[b,d]

[a] *Department of Physics, University of Houston, Houston, TX, USA 77204*
[b] *Department of Physics, Wuppertal University, Gaussstr. 20, D-42119 Wuppertal, Germany*
[c] *Inst. for Theoretical Physics, ELTE Eötvös Loránd University, Pázmány P. sétány 1/A, H-1117 Budapest, Hungary*
[d] *Jülich Supercomputing Centre, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[e] *Physics Department, UCSD, San Diego, CA 92093, USA*
[f] *University of Regensburg, Department of Physics, Regensburg D-93053, Germany*

E-mail: cratti@uh.edu

Abstract. We present our new results for the QCD transition line at finite chemical potential μ_B from first principle, lattice QCD simulations. We extrapolate our results from imaginary chemical potentials, up to $\mu_B \approx 300$ MeV. We obtain the most precise value for the transition temperature, the curvature of the QCD phase diagram and its fourth order correction. The results are continuum extrapolated, based on $N_t = 10,\ 12,\ 16$ lattices. We also study the height and width of the peak of the chiral susceptibility and how they change with increasing chemical potential. We find that both of them are consistent with a constant, which does not indicate any criticality in our explored

1. Introduction

Mapping out the whole phase diagram of strongly interacting matter is one of the main goals of high energy nuclear physics. From the experimental point of view, the deconfined phase of Quantum Chromodynamics (QCD) can be created in relativistic heavy-ion collisions. In particular, the LHC program aims at creating the hottest form of deconfined matter, while the RHIC program can vary the collision energy, to systematically scan the finite-density region. The second Beam Energy Scan (BESII) at RHIC is running between 2019 and 2021, its main goal being the search for the QCD critical point separating crossover from first order phase transition.

From the theoretical point of view, lattice simulations are the best tool to solve QCD in the regime of temperatures and densities where the transition occurs. However, at the moment finite-density simulations are hindered by the sign problem and cannot be directly performed. Two main methods have been proposed to circumvent this problem: Taylor expansion of thermodynamic observables in powers of μ_B [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] or simulations at imaginary μ_B (where the sign problem is absent) and analytical continuation of the results to real μ_B [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28].

In our recent manuscript [29], we considered the following expansion for the transition temperature of QCD:

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4 \dots \tag{1}$$

and we extracted the most precise values in the literature for $T_c(\mu_B = 0)$, κ_2 and κ_4 along the strangeness neutral crossover line. The latter is defined as the line in the T, μ_B plane at which the phase transition occurs, but with μ_S depending on T and μ_B so that the net-strangeness in the system is identically zero. This is imposed to match the experimental situation in which no net-strangeness is present in the system.

One can evaluate the coefficients κ_2 and κ_4 by either one of the standard extrapolation methods listed above. A direct evaluation of the μ_B-derivatives over $\mu_B = 0$ ensembles was used in Refs. [30, 31, 10]. We choose the analytical continuation from imaginary-μ_B method, which results in a larger signal-to-noise ratio [21, 22]. Results for κ_2 obtained with the two different methods were thoroughly compared in Ref. [27].

In Ref. [29], we also considered the height and width of the peak of the chiral susceptibility as functions of the chemical potential, as an indication of the strength of the crossover transition.

2. Methodology and results

We use the 4-stout staggered fermion action with temporal lattice sizes $N_t = 10,\ 12,\ 16$. We consider the following observables:

$$\begin{aligned} \langle\bar{\psi}\psi\rangle &= -\left[\langle\bar{\psi}\psi\rangle_T - \langle\bar{\psi}\psi\rangle_0\right] \frac{m_{\rm ud}}{f_\pi^4}\,, \\ \chi &= \left[\chi_T - \chi_0\right] \frac{m_{\rm ud}^2}{f_\pi^4}\,, \quad \text{with} \\ \langle\bar{\psi}\psi\rangle_{T,0} &= \frac{T}{V}\frac{\partial \log Z}{\partial m_{\rm ud}} \quad \chi_{T,0} = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_{\rm ud}^2}, \end{aligned} \tag{2}$$

where we assumed isospin symmetry, i.e. $m_u = m_d = m_{ud}$. In the above equations, the subscripts $T, 0$ indicate values at finite- and zero-temperature, respectively. In the following, $\langle\bar{\psi}\psi\rangle$ (the chiral condensate) and χ (the chiral susceptibility) are always shown after applying the correction to satisfy $n_s = 0$ with zero statistical error.

The left and middle panel of Fig. 1 show the chiral condensate and susceptibility as functions of the temperature, for different values of the imaginary chemical potential, respectively. The right panel shows that, when the chiral susceptibility is plotted as a function of the chiral condensate, all curves collapse on a common, smooth line. We exploit this feature in our analysis, which allows us a more precise determination of T_c and, as a consequence, a more precise determination of κ_2 and κ_4.

We proceed as follows:

i) We first determine $\langle\bar{\psi}\psi\rangle$ and χ as function of T and $\mathrm{Im}\mu_B$ through lattice simulations.
ii) We extract the curve of chiral susceptibility as a function of the chiral condensate.
iii) We search for the peak of $\chi(\langle\bar{\psi}\psi\rangle)$ through a low-order polynomial fit for each N_t and $\mathrm{Im}\mu_B$ obtaining $\langle\bar{\psi}\psi\rangle_c(N_t, \mathrm{Im}\mu_{\rm B})$.
iv) We interpolate $\langle\bar{\psi}\psi\rangle(T)$ to convert $\langle\bar{\psi}\psi\rangle_c$ to T_c for each $\mathrm{Im}\mu_{\rm B}/T$.
v) We perform a global fit of $T_c(N_t, \mathrm{Im}\mu_B/T_c)$ to determine its μ_B dependence. This yields κ_2 and κ_4 for $1/N_t^2 = 0$. For this step we use various functions – all containing an independent κ_6 – with coefficients depending linearly on $1/N_t^2$.

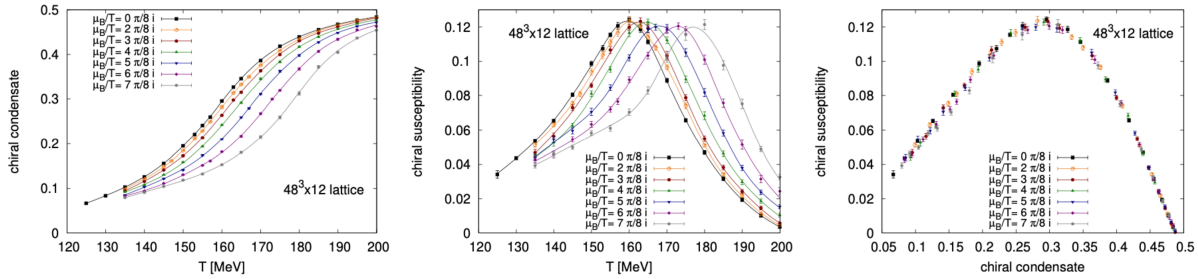

Figure 1. Renormalized chiral condensate $\langle\bar{\psi}\psi\rangle$ (left) and chiral susceptibility χ (middle) as functions of the temperature for the intermediate lattice spacing in this study. The black curves correspond to vanishing baryon density, while results for various imaginary values of the chemical potential are shown in other colors. Finally, in the right panel we show the susceptibility as a function of the condensate. In this representation the chemical potential dependence is very weak.

Each step listed above leads to a few possible choices for e.g. the fit functions, ranges etc. Combining all these choices we get $2^8 = 256$ different analyses. We take this into account in the systematic error. We obtain

$$\begin{aligned} T_c &= 158.0 \pm 0.6 \text{ MeV} \\ \kappa_2 &= 0.0153 \pm 0.0018 \\ \kappa_4 &= 0.00032 \pm 0.00067. \end{aligned} \tag{3}$$

We show the corresponding transition line in Fig. 2.

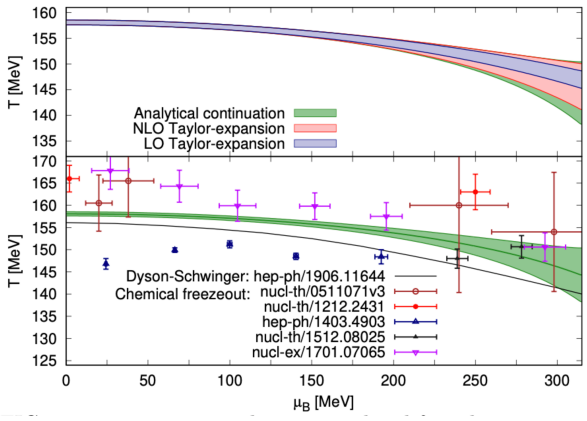

Figure 2. Top: Transition line extrapolated from lattice simulations at imaginary chemical potential using an analytical continuation with the ansätze used in step iv) of our analysis (green band) compared with an extrapolation using the formula in Eq. (1) up to the order of κ_4 (red band) or up to κ_2 (blue band). Bottom: Crossover line from the lattice compared with a prediction from truncated Dyson-Schwinger equations [32] and some estimates of the chemical freezeout parameters in heavy ion collisions [33, 34, 35, 36, 37].

We introduce a width parameter σ as a proxy for the natural definition of the width of the susceptibility peak, given by its second derivative:

$$\langle\bar{\psi}\psi\rangle(T_c \pm \sigma/2) = \langle\bar{\psi}\psi\rangle_c \pm \Delta\langle\bar{\psi}\psi\rangle/2 \ , \tag{4}$$

with $\langle\bar{\psi}\psi\rangle_c = 0.285$ and $\Delta\langle\bar{\psi}\psi\rangle = 0.14$. We show the half width of the transition in the left panel of Fig. 3. As evident from the figure, the half width is consistent with a constant for $\mu_B \approx 300$ MeV.

Finally, as a proxy for the strength of the crossover, we study the value of the chiral susceptibility at the crossover temperature, which corresponds to the height of the peak of the chiral susceptibility. We get this for each $\mathrm{Im}\mu_B$ and N_t as a byproduct of steps i-ii) of the analysis for κ_2 and κ_4. If one then performs a continuum extrapolation of the resulting values for fixed values of $\mathrm{Im}\mu_B$, one gets the right panel of Fig. 3. Again, we see a very mild $\hat{\mu}_B^2$ dependence, consistent with a constant.

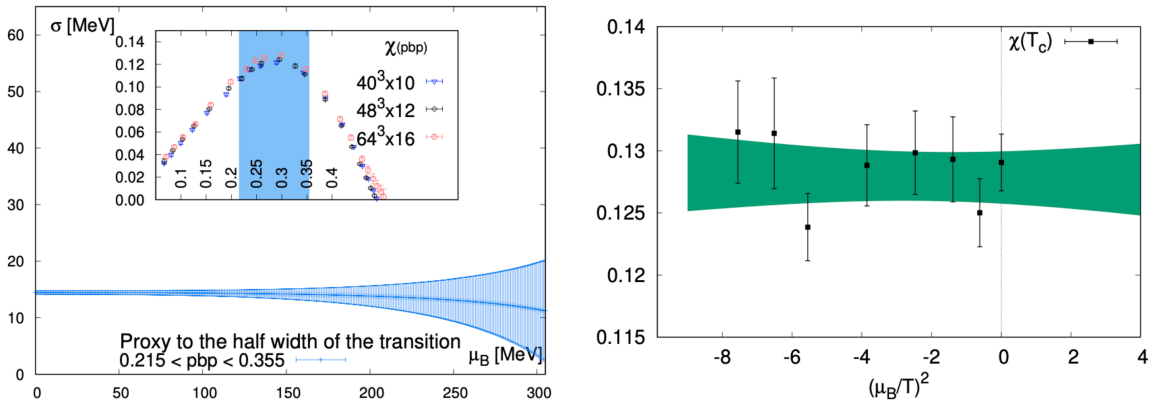

Figure 3. Left: Half width σ of the transition defined in Eq. (4) using the temperature difference of the contours $\langle\bar{\psi}\psi\rangle = 0.31$ and $\langle\bar{\psi}\psi\rangle = 0.19$. In the insert we show a plot of the $\chi(\langle\bar{\psi}\psi\rangle)$ peak, where the shaded region corresponds to $\langle\bar{\psi}\psi\rangle_c \pm \Delta\langle\bar{\psi}\psi\rangle/2$. Both are extrapolated to real μ_B. Right: Result of a μ_B-by-μ_B analysis for the value of the chiral susceptibility at the crossover temperature after continuum extrapolation and including the systematic errors for $LT_c = 4$. The green band shows a linear extrapolation in $\hat{\mu}_B^2$.

3. Conclusions

We have obtained the most precise values for the transition temperature at $\mu_B = 0$, the curvature κ_2 and the fourth-order coefficient κ_4 by studying the chiral condensate and susceptibility as functions of the temperature, for different values of imaginary chemical potential. We have also studied the height and width of the peak of the chiral susceptibility as a function of the chemical potential, and observed that they are both consistent with a constant. For this reason, we conclude that we see no sign of criticality in our explored μ_B range.

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