Adaptive Critic Learning and Experience Replay for Decentralized Event-Triggered Control of Nonlinear Interconnected Systems

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***Abstract*—In this paper, we develop a decentralized event-triggered control (ETC) strategy for a class of nonlinear systems with uncertain interconnections. To begin with, we show that the decentralized ETC policy for the whole system can be represented by a group of optimal ETC laws of auxiliary subsystems. Then, under the framework of adaptive critic learning, we construct the critic networks to solve the event-triggered Hamilton–Jacobi–Bellman equations related to these optimal ETC laws. The weight vectors used in the critic networks are updated by using the gradient descent approach and the experience replay (ER) technique together. With the aid of the ER technique, we can conquer the difficulty arising in the persistence of excitation condition. Meanwhile, by using classic Lyapunov approaches, we prove that the estimated weight vectors used in the critic networks are uniformly ultimately bounded. Moreover, we demonstrate that the obtained decentralized ETC can force the overall system to be asymptotically stable. Finally, we present an interconnected nonlinear plant to validate the proposed decentralized ETC scheme.**

***Index Terms*—Adaptive critic learning (ACL), adaptive dynamic programming (ADP), event-triggered control (ETC), experience replay (ER), interconnected systems, reinforcement learning (RL).**

I. Introduction

Adaptive critic learning (ACL), also known as adaptive critic design, has been a powerful technique to solve optimization problems [1]–[3]. The success of ACL in solving optimization problems mainly relies on an actor–critic structure. In this structure, the actor performs a control policy to systems or environments, and the critic evaluates the cost caused by that control policy and provides reward/punishment signals to the actor. A significant advantage of the actor–critic structure is that, by employing actor–critic dual neural networks (NNs), it can be utilized to avoid the well-known "curse of dimensionality." In the computational intelligence community, the actor–critic structure is a typical architecture used in adaptive dynamic programming (ADP) [4] and reinforcement learning (RL) [5]. Because ADP and RL have much in common with ACL (e.g., the same implementation structure), they are generally considered as synonyms for ACL. In this paper, we take ADP and RL as the members of ACL family. Over the past few decades, many kinds of ADP and RL have been introduced to handle optimal control problems, such as goal representative ADP [6], Hamiltonian-driven ADP [7], policy/value iteration ADP [8]–[10], robust ADP [11], [12], online RL [13]–[15], and off-policy RL [16]–[18]. Recently, based on the work of Lin [19] building a relationship between the robust control and the optimal control, ACL was successfully applied to solve the robust control problems [20]–[22]. However, when implementing these ACL algorithms, most of them required the controlled systems to be persistently exciting. Unfortunately, it is often intractable to verify the persistence of excitation (PE) condition, especially for nonlinear systems.

To conquer the difficulty arising in the PE condition, the experience replay (ER) technique was introduced [23]. The key of the ER technique is to use historical and current state data simultaneously. Owing to this feature, the ER technique is also called concurrent learning [24]. The early studies on relaxing PE conditions with the ER technique/concurrent learning included the works of Chowdhary [25] and Modares *et al.* [26]. Chowdhary [25] used concurrent learning to study the stabilization problem of nonlinear continuous-time (CT) systems. Modares *et al.* [26] employed the integral RL together with the ER technique to study optimal control problems of constrained nonlinear CT systems. The main difference between [25] and [26] was that, in [26], the optimality was taken into account. Recently, by using concurrent learning and ADP together, Vamvoudakis *et al.* [27] extended the work of [26] to design an optimal controller forcing constrained nonlinear systems to be asymptotically stable. Later, Zhao *et al.* [28] applied the ER technique combined with ADP to derive an optimal control of n-player nonzero-sum games. In all aforementioned works, the PE condition was replaced with an easy-checked rank condition. (*Note:* A similar rank condition has been provided in Remark 5.) This is an advantage of the ER technique/concurrent learning. Nevertheless, all the above-mentioned ACL algorithms were implemented without

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considering constrained resources, such as limited computation bandwidths and communication resources.

To take resource limitations into consideration, the event-triggered control (ETC) methods were proposed [29], [30]. In an event-triggering mechanism (ETM), the control policy is updated only when the deviation between the current state and the target state exceeds a prescribed threshold (*note:* in this case, the triggering condition is violated). Owing to this property, the ETC can decrease the computational load and keep a low frequency of communication between controlled systems and actuators [29]. Recently, applications of ACL to optimal ETC and robust ETC have been extensively reported. Vamvoudakis [31] proposed an optimal ETC law for nonlinear CT systems via an actor–critic structure within the framework of RL. By using a similar structure as [31], Dong *et al.* [32] presented an optimal ETC of constrained-input nonlinear CT systems. After that, Yang and He [33] developed a robust ETC scheme for constrained-input nonlinear systems through single network ACL. The key feature distinguishing [32] and [33] was whether the actor NN was taken into account. Generally speaking, if considering the action NN, it will increase the computational complexity as well as the errors caused by function approximations. Differing from Yang and He's work, Zhang *et al.* [34] derived the robust ETC via solving an event-triggered H_∞ control problem. By using ADP and concurrent learning together, they obtained the solution of the event-triggered H_∞ control problem without requiring the PE condition. However, as stated in [33], one had to judge the existence of the saddle point when solving the H_∞ control problems. This requirement is challengeable. To overcome the challenge, Zhang *et al.* [35] extended their previous work [34] to develop a robust ETC for nonlinear CT systems through the combination of an indirect method and ADP as well as the ER technique. More recently, under the framework of RL, Narayanan and Jagannathan [36] suggested a distributed approximate optimal ETC strategy for affine-input CT nonlinear interconnected systems. Owing to the use of σ-modification [37], they no longer needed the PE condition when tuning the weight vectors used in NN approximators. With a similar method utilized in [36], Narayanan and Jagannathan [38] further studied the distributed ETC problem of CT nonlinear interconnected systems. The difference between [36] and [38] was that, in [38], the state data generated from an intersampling time were reused, which aimed at getting a better performance in approximating the cost function than those regardless of these state data.

Motivated by the aforementioned literature, in this paper, we present a decentralized ETC scheme for a class of CT nonlinear interconnected systems. Initially, it is proved that the decentralized ETC policy for the whole system is composed of optimal ETC laws of auxiliary subsystems. Then, under the framework of ACL, critic networks are used to solve the event-triggered Hamilton–Jacobi–Bellman equations (HJBEs) which are corresponding to these optimal ETC laws. The weight vectors used in the critic networks are updated via the combination of the gradient descent approach and the ER technique. By using the ER technique, we can overcome the difficulty arising in the PE condition. Meanwhile, based on classic Lyapunov methods, the estimated weight vectors used in the critic networks are demonstrated to be uniformly ultimately bounded (UUB). In addition, with the obtained decentralized ETC, the overall closed-loop system can be kept asymptotically stable.

The novelties of this paper include the following three aspects.

1) Though the present method shares similar spirits as [35], a remarkable difference between this paper and [35] is that, in our case, the augmented control [see ϑ_i defined as in (7)] is tuned only in the ETM. (*Note:* In [35], one part of the augmented control was updated in a time-triggering mechanism, and the other part of the augmented control was tuned in the ETM.) Therefore, the present ETC approach has an advantage over [35] in decreasing the computational load.
2) Unlike [36] and [38] employing σ-modification to remove the PE condition, this paper applies the ER technique to convert the PE condition into an easy-checked rank condition [see (41) in the following Remark 5]. Moreover, the decentralized ETC policy developed in this paper can force the overall system to be asymptotically stable rather than locally UUB in [36] and [38] (see Theorem 1 later).
3) This paper extends our previous work [33] to obtain the decentralized ETC of uncertain nonlinear interconnected systems. It is always considered that developing ETC schemes for nonlinear interconnected systems, in particular nonlinear systems with *uncertain* interconnections, is much more difficult than those nonlinear plants regardless of interconnections.

The rest of this paper is structured as follows. Section II provides the problem descriptions and preliminaries. Section III develops the decentralized ETC of uncertain nonlinear interconnected systems. Section IV analyzes the stability of the closed-loop auxiliary systems with the obtained ETC policy. Section V presents an experiment to validate the established results. Finally, Section VI gives some concluding remarks.

Notation: $\mathbb{R}$, $\mathbb{N}$, $\mathbb{R}^{m_i}$, and $\mathbb{R}^{n_i\times m_i}$ represent the set of all real numbers, the set of all positive numbers, the Euclidean space of all real m_i-vectors, and the space of all $n_i\times m_i$ real matrices, respectively. T denotes the transposition. $\triangleq$ represents "equality relationship that is true by definition." Ω_i is a compact set of $\mathbb{R}^{n_i}$. For $\xi = (\xi_1, \xi_2, \ldots, \xi_{n_i})^\mathsf{T} \in \mathbb{R}^{n_i}$, its Euclidean norm is written as $\|\xi\| = \sqrt{\sum_{\jmath=1}^{n_i} |\xi_\jmath|^2}$. For $A \in \mathbb{R}^{n_i\times m_i}$, its Frobenius-norm is written as $\|A\| = \sqrt{\operatorname{tr}(A^\mathsf{T}A)}$ with $\operatorname{tr}(A^\mathsf{T}A)$ denoting the trace of $A^\mathsf{T}A$. $\nabla V_i^*(x_i) = \partial V_i^*(x_i)/\partial x_i$ represents the partial derivative of $V_i^*(x_i)$ with respect to $x_i \in \mathbb{R}^{n_i}$.

II. Problem Descriptions and Preliminaries

A. Problem Descriptions

Consider the nonlinear interconnected system consisting of N subsystems given in the form

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_i(t)) + g_i(x_i(t))u_i(t) + k_i(x_i(t))d_i(x(t)) \\ x_{i0} &= x_i(0), \quad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$ are the state and the control input of ith subsystem, respectively, $x = [x_1^\mathsf{T}, x_2^\mathsf{T}, \ldots, x_N^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^n$ $(n = \sum_{i=1}^N n_i)$ is the overall state, $f_i(x_i) \in \mathbb{R}^{n_i}$, $g_i(x_i) \in \mathbb{R}^{n_i \times m_i}$, and $k_i(x_i) \in \mathbb{R}^{n_i \times l_i}$ are known locally Lipschitz functions [*note:* $g_i(x_i) \neq k_i(x_i)$ if there exists $m_i = l_i$], $d_i(x) \in \mathbb{R}^{l_i}$ and $x_{i0} \in \mathbb{R}^{n_i}$ are, respectively, the *uncertain* interconnection and the initial state of ith subsystem.

Two basic assumptions are imposed to facilitate later discussion. These assumptions were used in [19], [39], and [40].

Assumption 1: The ith subsystem given as in (1) is controllable. $f_i(0) = 0$, $i = 1, 2, \ldots, N$, that is, $x_i = 0$ is the equilibrium point of the ith subsystem if letting $u_i = 0$ and $d_i(x) = 0$. Furthermore, $\text{rank}(g_i(x_i)) = m_i (m_i < n_i)$ and $g_i^\mathsf{T}(x_i)k_i(x_i) = 0$.

Remark 1: Admittedly, the condition $g_i^\mathsf{T}(x_i)k_i(x_i) = 0$ given in Assumption 1 is a strict restriction, which excludes some nonlinear interconnected systems. Nevertheless, if considering $g_i^\mathsf{T}(x_i)k_i(x_i) \neq 0$, then, we will find that more restrictive assumptions are required to be made, such as the boundedness of the Moore–Penrose pseudoinverse of $g_i(x_i)$ (see [41]). Generally speaking, it is computationally expensive to calculate the Moore–Penrose pseudoinverse of $g_i(x_i)$ when it has a large dimension. Therefore, for simplicity of discussion, we present $g_i^\mathsf{T}(x_i)k_i(x_i) = 0$ in Assumption 1. Actually, this condition can be easily checked [see analyses of later interconnected system (61) in Section V].

Assumption 2: The interconnection $d_i(x)$ is bounded as

$$\|d_i(x)\| \leq \sum_{s=1}^N b_{\text{is}} \alpha_{\text{is}}(\|x_s\|), \;\; i = 1, 2, \ldots, N \tag{2}$$

where b_{is}, $s = 1, 2, \ldots, N$, are non-negative constants and $\alpha_{\text{is}}(\cdot)$, $s = 1, 2, \ldots, N$, are the class $\mathcal{K}$ functions [42]. Moreover, $d_i(0) = 0$ and $\alpha_{\text{is}}(0) = 0$, $s = 1, 2, \ldots, N$.

Let

$$\alpha_i(\|x_i\|) = \max\{\alpha_{1i}(\|x_i\|), \alpha_{2i}(\|x_i\|), \ldots, \alpha_{Ni}(\|x_i\|)\}. \tag{3}$$

Then, we can further write (2) in the form

$$\|d_i(x)\| \leq \sum_{s=1}^N c_{\text{is}} \alpha_s(\|x_s\|), \;\; i = 1, 2, \ldots, N \tag{4}$$

where $c_{\text{is}} \geq b_{\text{is}} \alpha_{\text{is}}(\|x_s\|)/\alpha_s(\|x_s\|)$, $s = 1, 2, \ldots, N$, are the non-negative constants.

This paper aims at finding a proper state-feedback decentralized controller for interconnected system (1), which satisfies Assumptions 1 and 2, such that the overall closed-loop system is asymptotically stable. Because of the uncertain interconnections, it is challengeable to design such a decentralized controller. To overcome the challenge, we transform the decentralized stabilization problem into a group of optimal control problems of auxiliary subsystems related to interconnected system (1).

B. HJBE Related to the ith Auxiliary Subsystem

The auxiliary system related to the ith subsystem can be written in the form [*note:* in the subsequent discussion, we call the following system (5) the ith auxiliary subsystem for brevity]

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + \left(I_{n_i} - g_i(x_i)g_i^+(x_i)\right)k_i(x_i)\nu_i \tag{5}$$

with $g_i^+(x_i) \in \mathbb{R}^{m_i \times n_i}$ representing the Moore–Penrose pseudoinverse of $g_i(x_i)$ and $\nu_i \in \mathbb{R}^{l_i}$ the auxiliary control.

Because $\text{rank}(g_i(x_i)) = m_i$ (see Assumption 1) and $g_i(x_i)$ is a real matrix, we find

$$g_i^+(x_i) = \left(g_i^\mathsf{T}(x_i)g_i(x_i)\right)^{-1} g_i^\mathsf{T}(x_i).$$

Thus, we can further see from Assumption 1 that

$$g_i^+(x_i)k_i(x_i) = \left(g_i^\mathsf{T}(x_i)g_i(x_i)\right)^{-1} g_i^\mathsf{T}(x_i)k_i(x_i) = 0. \tag{6}$$

Let the augmented control $\vartheta_i \in \mathbb{R}^{m_i+l_i}$ and the augmented control matrix $G_i(x_i) \in \mathbb{R}^{n_i \times (m_i+l_i)}$ be separately defined as

$$\vartheta_i = \left[u_i^\mathsf{T}, \nu_i^\mathsf{T}\right]^\mathsf{T} \text{ and } G_i(x_i) = \left[g_i(x_i), k_i(x_i)\right]. \tag{7}$$

Then, by using (6) and (7), we can find that the ith auxiliary subsystem (5) becomes

$$\dot{x}_i = f_i(x_i) + G_i(x_i)\vartheta_i. \tag{8}$$

We introduce the cost function for the ith auxiliary subsystem (8) as follows:

$$V_i^{\vartheta_i}(x_i(t)) = \int_t^\infty (\Phi_i(x_i(\tau)) + r_i(x_i(\tau), \vartheta_i(\tau)))\mathrm{d}\tau \tag{9}$$

where $\Phi_i(x_i) = \pi_i(\alpha_i(\|x_i\|))^2$, π_i is an adjustable positive parameter, $\alpha_i(\|x_i\|)$ is defined as (3), and

$$r_i(x_i, \vartheta_i) = x_i^\mathsf{T} Q_i x_i + \vartheta_i^\mathsf{T} \mathcal{R}_i \vartheta_i \tag{10}$$

where $Q_i \in \mathbb{R}^{n_i \times n_i}$ is a symmetric positive-definite matrix and $\mathcal{R}_i \in \mathbb{R}^{(m_i+l_i)\times(m_i+l_i)}$ is the diagonal matrix defined as

$$\mathcal{R}_i = \text{diag}\left\{1_1, \ldots, 1_{m_i}, \rho_1, \ldots, \rho_{l_i}\right\}$$

with $1_q = 1 (q = 1, 2, \ldots, m_i)$ and $\rho_{\bar{q}} = \rho_i > 0 (\bar{q} = 1, 2, \ldots, l_i)$. From the expression $\mathcal{R}_i$, we can see that $\mathcal{R}_i = \mathcal{R}_i^{(1/2)}\mathcal{R}_i^{(1/2)}$.

According to [4] and [5], we can describe the optimal cost function as

$$V_i^*(x_i) = \min_{\vartheta_i \in \mathscr{B}(\Omega_i)} V_i^{\vartheta_i}(x_i) \tag{11}$$

with $\mathscr{B}(\Omega_i)$ representing the set of all admissible control laws defined on Ω_i. The time derivative of $V_i^*(x_i)$ in (11) satisfies

$$\left(\nabla V_i^*(x_i)\right)^\mathsf{T}(f_i(x_i) + G_i(x_i)\vartheta_i) + \Phi_i(x_i) + x_i^\mathsf{T} Q_i x_i + \vartheta_i^\mathsf{T} \mathcal{R}_i \vartheta_i = 0.$$

Define the Hamiltonian with respect to $V_i^*(x_i)$ and ϑ_i as

$$H\left(x_i, \nabla V_i^*(x_i), \vartheta_i\right) = \left(\nabla V_i^*(x_i)\right)^\mathsf{T}(f_i(x_i) + G_i(x_i)\vartheta_i) + \Phi_i(x_i) + x_i^\mathsf{T} Q_i x_i + \vartheta_i^\mathsf{T} \mathcal{R}_i \vartheta_i. \tag{12}$$

Then, according to [4] and [5], $V_i^*(x_i)$ can be derived via solving the HJBE

$$\min_{\vartheta_i \in \mathscr{B}(\Omega_i)} H\left(x_i, \nabla V_i^*(x_i), \vartheta_i\right) = 0 \tag{13}$$

with $V_i^*(0) = 0$. Meanwhile, the corresponding augmented optimal control can be written in the form

$$\begin{aligned}\vartheta_i^*(x_i) &= \arg\min_{\vartheta_i \in \mathscr{B}(\Omega_i)} H\big(x_i, \nabla V_i^*(x_i), \vartheta_i\big)\\ &= -\frac{1}{2}\mathcal{R}_i^{-1} G_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i). \end{aligned} \tag{14}$$

Based on the expressions ϑ_i and $G_i(x_i)$ given in (7), we can see from (14) that

$$\begin{cases} u_i^*(x_i) = -\frac{1}{2}g_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\\ \nu_i^*(x_i) = -\frac{1}{2\rho_i}k_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\end{cases} \tag{15}$$

where $u_i^*(x_i)$ and $\nu_i^*(x_i)$ are the optimum values of $u_i(x_i)$ and $\nu_i(x_i)$, respectively.

Inserting (14) into (13), we can rewrite the HJBE related to the ith auxiliary subsystem in the form

$$\begin{aligned}&\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} f_i(x_i) + x_i^{\mathsf{T}}Q_i x_i - \left\|\frac{1}{2}g_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\right\|^2\\ &- \left\|\frac{1}{2\sqrt{\rho_i}}k_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\right\|^2 + \Phi_i(x_i) = 0\end{aligned} \tag{16}$$

with $V_i^*(0) = 0$.

According to [39], the decentralized controller for interconnected system (1) can be derived through solving a group of HJBEs described as (16). Nevertheless, in [39], the decentralized controller was developed in a time-triggering mechanism. The time-triggered control laws (i.e., the periodically updated control policies) often result in low efficiency of using the communication resources between the controlled systems and the actuators [30]. More importantly, the time-triggered control policies often give rise to heavy computational loads. To overcome the above-mentioned shortcomings, we will propose a decentralized ETC scheme for interconnected system (1).

III. Decentralized ETC Scheme

First, we propose the HJBE related to the ith auxiliary subsystem in the ETM (i.e., the ith event-triggered HJBE). Then, we establish a theorem to illustrate the relationship between the decentralized ETC and the solutions of N event-triggered HJBEs. Specifically, for deriving the decentralized ETC of interconnected system (1), we need to solve N event-triggered HJBEs. To this end, we finally present ACL together with the ER technique to construct critic networks.

A. *ith Event-Triggered HJBE*

Let $\{t_j\}_{j=0}^{\infty}$ (*Note:* t_j represents the jth triggering instant) be the sequence of triggering instants, where $t_j < t_{j+1}, j \in \mathbb{N}$. At the triggering instant t_j, the sampled state of the ith auxiliary subsystem is written in the form

$$\bar{x}_{i,j} = x_i\big(t_j\big),\ j \in \mathbb{N}.$$

In general, there exists a gap between the sampled state $\bar{x}_{i,j}$ and the current state $x_i(t)$. To describe the gap, we introduce an error function as follows:

$$e_{i,j}(t) = \bar{x}_{i,j} - x_i(t),\ t \in \big[t_j, t_{j+1}\big). \tag{17}$$

We can see from (17) when the event is triggered. To be specific, if the event is triggered at $t = t_j$, then $e_{i,j}(t_j) = 0$. Based on the sequence of sampled states $\{\bar{x}_{i,j}\}_{j=0}^{\infty}$, we can derive the sequence of ETC policies $\{\vartheta_i(\bar{x}_{i,j})\}_{j=0}^{\infty}$. [*Note:* $\vartheta_i(\bar{x}_{i,j})$ is executed only at the triggering instant $t_j, j \in \mathbb{N}$.] Meanwhile, with the aid of zero-order hold [29], the discrete-time control signals in the sequence $\{\vartheta_i(\bar{x}_{i,j})\}_{j=0}^{\infty}$ will be converted into a CT input signal $\mu_i(\bar{x}_{i,j}, t)$, that is

$$\mu_i\big(\bar{x}_{i,j}, t\big) = \vartheta_i\big(\bar{x}_{i,j}\big) = \vartheta_i\big(x_i(t_j)\big),\ t \in \big[t_j, t_{j+1}\big).$$

Applying the aforementioned ETM to $\vartheta_i^*(x_i)$ in (14), we can obtain the augmented optimal ETC law for the ith auxiliary subsystem (8) and the associated cost function (9) as [for all $t \in [t_j, t_{j+1})$]

$$\mu_i^*\big(\bar{x}_{i,j}, t\big) = \vartheta_i^*\big(\bar{x}_{i,j}\big) = -\frac{1}{2}\mathcal{R}_i^{-1}G_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big) \tag{18}$$

where $\nabla V_i^*(\bar{x}_{i,j}) = (\partial V_i^*(x_i)/\partial x_i)|_{x_i=\bar{x}_{i,j}}$.

Remark 2: In subsequent discussion, we ignore the time variable t in $\mu_i^*(\bar{x}_{i,j}, t)$. That is, $\mu_i^*(\bar{x}_{i,j}, t)$ is written as $\mu_i^*(\bar{x}_{i,j})$ for brevity.

Replacing ϑ_i in (13) with $\mu_i^*(\bar{x}_{i,j})$ in (18), we derive the ith event-triggered HJBE as follows [for all $t \in [t_j, t_{j+1})$]:

$$\begin{aligned}&\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} f_i(x_i) - \frac{1}{2}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i)g_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big)\\ &- \frac{1}{2\rho_i}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} k_i(x_i)k_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big)\\ &+ \Phi_i(x_i) + x_i^{\mathsf{T}}Q_i x_i + \left\|\frac{1}{2}g_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big)\right\|^2\\ &+ \left\|\frac{1}{2\sqrt{\rho_i}}k_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big)\right\|^2 = 0\end{aligned} \tag{19}$$

with $V_i^*(0) = 0$.

B. *Relationship Between the Decentralized ETC and the Solutions of N Event-Triggered HJBEs*

According to the expressions ϑ_i and $G_i(x_i)$ given in (7), we can see that (18) implies [for every $t \in [t_j, t_{j+1})$]

$$u_i^*\big(\bar{x}_{i,j}\big) = -\frac{1}{2}g_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\nabla V_i^*\big(\bar{x}_{i,j}\big). \tag{20}$$

Before continuing discussion, we give the following necessary assumption which was used in [31], [33], and [34].

Assumption 3: $\vartheta_i^*(x_i)$ given in (14) satisfies the Lipschitz condition on Ω_i. Specifically, we can find a Lipschitz constant $K_{\vartheta_i^*} > 0$ such that, for all $x_i, \bar{x}_{i,j} \in \Omega_i$

$$\big\|\vartheta_i^*(x_i) - \vartheta_i^*\big(\bar{x}_{i,j}\big)\big\| \le K_{\vartheta_i^*}\big\|x_i - \bar{x}_{i,j}\big\| = K_{\vartheta_i^*}\big\|e_{i,j}\big\|.$$

Remark 3: We can see from (18) and Remark 2 that $\mu_i^*(\bar{x}_{i,j}) = \vartheta_i^*(\bar{x}_{i,j})$. Thus, from Assumption 3, we have

$$\big\|\vartheta_i^*(x_i) - \mu_i^*\big(\bar{x}_{i,j}\big)\big\| \le K_{\vartheta_i^*}\big\|e_{i,j}\big\|.$$

Moreover, by using (7) and the definition of the norm $\|\cdot\|$, we find that Assumption 3 also implies

$$\big\|u_i^*(x_i) - u_i^*\big(\bar{x}_{i,j}\big)\big\| \le K_{\vartheta_i^*}\big\|e_{i,j}\big\|. \tag{21}$$

Theorem 1: Take N auxiliary subsystems presented as in (8) and the corresponding cost functions described as in (9) into account. Let Assumptions 1–3 be valid. Then, there exist N positive constants π_i^*, $i = 1, 2, \ldots, N$, such that, for every $\pi_i \geq \pi_i^*$, the optimal ETC laws $u_i^*(\bar{x}_{i,j})$, $i = 1, 2, \ldots, N$, given as in (20) can force the interconnected system (1) to be asymptotically stable as long as $\nu_i^*(x_i)$, $i = 1, 2, \ldots, N$, satisfy

$$\left\|\nu_i^*(x_i(t))\right\|^2 \leq x_i^{\mathsf{T}}(t) Q_i x_i(t), \quad t \geq t_0 \tag{22}$$

where $t_0 \geq 0$ is the threshold, and the triggering condition is

$$\left\|e_{i,j}\right\|^2 \leq \frac{(1-2\rho_i)\lambda_{\min}(Q_i)}{2K_{\vartheta_i^*}^2}\|x_i\|^2 \triangleq \left\|e_{i,T}\right\|^2 \tag{23}$$

with $0 < \rho_i < 1/2$ being the adjustable parameter, $\lambda_{\min}(Q_i)$ being the minimum eigenvalue of Q_i, and $e_{i,T}$ being the triggering threshold.

Proof: Choose the Lyapunov function candidate as

$$\mathcal{L}(x) = \sum_{i=1}^{N} V_i^*(x_i) \tag{24}$$

with $V_i^*(x_i)$, $i = 1, 2, \ldots, N$, the optimal cost functions defined as (11). We can see from (11) that $V_i^*(x_i) > 0$ for every $x_i \neq 0$, and $V_i^*(x_i) = 0 \Leftrightarrow x_i = 0$, $i = 1, 2, \ldots, N$. That is, $V_i^*(x_i)$, $i = 1, 2, \ldots, N$, are the positive-definite functions [42]. We thus obtain that $\mathcal{L}(x)$ is positive definite.

Taking the time derivative of $\mathcal{L}(x)$ in (24) and using the solutions of N equations $\dot{x}_i = f_i(x_i) + g_i(x_i)u_i^*(\bar{x}_{i,j}) + k_i(x_i)d_i(x)$, $i = 1, 2, \ldots, N$, we have [*Note:* $\dot{\mathcal{L}}(x) = \mathrm{d}\mathcal{L}(x(t))/\mathrm{d}t$]

$$\begin{aligned}\dot{\mathcal{L}}(x) = \sum_{i=1}^{N}\Big\{&\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i) + g_i(x_i)u_i^*(\bar{x}_{i,j})\big) \\ &+ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} k_i(x_i) d_i(x)\Big\}.\end{aligned} \tag{25}$$

According to (12)–(15), it follows:

$$\begin{cases}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} f_i(x_i) = -\pi_i(\alpha_i(\|x_i\|))^2 - x_i^{\mathsf{T}} Q_i x_i \\ \qquad\qquad + \left\|u_i^*(x_i)\right\|^2 + \rho_i\left\|\nu_i^*(x_i)\right\|^2 \\ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i) = -2\big(u_i^*(x_i)\big)^{\mathsf{T}} \\ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} k_i(x_i) = -2\rho_i\big(\nu_i^*(x_i)\big)^{\mathsf{T}}.\end{cases} \tag{26}$$

Inserting (26) into (25), we have

$$\begin{aligned}\dot{\mathcal{L}}(x) = \sum_{i=1}^{N}\Bigg\{&-\pi_i(\alpha_i(\|x_i\|))^2 - x_i^{\mathsf{T}} Q_i x_i \\ &+ \underbrace{\left\|u_i^*(x_i)\right\|^2 - 2\big(u_i^*(x_i)\big)^{\mathsf{T}} u_i^*(\bar{x}_{i,j})}_{\beta_1(x_i,\bar{x}_{i,j})} \\ &+ \rho_i\left\|\nu_i^*(x_i)\right\|^2 \underbrace{-2\rho_i\big(\nu_i^*(x_i)\big)^{\mathsf{T}} d_i(x)}_{\beta_2(x_i,x)}\Bigg\}.\end{aligned} \tag{27}$$

Completing the squares with respect to $u_i^*(x_i) - u_i^*(\bar{x}_{i,j})$ and using (21), we find that $\beta_1(x_i, \bar{x}_{i,j})$ in (27) satisfies

$$\begin{aligned}\beta_1(x_i, \bar{x}_{i,j}) &= \left\|u_i^*(x_i) - u_i^*(\bar{x}_{i,j})\right\|^2 - \left\|u_i^*(\bar{x}_{i,j})\right\|^2 \\ &\leq K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2 - \left\|u_i^*(\bar{x}_{i,j})\right\|^2 \leq K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2.\end{aligned}$$

Applying Cauchy–Schwarz inequality [43, Th. 1.35] to $\beta_2(x_i, x)$ in (27) and using (4), we have

$$\begin{aligned}\beta_2(x_i, x) &\leq 2\rho_i\left\|\nu_i^*(x_i)\right\|\|d_i(x)\| \\ &\leq 2\rho_i\left\|\nu_i^*(x_i)\right\|\sum_{s=1}^{N} c_{is}\alpha_s(\|x_s\|).\end{aligned}$$

Thus, we can see from (27) that

$$\begin{aligned}\dot{\mathcal{L}}(x) \leq &-\sum_{i=1}^{N} 2\rho_i\Big(x_i^{\mathsf{T}} Q_i x_i - \left\|\nu_i^*(x_i)\right\|^2\Big) \\ &-\sum_{i=1}^{N}\Big\{(1-2\rho_i)x_i^{\mathsf{T}} Q_i x_i - K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2\Big\} \\ &-\sum_{i=1}^{N}\rho_i(1-\rho_i)\left\|\nu_i^*(x_i)\right\|^2 \\ &-\sum_{i=1}^{N}\Bigg\{\pi_i(\alpha_i(\|x_i\|))^2 + \rho_i^2\left\|\nu_i^*(x_i)\right\|^2 \\ &\qquad - 2\rho_i\left\|\nu_i^*(x_i)\right\|\sum_{s=1}^{N} c_{is}\alpha_s(\|x_s\|)\Bigg\}.\end{aligned} \tag{28}$$

For convenience, we denote

$$\begin{cases}\tilde{\boldsymbol{\pi}} = \mathrm{diag}\{\pi_1, \pi_2, \ldots, \pi_N\} \\ \tilde{\mathbf{1}} = \mathrm{diag}\{1_1, 1_2, \ldots, 1_N\} \quad (1_i = 1,\ i = 1, 2, \ldots, N) \\ z(x) = \big[-\alpha_1(\|x_1\|), -\alpha_2(\|x_2\|), \ldots, -\alpha_N(\|x_N\|) \\ \qquad \rho_1\left\|\nu_1^*(x_1)\right\|, \rho_2\left\|\nu_2^*(x_2)\right\|, \ldots, \rho_N\left\|\nu_N^*(x_N)\right\|\big]^{\mathsf{T}}.\end{cases}$$

Then, based on conditions (22) and (23) as well as the fact that $\lambda_{\min}(Q_i)\|x_i\|^2 \leq x_i^{\mathsf{T}} Q_i x_i$, we further find that (28) implies (for all $t \geq t_0$)

$$\dot{\mathcal{L}}(x) \leq -\frac{1}{2}\sum_{i=1}^{N}(1-2\rho_i)\lambda_{\min}(Q_i)\|x_i\|^2 - z^{\mathsf{T}}(x)Bz(x) \tag{29}$$

where

$$B = \begin{bmatrix}\tilde{\boldsymbol{\pi}} & C^{\mathsf{T}} \\ C & \tilde{\mathbf{1}}\end{bmatrix} \text{ and } C = \begin{bmatrix}c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & c_{N2} & \cdots & c_{NN}\end{bmatrix}. \tag{30}$$

Because π_i, $i = 1, 2, \ldots, N$, lie along the principal diagonal of the symmetric matrix B in (30), we can make B positive definite through selecting sufficiently large π_i, $i = 1, 2, \ldots, N$. Hence, there exist $\pi_i^* > 0$, $i = 1, 2, \ldots, N$, such that $\pi_i \geq \pi_i^*$, $i = 1, 2, \ldots, N$, guarantee $-z^{\mathsf{T}}(x)Bz(x) < 0$. In this situation, we thus find that (29) yields

$$\dot{\mathcal{L}}(x) \leq -\frac{1}{2}\sum_{i=1}^{N}(1-2\rho_i)\lambda_{\min}(Q_i)\|x_i\|^2, \ t \geq t_0. \tag{31}$$

Taking the integration on both sides of (31) with respect to t from t_0 to ∞ and noticing that $\mathcal{L}(x(\infty)) = 0$, we have

$$\frac{1}{2}\sum_{i=1}^{N}(1-2\rho_i)\lambda_{\min}(Q_i)\int_{t_0}^{\infty}\|x_i(t)\|^2\mathrm{d}t \le \mathcal{L}(x(t_0)).$$

Then, after performing some computations, we obtain

$$\int_{t_0}^{\infty}\|x_i(t)\|^2\mathrm{d}t \le \frac{2\mathcal{L}(x(t_0))}{(1-2\rho_i)\lambda_{\min}(Q_i)} \tag{32}$$

where $i = 1, 2, \ldots, N$. Noting that the right-hand side of (32) is finite and using the Barbalat's lemma [37], we derive

$$\lim_{t\to\infty}\|x_i(t)\| = 0,\ \ i = 1, 2, \ldots, N.$$

This proves that the interconnected system (1) is asymptotically stable with the N optimal ETC laws $u_i^*(\bar{x}_{i,j})$, $i = 1, 2, \ldots, N$. ■

Remark 4: Two notes on Theorem 1 are provided as follows.

1) Observing the expression $\nu_i^*(x_i)$ in (15), we can find that $\nu_i^*(x_i)$ has a strong connection with $\nabla V_i^*(x_i)$. Unfortunately, we often cannot obtain $\nabla V_i^*(x_i)$ analytically because of the intrinsic nonlinearity of the HJBE (16). Thus, the closed-form expression $\nu_i^*(x_i)$ is generally unavailable. In this case, we resort to checking the validity of (22) via simulations (see Section V).
2) The triggering instant t_j, $j \in \mathbb{N}$, can be derived through (23). Thus, the minimal intersampling time $(\Delta t_j)_{\min} \triangleq (t_{j+1} - t_j)_{\min}$, $j \in \mathbb{N}$, is available. If $(\Delta t_j)_{\min} = 0$, that is, the sampling system suffers from *Zeno behavior* [44], then the N optimal ETC laws $u_i^*(\bar{x}_{i,j})$, $i = 1, 2, \ldots, N$, have to be redesigned. Fortunately, $f_i(x_i)$, $g_i(x_i)$, and $k_i(x_i)$ satisfying the Lipschitz property can guarantee that $(\Delta t_j)_{\min} > 0$, $j \in \mathbb{N}$ (*note:* here we omit the proof, because similar proofs have been presented in [34] and [45]). Moreover, we can see from the simulation results provided in Section V that $(\Delta t_j)_{\min} > 0$, $j \in \mathbb{N}$.

Theorem 1 provides a promising way to find the decentralized ETC of interconnected system (1). That is, for deriving the decentralized ETC policy, we just need to obtain the N optimal ETC laws $u_i^*(\bar{x}_{i,j})$, $i = 1, 2, \ldots, N$. Because all these optimal ETC laws together constitute the decentralized ETC policy. However, as shown in (20), $u_i^*(\bar{x}_{i,j})$ is closely connected to $\nabla V^*_{\bar{x}_{i,j}}$, which is the solution of (19). Therefore, in order to obtain the decentralized ETC of interconnected system (1), we need to solve N event-triggered HJBEs described as (19).

C. Solving N Event-Triggered HJBEs Using ACL and the ER Technique Together

According to the approximation theory developed in [46], we can use a critic network to represent $V_i^*(x_i)$ over Ω_i as

$$V_i^*(x_i) = \omega_{c_i}^{\mathsf{T}}\sigma_{c_i}(x_i) + \varepsilon_{c_i}(x_i)$$

with $\omega_{c_i} \in \mathbb{R}^{\mathcal{N}_{c_i}}$ being the unknown ideal weight vector and $\sigma_{c_i}(x_i) = [\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\mathcal{N}_{c_i}}}(x_i)]^{\mathsf{T}} \in \mathbb{R}^{\mathcal{N}_{c_i}}$ being the vector activation function [*Note:* $\sigma_{c_{i\ell}}(x_i)$ is continuously differentiable over Ω_i with $\sigma_{c_{i\ell}}(0) = 0$, $\ell = 1, 2, \ldots, \mathcal{N}_{c_i}$. Meanwhile, for every $x_i \neq 0$, $\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\mathcal{N}_{c_i}}}(x_i)$ are linearly independent], $\mathcal{N}_{c_i} \in \mathbb{Z}^+$ being the number of neurons, and $\varepsilon_{c_i}(x_i) \in \mathbb{R}$ being the function reconstruction error.

The derivative of $V_i^*(x_i)$ at the sampled state $\bar{x}_{i,j}$ is

$$\nabla V_i^*(\bar{x}_{i,j}) = \nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,j})\omega_{c_i} + \nabla\varepsilon_{c_i}(\bar{x}_{i,j}) \tag{33}$$

with $\nabla\sigma_{c_i}(\bar{x}_{i,j}) = (\partial\sigma_{c_i}(x_i)/\partial x_i)|_{x_i=\bar{x}_{i,j}}$ and $\nabla\varepsilon_{c_i}(\bar{x}_{i,j}) = (\partial\varepsilon_{c_i}(x_i)/\partial x_i)|_{x_i=\bar{x}_{i,j}}$.

Inserting (33) into (18), we have [for all $t \in [t_j, t_{j+1})$]

$$\mu_i^*(\bar{x}_{i,j}) = -\frac{1}{2}\mathcal{R}_i^{-1}G_i^{\mathsf{T}}(\bar{x}_{i,j})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,j})\omega_{c_i} + \varepsilon_{\mu_i^*}(\bar{x}_{i,j}) \tag{34}$$

where $\varepsilon_{\mu_i^*}(\bar{x}_{i,j}) = -(1/2)\mathcal{R}_i^{-1}G_i^{\mathsf{T}}(\bar{x}_{i,j})\nabla\varepsilon_{c_i}(\bar{x}_{i,j})$.

Owing to the unavailability of ω_{c_i}, we cannot implement the ETC law $\mu_i^*(\bar{x}_{i,j})$ in (34). To tackle this problem, we replace ω_{c_i} with $\hat{\omega}_{c_i}$ in the critic network (*Note:* $\hat{\omega}_{c_i}$ is the current estimated value of ω_{c_i}.) Then, we can obtain the approximate cost function as

$$\hat{V}_i(x_i) = \hat{\omega}_{c_i}^{\mathsf{T}}\sigma_{c_i}(x_i).$$

Similar to (33), we have

$$\nabla\hat{V}_i(\bar{x}_{i,j}) = \nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,j})\hat{\omega}_{c_i}. \tag{35}$$

Replacing $\nabla V_i^*(\bar{x}_{i,j})$ in (18) with $\nabla\hat{V}_i(\bar{x}_{i,j})$ in (35), we can obtain the estimated value of $\mu_i^*(\bar{x}_{i,j})$ as

$$\hat{\mu}_i(\bar{x}_{i,j}) = -\frac{1}{2}\mathcal{R}_i^{-1}G_i^{\mathsf{T}}(\bar{x}_{i,j})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,j})\hat{\omega}_{c_i},\ \ t \in [t_j, t_{j+1}). \tag{36}$$

Let $V_i^*(x_i)$ and $\vartheta_i(x_i)$ in (12) be replaced with $\hat{V}_i(x_i)$ and $\hat{\mu}_i(\bar{x}_{i,j})$, respectively. Then, the approximate Hamiltonian can be written in the form

$$\begin{aligned}\hat{H}\Big(x_i, \nabla\hat{V}_i(x_i), \hat{\mu}_i(\bar{x}_{i,j})\Big) &= \hat{\omega}_{c_i}^{\mathsf{T}}\nabla\sigma_{c_i}(x_i)\big(f_i(x_i) + G_i(x_i)\hat{\mu}_i(\bar{x}_{i,j})\big)\\ &\quad + \Phi_i(x_i) + x_i^{\mathsf{T}}Q_ix_i + \hat{\mu}_i^{\mathsf{T}}(\bar{x}_{i,j})\mathcal{R}_i\hat{\mu}_i(\bar{x}_{i,j}).\end{aligned}$$

Because $\mu_i^*(\bar{x}_{i,j})$ is the discretized value of $\mu_i^*(x_i)$ at the triggering instant t_j, we can see from (13) that

$$H\big(x_i, \nabla V_i^*(x_i), \mu_i^*(\bar{x}_{i,j})\big) = 0.$$

Thus, the approximation error of Hamiltonian can be defined as follows:

$$\begin{aligned}e_{c_i} &= \hat{H}\Big(x_i, \nabla\hat{V}_i(x_i), \hat{\mu}_i(\bar{x}_{i,j})\Big) - H\big(x_i, \nabla V_i^*(x_i), \mu_i^*(\bar{x}_{i,j})\big)\\ &= \hat{\omega}_{c_i}^{\mathsf{T}}\phi_i + \Phi_i(x_i) + x_i^{\mathsf{T}}Q_ix_i + \hat{\mu}_i^{\mathsf{T}}(\bar{x}_{i,j})\mathcal{R}_i\hat{\mu}_i(\bar{x}_{i,j})\end{aligned} \tag{37}$$

where $\phi_i = \nabla\sigma_{c_i}(x_i)(f_i(x_i) + G_i(x_i)\hat{\mu}_i(\bar{x}_{i,j}))$.

To make e_{c_i} sufficiently small, we tend to find an appropriate weight vector $\hat{\omega}_{c_i}$ to minimize the target function

$$E\big(e_{c_i}, e_{c_{i,p}}\big) = \frac{1}{2}e_{c_i}^{\mathsf{T}}e_{c_i} + \frac{1}{2}\sum_{p=1}^{\mathcal{N}_{c_i}}e_{c_{i,p}}^{\mathsf{T}}e_{c_{i,p}} \tag{38}$$

where e_{c_i} is defined as (37), $p \in \{1, 2, \ldots, \mathcal{N}_{c_i}\}$ is the index of historical data $x(t_p)$, $t_p \in [t_j, t_{j+1})$, and $e_{c_{i,p}}$ is the recorded approximation error of Hamiltonian formulated as

$$\begin{aligned}e_{c_{i,p}} &= \hat{\omega}_{c_i}^{\mathsf{T}}\phi_{i,p} + \Phi_i\big(x_i(t_p)\big)\\ &\quad + x_i^{\mathsf{T}}(t_p)Q_ix_i(t_p) + \hat{\mu}_i^{\mathsf{T}}(\bar{x}_{i,j})\mathcal{R}_i\hat{\mu}_i(\bar{x}_{i,j})\end{aligned}$$

where

$$\phi_{i,p} = \nabla\sigma_{c_i}\big(x_i(t_p)\big)\big(f_i\big(x_i(t_p)\big) + G_i\big(x_i(t_p)\big)\hat{\mu}_i\big(\bar{x}_{i,j}\big)\big). \tag{39}$$

Applying the gradient descent approach to $E(e_{c_i}, e_{c_{i,p}})$ in (38) and employing the normalization terms $(1+\phi_i^{\mathsf{T}}\phi_i)^{-2}$ and $(1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p})^{-2}$, we can find that the weight update law used in the critic network is formulated as [*note:* $t, t_p \in [t_j, t_{j+1})$]

$$\dot{\hat{\omega}}_{c_i} = -\frac{\theta_{c_i}\phi_i}{\big(1+\phi_i^{\mathsf{T}}\phi_i\big)^2}e_{c_i} - \sum_{p=1}^{\mathcal{N}_{c_i}}\frac{\theta_{c_i}\phi_{i,p}}{\Big(1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p}\Big)^2}e_{c_{i,p}} \tag{40}$$

where θ_{c_i} is an adjustable positive parameter and $\phi_{i,p}$ is defined as (39).

Remark 5: As for (40), we present two following notes.

1) If neglecting the summation term in (40), a prosing noise must be provided to make $\phi_i/\big(1+\phi_i^{\mathsf{T}}\phi_i\big)$ satisfy the PE condition (see [31]–[33]). Nevertheless, it is often hard to find the appropriate prosing noise. To avoid this difficulty, we add the recorded approximation error of the Hamiltonian [i.e., the summation term in (38)] to the target function. As analyzed in [47, Remark 4], the summation term in (40) indeed can relax the PE condition.
2) To implement (40), we need to impose a requirement as follows. That is, the number of recorded state data is large enough, which guarantees

$$\operatorname{rank}\Big[\sigma_{c_i}(x_i(t_1)), \ldots, \sigma_{c_i}\Big(x_i\Big(t_{\mathcal{N}_{c_i}}\Big)\Big)\Big] = \mathcal{N}_{c_i}. \tag{41}$$

According to [47, Lemma 3], (41) can keep the set $\{\phi_{i,p}/(1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p})\}_1^{\mathcal{N}_{c_i}}$ linearly independent. We can see from [47, Remark 4] that the linear independence of the set $\{\phi_{i,p}/(1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p})\}_1^{\mathcal{N}_{c_i}}$ plays a crucial role in relaxing the PE condition.

Define the weight estimation error as $\tilde{\omega}_{c_i} = \omega_{c_i} - \hat{\omega}_{c_i}$. Then, we can see from (40) that the dynamics of $\tilde{\omega}_{c_i}$ satisfies

$$\begin{aligned}\dot{\tilde{\omega}}_{c_i} = &-\theta_{c_i}\left(\varphi_i\varphi_i^{\mathsf{T}} + \sum_{p=1}^{\mathcal{N}_{c_i}}\varphi_{i,p}\varphi_{i,p}^{\mathsf{T}}\right)\tilde{\omega}_{c_i} + \frac{\theta_{c_i}\varphi_i}{1+\phi_i^{\mathsf{T}}\phi_i}\varepsilon_{H_i}\\ &+\sum_{p=1}^{\mathcal{N}_{c_i}}\frac{\theta_{c_i}\varphi_{i,p}}{1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p}}\varepsilon_{H_{i,p}},\ \ t, t_p \in \big[t_j, t_{j+1}\big)\end{aligned} \tag{42}$$

where $\varphi_i = \phi_i/(1+\phi_i^{\mathsf{T}}\phi_i)$, $\varphi_{i,p} = \phi_{i,p}/(1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p})$, ε_{H_i}, and $\varepsilon_{H_{i,p}}$ are the residual errors arising in approximating the cost function with the critic network and formulated as [48]

$$\begin{aligned}\varepsilon_{H_i} &= -\nabla\varepsilon_{c_i}^{\mathsf{T}}(x_i)\big(f_i(x_i) + G_i(x_i)\hat{\mu}_i\big(\bar{x}_{i,j}\big)\big)\\ \varepsilon_{H_{i,p}} &= -\nabla\varepsilon_{c_i}^{\mathsf{T}}\big(x_i(t_p)\big)\big(f_i\big(x_i(t_p)\big) + G_i\big(x_i(t_p)\big)\hat{\mu}_i\big(\bar{x}_{i,j}\big)\big).\end{aligned}$$

Combining the closed-loop system

$$\dot{x}_i = f_i(x_i) + G_i(x_i)\hat{\mu}_i\big(\bar{x}_{i,j}\big)$$

with the dynamics of $\tilde{\omega}_{c_i}$ in (42), we can obtain an augmented hybrid system. That is, letting $y_i = [x_i^{\mathsf{T}}, \bar{x}_{i,j}^{\mathsf{T}}, \tilde{\omega}_{c_i}^{\mathsf{T}}]^{\mathsf{T}}$, we have the following.

1) *Continuous Dynamics:*

$$\dot{y}_i(t) = \begin{bmatrix} f_i(x_i) - \dfrac{1}{2}\Lambda\big(x_i, \bar{x}_{i,j}\big)\nabla\sigma_{c_i}^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\hat{\omega}_{c_i}\\ 0\\ -\theta_{c_i}\Pi\big(\varphi_i, \varphi_{i,p}\big)\tilde{\omega}_{c_i} + \Upsilon\big(\varphi_i, \varphi_{i,p}\big)\end{bmatrix} \tag{43}$$

for all $t \in [t_j, t_{j+1})$, $j \in \mathbb{N}$, where

$$\begin{aligned}\Lambda\big(x_i, \bar{x}_{i,j}\big) &= G_i(x_i)\mathcal{R}_i^{-1}G_i^{\mathsf{T}}\big(\bar{x}_{i,j}\big)\\ \Pi\big(\varphi_i, \varphi_{i,p}\big) &= \varphi_i\varphi_i^{\mathsf{T}} + \sum_{p=1}^{\mathcal{N}_{c_i}}\varphi_{i,p}\varphi_{i,p}^{\mathsf{T}}\\ \Upsilon\big(\varphi_i, \varphi_{i,p}\big) &= \frac{\theta_{c_i}\varphi_i}{1+\phi_i^{\mathsf{T}}\phi_i}\varepsilon_{H_i} + \sum_{p=1}^{\mathcal{N}_{c_i}}\frac{\theta_{c_i}\varphi_{i,p}}{1+\phi_{i,p}^{\mathsf{T}}\phi_{i,p}}\varepsilon_{H_{i,p}}.\end{aligned} \tag{44}$$

2) *Discrete Dynamics:*

$$y_i(t) = y_i\big(t^-\big) + \begin{bmatrix}0\\ x_i(t) - \bar{x}_{i,j}\\ 0\end{bmatrix}\ \ t = t_{j+1}, j \in \mathbb{N} \tag{45}$$

with $y_i(t^-) = \lim_{\eta\to 0^-} y_i(t+\eta)$, $\eta \in (t_j - t_{j+1}, 0)$.

IV. Stability Analysis

In this section, we analyze the stability of closed-loop auxiliary subsystems (43) and (45). Before continuing, we provide two following assumptions. Similar assumptions can be found in [39], [40], and [49].

Assumption 4: For every $x_i \in \Omega_i$, $\nabla\sigma_{c_i}(x_i)$ is bounded as $\|\nabla\sigma_{c_i}(x_i)\| \le b_{\sigma_{c_i}}$ with $b_{\sigma_{c_i}}$ being the known positive constant. Moreover, for every $x_i \in \Omega_i$, $\varepsilon_{\mu_i^*}(x_i)$ and ε_{H_i} are separately bounded as $\|\varepsilon_{\mu_i^*}(x_i)\| \le b_{\varepsilon_{\mu_i^*}}$ and $\|\varepsilon_{H_i}\| \le b_{\varepsilon_{H_i}}$ with $b_{\varepsilon_{\mu_i^*}}$ and $b_{\varepsilon_{H_i}}$ being the known positive constants.

Assumption 5: For every $x_i \in \mathbb{R}^{n_i}$, $G_i(x_i)$ given in (7) is bounded as $\|G_i(x_i)\| \le b_{G_i}$ with b_{G_i} the known positive constant.

Theorem 2: Take the ith auxiliary subsystem (8) and the corresponding event-triggered HJBE (19) into consideration. Suppose that Assumptions 1–5 are valid and take the ith augmented ETC given as (36). Meanwhile, let the initial control for the ith auxiliary subsystem (8) be admissible and let the critic network weight be updated via (40). Then, the ith closed-loop auxiliary subsystem (8) is stable in the sense of uniform ultimate boundedness and the weight estimation error $\tilde{\omega}_{c_i}$ is UUB as long as the triggering condition is proposed as

$$\big\|e_{i,j}\big\|^2 \le \frac{(1-2\gamma_i)\lambda_{\min}(Q_i)}{2K_{\vartheta_i^*}^2}\|x_i\|^2 \triangleq \big\|\bar{e}_{i,T}\big\|^2 \tag{46}$$

where $0 < \gamma_i < 1/2$ and $\bar{e}_{i,T}$ is the triggering threshold, and provided that the following inequality is valid:

$$\theta_{c_i}\lambda_{\min}\big(\Pi\big(\varphi_i, \varphi_{i,p}\big)\big) - 2b_{G_i}^2 b_{\sigma_{c_i}}^2\Big\|\mathcal{R}_i^{-1}\Big\|^2 > 0 \tag{47}$$

with $\lambda_{\min}(\Pi(\varphi_i, \varphi_{i,p}))$ denoting the minimum eigenvalue of $\Pi(\varphi_i, \varphi_{i,p})$ defined as (44).

Proof: Choose the Lyapunov function candidate as

$$L(t) = \underbrace{V_i^*\big(\bar{x}_{i,j}\big)}_{L_1(t)} + \underbrace{V_i^*(x_i(t))}_{L_2(t)} + \underbrace{(1/2)\tilde{\omega}_{c_i}^{\mathsf{T}}\tilde{\omega}_{c_i}}_{L_3(t)}. \tag{48}$$

As stated in Section III-C, the ith closed-loop auxiliary subsystem contains two parts: 1) the continuous dynamics (43) and 2) the discrete dynamics (45). Hence, we will study the stability of the ith closed-loop auxiliary subsystem from the following two cases.

Case I: Let $t \in [t_j, t_{j+1}), j \in \mathbb{N}$. Then, $\dot{L}_1(t) = \mathrm{d}V_i^*(\bar{x}_{i,j})/\mathrm{d}t = 0$. Differentiating $L_2(t)$ along the solution of $\dot{x}_i = f_i(x_i) + G_i(x_i)\hat{\mu}_i(\bar{x}_{i,j})$, we have

$$\begin{aligned}\dot{L}_2(t) &= \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}\left(f_i(x_i) + G_i(x_i)\hat{\mu}_i(\bar{x}_{i,j})\right)\\ &= \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}\left(f_i(x_i) + G_i(x_i)\vartheta_i^*(x_i)\right)\\ &\quad + \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} G_i(x_i)\left(\hat{\mu}_i(\bar{x}_{i,j}) - \vartheta_i^*(x_i)\right). \quad (49)\end{aligned}$$

According to (12)–(14), we find

$$\begin{cases}\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}\left(f_i(x_i) + G_i(x_i)\vartheta_i^*(x_i)\right)\\ \quad = -\Phi_i(x_i) - x_i^{\mathsf{T}} Q_i x_i - \left(\vartheta_i^*(x_i)\right)^{\mathsf{T}}\mathcal{R}_i\vartheta_i^*(x_i)\\ \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} G_i(x_i) = -2\left(\vartheta_i^*(x_i)\right)^{\mathsf{T}}\mathcal{R}_i.\end{cases}$$

Thus, we can rewrite (49) as

$$\begin{aligned}\dot{L}_2(t) &= -\Phi_i(x_i) - x_i^{\mathsf{T}} Q_i x_i - \left\|\mathcal{R}_i^{\frac{1}{2}}\hat{\mu}_i(\bar{x}_{i,j})\right\|^2\\ &\quad + \underbrace{\left\|\mathcal{R}_i^{\frac{1}{2}}\left(\vartheta_i^*(x_i) - \hat{\mu}_i(\bar{x}_{i,j})\right)\right\|^2}_{\Theta}. \quad (50)\end{aligned}$$

Owing to $0 < \rho_i < 1/2$ in (23), we can conclude $\|\mathcal{R}_i^{1/2}\| \leq 1$. Then, using the Young's inequality [50, Ch. 2.7] and Assumptions 3–5 as well as (34) and (36), we can see that $\Theta \in \mathbb{R}$ in (50) satisfies

$$\begin{aligned}\Theta &\leq \left\|\vartheta_i^*(x_i) - \hat{\mu}_i(\bar{x}_{i,j})\right\|^2\\ &= \left\|\left(\vartheta_i^*(x_i) - \mu_i^*(\bar{x}_{i,j})\right) + \left(\mu_i^*(\bar{x}_{i,j}) - \hat{\mu}_i(\bar{x}_{i,j})\right)\right\|^2\\ &\leq 2\left\|\mu_i^*(\bar{x}_{i,j}) - \hat{\mu}_i(\bar{x}_{i,j})\right\|^2 + 2\left\|\vartheta_i^*(x_i) - \mu_i^*(\bar{x}_{i,j})\right\|^2\\ &\leq 2\left\|-(1/2)\mathcal{R}_i^{-1} G_i^{\mathsf{T}}(\bar{x}_{i,j})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,j})\tilde{\omega}_{c_i} + \varepsilon_{\mu_i^*}(\bar{x}_{i,j})\right\|^2\\ &\quad + 2K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2\\ &\leq b_{G_i}^2 b_{\sigma_{c_i}}^2\left\|\mathcal{R}_i^{-1}\right\|^2\left\|\tilde{\omega}_{c_i}\right\|^2 + 2K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2 + 4b_{\varepsilon_{\mu_i^*}}^2. \quad (51)\end{aligned}$$

Combining (50) with (51) and observing that $\Phi_i(x_i)$ and $\|\mathcal{R}_i^{1/2}\hat{\mu}_i(\bar{x}_{i,j})\|^2$ are non-negative real-valued functions, we obtain

$$\begin{aligned}\dot{L}_2(t) &\leq -\lambda_{\min}(Q_i)\|x_i\|^2 + b_{G_i}^2 b_{\sigma_{c_i}}^2\left\|\mathcal{R}_i^{-1}\right\|^2\left\|\tilde{\omega}_{c_i}\right\|^2\\ &\quad + 2K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2 + 4b_{\varepsilon_{\mu_i^*}}^2. \quad (52)\end{aligned}$$

Taking the time derivative of $L_3(t)$ and using (42), we have

$$\begin{aligned}\dot{L}_3(t) &= -\theta_{c_i}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(\varphi_i\varphi_i^{\mathsf{T}} + \sum_{p=1}^{\mathcal{N}_{c_i}}\varphi_{i,p}\varphi_{i,p}^{\mathsf{T}}\right)\tilde{\omega}_{c_i}\\ &\quad + \frac{\theta_{c_i}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i}{1 + \phi_i^{\mathsf{T}}\phi_i}\varepsilon_{H_i} + \sum_{p=1}^{\mathcal{N}_{c_i}}\frac{\theta_{c_i}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_{i,p}}{1 + \phi_{i,p}^{\mathsf{T}}\phi_{i,p}}\varepsilon_{H_{i,p}}. \quad (53)\end{aligned}$$

Applying Young's inequality [50, Ch. 2.7] to the second term in the right-hand side of (53), we get

$$\begin{aligned}\frac{\theta_{c_i}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varepsilon_{H_i}}{1 + \phi_i^{\mathsf{T}}\phi_i} &\leq \frac{\theta_{c_i}}{1 + \phi_i^{\mathsf{T}}\phi_i}\left(\frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \frac{1}{2}\varepsilon_{H_i}^{\mathsf{T}}\varepsilon_{H_i}\right)\\ &\leq \frac{\theta_{c_i}}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \frac{\theta_{c_i}}{2}\varepsilon_{H_i}^{\mathsf{T}}\varepsilon_{H_i}. \quad (54)\end{aligned}$$

Similarly, the third term in the right-hand side of (53) satisfies

$$\begin{aligned}\sum_{p=1}^{\mathcal{N}_{c_i}}\frac{\theta_{c_i}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_{i,p}}{1 + \phi_{i,p}^{\mathsf{T}}\phi_{i,p}}\varepsilon_{H_{i,p}} &\leq \frac{\theta_{c_i}}{2}\sum_{p=1}^{\mathcal{N}_{c_i}}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_{i,p}\varphi_{i,p}^{\mathsf{T}}\tilde{\omega}_{c_i}\\ &\quad + \frac{\theta_{c_i}}{2}\sum_{p=1}^{\mathcal{N}_{c_i}}\varepsilon_{H_{i,p}}^{\mathsf{T}}\varepsilon_{H_{i,p}}. \quad (55)\end{aligned}$$

Using (53)–(55) and making some calculations, we obtain

$$\dot{L}_3(t) \leq -\frac{\theta_{c_i}}{2}\lambda_{\min}\left(\Pi(\varphi_i, \varphi_{i,p})\right)\left\|\tilde{\omega}_{c_i}\right\|^2 + \frac{\theta_{c_i}\left(\mathcal{N}_{c_i} + 1\right)}{2}b_{\varepsilon_{H_i}}^2 \quad (56)$$

with $\Pi(\varphi_i, \varphi_{i,p})$ defined as (44).

Combining (52) with (56), we find that the time derivative of $L(t)$ in (48) yields

$$\begin{aligned}\dot{L}(t) &\leq -2\gamma_i\lambda_{\min}(Q_i)\|x_i\|^2 - (1 - 2\gamma_i)\lambda_{\min}(Q_i)\|x_i\|^2\\ &\quad + 2K_{\vartheta_i^*}^2\left\|e_{i,j}\right\|^2 - \frac{1}{2}\Bigg(\theta_{c_i}\lambda_{\min}\left(\Pi(\varphi_i, \varphi_{i,p})\right)\\ &\qquad - 2b_{G_i}^2 b_{\sigma_{c_i}}^2\left\|\mathcal{R}_i^{-1}\right\|^2\Bigg)\left\|\tilde{\omega}_{c_i}\right\|^2\\ &\quad + 4b_{\varepsilon_{\mu_i^*}}^2 + \frac{\theta_{c_i}}{2}\left(\mathcal{N}_{c_i} + 1\right)b_{\varepsilon_{H_i}}^2. \quad (57)\end{aligned}$$

Thus, with (46) and (47) held, we can see from (57) that $\dot{L}(t) < 0$ only if we can make $x_i \notin \Omega_{x_i}$ or $\tilde{\omega}_{c_i} \notin \Omega_{\tilde{\omega}_{c_i}}$, where Ω_{x_i} and $\Omega_{\tilde{\omega}_{c_i}}$ are given as

$$\Omega_{x_i} = \left\{x_i \colon \|x_i\| \leq \sqrt{\frac{2b_{\varepsilon_{\mu_i^*}}^2 + \left(\theta_{c_i}/4\right)\left(\mathcal{N}_{c_i} + 1\right)b_{\varepsilon_{H_i}}^2}{\gamma_i\lambda_{\min}(Q_i)}}\right\}$$

$$\Omega_{\tilde{\omega}_{c_i}} = \left\{\tilde{\omega}_{c_i} \colon \left\|\tilde{\omega}_{c_i}\right\| \leq \sqrt{\frac{8b_{\varepsilon_{\mu_i^*}}^2 + \theta_{c_i}\left(\mathcal{N}_{c_i} + 1\right)b_{\varepsilon_{H_i}}^2}{\theta_{c_i}\lambda_{\min}\left(\Pi(\varphi_i, \varphi_{i,p})\right) - a_0}}\right\}$$

with $a_0 = 2b_{G_i}^2 b_{\sigma_{c_i}}^2\|\mathcal{R}_i^{-1}\|^2$.

According to the Lyapunov extension theorem [51], this proves that x_i and $\tilde{\omega}_{c_i}$ are UUB. The ultimate bounds of x_i and $\tilde{\omega}_{c_i}$ are the same as the bounds of Ω_{x_i} and $\Omega_{\tilde{\omega}_{c_i}}$, respectively.

Case II: Let $t = t_{j+1}, j \in \mathbb{N}$. Then, we consider the difference of Lyapunov function candidate given in (48), i.e.,

$$\Delta L(t_{j+1}) = V_i^*(\bar{x}_{i,j+1}) - V_i^*(\bar{x}_{i,j}) + \Delta\Sigma_i \quad (58)$$

where

$$\begin{aligned}\Delta\Sigma_i &= V_i^*\left(x_i(t_{j+1})\right) - V_i^*\left(x_i\left(t_{j+1}^-\right)\right)\\ &\quad + \frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}(t_{j+1})\tilde{\omega}_{c_i}(t_{j+1}) - \frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(t_{j+1}^-\right)\tilde{\omega}_{c_i}\left(t_{j+1}^-\right)\end{aligned}$$

with $x_i(t_{j+1}^-) = \lim_{\eta\to 0^-} x_i(t_{j+1} + \eta)$, $\tilde{\omega}_{c_i}(t_{j+1}^-) = \lim_{\eta\to 0^-}\tilde{\omega}_{c_i}(t_{j+1} + \eta)$, and $\eta \in (t_j - t_{j+1}, 0)$.

According to case I, if $x_i \notin \Omega_{x_i}$ or $\tilde{\omega}_{c_i} \notin \Omega_{\tilde{\omega}_{c_i}}$, then $\dot{L}(t) < 0$ for all $t \in [t_j, t_{j+1})$. Therefore, $L(t)$ in (48) is strictly monotonically decreasing over the interval $[t_j, t_{j+1})$. Owing to $t_{j+1} > t_{j+1} + \eta$, for all $\eta \in (t_j - t_{j+1}, 0)$, we thus have

$$L\big(t_{j+1}\big) < L\big(t_{j+1} + \eta\big), \ \ \eta \in \big(t_j - t_{j+1}, 0\big). \tag{59}$$

Letting $\eta \to 0^-$ in (59) and using the property of the limit [43, Ch. 4], we obtain

$$L\big(t_{j+1}\big) \le \lim_{\eta \to 0^-} L\big(t_{j+1} + \eta\big) = L\Big(t_{j+1}^-\Big). \tag{60}$$

Then, we can see from (60) that

$$\begin{aligned} &V_i^*\big(x_i\big(t_{j+1}\big)\big) + (1/2)\tilde{\omega}_{c_i}^{\mathsf{T}}\big(t_{j+1}\big)\tilde{\omega}_{c_i}\big(t_{j+1}\big) \\ &\quad \le V_i^*\Big(x_i\Big(t_{j+1}^-\Big)\Big) + (1/2)\tilde{\omega}_{c_i}^{\mathsf{T}}\Big(t_{j+1}^-\Big)\tilde{\omega}_{c_i}\Big(t_{j+1}^-\Big). \end{aligned}$$

This verifies that $\Delta\Sigma_i \le 0$. On the other hand, based on the conclusion that $x_i(t)$ is UUB in case I, we can derive

$$V_i^*\big(\bar{x}_{i,j+1}\big) \le V_i^*\big(\bar{x}_{i,j}\big).$$

Accordingly, if $x_i \notin \Omega_{x_i}$ or $\tilde{\omega}_{c_i} \notin \Omega_{\tilde{\omega}_{c_i}}$, then $\Delta L(t_{j+1})$ defined as (58) satisfies $\Delta L(t_{j+1}) < 0$. This proves that x_i and $\tilde{\omega}_{c_i}$ are UUB by using the Lyapunov extension theorem [51]. The ultimate bounds of x_i and $\tilde{\omega}_{c_i}$ are the same as the bounds of Ω_{x_i} and $\Omega_{\tilde{\omega}_{c_i}}$, respectively. ■

V. Experimental Study

We study the CT nonlinear interconnected system described by the following equations:

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} -x_{11} + x_{12} \\ -0.5(x_{11} + x_{12}) + 0.5x_{11}^2 x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \sin(x_{11}) \end{bmatrix} u_1 \\ &\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (x_{11} + x_{22}) \sin^2(\varrho_1 x_{12}) \cos(0.5x_{21}) \\ \dot{x}_2 &= \begin{bmatrix} 0.5x_{22} \\ -x_{21} - 0.5x_{22} + 0.5x_{21}\cos^2(x_{22}) \end{bmatrix} + \begin{bmatrix} 0 \\ x_{21} \end{bmatrix} u_2 \\ &\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Big(0.5(x_{12} + x_{22}) \cos\Big(\varrho_2 e^{x_{21}^2}\Big)\Big) \end{aligned} \tag{61}$$

where $x_1 = [x_{11}, x_{12}]^{\mathsf{T}} \in \mathbb{R}^2$ and $x_2 = [x_{21}, x_{22}]^{\mathsf{T}} \in \mathbb{R}^2$ are the state vectors of subsystems 1 and 2, respectively, $u_1 \in \mathbb{R}$ and $u_2 \in \mathbb{R}$ are the control inputs of subsystems 1 and 2, respectively, and $\varrho_1 \in \mathbb{R}$ and $\varrho_2 \in \mathbb{R}$ are the unknown parameters. To simplify discussion, we randomly choose $\varrho_i \in [-1, 1]$, $i = 1, 2$. From expression (61), it can be seen that $g_i^{\mathsf{T}}(x_i)k_i(x_i) = 0$ and $\mathrm{rank}(g_i(x_i)) < 2$, $i = 1, 2$. Meanwhile, the equilibrium points of subsystems 1 and 2 are both zero, i.e., $x_i = 0$, $i = 1, 2$. Thus, Assumption 1 holds. Moreover, we can see from (61) that

$$\begin{aligned} d_1(x) &= (x_{11} + x_{22}) \sin^2(\varrho_1 x_{12}) \cos(0.5x_{21}) \\ d_2(x) &= 0.5(x_{12} + x_{22}) \cos\Big(\varrho_2 e^{x_{21}^2}\Big). \end{aligned}$$

To satisfy Assumption 2 [or rather, the inequality (4)], we choose $\alpha_1(\|x_1\|) = \|x_1\|$, $\alpha_2(\|x_2\|) = \|x_2\|$, and let the relevant parameters be presented as follows: $c_{11} = 1$, $c_{12} = 1$, $c_{21} = 0.5$, and $c_{22} = 0.5$. The initial state of interconnected system (61) is $x_0 = [0.5, -0.5, 1, -1]^{\mathsf{T}}$.

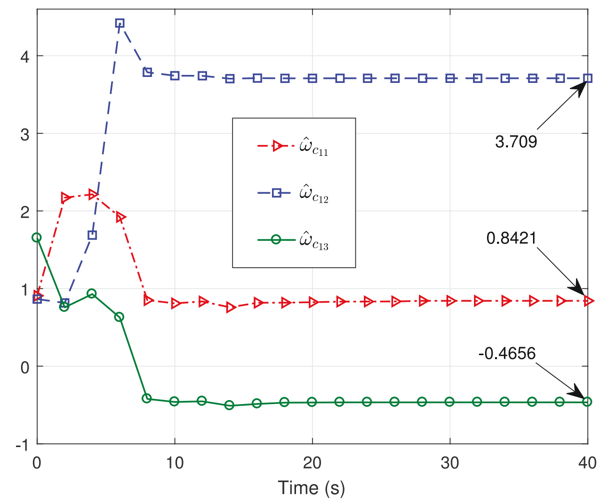

Fig. 1. Convergence of the weight vector $\hat{\omega}_{c_1} = [\hat{\omega}_{c_{11}}, \hat{\omega}_{c_{12}}, \hat{\omega}_{c_{13}}]^{\mathsf{T}}$.

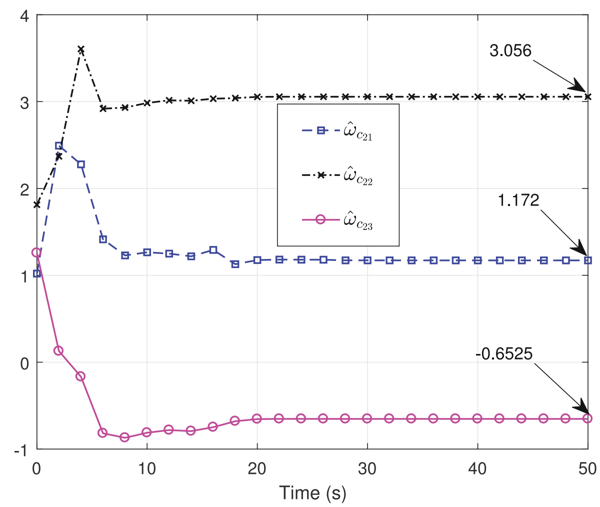

Fig. 2. Convergence of the weight vector $\hat{\omega}_{c_2} = [\hat{\omega}_{c_{21}}, \hat{\omega}_{c_{22}}, \hat{\omega}_{c_{23}}]^{\mathsf{T}}$.

Based on (5), (7), and (8), we can derive the auxiliary subsystems 1 and 2 for interconnected system (61). According to Theorem 1, for obtaining the decentralized ETC of interconnected system (61), we have to solve the event-triggered HJBEs associated with the auxiliary subsystems 1 and 2 [such as (19)]. We let $\rho_1 = 0.3$ and $\rho_2 = 0.3$. Meanwhile, we set $\pi_1 = 3$ and $\pi_2 = 3$ to make the matrix B in (30) positive definite. Let $Q_1 = Q_2 = I_2$, and I_2 is the identity matrix with $\mathrm{rank}(I_2) = 2$. Then, we can see from (9) and (10) that the cost functions for auxiliary subsystems 1 and 2 are, respectively, given in the form (*note:* $\vartheta_i^{\mathsf{T}} = [u_i^{\mathsf{T}}, \nu_i^{\mathsf{T}}]$, $i = 1, 2$)

$$\begin{aligned} V_1^{\vartheta_1}(x_1) &= \int_t^{\infty} \Big(4\|x_1\|^2 + u_1^{\mathsf{T}}u_1 + 0.3\nu_1^{\mathsf{T}}\nu_1\Big) \mathrm{d}s \\ V_2^{\vartheta_2}(x_2) &= \int_t^{\infty} \Big(4\|x_2\|^2 + u_2^{\mathsf{T}}u_2 + 0.3\nu_2^{\mathsf{T}}\nu_2\Big) \mathrm{d}s. \end{aligned}$$

The vector activation functions used in the critic networks are selected as follows (*note:* $\mathcal{N}_{c_1} = 3$ and $\mathcal{N}_{c_2} = 3$):

$$\begin{aligned} \sigma_{c_1}(x_1) &= \Big[x_{11}^2, x_{12}^2, x_{11}x_{12}\Big]^{\mathsf{T}} \\ \sigma_{c_2}(x_2) &= \Big[x_{21}^2, x_{22}^2, x_{21}x_{22}\Big]^{\mathsf{T}}. \end{aligned}$$

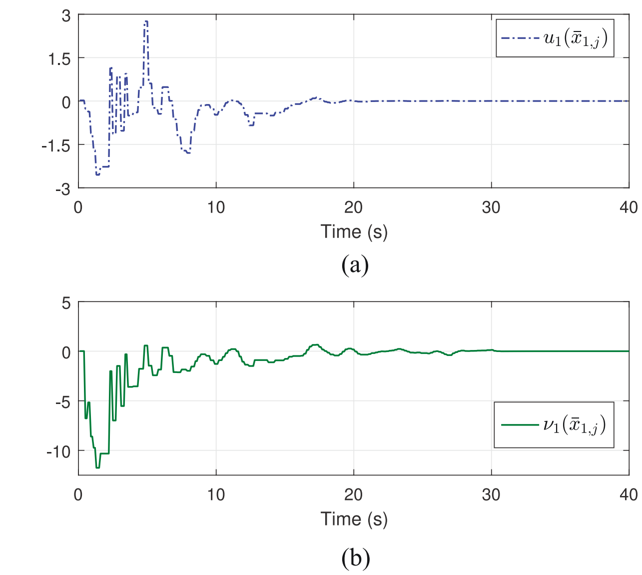

Fig. 3. (a) ETC $u_1(\bar{x}_{1,j})$. (b) Auxiliary ETC $\nu_1(\bar{x}_{1,j})$ [*Note:* $u_1(\bar{x}_{1,j})$ and $\nu_1(\bar{x}_{1,j})$ together constitute the augmented ETC $\mu_1(\bar{x}_{1,j})$ or $\vartheta_1(\bar{x}_{1,j})$].

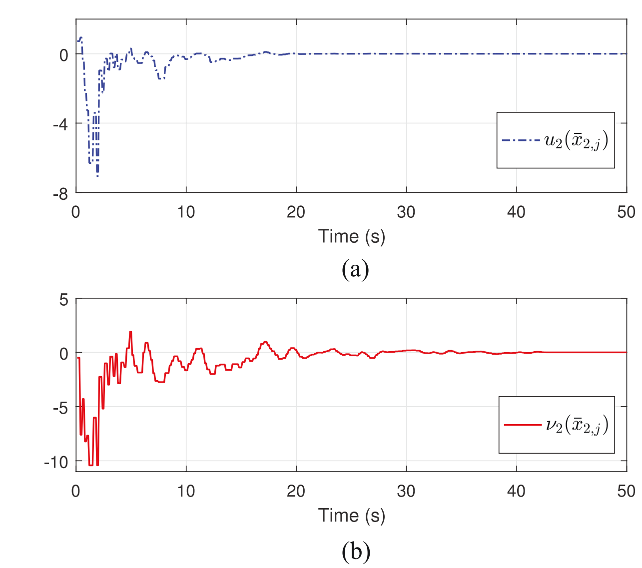

Fig. 4. (a) ETC $u_2(\bar{x}_{2,j})$. (b) Auxiliary ETC $\nu_2(\bar{x}_{2,j})$ [*Note:* $u_2(\bar{x}_{2,j})$ and $\nu_2(\bar{x}_{2,j})$ together constitute the augmented ETC $\mu_2(\bar{x}_{2,j})$ or $\vartheta_2(\bar{x}_{2,j})$].

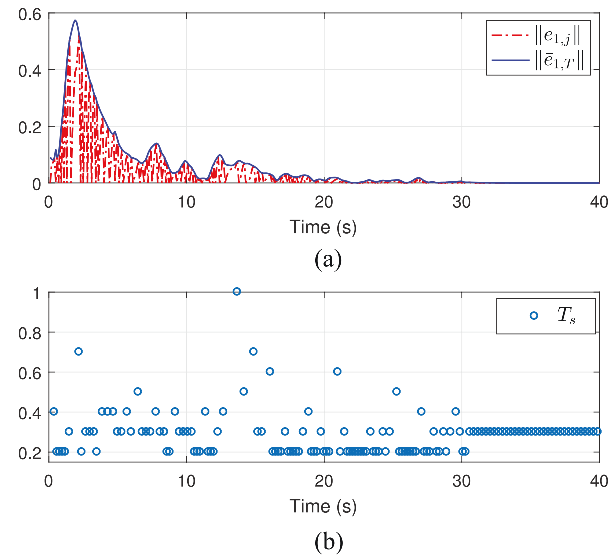

Fig. 5. (a) Norms of event-triggering condition $e_{1,j}$ and event-triggering threshold $\bar{e}_{1,T}$ (i.e., $\|e_{1,j}\|$ and $\|\bar{e}_{1,T}\|$). (b) Intersampling time T_s.

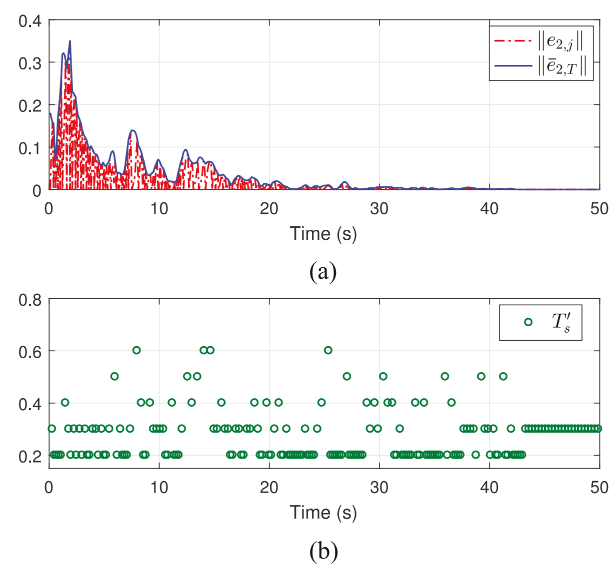

Fig. 6. (a) Norms of event-triggering condition $e_{2,j}$ and event-triggering threshold $\bar{e}_{2,T}$ (i.e., $\|e_{2,j}\|$ and $\|\bar{e}_{2,T}\|$). (b) Intersampling time T'_s.

Associated with $\sigma_{c_1}(x_1)$ and $\sigma_{c_2}(x_2)$, the weight vectors utilized in the critic networks are, respectively, denoted as $\hat{\omega}_{c_1} = [\hat{\omega}_{c_{11}}, \hat{\omega}_{c_{12}}, \hat{\omega}_{c_{13}}]^{\mathsf{T}}$ and $\hat{\omega}_{c_2} = [\hat{\omega}_{c_{21}}, \hat{\omega}_{c_{22}}, \hat{\omega}_{c_{23}}]^{\mathsf{T}}$. The relevant parameters in the weight tuning rule (40) and the event triggering condition (46) are designed as follows: $\theta_{c_i} = 0.75$, $\gamma_i = 0.3$, and $K_{\vartheta_i^*} = 3.5$, $i = 1, 2$.

We perform the experimental study through the MATLAB (R2016b) software package and illustrate the simulation results in Figs. 1–10. Figs. 1 and 2 describe the convergence of weight vectors $\hat{\omega}_{c_1}$ and $\hat{\omega}_{c_2}$ used in approximating the cost functions related to auxiliary subsystems 1 and 2. As displayed in Figs. 1 and 2, $\hat{\omega}_{c_1}$ converges to $\hat{\omega}_{c_1}^{\text{final}} = [0.8421, 3.709, -0.4656]^{\mathsf{T}}$ after 30 s, and $\hat{\omega}_{c_2}$ converges to $\hat{\omega}_{c_2}^{\text{final}} = [1.172, 3.056, -0.6525]^{\mathsf{T}}$ after 43 s. Fig. 3(a) and (b) shows the ETC $u_1(\bar{x}_{1,j})$ and the auxiliary ETC $\nu_1(\bar{x}_{1,j})$ for subsystem 1. Fig. 4(a) and (b) depicts the ETC $u_2(\bar{x}_{2,j})$ and the auxiliary ETC $\nu_2(\bar{x}_{2,j})$ for subsystem 2. According to (7) and (18), $u_i(\bar{x}_{i,j})$ and $\nu_i(\bar{x}_{i,j})$ together constitute the augmented ETC $\mu_i(\bar{x}_{i,j})$ or $\vartheta_i(\bar{x}_{i,j})$, where $i = 1, 2$, $j \in \mathbb{N}$. Fig. 5(a) describes the norms of event-triggering condition $e_{i,j}$ and event-triggering threshold $\bar{e}_{i,T}$ given in (46) when considering subsystem 1 (i.e., $\|e_{1,j}\|$ and $\|\bar{e}_{1,T}\|$). Meanwhile, Fig. 5(b) shows the intersampling time T_s, where $T_s = t_{j+1} - t_j$, $j \in \mathbb{N}$. (*Note:* During the last 10 s, the intersampling time T_s turns out to be constant, i.e., $T_s = 0.3$ s. This is because the weight vector of the critic network has been convergent. In other words, the update of the critic network ended before the last 10 s.) Similarly, Fig. 6(a) presents the norms of event-triggering condition $e_{i,j}$ and event-triggering threshold $\bar{e}_{i,T}$ given in (46) when considering subsystem 2 (i.e., $\|e_{2,j}\|$ and

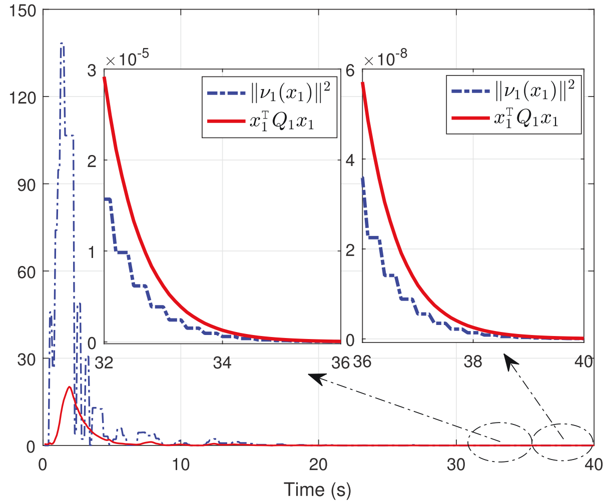

Fig. 7. Verifying the validity of (22) when considering subsystem 1.

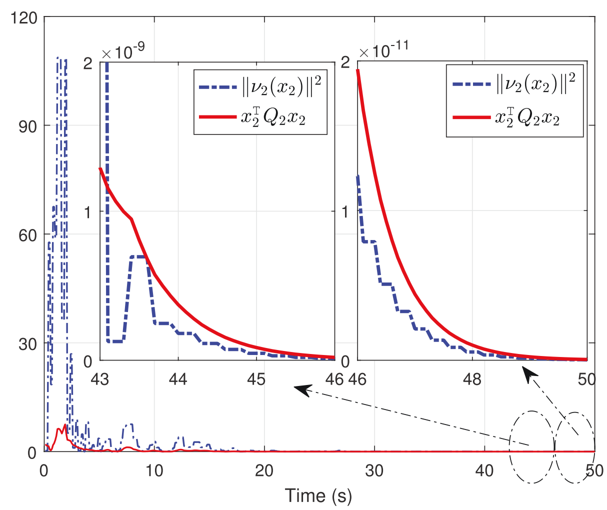

Fig. 8. Verifying the validity of (22) when considering subsystem 2.

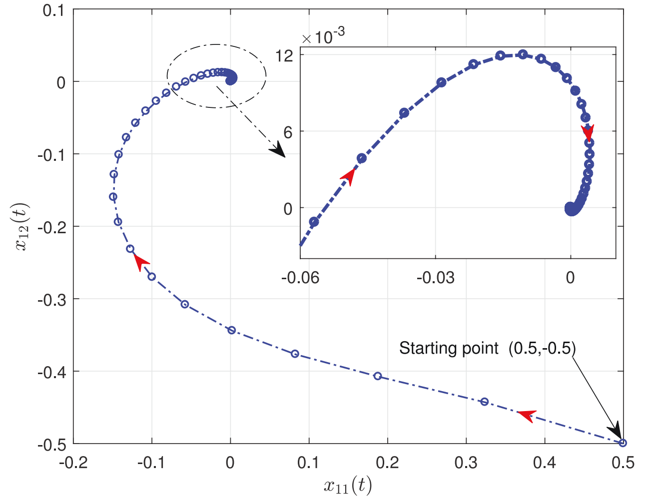

Fig. 9. States $x_{11}(t)$ and $x_{12}(t)$ of subsystem 1 in (61).

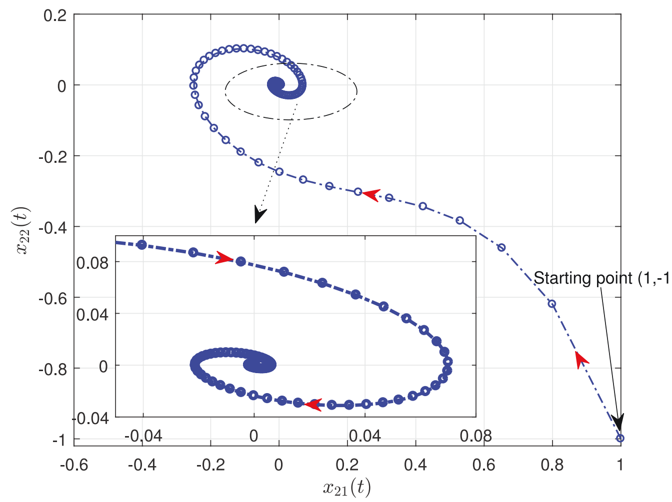

Fig. 10. States $x_{21}(t)$ and $x_{22}(t)$ of subsystem 2 in (61).

$\|\bar{e}_{2,T}\|$). Meanwhile, Fig. 6(b) illustrates the intersampling time T_s'. (*Note:* T_s' shares the similar definition as T_s. Because of the above-mentioned reason, T_s' also turns out to be constant during the last 7 s). We can see from Figs. 5(b) and 6(b) that $\min\{T_s, T_s'\} = 0.2$ s. Thus, the *Zeno behavior* is excluded. On the other hand, we can observe from Fig. 5(b) [or Fig. 6(b)] that there are actually 139 (or 181) state samples. In other words, it is only necessary to use 139 (or 181) state samples to implement the present ETC strategy. However, for implementing the corresponding time-triggering control scheme, we need 400 (or 500) state samples. Thus, we can reduce the controller updates up to 65.25% (or 63.8%), which indicates that the present ETC strategy can remarkably decrease the computational burden.

To check the validity of (22), we present Figs. 7 and 8 for subsystems 1 and 2, respectively. It can be seen from Fig. 7 that (22) holds when $t \geq 32$ s. Meanwhile, we can see from Fig. 8 that (22) holds when $t \geq 44$ s. Thus, we can choose $t_0 = \max\{32, 44\} = 44$ s in Theorem 1 to make (22) valid. Let the aforementioned weight vectors $\hat{\omega}_{c1}^{\text{final}}$ and $\hat{\omega}_{c2}^{\text{final}}$ be separately inserted into (36). Then, we can derive the approximate optimal ETC laws for auxiliary subsystems 1 and 2, which together constitute the decentralized ETC for interconnected system (61). With the obtained decentralized ETC, the closed-loop system (61) is forced to be asymptotically stable (see Figs. 9 and 10).

VI. Conclusion

We have presented a decentralized ETC strategy for a class of uncertain nonlinear interconnected systems via ACL combined with the ER technique. Rather than directly develop the decentralized ETC policy, we resort to obtaining a group of optimal ETC laws of auxiliary subsystems. It has been demonstrated that these optimal ETC laws together constitute the decentralized ETC policy. Owing to the ER technique, we can avoid the difficulty arising in the PE condition when deriving the optimal ETC laws of auxiliary subsystems.

It is worth mentioning here that a limitation in implementing the decentralized ETC scheme is that prior knowledge of the interconnected system (1) must be known. In general, prior information of plants is unavailable in engineering industries. Recently, ACL together with the ER technique has been used to design optimal controllers for nonlinear systems with totally unknown dynamics [52], [53]. Therefore, in our consecutive study, we will work on developing the decentralized ETC scheme for nonlinear interconnected systems subject to completely unknown dynamics.

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