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Corrigendum

Corrigendum to "Optimal decay rates for a chemotaxis model with logistic growth, logarithmic sensitivity and density-dependent production/consumption rate" [J. Differential Equations (2020) 1379–1411]

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In Section 5 of our previous paper [1] (the proof of Theorem 2.2), it is presumed $\varepsilon > 0$ in (5.14) therein. While it works for $\varepsilon > 0$, the approach needs to be modified for the case $\varepsilon = 0$. The purpose of this corrigendum is to provide an alternative, which is to replace the proofs of Lemma 5.2 and Lemma 5.3, and is valid for both $\varepsilon = 0$ and $\varepsilon > 0$.

From (5.11), (5.10), (5.4) and (1.13) of [1],

$$\phi(x,t) = \tilde{s}(x,t) - \bar{s} = \bar{s}\left[\exp\left(\frac{D}{\chi}\psi\left(\frac{\sqrt{\chi\mu K}}{D}x, \frac{\chi\mu K}{D}t\right)\right) - 1\right].$$

Thus by the mean value theorem,

$$|\phi(x,t)| \le C\left|\psi\left(\frac{\sqrt{\chi\mu K}}{D}x, \frac{\chi\mu K}{D}t\right)\right|\exp\left(\frac{D}{|\chi|}\|\psi\|_{L^\infty}\left(\frac{\chi\mu K}{D}t\right)\right), \tag{1}$$

where $C = \bar{s}D/|\chi|$ is a constant. Our goal here is to prove

$$\|\psi\|(t) \le C(t+1)^{-\frac{1}{4}}, \quad t \ge 2, \tag{2}$$

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with a constant $C > 0$ depending on the system parameters and initial data. Once we have proved (2), the Sobolev inequality and the estimates on v, see (4.11), (1.13), (2.3) and (2.4) in [1], imply

$$\|\psi\|_{L^\infty}(t) \le C\|\psi\|^{\frac{1}{2}}\|v\|^{\frac{1}{2}} \le C(t+1)^{-\frac{1}{2}}, \quad t \ge 2. \tag{3}$$

Substituting (2) and (3) into (1) gives us

$$\|\phi\|(t) \le C\|\psi\|\left(\frac{\chi\mu K}{D}t\right) \le C(t+1)^{-\frac{1}{4}} \tag{4}$$

for $t \ge 2D/(\chi\mu K)$. The case $t \le 2D/(\chi\mu K)$ is trivial since $\|\phi\|(t)$ is bounded by Lemma 5.1 in [1]. Equation (4) is (5.24) in [1] hence Lemma 5.3 therein is justified. Lemma 5.2 is also justified in view of (1) and (3). The rest of the proof (after Lemma 5.3) in [1] stays valid.

To prove (2) we have the following from (1.13) and (2.1) in [1]:

$$\begin{aligned}\psi_t &= 1-u+\varepsilon_2 v^2+\varepsilon_1 v_x = \frac{1}{r}[u_t+(uv)_x-u_{xx}+r(u-1)^2]+\varepsilon_2 v^2+\varepsilon_1 v_x\\ &= \beta\psi_{xx}+\sum_{i=1}^{4}R_i,\end{aligned} \tag{5}$$

where, as in (2.1) of [1], $\varepsilon_1 = \varepsilon/D$, $\varepsilon_2 = \varepsilon/\chi$, $r = aD/(\chi\mu K)$ and

$$\begin{aligned}&\beta = \frac{1}{r}+\varepsilon_1 > 0, \quad R_1 = \frac{1}{r}u_t, \quad R_2 = \frac{1}{r}[(u-1)v]_x,\\ &R_3 = -\frac{1}{r}u_{xx}, \quad R_4 = (u-1)^2+\varepsilon_2 v^2.\end{aligned} \tag{6}$$

By Duhamel's principle and (5), we write

$$\begin{aligned}&H(x,t) \equiv \frac{1}{\sqrt{4\beta\pi t}}e^{-\frac{x^2}{4\beta t}},\\ &\psi(x,t) = \int_{\mathbb{R}} H(x-y,t)\psi_0(y)\,dy + \sum_{i=1}^{4}\int_0^t\int_{\mathbb{R}} H(x-y,t-\tau)R_i(y,\tau)\,dyd\tau.\end{aligned}$$

Thus by the triangle inequality,

$$\begin{aligned}\|\psi\|(t) &\le \|\int_{\mathbb{R}} H(x-y,t)\psi_0(y)\,dy\| + \sum_{i=1}^{4}\|\int_0^t\int_{\mathbb{R}} H(x-y,t-\tau)R_i(y,\tau)\,dyd\tau\|\\ &\equiv I_0+\sum_{i=1}^{4}I_i.\end{aligned} \tag{7}$$

Noting $t \ge 2$ and by Young's inequality and (2.3) and (2.4) in [1], (6) and (7) imply the following:

$$I_0 \le \|H\|(t)\|\psi_0\|_{L^1} \le C(t+1)^{-\frac{1}{4}}, \tag{8}$$

$$\begin{aligned} I_2 &\le \int_0^t \left\| \int_{\mathbb{R}} H_x(x-y,t-\tau)\frac{1}{r}[(u-1)v](y,\tau)\,dy \right\| d\tau \\ &\le \frac{1}{r}\int_0^t \|H_x\|(t-\tau)\|(u-1)v\|_{L^1}(\tau)\,d\tau \le C\int_0^t (t-\tau)^{-\frac{3}{4}}(\|u-1\|\|v\|)(\tau)\,d\tau \\ &\le C\int_0^t (t-\tau)^{-\frac{3}{4}}(\tau+1)^{-2}\,d\tau \le C(t+1)^{-\frac{3}{4}}, \end{aligned} \tag{9}$$

$$\begin{aligned} I_3 &\le \int_0^t \left\| \int_{\mathbb{R}} H_x(x-y,t-\tau)\frac{1}{r}u_x(y,\tau)\,dy \right\| d\tau \le \frac{1}{r}\int_0^t \|H_x\|_{L^1}(t-\tau)\|u_x\|(\tau)\,d\tau \\ &\le C\int_0^t (t-\tau)^{-\frac{1}{2}}(\tau+1)^{-\frac{7}{4}}\,d\tau \le C(t+1)^{-\frac{1}{2}}, \end{aligned} \tag{10}$$

$$\begin{aligned} I_4 &\le \int_0^t \left\| \int_{\mathbb{R}} H(x-y,t-\tau)[(u-1)^2+\varepsilon_2 v^2](y,\tau)\,dy \right\| d\tau \\ &\le \int_0^t \|H\|(t-\tau)(\|(u-1)^2\|_{L^1}+\varepsilon_2\|v^2\|_{L^1})(\tau)\,d\tau \\ &\le C\int_0^t (t-\tau)^{-\frac{1}{4}}(\tau+1)^{-\frac{3}{2}}\,d\tau \le C(t+1)^{-\frac{1}{4}}. \end{aligned} \tag{11}$$

For the estimate on I_1 we have

$$\begin{aligned} I_1 \le & \left\| \int_0^{t-1}\int_{\mathbb{R}} H(x-y,t-\tau)\frac{1}{r}u_t(y,\tau)\,dyd\tau \right\| \\ & + \left\| \int_{t-1}^{t}\int_{\mathbb{R}} H(x-y,t-\tau)\frac{1}{r}u_t(y,\tau)\,dyd\tau \right\| \equiv I_{11}+I_{12}. \end{aligned} \tag{12}$$

Here I_{11} is treated by integration by parts with respect to t, and following the same strategy as in the estimate of I_3.

$$
\begin{aligned}
I_{11} \le & \left\| \frac{1}{r} \int_{\mathbb{R}} H(x-y,1)[u(y,t-1)-1]\,dy \right\| + \left\| \frac{1}{r} \int_{\mathbb{R}} H(x-y,t)[u_0(y)-1]\,dy \right\| \\
& + \left\| \frac{1}{r} \int_0^{t-1} \int_{\mathbb{R}} H_t(x-y,t-\tau)[u(y,\tau)-1]\,dy d\tau \right\| \\
\le & C\|H\|_{L^1}(1)\|u-1\|(t-1) + C\|H\|(t)\|u_0-1\|_{L^1} \\
& + C\int_0^{t-1} \|H_t\|_{L^1}(t-\tau)\|u-1\|(\tau)\,d\tau \\
\le & C(t+1)^{-\frac{5}{4}} + C(t+1)^{-\frac{1}{4}} + C\int_0^{t-1}(t-\tau)^{-1}(\tau+1)^{-\frac{5}{4}}\,d\tau \le C(t+1)^{-\frac{1}{4}}.
\end{aligned}
\tag{13}
$$

For the estimate of I_{12} we use the second equation in (2.1) of [1] to convert u_t as follows.

$$
\begin{aligned}
I_{12} \le & \int_{t-1}^{t} \left\| \int_{\mathbb{R}} H(x-y,t-\tau)\frac{1}{r}[-(uv)_x + u_{xx} + ru(1-u)](y,\tau)\,dy \right\| d\tau \\
\le & C\int_{t-1}^{t} \|H\|_{L^1}(t-\tau)(\|u_x v\| + \|uv_x\| + \|u(1-u)\|)(\tau)\,d\tau \\
& + C\int_{t-1}^{t} \|H_x\|_{L^1}(t-\tau)\|u_x\|(\tau)\,d\tau \\
\le & C\int_{t-1}^{t}(\tau+1)^{-\frac{5}{4}}\,d\tau + C\int_{t-1}^{t}(t-\tau)^{-\frac{1}{2}}(\tau+1)^{-\frac{7}{4}}\,d\tau \le C(t+1)^{-\frac{5}{4}}.
\end{aligned}
\tag{14}
$$

Combining (12)-(14) gives us

$$
I_1 \le C(t+1)^{-\frac{1}{4}}. \tag{15}
$$

Substituting (8)-(11) and (15) into (7), we arrive at (2).

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