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A PRACTICAL SAFETY FACTOR METHOD FOR RELIABILITY-BASED COMPONENT DESIGN

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ABSTRACT

Reliability-based design (RBD) identifies design variables that maintain reliability at a required level. For many routine component design jobs, RBD may not be practical as it requires nonlinear optimization and specific reliability methods, especially for those design jobs which are performed manually or with a spreadsheet. This work develops a practical approach to reliability-based component design so that the reliability target can be achieved by conducting traditional component design repeatedly using a deterministic safety factor. The new component design is based on the First Order Reliability Method, which iteratively assigns the safety factor during the design process until the reliability requirement is satisfied. In addition to a number of iterations of deterministic component design, the other additional work is the calculation of the derivatives of the design margin with respect to the random input variables. The proposed method can be used for a wide range of component design applications. For example, if a deterministic component design is performed manually or with a spreadsheet, so it the reliability-based component design. Three examples are used to demonstrate the practicality of the new design method.

Keywords: Reliability in design, Design of machine elements, Design methodologies, Algorithms

1. INTRODUCTION

Safety factors are routinely used in mechanical design to account for uncertainty [1-6]. They are particularly useful when complete distributions of random variables are unknown. When such distributions are available, the safety factor-based design can be replaced by the reliability-based design (RBD) [7-16]. RBD solves an optimization problem by identifying optimal design variables that minimize a cost-type objective function while satisfying reliability constraints. The reliability in RBD is the probability that a design requirement is satisfied [17].

There are many RBD methodologies. The most common ones employ the First Order Reliability Method (FORM) [18-20] to evaluate reliability constraints during the optimization process. FORM can not only provide a good balance between accuracy and efficiency, but also make it possible to decouple deterministic optimization from reliability analysis, thereby further reducing the computational cost. RBD has been successfully used in many applications, for example, design of composite over-wrapped tanks [21], B-pillar design for side impact [22], crashworthiness of vehicle side impact [23], and engine piston design for secondary motion [24].

The concept of safety factor, with which engineers are familiar, can also be incorporated in RBD. The safety-factor based approach for RBD [2, 5] is such a method. This method employs nonlinear optimization and FORM, calling deterministic optimization and FORM sequentially until all the reliability constraints are satisfied. During this process, partial safety factors are applied to all the input random variables.

The RBD methodologies [25-29], however, may not be applicable for many component design problems. There are several reasons for this. First, optimization may not be needed for routine mechanical component design. Design variables can be determined using safety factors by following design codes and standards. Second, many engineers are not readily equipped with knowledge of optimization, and they do not have access to nonlinear optimization algorithms and software. Third, optimization may not be performed at the component design level, but at higher levels. Fourth, many engineers perform their routine component design jobs manually or semi-manually with the help of spreadsheets or simple programming. Some of the

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design variables are chosen from tables and graphs and are integers or discrete values, and human interference is also likely needed. Optimization cannot be conveniently used for this kind of routine design. Last, RBD requires reliability analysis. If FORM is used, the Most Probable Point (MPP) [30-32] should be found. The MPP search itself is also an optimization problem.

It is therefore desirable to design a practical RBD approach that relies on only the routine deterministic component design. One approach, which satisfies this requirement is the mechanical design approach using the First Order Second Moment (FOSM) method [33-38]. This method can find design variables for a given reliability target with only the minimal extra work: the calculation of derivatives of a response variable with respect to input random variables. It is therefore very practical and can be used for routine component design. The accuracy of the reliability produced by the design variables, however, may be poor. This means that the designed reliability may be far away from the required reliability. The reason is that FOSM uses a first order approximation around the means of input random variables and only the first two moments (means and standard deviations).

This work develops a practical approach to reliability-based component design, which does not need to specify a cost-type objective. And it uses FORM and produces higher accuracy than FOSM. During the design process, the method iteratively updates a safety factor for the deterministic component design until the reliability requirement is satisfied. In addition to a number of iterations of the deterministic component design, the only additional work is the calculation of the derivatives of the design margin with respect to the random input variables. The major advantage of this approach is that engineers can use it in the same way as they perform their deterministic routine component design, either manually or by other means.

Reliability-based design and the safety factor are reviewed in Section 2, and the new component design approach is presented in Section 3, followed by three examples in Section 4. Conclusions are given in Section 5.

2. Review of RBD and Safety Factor

Reliability-based design (RBD) is a design methodology that minimizes a cost-type objective and maintains reliability requirements when uncertainty (randomness) presents. Uncertainty can also be accommodated deterministically by using a safety factor. Both of the design methodologies are briefly reviewed here.

2.1 Reliability-based design

A typical RBD model is given by

$$\begin{cases} \underset{\boldsymbol{d}}{\text{Min}} & f(\boldsymbol{d}) \\ s.t. & \Pr\{G_i(\boldsymbol{d},\boldsymbol{X}) > 0\} \geq [R_i], i = 1,2,\dots,n_g \\ & \boldsymbol{d}^L \leq \boldsymbol{d} \leq \boldsymbol{d}^U \end{cases} \tag{1}$$

In the above model, $\boldsymbol{d}$ is the vector of design variables with their lower and upper bounds $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$, respectively. $\boldsymbol{X} = (X_1, X_2, \dots, X_n)$ is the vector of random variables. $f(\cdot)$ is a cost-type objective function, and $G_i(\boldsymbol{d},\boldsymbol{X})$ is a limit-state function. The requirement is $G_i(\boldsymbol{d},\boldsymbol{X}) > 0$, and the probability of satisfying the requirement is called reliability, denoted by R_i; namely

$$R_i = \Pr\{G_i(\boldsymbol{d},\boldsymbol{X}) > 0\} \tag{2}$$

The constraint associated with $G_i(\boldsymbol{d},\boldsymbol{X})$ is that R_i should be greater than or equal to the desired reliability $[R_i]$ or $1 - [p_{f_i}]$, where $[p_{f_i}]$ is the allowable probability of failure.

The reliability R_i is obtained by

$$R_i = \Pr\{G_i(\boldsymbol{d},\boldsymbol{X}) > 0\} = \int_{G_i(\boldsymbol{d},\boldsymbol{X})>0} f_{\boldsymbol{x}}(\boldsymbol{x})\, d\boldsymbol{x} \tag{3}$$

where $f_{\boldsymbol{x}}(\boldsymbol{x})$ is the joint probability density function (PDF) of $\boldsymbol{X}$. The First Order Reliability Method (FORM) is commonly used to calculate R_i. FORM first transforms $\boldsymbol{X}$ into independent standard normal variables $\boldsymbol{U}$ with $\boldsymbol{X} = T(\boldsymbol{U})$ [31, 32], where $T(\cdot)$ denotes the transformation operation. The limit-state function then becomes

$$G(\boldsymbol{d},\boldsymbol{X}) = G(\boldsymbol{d}, T(\boldsymbol{U})) \tag{4}$$

Then R_i is approximated by

$$R_i = \Phi(\beta) \tag{5}$$

where β is the reliability index, which is the shortest distance from the origin of the U-space to the limit-state contour $g(\boldsymbol{d}, T(\boldsymbol{U})) = 0$. The distance is obtained by solving the following optimization model:

$$\begin{cases} \text{Min } \|\boldsymbol{u}\| \\ \text{s.t. } G(\boldsymbol{d}, T(\boldsymbol{u})) = 0 \end{cases} \tag{6}$$

The solution $\boldsymbol{u}^*$ is called the most probable point (MPP), whose norm is the reliability index.

$$\beta = \|\boldsymbol{u}^*\| \tag{7}$$

where $\|\cdot\|$ stands for the norm of a vector.

2.2 Safety factor

A safety factor is the ratio of the maximum mechanical strength divided by the maximum load employed to specify component or structure. For example, if the yield strength of the component is S and the maximum load carried by the component is L, for a yield failure mode, the safety factor is given by

$$S_F = \frac{S}{L} \tag{8}$$

As indicated by Eq. (8), the safety factor is a random variable if both S and L are random. We call it a random safety factor. It must be greater than 1. The design task is to identify design variables $\boldsymbol{d}$ so that the random safety factor is greater than 1 or the following design function holds:

$$g(\boldsymbol{d}) = \frac{S}{S_F} - L(\boldsymbol{d}) > 0 \tag{9}$$

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The strength and load used in this work are in a general sense. A general strength could be anything that is related to the capacity of a component, for example, a yield strength, permitted deflection, or required fatigue life; a general load could be anything that related to demand of the component or the loading acting on or generated in the component, such a normal stress, force, deflection, and fatigue damage accumulation.

3. A Practical Method for Reliability-Based Component Design

As discussed in Section 1, RBD requires a cost-type function in its design model and an optimizer to solve the model. A cost-type function may not exist and optimization may not be needed for a regular component design. In the routine mechanical component design process, engineers may perform their design job with the help of computer programs such as spreadsheets or even manually while following professional or corporation design codes and procedures. In this case, it is difficult to perform RBD rigorously even though distributions of random variables are available.

This work develops a practical reliability-based component design method using FORM and the safety factor. The method called reliability-based component design with safety factor, or RBD/SF for short. For a given reliability target, RBD/SF allows design engineers to update the safety factor by repeatedly performing their routine deterministic design method until the reliability target is reached. Engineers can therefore quickly obtain a feasible design solution with satisfied reliability. Next, we use an example in Section 3.1 to highlight the deterministic design procedure then discuss how it can be extended to achieve the required reliability by RBD/SF in Section 3.2. The implementation procedure of RBD/SF is given in Section 3.3.

3.1 An example of traditional deterministic design

A force $P = 1.2$ kN is applied to a cantilever bar as shown in Fig. 1. A failure occurs when the von Mises stress σ' is greater than the yield strength s_y. Then the factor of safety s_y/σ' should be greater than 1. The deterministic design function is then given by

$$g(\boldsymbol{d}) = \frac{s_y}{S_F} - \sigma' = \frac{s_y}{S_F} - \sqrt{\left[\frac{32P(a+b)}{\pi d^3}\right]^2 + 3\left(\frac{16Pe}{\pi d^3}\right)^2} \quad (10)$$

where σ' is

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{zx}^2} \quad (11)$$

in which

$$\sigma_x = \frac{32P(a+b)}{\pi d^3} \quad (12)$$

$$\tau_{zx} = \frac{16Pe}{\pi d^3} \quad (13)$$

The design variable is the diameter of the shaft d. The required safety factor S_F is 2. Given $a = 300$ mm, $b = 50$ mm, $e = 350$ mm, and $s_y = 530$ MPa, the design variable can be obtained by solving the design function $\frac{s_y}{S_F} - \sigma' > 0$, or $G(\boldsymbol{d}) = s_y - S_F\sigma'$, which yields $d > 32.77$ mm. The designer may finally choose $d = 33$ mm. This example demonstrates that the deterministic design does not need any optimization. Only one design variable is involved here, but there may be more design variables in a general problem.

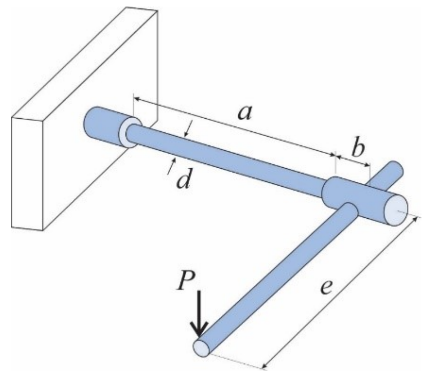

Fig. 1 A cantilever bar

3.2 The RBD/SF method

The proposed method is based on FORM. The random variables and their nominal values (means) are $\boldsymbol{X} = (X_1, X_2, \dots, X_n)$ and $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$, respectively. Let the cumulative distribution function of (CDF) X_i be $F_i(X_i)$, $i = 1,2,\dots,n$, and assume all the variables in $\boldsymbol{X}$ are independent. The general strength of the component is S, which is the first element of $\boldsymbol{X}$, namely, $X_1 = S$. S could be a yield strength, permissible deflection, or capacity. Let the rest of $\boldsymbol{X}$ be $\boldsymbol{Y} = (X_2, X_3, \dots, X_n)$. The general load L of the component is given by $L(\boldsymbol{d}, \boldsymbol{Y})$, where $\boldsymbol{d}$ and $\boldsymbol{Y}$ are vectors to represent multiple design variables and parameters, respectively. The general load could be a force, moment, and stress. For the example above, the general strength is the yield strength; namely, $S = S_y$; and the general load is von Mises stress σ', namely, $L = \sigma'$, which is a function of the design variable or the diameter d.

If we use the nominal values of general strength and general load to calculate the safety factor given in Eq. (8), we get a deterministic safety factor S_F.

$$S_F = \frac{s}{l} = \frac{s}{L(\boldsymbol{d}, \boldsymbol{y})} \quad (14)$$

where s and l are nominal values of the strength and load, respectively. We simply call S_F a safety factor. Note that the nominal value of a random variable is the median of a random variable or its mean value if its distribution is symmetric. The deterministic design function is $g(\boldsymbol{d}) = \frac{s}{S_F} - L(\boldsymbol{d}) > 0$ as already been given in Eq. (9).

The design margin, or the difference between the general strength and general load, is given by

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$$G(\boldsymbol{d},\boldsymbol{X}) = S - L(\boldsymbol{d},\boldsymbol{Y}) > 0 \tag{15}$$

As we have discussed, the probability of satisfying a nonnegative design margin $R = \Pr\{G(\boldsymbol{d},\boldsymbol{X}) > 0\}$ is the component reliability. If the required reliability is $[R]$, from Eq. (5), the reliability index is

$$\beta = \Phi^{-1}([R]) \tag{16}$$

Many studies [2, 5, 8, 16, 39] have shown that the reliability requirement $R = \Pr\{G(\boldsymbol{d},\boldsymbol{X}) > 0\} > [R]$ is equivalent to

$$G(\boldsymbol{d},\boldsymbol{x}^*) = S^* - L(\boldsymbol{d},\boldsymbol{y}^*) > 0 \tag{17}$$

where $\boldsymbol{x}^* = (S^*, \boldsymbol{y}^*)$ is the MPP in the X-space, and it is transformed from the MPP $\boldsymbol{u}^* = (u_i^*)_{i=1,n}$ in the U-space. We rewrite Eq. (17) by

$$\frac{S^*}{s}\frac{s}{L(\boldsymbol{d},\boldsymbol{y})} - \frac{L(\boldsymbol{d},\boldsymbol{y}^*)}{L(\boldsymbol{d},\boldsymbol{y})} > 0 \tag{18}$$

The X- to U space transformation is given by

$$F_i(x_i^*) = \Phi(u_i^*) \tag{19}$$

Then

$$x_i^* = F_i^{-1}[\Phi(u_i^*)] = T(u_i^*) \tag{20}$$

It can be proved from the optimization model in Eq. (7) that at the MPP [40]

$$u_i^* = -\beta\alpha_i \tag{21}$$

where

$$\alpha_i = \frac{\dfrac{\partial G(\boldsymbol{d}, T(\boldsymbol{u}^*))}{\partial u_i^*}}{\|\nabla_G\|} \tag{22}$$

where ∇_G is the gradient of $G(\cdot)$ and is given by

$$\nabla_G = \left(\frac{\partial G(\boldsymbol{d}, T(\boldsymbol{u}^*))}{\partial u_i^*}\right)_{i=1,\dots,n} \tag{23}$$

$$\frac{\partial G(\boldsymbol{d}, T(\boldsymbol{u}^*))}{\partial u_i^*} = \frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*))}{\partial x_i^*}\frac{dx_i^*}{du_i^*} \tag{24}$$

From Eq. (20)

$$w_i = \frac{dx_i^*}{du_i^*} = \frac{\phi\left(\Phi^{-1}(F_i(x_i^*))\right)}{f_i(x_i^*)} \tag{25}$$

where $\phi(\cdot)$ and $f_i(\cdot)$ are the probability density function (PDF) of a standard normal variable and X_i, respectively. For commonly used distributions, w_i is listed in the appendix.

$$\frac{\partial G(\boldsymbol{d}, T(\boldsymbol{u}^*))}{\partial u_i^*} = w_i\frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_i^*} \tag{26}$$

$$\nabla_G = \left(w_i\frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_i^*}\right)_{i=1,\dots,n}$$

$$= \left(w_1\frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_1^*}, w_2\frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_2^*}, \cdots, w_n\frac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_n^*}\right) \tag{27}$$

From Eq. (22), at the MPP $\boldsymbol{x}^*$

$$\alpha_i = \frac{w_i\dfrac{\partial G(\boldsymbol{d},\boldsymbol{x}^*)}{\partial x_i^*}}{\|\nabla_G\|} \tag{28}$$

By substituting α_i into Eq. (21), we obtain the value of u_i^*. Then, we can obtain x_i^* by substituting u_i^* into Eq. (20). Let

$$\lambda_S = \frac{S^*}{s} \tag{29}$$

and

$$\lambda_L = \frac{L(\boldsymbol{d},\boldsymbol{y}^*)}{L(\boldsymbol{d},\boldsymbol{y})} \tag{30}$$

Substituting Eqs. (29) and (30) into Eq. (18), we have

$$\lambda_S S_F - \lambda_L > 0 \tag{31}$$

By solving the inequality equation, we have the range for design variables. Once we specify the design variables, the safety factor for the given design is

$$S_F = \frac{\lambda_L}{\lambda_S} \tag{32}$$

To design the component with the reliability target, we can then use the deterministic design function, which is rewritten here.

$$g(\boldsymbol{d}) = \frac{1}{S_F}s - L(\boldsymbol{d}) > 0 \tag{33}$$

In the above deterministic design function, only the nominal values $\boldsymbol{y}$ of $\boldsymbol{Y}$ are involved. No random variables appear in the function. If the MPP $\boldsymbol{x}^*$ is given, solving for $\boldsymbol{d}$ needs just one deterministic design as discussed in Section 3.1. We have therefore converted a reliability-based design into a deterministic design. To determine final design variables $\boldsymbol{d}$, we need to repeat this process iteratively since the MPP $\boldsymbol{x}^*$ depends on $\boldsymbol{d}$. The true MPP is found upon the convergence of the design. The result of the true MPP is the same as the result solved by FORM.

Note that the proposed approach relies on the MPP, and the MPP search, which can be considered as an optimization problem, is performed implicitly. The approach, however, does not require an explicit optimization model and is therefore easy to implement. The proposed approach is not optimization, and its execution may not be automatic, totally depending on how the deterministic design is performed.

3.3 The procedure

The design margin function $G(\boldsymbol{d},\boldsymbol{X}) = S - L(\boldsymbol{d},\boldsymbol{Y})$ and deterministic design function $g(\boldsymbol{d}) = \frac{1}{S_F}s - L(\boldsymbol{d})$ are usually nonlinear functions. As the safety factor S_F depends on $\boldsymbol{d}$, directly solving for $\boldsymbol{d}$ from $g(\boldsymbol{d}) > 0$ requires a numerical

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procedure, which diminishes the practicality of the design. We develop a straightforward procedure so that the design variables can be obtained iteratively by performing deterministic design a number of times. The procedure is discussed below.

Initial design

1) Perform the initial deterministic design by using $S_F = 1$ or other value of $S_F > 1$. From $g(\boldsymbol{d}) = \frac{1}{S_F}s - L(\boldsymbol{d}) > 0$, initial deterministic design variables $\boldsymbol{d}$ are obtained. Then the initial design is completed.
Since the safety factor used here may not satisfy the reliability requirement, it will be updated iteratively next. To prepare for the iterations, set $\boldsymbol{d}$ to be the current design, and set the MPP $\boldsymbol{x}^*$ to be the means of all random input variables.

Iterative design

2) At the current design point $\boldsymbol{d}$ and $\boldsymbol{x}^*$, calculate the gradient of the design margin function $G(\boldsymbol{d}, \boldsymbol{X})$ and update the MPP following the procedures in Fig. 3.
3) Update λ_S and λ_L using Eqs. (29) and (30), and solve for the safety factor S_F using Eq. (32).
4) Solve for new design point $\boldsymbol{d}$ by plugging the new S_F into the deterministic design function $g(\boldsymbol{d}) = \frac{1}{S_F}s - L(\boldsymbol{d})$.
5) Check convergence. The criterion is that the distance of the design point $\boldsymbol{d}$ between two consecutive designs is sufficiently small, which is given by

$$\frac{\left\|\boldsymbol{d}_{current} - \boldsymbol{d}_{previous}\right\|}{\boldsymbol{d}_{current}} \leq \varepsilon \tag{34}$$

where ε is a small positive quantity. $\varepsilon = 0.1\%$, $\varepsilon = 0.01\%$, or other values could be used. If convergence is not achieved, go to step 2); otherwise, go to step 6).

Final design

6) Based on $\boldsymbol{d}$ obtained, choose appropriate final design variables.

The MPP is updated after a new design $\boldsymbol{d}$ is identified. $\boldsymbol{u}^*$ obtained during each iteration before convergence is not the true MPP for a given design $\boldsymbol{d}$. Upon convergence of the entire design process, $\boldsymbol{u}^*$ will be the true MPP for the final design. This will not only save design time but also guarantee the target reliability is achieved.

The flowcharts of the proposed approach are provided in Figs. 2 and 3.

4. Examples

In this section, we provide three examples. Example 1 is the shaft design problem we have discussed previously in Sec. 3.1. All details of using RBD/SF are given so that an interested reader could easily repeat the process and reproduce the result. Example 2 has more than one discrete design variable selected from the preferred values in a table. This example shows the capability and feasibility of RBD/SF for solving practical design problems.

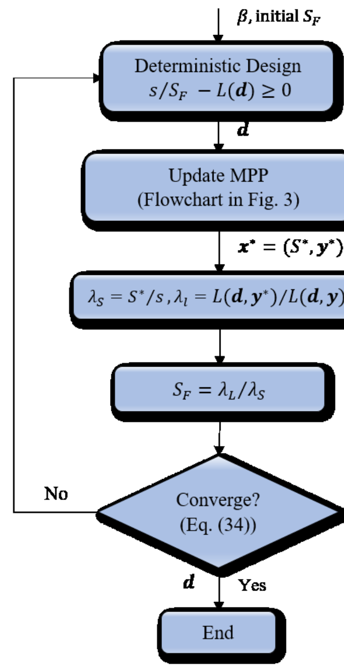

Fig. 2 Flowchart of reliability-based component design

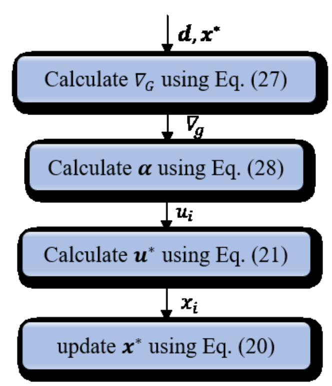

Fig. 3 Flowchart of MPP updating

Example 3 demonstrates that RBD/SF can also be used to design a component with multiple failure modes.

4.1 A shaft design

A cantilever shaft is shown in Fig. 1. The design margin function has been given in Eq. (10). The yield strength and the applied force follow normal distributions $S_y \sim N(530, 20^2)$ MPa and $P \sim N(1200, 100^2)$ N, respectively. S_y and P are independent. The random variables are therefore $\boldsymbol{X} = (S_y, P)$. All other parameters have also been given in Section 3.1. The design task is to determine the diameter of the shaft d so that

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the reliability of the shaft is no less than $[R] = 0.9999$. The design margin function is

$$G(\boldsymbol{d}, \boldsymbol{X}) = S_y - L(\boldsymbol{d}, \boldsymbol{Y}) = \sqrt{\left[\frac{32P(a+b)}{\pi d^3}\right]^2 + 3\left(\frac{16Pe}{\pi d^3}\right)^2} \quad (35)$$

And the deterministic design function in Eq. (10) is rewritten as

$$g(\boldsymbol{d}) = \frac{s_y}{s_F} - L(\boldsymbol{d}) = \sqrt{\left[\frac{32p(a+b)}{\pi d^3}\right]^2 + 3\left(\frac{16pe}{\pi d^3}\right)^2}$$

where p is the nominal value of P.

Design process

Determine the reliability index

$$\beta = \Phi^{-1}([\mathrm{R}]) = \Phi^{-1}([0.9999]) = 3.7190$$

Derive the gradient

$$\nabla_G = \left(w_i \frac{\partial G(\boldsymbol{d}, \boldsymbol{X})}{\partial X_i}\right)_{i=1,\ldots,n} = \left(w_1 \frac{\partial G}{\partial X_1}, w_2 \frac{\partial G}{\partial X_2}\right)$$

$$\frac{\partial G}{\partial X_1} = \frac{\partial G}{\partial S_y} = 1$$

$$\frac{\partial G}{\partial X_2} = \frac{\partial G}{\partial P} = -\frac{16\sqrt{4(a+b)^2 + 3e^2}}{\pi d^3}$$

From Table A1, we have

$$w_1 = \sigma_1 = 20 \text{ MPa}, w_2 = \sigma_2 = 1.2 \text{ kN}$$

Iteration 1

Start from the deterministic design by setting $S_F = 1.0$. Then plug the nominal values of S_y and P, which are $s_y = 530$ MPa and $p = 1200$ N, respectively, into

$$g(\boldsymbol{d}) = \frac{s_y}{S_F} - \sqrt{\left[\frac{32p(a+b)}{\pi d^3}\right]^2 + 3\left(\frac{16pe}{\pi d^3}\right)^2} > 0$$

We have

$$530(10)^6 - \sqrt{A_1^2 + 3B_1^2} > 0$$

where

$$A_1 = \frac{32(1.2)(10)^3(300+50)(10)^{-3}}{\pi d^3}$$

$$B_1 = \frac{16(1.2)(10)^3(350)(10)^{-3}}{\pi d^3}$$

which yields the initial design $d > 22.02$ mm. Substituting d into A_1 and B_1, the general load (normal stress) at the design point $d = 22.02$ mm is

$$L(\boldsymbol{d}, \boldsymbol{y}) = \sqrt{A_1^2 + 3B_1^2} = 530.0 \text{ MPa}$$

Iteration 2

At $d = 22.02$ mm, using Eq. (27) we obtain the gradient

$$\nabla_G = \left(w_1 \frac{\partial G}{\partial x_1}, w_2 \frac{\partial G}{\partial x_2}\right) = (2.0 \times 10^7, -4.4167 \times 10^7)$$

$$\boldsymbol{\alpha} = (\alpha_1, \alpha_2) = \left(\frac{w_1 \frac{\partial G}{\partial x_1}}{\|\nabla_G\|}, \frac{w_2 \frac{\partial G}{\partial x_2}}{\|\nabla_G\|}\right) = (0.4125, -0.9110)$$

$$\boldsymbol{u}^* = (u_1^*, u_2^*) = (-\beta\alpha_1, -\beta\alpha_2) = (-1.5341, 3.3879)$$

$$\boldsymbol{x}^* = (x_1^*, x_2^*) = (F_1^{-1}[\Phi(u_1)],\ F_2^{-1}[\Phi(u_2)]) = (499.3176 \text{ MPa}, 1.5388 \text{ kN})$$

and the general strength $S^* = x_1^* = 499.3176$ MPa.

$$\lambda_S = \frac{S^*}{s} = \frac{499.3176}{530} = 0.9421$$

The general load at $\boldsymbol{y}^* = (x_2^*)$ is

$$L(\boldsymbol{d}, \boldsymbol{y}^*) = \sqrt{A_2^2 + 3B_2^2} = 679.6302 \text{ MPa}$$

where

$$A_2 = \frac{32(1.5388)(10)^3(300+50)(10)^{-3}}{\pi(22.02 \times 10^{-3})^3}$$

$$B_2 = \frac{16(1.5388)(10)^3(350)(10)^{-3}}{\pi(22.02 \times 10^{-3})^3}$$

$$\lambda_L = \frac{L(\boldsymbol{d}, \boldsymbol{y}^*)}{L(\boldsymbol{d}, \boldsymbol{y})} = \frac{679.6302}{530.0} = 1.2813$$

Then the updated safety factor is

$$S_F = \frac{\lambda_L}{\lambda_S} = \frac{1.2813}{0.9421} = 1.3611$$

Plugging the new S_F into the deterministic design function in Eq. (33), we have

$$\frac{530(10)^6}{1.3611} - \sqrt{C_2^2 + 3D_2^2} > 0$$

where

$$C_2 = \frac{32(1.2)(10)^3(300+50)(10)^{-3}}{\pi d^3}$$

$$D_2 = \frac{16(1.2)(10)^3(350)(10)^{-3}}{\pi d^3}$$

which yields

$$d > 24.40 \text{ mm}$$

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At $d = 24.40$ mm, $L(\boldsymbol{d}, \boldsymbol{y}) = 389.3857$ MPa. Check the convergence using Eq. (34) and we obtain

$$\varepsilon = \frac{|S_{F,current} - S_{F,previous}|}{S_{F,previous}} = \frac{|1.3611 - 1.0|}{1.0} = 36.11\%$$

It is greater than the tolerance 0.01%, and the process continues.

Iteration 3

At $d = 24.40$ mm, we have

$$\nabla_G = \left(w_1 \frac{\partial G}{\partial x_1^*}, w_2 \frac{\partial G}{\partial x_2^*}\right) = (2.0 \times 10^7, -3.2449 \times 10^7)$$

$$\boldsymbol{\alpha} = (\alpha_1, \alpha_2) = \left(\frac{w_1 \frac{\partial G}{\partial x_1^*}}{\|\nabla_G\|}, \frac{w_2 \frac{\partial G}{\partial x_2^*}}{\|\nabla_G\|}\right) = (0.5247, -0.8513)$$

$$\boldsymbol{u}^* = (u_1^*, u_2^*) = (-\beta\alpha_1, -\beta\alpha_2) = (-1.9514, 3.1660)$$

$$\boldsymbol{x}^* = (x_1^*, x_2^*) = (F_1^{-1}[\Phi(u_1)], F_2^{-1}[\Phi(u_2)]) = (490.9729 \text{ MPa}, 1.5166 \text{ kN})$$

and the general strength $S^* = x_1^* = 490.9729$ MPa.

$$\lambda_S = \frac{S^*}{s} = \frac{490.9729}{530} = 0.9264$$

The general load at $\boldsymbol{y}^* = (x_2^*)$ is

$$L(\boldsymbol{d}, \boldsymbol{y}^*) = \sqrt{A_3^2 + 3B_3^2} = 492.1173 \text{ MPa}$$

where

$$A_3 = \frac{32(1.5166)(10)^3(300 + 50)(10)^{-3}}{\pi(24.40 \times 10^{-3})^3}$$

$$B_3 = \frac{16(1.5166)(10)^3(350)(10)^{-3}}{\pi(24.40 \times 10^{-3})^3}$$

$$\lambda_L = \frac{L(\boldsymbol{d}, \boldsymbol{y}^*)}{L(\boldsymbol{d}, \boldsymbol{y})} = \frac{492.1173}{389.3857} = 1.2638$$

Then the updated safety factor is

$$S_F = \frac{\lambda_L}{\lambda_S} = \frac{1.2638}{0.9264} = 1.3643$$

Plugging the new S_F into the deterministic limit-state function in Eq. (33), we have

$$\frac{530(10)^6}{1.3643} - \sqrt{C_3^2 + 3D_3^2} > 0$$

where

$$C_3 = \frac{32(1.2)(10)^3(300 + 50)(10)^{-3}}{\pi d^3}$$

$$D_3 = \frac{16(1.2)(10)^3(350)(10)^{-3}}{\pi d^3}$$

which yields

$$d > 24.42 \text{ mm}$$

Check the convergence using Eq. (34) and we obtain

$$\varepsilon = \frac{|S_{F,current} - S_{F,previous}|}{S_{F,previous}} = \frac{|1.3643 - 1.3611|}{1.3611} = 0.22\%$$

which is greater than the convergence tolerance 0.01%. After one more iteration, the process converges and the final design variable is $d > 24.42$ mm. This design will meet the reliability target 0.9999, which is equivalent to a probability of failure 10^{-4}. To verify this, Monte Carlo simulation (MCS) is performed with a large sample size of 10^8. The probability of failure produced by MCS is 1.01×10^{-4}, very close to the required probability of failure. For a manufacturability consideration, we can set the final design $d = 24.5$ mm, which ensures higher reliability than the required one. The entire design process is summarized in Table 1.

Table 1 Design Process of the Shaft Design

Iteration	∇_G	S_F	d (mm)	ε(%)
1	–	1.0	22.6	–
2	$(2.0 \times 10^7, -4.4167 \times 10^7)$	1.3611	25.07	36.11
3	$(2.0 \times 10^7, -3.2449 \times 10^7)$	1.3643	25.09	0.22
4	$(2.0 \times 10^7, -3.2373 \times 10^7)$	1.3643	25.09	0.00

4.2 Reliability-based design for a cantilever tube

The design task is to select a tube (Fig. 4) so that it can withstand random forces F and P; and a random torque T, with the reliability greater than or equal to $[R] = 0.99998$. The random variables are $X = (S_y, P, F, T, L)$, where L is the length of the tube, and S_y is the yield strength of the material. All the random variables are independent, and their distributions are given in Table 2. The design variables are $d = (d_0, t)$, which can be chosen only from the following list of preferred sizes for $d_0 \times t$ (mm): 12×2, 16×2, 16×3, 20×4, 24×4, 25×5, 30×4, 30×5, 42×4, 42×5, 50×4, 50×5.

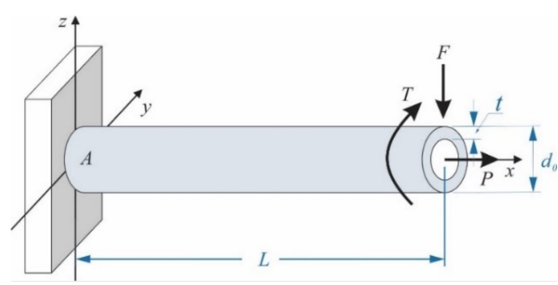

Fig. 4 A cantilever tube

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This problem is more general than Example 1 because it involves a non-normally distributed random variable and more than one design variable, and design variables are discrete.

Table 2 Distributions of the random variables in Example 2

Random Variable	Distribution	Mean	Standard Deviation
S_y (MPa)	Normal	250	20
P (N)	Normal	80000	9000
F (N)	Lognormal	1500	100
T (N·m)	Normal	75	10
L (m)	Normal	0.15	0.001

Design Process

The design margin is defined with

$$G(\boldsymbol{d},\boldsymbol{X}) = S_y - L(\boldsymbol{d},\boldsymbol{Y}) = S_y - \sqrt{\sigma_X^2 + 3\tau_{ZX}^2} \quad (36)$$

And the deterministic design function is

$$g(\boldsymbol{d}) = \frac{s_y}{S_F} - L(\boldsymbol{d}) = \frac{s_y}{S_F} - \sqrt{\sigma_x^2 + 3\tau_{zx}^2} \quad (37)$$

where s_y is the mean value of S_y. The random normal stress and its mean value are given by

$$\sigma_X = \frac{P}{\frac{\pi(d_0^2 - (d_0-2t)^2)}{4}} + \frac{FL\left(\frac{d_0}{2}\right)}{\frac{\pi(d_0^4 - (d_0-2t)^4)}{64}} \quad (38)$$

$$\sigma_x = \frac{p}{\frac{\pi(d_0^2 - (d_0-2t)^2)}{4}} + \frac{fl\left(\frac{d_0}{2}\right)}{\frac{\pi(d_0^4 - (d_0-2t)^4)}{64}} \quad (39)$$

where p, f, l are the means of P, F, L. The random shear stress and its mean value are given by

$$\tau_{XZ} = \frac{T\left(\frac{d_0}{2}\right)}{\frac{\pi(d_0^4 - (d_0-2t)^4)}{32}} \quad (40)$$

$$\tau_{zx} = \frac{t\left(\frac{d_0}{2}\right)}{\frac{\pi(d_0^4 - (d_0-2t)^4)}{32}} \quad (41)$$

where t is the mean value of T. Plugging Eqs. (38) and (40) into Eq. (36), we obtain

$$G(\boldsymbol{d},\boldsymbol{X}) = S_y - \frac{1}{\pi}\sqrt{\left(\frac{4P}{d_0^2-(d_0-2t)^2} + \frac{32FLd_0}{d_0^4-(d_0-2t)^4}\right)^2 + 3\left(\frac{16Td_0}{d_0^4-(d_0-2t)^4}\right)^2} \quad (42)$$

Determine the reliability index

$$\beta = \Phi^{-1}([R]) = \Phi^{-1}([0.99998]) = 4.1074$$

Derive the gradient

$$\nabla_G = \left(w_i \frac{\partial G(\boldsymbol{d},\boldsymbol{X})}{\partial X_i}\right)_{i=1,\dots,n} = \left(w_1\frac{\partial G}{\partial X_1},\ w_2\frac{\partial G}{\partial X_2},\ w_3\frac{\partial G}{\partial X_3},\ w_4\frac{\partial G}{\partial X_4},\ w_5\frac{\partial G}{\partial X_5}\right)$$

$$\frac{\partial G}{\partial X_1} = \frac{dG}{dS_y} = 1$$

$$\frac{\partial G}{\partial X_2} = \frac{dG}{dP} = -\frac{4P(d_0^2 + (d_0-2t)^2) + 32d_0FL}{A\pi(d_0^2 - (d_0-2t)^2)}$$

$$\frac{\partial G}{\partial X_3} = \frac{dG}{dF} = -\frac{8d_0L((d_0^2 + (d_0-2t)^2) + 32d_0FL)}{A\pi(d_0^4 - (d_0-2t)^4)}$$

$$\frac{\partial G}{\partial X_4} = \frac{dG}{dT} = -\frac{192Td_0^2}{A\pi(d_0^4 - (d_0-2t)^4)}$$

$$\frac{\partial G}{\partial X_5} = \frac{dG}{dL} = -\frac{8d_0F((d_0^2 + (d_0-2t)^2) + 32d_0FL)}{A\pi\left({d_0}^4 - (d_0-2t)^4\right)}$$

where

$$A = \sqrt{(P(d_0^2 + (d_0-2t)^2) + 8d_0FL)^2 + 48T^2d_0^2}$$

From Table A1, we have

$$\boldsymbol{w} = (w, w_2, w_3, w_4, w_5) = (2\times10^7, 9\times10^3, 6.6\times10^{-2}, 10, 1\times10^{-3})$$

Iteration 1

Start from the deterministic design by setting $S_F = 1.0$. Plugging the mean values into Eq. (37), we find that the smallest size for $g(\boldsymbol{d}) > 0$ is 42×4; namely, $d_0 = 42$ mm, $t = 4$ mm. At this design point, the general load (normal stress) is 222.3 MPa.

Iteration 2

Update the MPP following the procedures in Fig. 3, we have

$$\boldsymbol{x}^* = (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = (190.15\text{ MPa}, 105317.11\text{ N}, 1496.71\text{ N}, 75.22\text{ N}\cdot\text{m}, 0.15\text{ m})$$

The general strength is $S^* = x_1^* = 190.15$ MPa. Update λ_S by Eq. (29), we have $\lambda_S = 0.7606$. The general load at $\boldsymbol{y}^*$ is 275.12 MPa. Updating λ_L by Eq. (30), we have $\lambda_L = 1.2376$. Then the updated safety factor is 1.6271 by Eq. (32). Plugging the new S_F into the deterministic design function in Eq. (37), we have the new design point 50×5; namely, $d_0 = 50$ mm, $t = 5$ mm. Check the convergence using Eq. (34), and we obtain $\varepsilon = 62.71\%$, which is larger than the tolerance 0.01%, and the process continues.

After two more iterations, the process converges and the final design variable is $\boldsymbol{d} = 50\times5$ mm. Since the final solution $d_0 = 50$ mm, $t = 5$ mm satisfies $Y > 0$, we expect

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the actual reliability is greater than the require reliability $[R] = 0.99998$, or the actual probability of failure is less than 2×10^{-5}. This is confirmed by MCS, which produces 4.6700×10^{-6}, less than 2×10^{-5}. The calculations are summarized in Table 3.

Table 3 Design Process of the Cantilever Tube

Iteration	∇_G	S_F	$d_0 \times t$ (mm)	ε (%)
1	-	1	42×4	-
2	$(2.00 \times 10^7, -1.88 \times 10^7, -2.40 \times 10^3, -1.47 \times 10^5, -3.61 \times 10^5)$	1.6271	50×5	62.71
3	$(2.00 \times 10^7, -1.27 \times 10^7, -1.38 \times 10^3, -7.42 \times 10^4, -2.07 \times 10^5)$	1.6513	50×5	1.49
4	$(2.00 \times 10^7, -1.27 \times 10^7, -1.38 \times 10^3, -7.42 \times 10^4, -2.07 \times 10^5)$	1.6513	50×5	0.00

4.3 A key design

The task is to design a key (Fig. 5) for a shaft with a diameter of 22 mm so that its hub can withstand compression and shearing stress induced by the transmission power P. The target reliability is $[R] = 0.999999$. The width and height are determined given by shaft diameter according ANSI Standard, which are 8 mm and 7 mm, respectively. The random variables are $\boldsymbol{x} = (S_y, S_{sy}, P, \omega)$, where S_y is the compression (crushing) yield strength of the material, $S_{sy} = 0.577 S_y$ is the shearing strength of the material, P is the transmission power, and ω is the angular velocity of the shaft. All the random variables are independent, and their distributions are given in Table 4. The design variable E is the length of the key, namely, $\boldsymbol{d} = (E)$, which should be less than 30 mm because the diameter of the shaft is 22 mm.

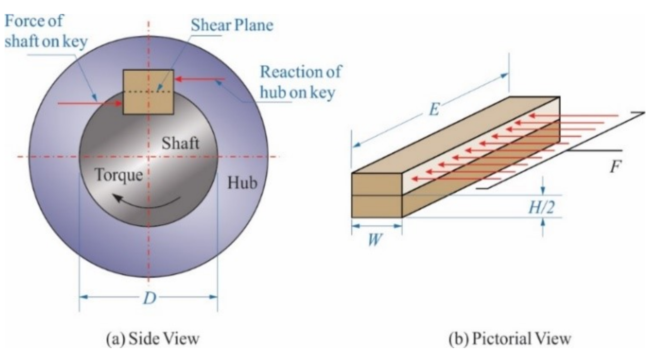

Fig. 5 A key of shaft-hub gear

Table 4 Distributions of the random variables in Example 3

Random Variable	Distribution	Mean	Standard Deviation
S_y (MPa)	Normal	450	30
S_{sy}(MPa)	Normal	0.577×450	0.577×30
P (Watt)	Normal	20000	1200
ω(rpm)	Normal	650	32.5

There are two failure modes existed because the key needs to withstand compression and shearing stress induced by the transmission power. Therefore, the design margin function and deterministic design function are defined by

$$\begin{cases} G_1(E, \boldsymbol{X}) = S_y - L_1(E, \boldsymbol{Y}) = \dfrac{S_y}{S_{F_1}} - \dfrac{4P}{DHE\omega} > 0 \\ G_2(E, \boldsymbol{X}) = S_{sy} - L_2(E, \boldsymbol{Y}) = \dfrac{S_{sy}}{S_{F_2}} - \dfrac{2P}{DWE\omega} > 0 \end{cases} \quad (43)$$

$$\begin{cases} g_1(E) = \dfrac{s_y}{S_F} - L_1(E) = \dfrac{s_y}{S_{F_1}} - \dfrac{4p}{DHE\omega_\mu} > 0 \\ g_2(E) = \dfrac{s_{sy}}{S_F} - L_2(E) = \dfrac{s_{sy}}{S_{F_2}} - \dfrac{2p}{DWE\omega_u} > 0 \end{cases} \quad (44)$$

where s_y, s_{sy}, p, ω_u are the means of S_y, S_{sy}, P, ω, respectively.

Determine the reliability index

$$\beta = \Phi^{-1}([0.999999]) = 4.7534$$

Derive the gradient

$$\nabla_{G_1} = (w_1^1 \frac{\partial G_1}{\partial X_1}, w_2^1 \frac{\partial G_1}{\partial X_2}, w_3^1 \frac{\partial G_1}{\partial X_3})$$

$$\nabla_{G_2} = (w_1^2 \frac{\partial G_2}{\partial X_1}, w_2^2 \frac{\partial G_2}{\partial X_2}, w_3^2 \frac{\partial G_2}{\partial X_3})$$

where

$$\frac{\partial G_1}{\partial X_1} = \frac{\partial G_1}{\partial S_y} = 1$$

$$\frac{\partial G_1}{\partial X_2} = \frac{\partial G_1}{\partial P} = -\frac{4}{DHE\omega}$$

$$\frac{\partial G_1}{\partial X_3} = \frac{\partial G_1}{\partial \omega} = \frac{4}{DHE\omega^2}$$

$$\frac{\partial G_2}{\partial X_1} = \frac{\partial G_2}{\partial S_{sy}} = 1$$

$$\frac{\partial G_2}{\partial X_2} = \frac{\partial G_2}{\partial P} = -\frac{2}{DHE\omega}$$

$$\frac{\partial G_2}{\partial X_3} = \frac{\partial G_2}{\partial \omega} = \frac{2}{DHE\omega^2}$$

From Table A1, we have

$$\boldsymbol{w_1} = (w_1^1, w_2^1, w_3^1) = (3 \times 10^7, 1200, 32.5)$$

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Table 5 Design Process of the Key Design

Iteration	$\nabla_{G_1}, \nabla_{G_2}$	$S_{F_1},\ S_{F_2}$	E_1, E_2 (mm)	$\varepsilon_1, \varepsilon_2$ (%)
1	–	1.0, 1.0	17.0, 12.8	–
2	$(3\times10^7, -2.7\times10^7, 2.25\times10^7)$, $(1.73\times10^7, -1.56\times10^7, 1.30\times10^7)$	1.6611, 1.6611	28.2, 21.4	66.11, 66.11
3	$(3\times10^7, -1.8\times10^7, 2.02\times10^7)$, $(1.73\times10^7, -1.06\times10^7, 1.17\times10^7)$	1.6728, 1.6728	28.4, 21.5	0.7, 0.7
4	$(3\times10^7, -1.8\times10^7, 1.95\times10^7)$, $(1.73\times10^7, -1.06\times10^7, 1.13\times10^7)$	1.6729, 1.6729	28,4, 21.5	6.4×10^{-3}, 6.4×10^{-3}

$$\boldsymbol{w_2} = (w_1^2, w_2^2, w_3^2) = (\,17.31\times10^6, 1200, 32.5)$$

Iteration 1

Start from the deterministic design by setting $S_F = 1.0$. Plugging the means into Eq. (44), We find that the smallest sizes for $g_1(E) > 0$ and $g_2(E) > 0$ are $E_1 = 17.0$ mm and $E_2 = 12.8$ mm, respectively. At these design points, the general loads (normal stress) are $L_1(E_1, \boldsymbol{y}) = 450$ MPa , $L_2(E_2, \boldsymbol{y}) = 259.65$ MPa, respectively.

Iteration 2

Following the procedures in Fig. 3, we have

$$\boldsymbol{x_1^*} = (x_1^{1*}, x_2^{1*}, x_3^{1*}) = (357\text{ MPa}, 23333\text{ Watt}, 60.19\text{ rad/s})$$

$$\boldsymbol{x_2^*} = (x_1^{2*}, x_2^{2*}, x_3^{2*}) = (206\text{ MPa}, 23333\text{ Watt}, 60.19\text{ rad/s})$$

The general strength is $S_y^* = x_1^{1*} = 357$ MPa , $S_{sy}^* = x_1^{2*} = 206$ MPa. Updating λ_{S_1} and λ_{S_2} by Eq. (29), we have $\lambda_{S_1} = 0.7943$ and $\lambda_{S_2} = 0.7643$. The general load at $\boldsymbol{y}_1^* = (x_2^{1*}, x_3^{1*})$, $\boldsymbol{y}_2^* = (x_2^{2*}, x_3^{2*})$ are $L_1(\boldsymbol{d}, \boldsymbol{y}_1^*) = 594$ MPa , $L_2(\boldsymbol{d}, \boldsymbol{y}_2^*) = 343$ MPa, repectively. Updating λ_{L_1} and λ_{L_2} by Eq. (30), we have $\lambda_{L_1} = 1.3193$ and $\lambda_{L_2} = 1.3193$. Then the safety factors are obtained by Eq. (32) that $S_{F_1} = 1.6611$ and $S_{F_2} = 1.6611$. Plugging the new S_{F_1}, S_{F_2} into the deterministic design function in Eq. (44), we have the new design $E_1 = 28.2$ mm and $E_2 = 21.4$ mm. Checking the convergence using Eq. (34), we obtain $\varepsilon_1 = 66.11\%$ and $\varepsilon_2 = 66.11\%$, which are larger than the tolerance 0.01%, and the iterative process continues.

After one more iteration, the process converges and the final design variables are $E_1 = 28.4$ mm and $E_2 = 21.5$ mm . This design will meet the reliability target 0.999999, which is equivalent to a probability of failure 10^{-6}. To verify this, Monte Carlo simulation (MCS) is performed with a large sample size of 10^8. The probability of failure produced by MCS is 1.09×10^{-6}, very close to the required probability of failure. For a manufacturability and safety consideration, we can set the final design $E = 29$ mm, which ensure higher reliability than the required one. The entire design process is summarized in Table 5.

The three examples demonstrate that the deterministic design is performed several times with the additional computations for the derivatives of the design margin with respect to random variables. In the examples, the deterministic design is conducted manually, and so is the proposed reliability-based design method.

5. CONCLUSION

This work develops a practical approach to reliability-based component design. The approach is practical because it is essentially the traditional safety factor design approach with which engineers are familiar. The safety factor is determined by the specified reliability of the component. The First Order Reliability Method (FORM) is used to link the safety factor and component reliability. Since the safety factor for the required reliability also depends on design variables, the design process is iterative, and the proposed efficient numerical procedure ensures that the design process can converge with a few iterations.

The prerequisites of the practical reliability-based component design approach are as follows: the availability of derivatives of the design margin function with respect to basic input variables and the availability of distributions of the basic input variables. In addition to the derivative calculation, the traditional safety factor design method is performed repeatedly several times. The new approach can be therefore conducted in the same manner as the traditional safety factor design method, manually, numerically, or with the help of computer software such as a spreadsheet. No optimization is needed.

Note that the proposed approach is not optimization and cannot make decisions (find design variables) automatically. It provides a safety factor for engineers to meet their reliability target. How to get the design variables from the safety factor largely depends on how engineers perform their deterministic component design. If the deterministic component design can deal with black-box models, so can the proposed approach.

The proposed approach is based on the first order reliability method (FORM), and it performs a complete MPP search. It is possible, however, the proposed approach does not converge, especially when the design margin function is highly nonlinear in the transformed normal space. The approach may produce a large error if multiple MPPs exist. Our future research will investigate possible ways to avoid divergence and to deal with multiple MPPs.

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Appendix

Table A1 *w* for distributions

Distribution	PDF	w
Normal	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ μ: mean, σ: standard deviation	σ
Lognormal	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)$ μ: mean of $\ln x$, σ: standard deviation of $\ln x$	$\frac{\phi\left[\Phi^{-1}\left(\frac{1}{2}\left(1+\mathrm{erf}\left(\frac{\ln x-\mu}{\sqrt{2}\sigma}\right)\right)\right)\right]}{\frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)}$
Gumbel	$f(x)=\frac{1}{\beta}\exp\left(-\left(\frac{x-\mu}{\beta}+\exp\left(-\frac{x-\mu}{\beta}\right)\right)\right)$ μ: location parameter, β: scale parameter	$\frac{\phi\left[\Phi^{-1}\left(\exp\left(-\exp\left(-\frac{x-\mu}{\beta}\right)\right)\right)\right]}{\frac{1}{\beta}\exp\left(-\left(\frac{x-\mu}{\beta}+\exp\left(-\frac{x-\mu}{\beta}\right)\right)\right)}$
Exponential	$f(x)=\begin{cases}\frac{1}{\beta}\exp\left(-\frac{1}{\beta}x\right) & x\geq 0,\\ 0 & x<0.\end{cases}$ β: mean, β^2: variance	$\frac{\phi\left[\Phi^{-1}\left(1-\exp\left(-\frac{1}{\beta}x\right)\right)\right]}{\frac{1}{\beta}\exp\left(-\frac{1}{\beta}x\right)}$
Weibull	$f(x)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp\left(-\left(\frac{x}{\lambda}\right)^k\right) & x\geq 0,\\ 0 & x<0.\end{cases}$ λ: scale parameter, k: shape parameter	$\frac{\phi\left[\Phi^{-1}(1-\exp(-(x/\lambda)^k))\right]}{\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp(-(x/\lambda)^k)}$
Uniform	$f(x)=\begin{cases}\frac{1}{b-a} & a\leq x\leq b,\\ 0 & \text{otherwise.}\end{cases}$ $\frac{a+b}{2}$: mean, $\frac{1}{12}(b-a)^2$: variance	$\frac{\phi\left[\Phi^{-1}\left(\frac{x-a}{b-a}\right)\right]}{\frac{1}{b-a}}$

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