Global Stabilization of Fuzzy Memristor-Based Reaction–Diffusion Neural Networks

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***Abstract*—This article investigates the global stabilization problem of Takagi–Sugeno fuzzy memristor-based neural networks with reaction–diffusion terms and distributed time-varying delays. By using the Green formula and proposing fuzzy feedback controllers, several algebraic criteria dependent on the diffusion coefficients are established to guarantee the global exponential stability of the addressed networks. Moreover, a simpler stability criterion is obtained by designing an adaptive fuzzy controller. The results derived in this article are generalized and include some existing ones as special cases. Finally, the validity of the theoretical results is verified by two examples.**

***Index Terms*—Distributed delays, fuzzy memristor-based neural networks (NNs), reaction–diffusion, stabilization.**

I. Introduction

MEMRISTOR, first predicted by Chua in 1971 [1], has found its ever-increasing practical values since it was successfully invented by the HP laboratory in 2008 [2]. It has been proved to be an ideal element to act as the neural synapse in circuits of neural networks (NNs), in view of its superiorities of nanoscale dimension, unified logical operation and information storage, and memory characteristic [3]. Recently, the circuit as well as the model of memristor-based NNs (MNNs) have been proposed to replace the conventional NNs in applications, such as optimization problems [4]–[7], memristor-based learning [8]–[11], and signal processing [12]–[16].

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As pointed out in [17], the dynamics research of MNNs is essential and significant for their successful applications. These days, the dynamical behaviors of MNNs have been widely investigated, including stability [18]–[20], dissipativity [21]–[23], and synchronization [24]–[27]. Besides, the stabilization of MNNs has also attracted increasing concern and several excellent results have been reported [28]–[32]. Specifically, the exponential stabilization of MNNs was studied under various control approaches [28]–[30]. Then, the Mittag–Leffler stabilization was analyzed for fractional-order MNNs [31]. Later, Wang *et al.* [32] extended the asymptotic stabilization to finite-time stabilization for delayed MNNs.

The Takagi–Sugeno (T–S) fuzzy system, first introduced in [33], had played a vital role in applications of modeling and control [34]–[36]. Recently, the T–S fuzzy rules have been applied to MNNs and lots of accomplishments have been made. For instance, the fuzzy method was first adopted in [37] to investigate the adaptive lag synchronization for MNNs and an application was presented in pseudorandom number generators. In [38], the exponential stabilization and synchronization were studied for T–S fuzzy MNNs (FMNNs) under intermittent control. In [39], the Lagrange stability was analyzed for FMNNs on time scales. Further, the stabilization of FMNNs was fully addressed with bounded and unbounded distributed time-varying delays in [40]. Then based on the comparison strategy in [40], Sheng *et al.* [41] further studied the Lagrange exponential stability and finite-time stabilization for FMNNs with discrete and distributed time-varying delays. In [42], the synchronization control problem of FMNNs with distributed delays was fully discussed. Nevertheless, the reaction–diffusion is not involved in the above-mentioned results.

In fact, the diffusion phenomenon is unavoidable due to the fact that dynamical behaviors of MNNs typically rely on the evolution time and space of the system states. For instance, due to the information transmission in a heterogeneous electromagnetic field, the circuits of MNNs appear as the effect of the space-distributed structure [43]–[45], which is commonly presented as the reaction–diffusion terms [46]–[49]. Considering this fact, it is essential and critical to consider the reaction–diffusion terms in qualitative analysis of MNNs. By taking into account the reaction–diffusion terms, Tu *et al.* [50] studied the synchronization of memristor-based reaction–diffusion NNs (MRDNNs) via utilizing the adaptive control method. In [51], the passivity was addressed for delayed MRDNNs and several conditions were derived in the

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form of linear matrix inequalities. Further, the robust stability of uncertain MRDNNs with leakage delays was discussed in [52]. It is worth noting that the distributed time delays are neglected in the above results of MRDNNs [50]–[52]. Due to the parallel pathways with axon sized and lengths in circuits of MNNs, the distributed time delays are inevitable and it is of great necessity to involve them in stability analysis of MNNs or MRDNNs [53].

From the above analysis, it is necessary and important to investigate the dynamics for FMNNs considering both the reaction–diffusion terms and distributed delays. However, up to now, there is little work on this topic. Actually, by taking into account the fuzzy rules, the reaction–diffusion terms, and the distributed delays, the considered fuzzy MRDNNs (FMRDNNs) are viewed as a class of delayed fuzzy partial differential systems. Thus, it is difficult to analyze the dynamical behaviors for these kinds of complicated systems. Specifically, it remains unsolved and challenging for the stability analysis of FMRDNNs with distributed delays.

Motivated by the above discussions, this article aims to explore the stabilization problem for FMNNs with the reaction–diffusion terms and distributed delays. By virtue of the Green formula and inequality technique, several algebraic criteria are established to guarantee the global exponential stability of the addressed networks using designed fuzzy feedback controllers. The main contributions are three-fold.

1) Since the fuzzy rules, reaction–diffusion terms, and distributed delays are all considered, the FMRDNNs in this article complement and extend those without fuzzy rules in [11] and [17]–[32], FMNNs without reaction–diffusion terms [37]–[40] and MNNs without distributed delays [11], [17], [19], [20], [22], [24]–[29], [32].
2) The derived algebraic criteria in this article can be easily checked due to the introduction of a large number of parameters. They also show superiority over those in [40], on account of the fact that the criteria in [40] rely on the time and may bring constraint in practical testification.
3) The obtained results in this article are general and include existing ones in [24] and [54] as special cases.

The remainder of this article is given as follows. The preliminaries, including the model formulation and the problem description, are introduced in Section II. In Section III, the main stabilization results of FMRDNNs are presented and some discussions and comparisons with existing work are provided. Then, two numerical examples are shown in Section IV. Finally, Section V draws the conclusions.

II. Preliminaries

In this article, the solutions of all systems are understood in the sense of Filippov. $\mathcal{R}_+$ and $\mathcal{R}^n$ denote the set of all non-negative real numbers and the n-dimensional Euclidean space, respectively. Define a set as $\mathbb{A} = \{1, 2, \ldots, A\}$, where A is a constant, and $\mathbb{A}$ turns out to be $\mathbb{N}, \mathbb{Q}, \mathbb{L}, \mathbb{J}, \mathbb{T}$ when constant A is chosen as constants $n, Q, L, J,$ and τ. $\Delta = \{(z_1, z_2, \ldots, z_Q)^T \| |z_q| < \Upsilon_q, q \in \mathbb{Q}\}$ is bounded compact set with smooth boundary $\partial\Delta$ and the measure $\text{mes}\Delta > 0$. $\mathcal{C}([-\mu, 0] \times \Delta, \mathcal{R}^n)$ denotes the Banach space of continuous functions, and define the norm $\|\omega(\theta, z)\|_\tau = (\int_\Delta \sup_{-\mu\le\theta\le 0} \sum_{p=1}^n |\omega_p(\theta, z)|^\tau \mathrm{d}z)^{1/\tau}$ for $\omega(\theta, z) \in \mathcal{C}, \tau \ge 2$. For any $w(t, z) \in \mathcal{R}^n$, define $\|w(t, z)\|_\tau = (\int_\Delta \sum_{p=1}^n |w_p(t, z)|^\tau \mathrm{d}z)^{1/\tau}$. Define two sets $\Omega(x_p) = \{x_p \| |x_p| \le \chi_p\}$, $\bar{\Omega}(x_p) = \{x_p \| |x_p| > \chi_p\}$ with constants $\chi_p > 0, p \in \mathbb{N}$. $\breve{a}_{pq} = \max\{|a^+_{pq}|, |a^-_{pq}|\}$, $\breve{b}_{pq} = \max\{|b^+_{pq}|, |b^-_{pq}|\}$, $\breve{c}_{pq} = \max\{|c^+_{pq}|, |c^-_{pq}|\}$.

A. Model

Consider the following FMRDNNs with discrete and distributed delays.

Fuzzy Rule l: If $\epsilon_1(t)$ is $\Xi^l_1, \ldots,$ and $\epsilon_J(t)$ is Ξ^l_J THEN

$$\begin{aligned}\frac{\partial w_p(t,z)}{\partial t} &= \sum_{q=1}^{Q}\frac{\partial}{\partial z_q}\left(\varepsilon_{pq}\frac{\partial w_p(t,z)}{\partial z_q}\right) - d_p^{<l>} w_p(t,z)\\ &\quad + \sum_{k=1}^{n} a_{pk}\big(w_p(t,z)\big) f_k(w_k(t,z))\\ &\quad + \sum_{k=1}^{n} b_{pk}\big(w_p(t,z)\big) g_k(w_k(t-\rho_k(t),z))\\ &\quad + \sum_{k=1}^{n} c_{pk}\big(w_p(t,z)\big)\int_{t-\varrho_k(t)}^{t} h_k(w_k(\theta,z))\mathrm{d}\theta \quad (1)\end{aligned}$$

where $p \in \mathbb{N}, k \in \mathbb{N}, l \in \mathbb{L}$. $z = \big(z_1, z_2, \ldots, z_Q\big)^T \in \Delta \subset \mathcal{R}^Q$. $\varepsilon_{pq} \ge 0$ is the transmission diffusion parameters with the pth neuron. The real value L is the number of fuzzy IF-THEN rules, ϵ_j and $\Xi^l_j, j \in \mathbb{J}$ are, respectively, the premise variables and fuzzy sets. $w_p(t, z)$ is the state variable at time t and space z. $\rho_k(t)$ is the discrete delay and $\varrho_k(t)$ is the distributed delay. $f_k, g_k, h_k \in \mathcal{R}$ are the neuron activation functions and $f_k(0) = g_k(0) = h_k(0) = 0$. $d_p^{<l>} > 0$ is the self-feedback coefficient, $a_{pk}\big(w_p(t, z)\big)$, $b_{pk}\big(w_p(t, z)\big)$, and $c_{pk}\big(w_p(t, z)\big)$ are the memristor-based weights and take values as a^+_{pk}, b^+_{pk}, and c^+_{pk} if $w_p(t, z) \in \Omega\big(w_p(t, z)\big)$, or otherwise a^-_{pk}, b^-_{pk}, and c^-_{pk} if $w_p(t, z) \in \bar{\Omega}\big(w_p(t, z)\big)$.

Then, we present the following assumptions for system (1).

Assumption 1: The nonlinear functions f_k, g_k, and h_k are continuous, and there exist positive constants F_k, G_k, and H_k such that

$$\begin{aligned}|f_k(x_k) - f_k(y_k)| &\le F_k|x_k - y_k|\\ |g_k(x_k) - g_k(y_k)| &\le G_k|x_k - y_k|\\ |h_k(x_k) - h_k(y_k)| &\le H_k|x_k - y_k| \quad (2)\end{aligned}$$

for any $x_k, y_k \in \mathcal{R}, k \in \mathbb{N}$.

Assumption 2: The discrete delays $\rho_k(t) (k \in \mathbb{N})$ are bounded, and there exist constants ρ_1 and ρ_2 such that

$$0 \le \rho_k(t) \le \rho_1, \dot{\rho}_k(t) \le \rho_2 < 1. \quad (3)$$

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Assumption 3: The distributed delays $\varrho_k(t) (k \in \mathbb{N})$ are bounded and there exist constants ϱ_1 and ϱ_2 such that

$$0 \le \varrho_k(t) \le \varrho_1, \dot{\varrho}_k(t) \le \varrho_2 < 1. \tag{4}$$

Then by the fuzzy blending, system (1) can be transferred to the following differential system:

$$\begin{aligned}\frac{\partial w_p(t,z)}{\partial t} &= \sum_{l=1}^{L} \gamma_l(\epsilon(t)) \Bigg[\sum_{q=1}^{Q} \frac{\partial}{\partial z_q}\left(\varepsilon_{pq} \frac{\partial w_p(t,z)}{\partial z_q}\right) - d_p^{<l>} w_p(t,z) \\ &\quad + \sum_{k=1}^{n} a_{pk}\big(w_p(t,z)\big) f_k(w_k(t,z)) \\ &\quad + \sum_{k=1}^{n} b_{pk}\big(w_p(t,z)\big) g_k(w_k(t-\rho_k(t),z)) \\ &\quad + \sum_{k=1}^{n} c_{pk}\big(w_p(t,z)\big) \int_{t-\varrho_k(t)}^{t} h_k(w_k(\theta,z)) d\theta \Bigg]\end{aligned} \tag{5}$$

where

$$\gamma_l(\epsilon(t)) = \frac{\prod_{j=1}^{J} \Xi_j^l(\epsilon_j(t))}{\sum_{l=1}^{L} \prod_{j=1}^{J} \Xi_j^l(\epsilon_j(t))} \tag{6}$$

and $\Xi_j^l(\epsilon_j(t))$ is the grade of membership of $\epsilon_j(t)$ in Ξ_j^l. Then, it follows that $\gamma_l(\epsilon(t)) \ge 0 (l \in \mathbb{L}), \sum_{l=1}^{L} \gamma_l(\epsilon(t)) = 1$. System has the initial conditions $w_p(t,z) = 0$ for $(t,z) \in [-\mu, +\infty) \times \partial\Delta$, and $w_p(t,z) = \omega_p(\theta,z)$ for $(\theta,z) \in [-\mu, 0] \times \Delta$, where $\mu = \max\{\rho_1, \varrho_1\}$, $\omega(\theta,z) = (\omega_1(\theta,z), \omega_2(\theta,z), \ldots, \omega_n(\theta,z))^T \in \mathcal{C}$ is bounded.

Remark 1: The FMRDNNs model in this article is general and it contains the MNNs in [11], [17], [19], [21], [23]–[25], [27]–[29], [32], and [42] and MRDNNs in [50]–[53] as special cases. In addition, the FMRDNNs reduce to the conventional RDNNs in [46]–[48] if the distributed delays are unbounded and the fuzzy logics and memristors are not involved.

B. Problem Description

Since the reaction–diffusion terms, fuzzy logics, and memristors are all involved in system (1), it is general and definitely performs more complex dynamical behaviors compared with most of the existing models. It then comes out the natural question: how to guarantee the stability and stabilizability of such kind of complicated systems. Suppose the trajectories of system (1) or (5) do not converge to the origin, then we design the following fuzzy controller:

$$u_p(t,z) = \sum_{l=1}^{L} \sum_{k=1}^{n} \gamma_l(\epsilon(t)) \lambda_{pk}^{<l>} w_k(t,z) \tag{7}$$

where $\lambda_{pk}^{<l>} \in \mathcal{R}$ and $\lambda_{pp}^{<l>} < 0, p \in \mathbb{N}, k \in \mathbb{N}, l \in \mathbb{L}$.

Then together with (5) and (7), it follows that:

$$\begin{aligned}\frac{\partial w_p(t,z)}{\partial t} &= \sum_{l=1}^{L} \gamma_l(\epsilon(t)) \Bigg[\sum_{q=1}^{Q} \frac{\partial}{\partial z_q}\left(\varepsilon_{pq} \frac{\partial w_p(t,z)}{\partial z_q}\right) \\ &\quad - d_p^{<l>} w_p(t,z) + \sum_{k=1}^{n} a_{pk}\big(w_p(t,z)\big) f_k(w_k(t,z)) \\ &\quad + \sum_{k=1}^{n} b_{pk}\big(w_p(t,z)\big) g_k(w_k(t-\rho_k(t),z)) \\ &\quad + \sum_{k=1}^{n} c_{pk}\big(w_p(t,z)\big) \int_{t-\varrho_k(t)}^{t} h_k(w_k(\theta,z)) d\theta \\ &\quad + \sum_{k=1}^{n} \lambda_{pk}^{<l>} w_k(t,z) \Bigg].\end{aligned} \tag{8}$$

Then, the global stabilization problem of system (1) can be turned into the stability problem of system (8). Finally, a useful lemma is presented as follows.

Lemma 1 [46]: Given $\Delta = \{z = (z_1, z_2, \ldots, z_Q)^T \| |z_q| < \Upsilon_q, q \in \mathbb{Q}\}$ with smooth boundary $\partial\Delta$, constant $\tau \ge 2$ and function $\pi(z) \in \mathcal{C}^1(\Delta)$ with $\pi(z)|_{\partial\Delta} = 0$, then for $q \in \mathbb{Q}$

$$\int_{\Delta} |\pi(z)|^{\tau} dz \le \frac{\tau^2 \Upsilon_q^2}{4} \int_{\Delta} |\pi(z)|^{\tau-2} |\frac{\partial \pi}{\partial z_q}|^2 dz. \tag{9}$$

III. Main Results

In this section, we establish the conditions to guarantee the global stabilization of FMRDNNs under designed fuzzy controllers (7) and (28). On the other hand, we also make some comparisons between our results and those in two published papers. For convenience, we first define functions

$$\begin{aligned}\alpha_p(\nu) &= -\frac{4(\tau-1)}{\tau} \sum_{q=1}^{Q} \frac{\varepsilon_{pq}}{\Upsilon_q^2} - \tau d_p^{<l>} \\ &\quad + \sum_{k=1}^{n} \Bigg[\sum_{m=1}^{\tau-1} \left(\breve{a}_{pk}^{\tau \acute{\upsilon}_{mpk}} F_k^{\tau \grave{\upsilon}_{mpk}} + \breve{b}_{pk}^{\tau \acute{\varpi}_{mpk}} G_k^{\tau \grave{\varpi}_{mpk}} + \breve{c}_{pk}^{\tau \acute{\kappa}_{mpk}} H_k^{\tau \grave{\kappa}_{mpk}} \varrho_1 \right) \\ &\quad + \frac{\xi_k}{\xi_p} \Bigg(\breve{a}_{kp}^{\tau \acute{\upsilon}_{\tau kp}} F_p^{\tau \grave{\upsilon}_{\tau kp}} + \frac{\breve{b}_{kp}^{\tau \acute{\varpi}_{\tau kp}} G_p^{\tau \grave{\varpi}_{\tau kp}} e^{\nu \rho_1}}{1-\rho_2} \\ &\quad + \frac{\breve{c}_{kp}^{\tau \acute{\kappa}_{\tau kp}} H_p^{\tau \grave{\kappa}_{\tau kp}} \varrho_1 e^{\nu \varrho_1}}{1-\varrho_2} \Bigg) \Bigg]\end{aligned}$$

where $\xi_p > 0, \tau \ge 2, \acute{\upsilon}_{mpk}, \grave{\upsilon}_{mpk}, \acute{\varpi}_{mpk}, \grave{\varpi}_{mpk}, \acute{\kappa}_{mpk}$, and $\grave{\kappa}_{mpk}$ are non-negative constants with $\sum_{m=1}^{\tau} \acute{\upsilon}_{mpk} = \sum_{m=1}^{\tau} \grave{\upsilon}_{mpk} = \sum_{m=1}^{\tau} \acute{\varpi}_{mpk} = \sum_{m=1}^{\tau} \grave{\varpi}_{mpk} = \sum_{m=1}^{\tau} \acute{\kappa}_{mpk} = \sum_{m=1}^{\tau} \grave{\kappa}_{mpk} = 1$ for $m \in \mathbb{T}, p, k \in \mathbb{N}$.

Then, the following two sections show the derived results and comparisons, respectively.

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A. Global Stabilization Results of FMRDNNs

Theorem 1: Suppose that Assumptions 1–3 hold. Given constant $\tau \geq 2$, system (1) is globally exponentially stabilizable under controller (7), if there exist non-negative constants $\eta_{mpk}, m \in \mathbb{T}, p, k \in \mathbb{N}$, with $\sum_{m=1}^{\tau} \eta_{mpk} = 1$, and constant $\xi_p > 0$ such that for any $l \in \mathbb{L}$

$$\alpha_p(0) + \tau\lambda_{pp}^{<l>} + \sum_{k=1,k\neq p}^{n}\sum_{m=1}^{\tau-1}|\lambda_{pk}^{<l>}|^{\tau\eta_{mpk}} + \sum_{k=1,k\neq p}^{n}\frac{\xi_k}{\xi_p}|\lambda_{kp}^{<l>}|^{\tau\eta_{\tau kp}} < 0. \tag{10}$$

Proof: Consider the following functions:

$$\Gamma_p(\vartheta_p) = \vartheta_p + \tau\lambda_{pp}^{<l>} + \alpha_p(\vartheta_p) + \sum_{k=1,k\neq p}^{n}\sum_{m=1}^{\tau-1}|\lambda_{pk}^{<l>}|^{\tau\eta_{mpk}} + \sum_{k=1,k\neq p}^{n}\frac{\xi_k}{\xi_p}|\lambda_{kp}^{<l>}|^{\tau\eta_{\tau kp}} \tag{11}$$

where $\vartheta_p \geq 0$ for $p \in \mathbb{N}$. From (11), we can see that $\Gamma_p(\vartheta_p)$ is increasing with ϑ_p on $[0, +\infty)$. Then, there exists φ_p such that $\Gamma_p(\varphi_p) = 0$ in light of $\Gamma_p(0) < 0$ and $\lim_{\vartheta_p\to+\infty}\Gamma_p(\vartheta_p) = +\infty$. Choose $\varphi = \min_{p\in\mathbb{N}}\{\varphi_p\}$, then

$$\Gamma_p(\varphi) = \varphi + \tau\lambda_{pp}^{<l>} + \alpha_p(\varphi) + \sum_{k=1,k\neq p}^{n}\sum_{m=1}^{\tau-1}|\lambda_{pk}^{<l>}|^{\tau\eta_{mpk}} + \sum_{k=1,k\neq p}^{n}\frac{\xi_k}{\xi_p}|\lambda_{kp}^{<l>}|^{\tau\eta_{\tau kp}} \leq 0. \tag{12}$$

Consider the Lyapunov–Krasovskii functional

$$\begin{aligned} V(t) &= \int_\Delta \sum_{p=1}^{n}\xi_p\Bigg(W_p(t,z) \\ &\quad + \sum_{k=1}^{n}\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}}{1-\rho_2}\int_{t-\rho_k(t)}^{t}W_k(\theta,z)\mathrm{d}\theta \\ &\quad + \sum_{k=1}^{n}\frac{\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}e^{\varphi\varrho_1}}{1-\varrho_2} \\ &\quad \times \int_{-\varrho_k(t)}^{0}\int_{t+\varsigma}^{t}W_k(\theta,z)\mathrm{d}\theta\mathrm{d}\varsigma\Bigg)\mathrm{d}z \end{aligned} \tag{13}$$

where $W_p(t,z) = e^{\varphi t}|w_p(t,z)|^\tau$.

Since

$$\begin{aligned} &\int_\Delta \frac{\tau}{2}|w_p(t,z)|^{\tau-2}\frac{\partial w_p^2(t,z)}{\partial t}\mathrm{d}z \\ &= \int_\Delta \frac{\partial |w_p(t,z)|^\tau}{\partial t}\mathrm{d}z \\ &\leq \int_\Delta \sum_{l=1}^{L}\gamma_l(\epsilon(t)) \\ &\quad\times\Bigg[\tau|w_p(t,z)|^{\tau-2}w_p(t,z) \times \sum_{q=1}^{Q}\frac{\partial}{\partial z_q}\left(\varepsilon_{pq}\frac{\partial w_p(t,z)}{\partial z_q}\right) \\ &\qquad - \tau d_p^{<l>}|w_p(t,z)|^\tau \\ &\qquad + \sum_{k=1}^{n}\tau\breve{a}_{pk}F_k|w_p(t,z)|^{\tau-1}|w_k(t,z)| \\ &\qquad + \sum_{k=1}^{n}\tau\breve{b}_{pk}G_k|w_p(t,z)|^{\tau-1}|w_k(t-\rho_k(t),z)| \\ &\qquad + \sum_{k=1}^{n}\tau\breve{c}_{pk}H_k|w_p(t,z)|^{\tau-1}\int_{t-\varrho_k(t)}^{t}|w_k(\theta,z)|\mathrm{d}\theta \\ &\qquad + \sum_{k=1}^{n}\tau\lambda_{pk}^{<l>}|w_p(t,z)|^{\tau-2}w_p(t,z)w_k(t,z)\Bigg]\mathrm{d}z. \end{aligned} \tag{14}$$

Then based on the Green formula [55] and Lemma 1, it follows:

$$\begin{aligned} &\int_\Delta \tau|w_p(t,z)|^{\tau-2}w_p(t,z)\sum_{q=1}^{Q}\frac{\partial}{\partial z_q}\left(\varepsilon_{pq}\frac{\partial w_p(t,z)}{\partial z_q}\right)\mathrm{d}z \\ &= \int_\Delta \frac{\tau}{2}|w_p(t,z)|^{\tau-2}\sum_{q=1}^{Q}\left[\frac{\partial}{\partial z_q}\left(\varepsilon_{pq}\frac{\partial|w_p(t,z)|^2}{\partial z_q}\right) - 2\varepsilon_{pq}\left(\frac{\partial w_p(t,z)}{\partial z_q}\right)^2\right]\mathrm{d}z \\ &= \int_\Delta \tau|w_p(t,z)|^{\tau-1}\sum_{q=1}^{Q}\frac{\partial}{\partial z_q}\left(\varepsilon_{pq}\frac{\partial|w_p(t,z)|}{\partial z_q}\right)\mathrm{d}z \\ &= \int_\Delta \tau|w_p(t,z)|^{\tau-1}\mathrm{div}\left(\varepsilon_{pq}\frac{\partial|w_p(t,z)|}{\partial z_q}\right)_{q=1}^{Q}\mathrm{d}z \\ &= \tau\int_{\partial\Delta}\left(|w_p(t,z)|^{\tau-1}\varepsilon_{pq}\frac{\partial|w_p(t,z)|}{\partial z_q}\right)_{q=1}^{Q}\mathrm{d}S \\ &\quad - \tau(\tau-1)\int_\Delta |w_p(t,z)|^{\tau-2}\sum_{q=1}^{Q}\varepsilon_{pq}\left(\frac{\partial|w_p(t,z)|}{\partial z_q}\right)^2\mathrm{d}z \\ &\leq -\sum_{q=1}^{Q}\frac{4(\tau-1)\varepsilon_{pq}}{\tau\Upsilon_q^2}\int_\Delta |w_p(t,z)|^\tau\mathrm{d}z \end{aligned} \tag{15}$$

where div is the divergence operator and

$$\left(\varepsilon_{pq}\frac{\partial|w_p(t,z)|}{\partial z_q}\right)_{q=1}^{Q} = \left(\varepsilon_{p1}\frac{\partial|w_p(t,z)|}{\partial z_1}, \ldots, \varepsilon_{pQ}\frac{\partial|w_p(t,z)|}{\partial z_Q}\right).$$

Considering that $\sum_{m=1}^{\tau}\acute{\upsilon}_{mpk} = \sum_{m=1}^{\tau}\grave{\upsilon}_{mpk} = 1, p, k \in \mathbb{N}$, then

$$\begin{aligned} &\sum_{k=1}^{n}\tau\breve{a}_{pk}F_k|w_p(t,z)|^{\tau-1}|w_k(t,z)| \\ &\leq \sum_{k=1}^{n}\tau\breve{a}_{pk}^{\acute{\upsilon}_{1pk}+\acute{\upsilon}_{2pk}+\cdots+\acute{\upsilon}_{\tau pk}}F_k^{\grave{\upsilon}_{1pk}+\grave{\upsilon}_{2pk}+\cdots+\grave{\upsilon}_{\tau pk}} \\ &\quad\times|w_p(t,z)|^{\tau-1}|w_k(t,z)| \\ &\leq \sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{a}_{pk}^{\tau\acute{\upsilon}_{mpk}}F_k^{\tau\grave{\upsilon}_{mpk}}|w_p(t,z)|^\tau \\ &\quad + \sum_{k=1}^{n}\breve{a}_{pk}^{\tau\acute{\upsilon}_{\tau pk}}F_k^{\tau\grave{\upsilon}_{\tau pk}}|w_k(t,z)|^\tau. \end{aligned} \tag{16}$$

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Then, similar to (16), the following three inequalities are derived under $\sum_{m=1}^{\tau}\acute{\varpi}_{mpk}=\sum_{m=1}^{\tau}\grave{\varpi}_{mpk}=\sum_{m=1}^{\tau}\acute{\kappa}_{mpk}=\sum_{m=1}^{\tau}\grave{\kappa}_{mpk}=\sum_{m=1}^{\tau}\eta_{mpk}=1, p, k\in\mathbb{N}$:

$$
\begin{aligned}
&\sum_{k=1}^{n}\tau\breve{b}_{pk}G_k|w_p(t,z)|^{\tau-1}|w_k(t-\rho_k(t),z)| \\
&\quad\le\sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}}G_k^{\tau\grave{\varpi}_{mpk}}|w_p(t,z)|^{\tau} \\
&\qquad+\sum_{k=1}^{n}\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}|w_k(t-\rho_k(t),z)|^{\tau}
\end{aligned}\tag{17}
$$

$$
\begin{aligned}
&\sum_{k=1}^{n}\tau\breve{c}_{pk}H_k|w_p(t,z)|^{\tau-1}\int_{t-\varrho_k(t)}^{t}|w_k(\theta,z)|\mathrm{d}\theta \\
&\quad\le\sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{c}_{pk}^{\tau\acute{\kappa}_{mpk}}H_k^{\tau\grave{\kappa}_{mpk}}\varrho_k(t)|w_p(t,z)|^{\tau} \\
&\qquad+\sum_{k=1}^{n}\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}\int_{t-\varrho_k(t)}^{t}|w_k(\theta,z)|^{\tau}\mathrm{d}\theta
\end{aligned}\tag{18}
$$

$$
\begin{aligned}
&\sum_{k=1}^{n}\tau\lambda_{pk}^{<l>}|w_p(t,z)|^{\tau-2}w_p(t,z)w_k(t,z) \\
&\quad\le\tau\lambda_{pp}^{<l>}|w_p(t,z)|^{\tau}+\sum_{k=1,k\neq p}^{n}\sum_{m=1}^{\tau-1}|\lambda_{pk}^{<l>}|^{\tau\eta_{mpk}}|w_p(t,z)|^{\tau} \\
&\qquad+\sum_{k=1,k\neq p}^{n}|\lambda_{pk}^{<l>}|^{\tau\eta_{\tau pk}}|w_k(t,z)|^{\tau}.
\end{aligned}\tag{19}
$$

Then, we can obtain the upper right derivation of $V(t)$ along trajectories of system (8) that

$$
\begin{aligned}
D^{+}V(t)\le&\int_{\Delta}\sum_{p=1}^{n}\xi_p e^{\varphi t}\left(\frac{\partial|w_p(t,z)|^{\tau}}{\partial t}+\varphi|w_p(t,z)|^{\tau}\right)\mathrm{d}z \\
&+\int_{\Delta}\sum_{p=1}^{n}\sum_{k=1}^{n}\xi_p \\
&\times\left[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}}{1-\rho_2}W_k(t,z)\right. \\
&\quad-\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}W_k(t-\rho_k(t),z) \\
&\quad+\frac{\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}e^{\varphi\varrho_1}}{1-\varrho_2} \\
&\quad\times\left(\dot{\varrho}_k(t)\int_{t-\varrho_k(t)}^{t}W_k(\theta,z)\mathrm{d}\theta+\varrho_k(t)W_k(t,z)\right. \\
&\quad\left.\left.-\int_{-\varrho_k(t)}^{0}W_k(t+\varsigma,z)\mathrm{d}\varsigma\right)\right]\mathrm{d}z.
\end{aligned}\tag{20}
$$

Combining with (14)–(20), it gives

$$
\begin{aligned}
D^{+}V(t)\le&\int_{\Delta}\sum_{p=1}^{n}\sum_{l=1}^{L}\gamma_l(\epsilon(t)) \\
&\times\left[\xi_p\left(\varphi-\sum_{q=1}^{Q}\frac{4(\tau-1)\varepsilon_{pq}}{\tau\Upsilon_q^2}\right.\right. \\
&\qquad-\tau d_p^{<l>}+\tau\lambda_{pp}^{<l>}+\sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{a}_{pk}^{\tau\acute{\upsilon}_{mpk}}F_k^{\tau\grave{\upsilon}_{mpk}} \\
&\qquad+\sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}}G_k^{\tau\grave{\varpi}_{mpk}} \\
&\qquad+\sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{c}_{pk}^{\tau\acute{\kappa}_{mpk}}H_k^{\tau\grave{\kappa}_{mpk}}\varrho_1 \\
&\qquad\left.+\sum_{k=1,k\neq p}^{n}\sum_{m=1}^{\tau-1}|\lambda_{pk}^{<l>}|^{\tau\eta_{mpk}}\right) \\
&\left.\quad+\xi_k\left(\sum_{k=1}^{n}\breve{a}_{kp}^{\tau\acute{\upsilon}_{\tau kp}}F_p^{\tau\grave{\upsilon}_{\tau kp}}+\sum_{k=1,k\neq p}^{n}|\lambda_{kp}^{<l>}|^{\tau\eta_{\tau kp}}\right)\right] \\
&\times W_p(t,z)\mathrm{d}z+\int_{\Delta}\sum_{p=1}^{n}\xi_p e^{\varphi t} \\
&\times\left(\sum_{k=1}^{n}\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}\times|w_k(t-\rho_k(t),z)|^{\tau}\right. \\
&\qquad\left.+\sum_{k=1}^{n}\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}\int_{t-\varrho_k(t)}^{t}|w_k(\theta,z)|^{\tau}\mathrm{d}\theta\right)\mathrm{d}z \\
&+\int_{\Delta}\sum_{p=1}^{n}\sum_{k=1}^{n}\xi_p \\
&\times\left[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}}{1-\rho_2}W_k(t,z)\right. \\
&\qquad-\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}W_k(t-\rho_k(t),z) \\
&\qquad+\frac{\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}\varrho_k(t)e^{\varphi\varrho_1}}{1-\varrho_2}W_k(t,z) \\
&\qquad\left.-\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}e^{\varphi\varrho_1}\int_{t-\varrho_k(t)}^{t}W_k(\theta,z)\mathrm{d}\theta\right]\mathrm{d}z \\
\le&\int_{\Delta}\sum_{p=1}^{n}\sum_{l=1}^{L}\gamma_l(\epsilon(t))\xi_p\Gamma_p((\varphi))W_p(t,z)\mathrm{d}z.
\end{aligned}\tag{21}
$$

Then, it follows from (12) and (21) that $D^{+}V(t)\le 0$, which means for any $t\in\mathcal{R}_{+}$

$$
\int_{\Delta}\sum_{p=1}^{n}\xi_p W_p(t,z)\mathrm{d}z\le V(t)\le V(0)
$$

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then

$$\begin{aligned}&\underline{\xi}\int_{\Delta}\sum_{p=1}^{n}e^{\varphi t}|w_p(t,z)|^{\tau}\mathrm{d}z\\&\le\bar{\xi}\int_{\Delta}\sum_{p=1}^{n}\Bigg[|w_p(0,z)|^{\tau}+\sum_{k=1}^{n}\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}G_k^{\tau\grave{\varpi}_{\tau pk}}e^{\varphi\rho_1}}{1-\rho_2}\\&\qquad\times\int_{-\rho_1}^{0}e^{\varphi\theta}|w_k(\theta,z)|^{\tau}\mathrm{d}\theta+\sum_{k=1}^{n}\frac{\breve{c}_{pk}^{\tau\acute{\kappa}_{\tau pk}}H_k^{\tau\grave{\kappa}_{\tau pk}}e^{\varphi\varrho_1}}{1-\varrho_2}\\&\qquad\times\int_{-\varrho_1}^{0}\int_{\varsigma}^{0}e^{\varphi\theta}|w_k(\theta,z)|^{\tau}\mathrm{d}\theta\mathrm{d}\varsigma\Bigg]\mathrm{d}z\end{aligned}$$

which yields that

$$\begin{aligned}&\int_{\Delta}\sum_{p=1}^{n}e^{\varphi t}|w_p(t,z)|^{\tau}\mathrm{d}z\\&\le\frac{\bar{\xi}}{\underline{\xi}}\int_{\Delta}\sum_{p=1}^{n}\Bigg[|w_p(0,z)|^{\tau}+\sum_{k=1}^{n}\frac{\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}}G_p^{\tau\grave{\varpi}_{\tau kp}}e^{\varphi\rho_1}}{1-\rho_2}\\&\qquad\times\int_{-\mu}^{0}|w_p(\theta,z)|^{\tau}\mathrm{d}\theta+\sum_{k=1}^{n}\frac{\breve{c}_{kp}^{\tau\acute{\kappa}_{\tau kp}}H_p^{\tau\grave{\kappa}_{\tau kp}}e^{\varphi\varrho_1}}{1-\varrho_2}\\&\qquad\times\int_{-\mu}^{0}\int_{\varsigma}^{0}e^{\varphi\theta}|w_p(\theta,z)|^{\tau}\mathrm{d}\theta\mathrm{d}\varsigma\Bigg]\mathrm{d}z\\&\le\phi\int_{\Delta}\sup_{-\mu\le\theta\le0}\sum_{p=1}^{n}|\omega_p(\theta,z)|^{\tau}\mathrm{d}z\end{aligned}\tag{22}$$

where $\bar{\xi}=\max_p\{\xi_p\}$, $\underline{\xi}=\min_p\{\xi_p\}$, and $\phi=\bar{\xi}/\underline{\xi}\max_p\{1+\sum_{k=1}^{n}(\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}}G_p^{\tau\grave{\varpi}_{\tau kp}}e^{\varphi\mu}\mu)/(1-\rho_2)+\sum_{k=1}^{n}(\breve{c}_{kp}^{\tau\acute{\kappa}_{\tau kp}}H_p^{\tau\grave{\kappa}_{\tau kp}}e^{\varphi\mu}\mu^2)/(1-\varrho_2)\}$.

Then, from (22), one obtains

$$\|w(t,z)\|_{\tau}\le\phi^{1/\tau}\|\omega(\theta,z)\|_{\tau}\exp\Big\{-\frac{\varphi}{\tau}t\Big\}\tag{23}$$

which means system (8) is globally exponentially stable on account of definition in [46]. The proof is completed. ■

Remark 2: Theorem 1 presents the criteria for global stabilization of FMRDNNs via the Green formula and some inequality techniques. The criteria are in the algebraic form and can be easily checked due to the introduction of a large number of parameters. Besides, they are better compared to those in [40] and [47], where the former ones rely on the time and bring difficulties in verifying, and the latter ones adopt many complicated matrices.

Remark 3: From (11), it is not hard to see that the criteria related to the fuzzy interconnected control gains of controller (7). To simplify the calculation, we introduce another special case of the controller (7) and thus obtain Corollaries 1 and 2.

If the fuzzy controller (7) reduces to the following form:

$$u_p(t,z)=\sum_{l=1}^{L}\gamma_l(\epsilon(t))\lambda_p^{<l>}w_p(t,z)\tag{24}$$

where $\lambda_p^{<l>}<0, p\in\mathbb{N}, l\in\mathbb{L}$. Then, Corollary 1 is the direct result of Theorem 1.

Corollary 1: Suppose that Assumptions 1–3 hold. Given constant $\tau\ge2$, system (1) is globally exponentially stabilizable under controller (24), if there exists constant $\xi_p>0$ such that for any $p\in\mathbb{N}, l\in\mathbb{L}$

$$\alpha_p(0)+\tau\lambda_p^{<l>}<0.\tag{25}$$

Corollary 2: Suppose that Assumptions 1 and 2 hold and $\varrho_k(t)=0$. Given constant $\tau\ge2$, system (1) is globally exponentially stabilizable under controller (24), if there exist non-negative constants $\xi_p>0, \acute{\upsilon}_{mpk}, \grave{\upsilon}_{mpk}, \acute{\varpi}_{mpk}, \grave{\varpi}_{mpk}, m\in\mathbb{T}, p,k\in\mathbb{N}$, with $\sum_{m=1}^{\tau}\acute{\upsilon}_{mpk}=\sum_{m=1}^{\tau}\grave{\upsilon}_{mpk}=\sum_{m=1}^{\tau}\acute{\varpi}_{mpk}=\sum_{m=1}^{\tau}\grave{\varpi}_{mpk}=1$, such that for any $l\in\mathbb{L}$

$$\begin{aligned}&-\frac{4(\tau-1)}{\tau}\sum_{q=1}^{Q}\frac{\varepsilon_{pq}}{\Upsilon_q^2}-\tau d_p^{<l>}+\tau\lambda_p^{<l>}\\&+\sum_{k=1}^{n}\Bigg[\sum_{m=1}^{\tau-1}\Big(\breve{a}_{pk}^{\tau\acute{\upsilon}_{mpk}}F_k^{\tau\grave{\upsilon}_{mpk}}+\breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}}G_k^{\tau\grave{\varpi}_{mpk}}\Big)\\&\qquad+\frac{\xi_k}{\xi_p}\Bigg(\breve{a}_{kp}^{\tau\acute{\upsilon}_{\tau kp}}F_p^{\tau\grave{\upsilon}_{\tau kp}}+\frac{\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}}G_p^{\tau\grave{\varpi}_{\tau kp}}}{1-\rho_2}\Bigg)\Bigg]<0.\end{aligned}\tag{26}$$

By using Young's inequality to handle the cross terms in (14), Corollary 3 can be obtained.

Corollary 3: Suppose that Assumptions 1–3 hold. Given constant $\tau\ge2$, system (1) is globally exponentially stabilizable under controller (24), if there exists constant $\xi_p>0$ such that for any $p\in\mathbb{N}, l\in\mathbb{L}$

$$\begin{aligned}&-\frac{4(\tau-1)}{\tau}\sum_{q=1}^{Q}\frac{\varepsilon_{pq}}{\Upsilon_q^2}-\tau d_p^{<l>}+\tau\lambda_p^{<l>}\\&+\sum_{k=1}^{n}\Bigg[(\tau-1)\Big(\breve{a}_{pk}F_k+\breve{b}_{pk}G_k+\varrho_1\breve{c}_{pk}H_k\Big)\\&\qquad+\frac{\xi_k}{\xi_p}\Bigg(\breve{a}_{kp}F_p+\frac{\breve{b}_{kp}G_p}{1-\rho_2}+\frac{\breve{c}_{kp}H_p\varrho_1}{1-\varrho_2}\Bigg)\Bigg]<0.\end{aligned}\tag{27}$$

Remark 4: Under the linear feedback controller (7) or (24), it is obvious that the conditions in Theorem 1 and Corollaries 1–3 always hold if the control gains are chosen to be large enough. Thus, to efficiently adjust the control gains so that save control cost, we introduce another adaptive control approach in (28). Moreover, we will show that the criteria under the proposed adaptive controller in Theorem 2 is more simple than those in Theorem 1.

The adaptive controller is presented as follows:

$$\begin{aligned}u_p(t,z)&=\sum_{l=1}^{L}\gamma_l(\epsilon(t))\lambda_p^{<l>}(t,z)w_p(t,z)\\\frac{\partial\lambda_p^{<l>}(t,z)}{\partial t}&=-\zeta_p^{<l>}|w_p(t,z)|^{\tau}\end{aligned}\tag{28}$$

where $\zeta_p^{<l>}>0, p\in\mathbb{N}, l\in\mathbb{L}$.

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Theorem 2: Suppose that Assumptions 1–3 hold. Given constant $\tau \geq 2$, then system (1) is globally asymptotically stabilizable under controller (28).

Proof: Consider the Lyapunov–Krasovskii functional

$$V(t) = \int_{\Delta} \sum_{p=1}^{n} \xi_p \Bigg[|w_p(t,z)|^{\tau} + \sum_{l=1}^{L} \frac{\tau \gamma_l(\epsilon(t))}{2\zeta_p^{<l>}} \left(\lambda_p^{<l>}(t,z) + \tilde{\zeta}_p^{<l>} \right)^2 + \sum_{k=1}^{n} \frac{\breve{b}_{pk}^{\tau \acute{\varpi}_{\tau pk}} G_k^{\tau \grave{\varpi}_{\tau pk}}}{1-\rho_2} \int_{t-\rho_k(t)}^{t} |w_k(\theta,z)|^{\tau} d\theta + \sum_{k=1}^{n} \frac{\breve{c}_{pk}^{\tau \acute{\kappa}_{\tau pk}} H_k^{\tau \grave{\kappa}_{\tau pk}}}{1-\varrho_2} \times \int_{-\varrho_k(t)}^{0} \int_{t+\varsigma}^{t} |w_k(\theta,z)|^{\tau} d\theta d\varsigma \Bigg] dz \quad (29)$$

where $\tilde{\zeta}_p^{<l>}, p \in \mathbb{N}, l \in \mathbb{L}$ are constants defined by

$$\alpha_p(0) - \tau \tilde{\zeta}_p^{<l>} + \beta_p^{<l>} = 0 \quad (30)$$

with $\beta_p^{<l>} > 0, p \in \mathbb{N}, l \in \mathbb{L}$.

Then similar to the proof of Theorem 1, it follows:

$$D^+V(t) \leq -\int_{\Delta} \sum_{p=1}^{n} \sum_{l=1}^{L} \gamma_l(\epsilon(t)) \xi_p \beta_p^{<l>} |w_p(t,z)|^{\tau} dz \leq 0 \quad (31)$$

which means system (1) is globally asymptotically stabilizable under controller (28). The proof is completed. ■

Remark 5: In [50], the synchronization of MRDNNs with discrete delays was studied under the adaptive controller. If the distributed delays and fuzzy rules are not involved in system (1), then [50, Th. 3.1] is the direct result of Theorem 2.

Remark 6: The results in this article are more general compared to those in [50]–[53], where the stability and synchronization of MRDNNs were widely discussed. On the one hand, the distributed delays are neglected in [50]–[52] and the fuzzy rules are not considered in [53]. On the other hand, all conditions [50]–[53] are in terms of 2-norm while τ-norm ($\tau \geq 2$) is adopted in this article.

B. Comparisons

In this part, to show the improvement of our results compared with existing ones, we give the following comparisons.

If system (1) acts without reaction–diffusion terms, distributed delays, and fuzzy rules [24], that is

$$\dot{e}_p(t) = -d_p e_p(t) + \sum_{k=1}^{n} a_{pk}\big(e_p(t)\big) f_k(e_k(t)) + \sum_{k=1}^{n} b_{pk}\big(e_p(t)\big) g_k(e_k(t-\rho_k(t))) \quad (32)$$

its stabilizability can be addressed under

$$u_p(t) = \pi_p e_p(t) \quad (33)$$

with the control gain $\pi_p, p \in \mathbb{N}$, which leads to Corollary 4.

Corollary 4: Suppose that Assumptions 1 and 2 hold. Given constant $\tau > 1$, system (32) is globally exponentially stabilizable under controller (33), if there exist constants $\pi_p < 0$, $\xi_p > 0$ and non-negative constants $\acute{\upsilon}_{mpk}, \grave{\upsilon}_{mpk}, \acute{\varpi}_{mpk}, \grave{\varpi}_{mpk}, m \in \mathbb{T}, p, k \in \mathbb{N}$, with $\sum_{m=1}^{\tau} \acute{\upsilon}_{mpk} = \sum_{m=1}^{\tau} \grave{\upsilon}_{mpk} = \sum_{m=1}^{\tau} \acute{\varpi}_{mpk} = \sum_{m=1}^{\tau} \grave{\varpi}_{mpk} = 1$, such that

$$-\tau\left(d_p - \pi_p\right) + \sum_{k=1}^{n} \Bigg[\sum_{m=1}^{\tau-1} \left(\breve{a}_{pk}^{\tau \acute{\upsilon}_{mpk}} F_k^{\tau \grave{\upsilon}_{mpk}} + \breve{b}_{pk}^{\tau \acute{\varpi}_{mpk}} G_k^{\tau \grave{\varpi}_{mpk}} \right) + \frac{\xi_k}{\xi_p} \left(\breve{a}_{kp}^{\tau \acute{\upsilon}_{\tau kp}} F_p^{\tau \grave{\upsilon}_{\tau kp}} + \frac{\breve{b}_{kp}^{\tau \acute{\varpi}_{\tau kp}} G_p^{\tau \grave{\varpi}_{\tau kp}}}{1-\rho_2} \right) \Bigg] < 0. \quad (34)$$

Remark 7: Zhang and Shen [24] studied the exponential synchronization of MNNs with discrete delays. The advantages of our results compared to those in [24] are two-fold.

1) MNNs in [24] do not involve the reaction–diffusion terms, distributed delays, and fuzzy rules, while system (1) in this article consider the three factors. To this extent, our system is more general.
2) Note that Corollary 4 includes [24, Th. 1] as special case. On the basis of condition (34), if we choose $d_p = 1$, $\acute{\upsilon}_{mpk} = (\tau - \mathfrak{A}_{pk})/(\tau(\tau-1))$, $\grave{\upsilon}_{mpk} = (\tau - \mathfrak{B}_{pk})/(\tau(\tau-1))$, $\acute{\varpi}_{mpk} = (\tau - \mathfrak{C}_{pk})/(\tau(\tau-1))$, $\grave{\varpi}_{mpk} = (\tau - \mathfrak{D}_{pk})/(\tau(\tau-1))$, $\acute{\upsilon}_{\tau kp} = \mathfrak{A}_{kp}/\tau$, $\grave{\upsilon}_{\tau kp} = \mathfrak{B}_{kp}/\tau$, $\acute{\varpi}_{\tau kp} = \mathfrak{C}_{kp}/\tau$, $\grave{\varpi}_{\tau kp} = \mathfrak{D}_{kp}/\tau$, $m \in \mathbb{T}, p, k \in \mathbb{N}$, then Corollary 4 turns into [24, Th. 1].

If system (1) acts without memristors, reaction–diffusion terms, distributed delays, and fuzzy rules, that is

$$\dot{e}_p(t) = -d_p e_p(t) + \sum_{k=1}^{n} a_{pk} f_k(e_k(t)) + \sum_{k=1}^{n} b_{pk} g_k(e_k(t-\rho_k(t))) \quad (35)$$

its stability can be solved and the result is given in Corollary 5.

Corollary 5: Suppose that Assumptions 1 and 2 hold. Given constant $\tau \geq 1$, system (35) is globally exponentially stable, if there exists constant $\xi_p > 0$ such that for any $p \in \mathbb{N}$

$$-d_p\tau + \sum_{k=1}^{n} (\tau-1)\big(|a_{pk}|F_k + |b_{pk}|G_k\big) + \sum_{k=1}^{n} \frac{\xi_k}{\xi_p} \left(|a_{kp}|F_p + \frac{|b_{kp}|G_p}{1-\rho_2} \right) < 0. \quad (36)$$

Remark 8: If we choose $\tau = 1$ and $\tau = 2$, then Corollary 5 turns into [54, Ths. 4 and 5], respectively.

IV. Numerical Simulations

Two examples are provided to show the effectiveness of the results obtained in the previous section.

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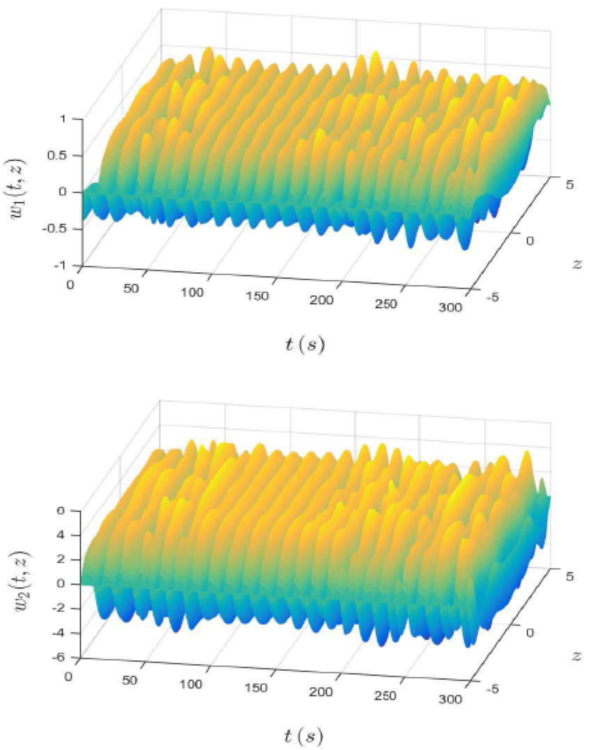

Fig. 1. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (37).

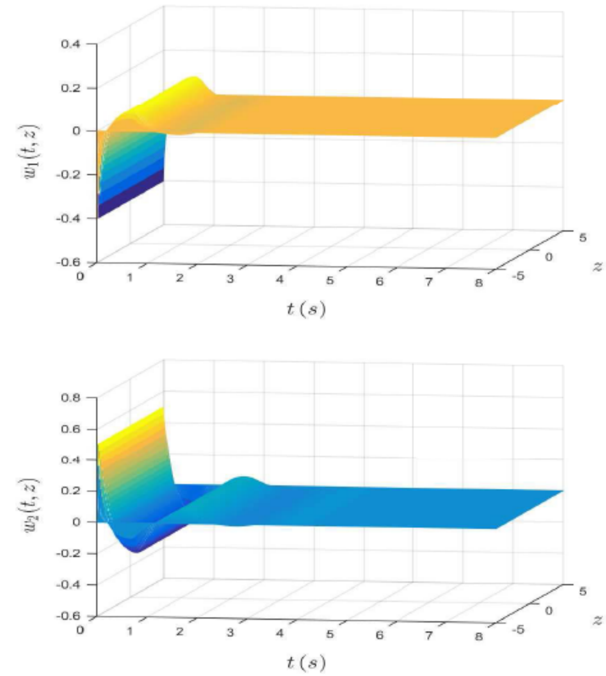

Fig. 2. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (37) under controller (38).

Example 1: Consider the FMRDNNs with discrete and distributed delays

$$\begin{aligned}\frac{\partial w_p(t,z)}{\partial t} &= \sum_{l=1}^{2}\gamma_l(\epsilon(t))\Bigg[\varepsilon_p\frac{\partial^2 w_p(t,z)}{\partial z^2} - d_p^{<l>}w_p(t,z)\\ &\quad + \sum_{k=1}^{2}a_{pk}\big(w_p(t,z)\big)f_k(w_k(t,z))\\ &\quad + \sum_{k=1}^{2}b_{pk}\big(w_p(t,z)\big)g_k(w_k(t-\rho_k(t),z))\\ &\quad + \sum_{k=1}^{2}c_{pk}\big(w_p(t,z)\big)\int_{t-\varrho_k(t)}^{t}h_k(w_k(\theta,z))\mathrm{d}\theta\Bigg]\end{aligned} \tag{37}$$

where $p, l = 1, 2, \varepsilon_1 = \varepsilon_2 = 0.1$, $z \in \Delta = [-5, 5]$, $d_1^{<1>} = d_2^{<2>} = 1, d_1^{<2>} = 1.2, d_2^{<1>} = 0.9, \chi_1 = \chi_2 = 1, a_{11}^{+} = 1.8, a_{11}^{-} = 1.7, a_{12}^{+} = -0.15, a_{12}^{-} = -0.2, a_{21}^{+} = -5.2, a_{21}^{-} = -4.8, a_{22}^{+} = 3.5, a_{22}^{-} = 2.8, b_{11}^{+} = -1.7, b_{11}^{-} = -1.5, b_{12}^{+} = -0.12, b_{12}^{-} = -0.1, b_{21}^{+} = -0.25, b_{21}^{-} = -0.2, b_{22}^{+} = -2.3, b_{22}^{-} = -2.5, c_{11}^{+} = 0.6, c_{11}^{-} = 0.5, c_{12}^{+} = 0.15, c_{12}^{-} = 0.18, c_{21}^{+} = -2, c_{21}^{-} = -2.1, c_{22}^{+} = -0.1, c_{22}^{-} = -0.2, \gamma_1(w) = 1 - 0.1\sin(w)^4, \gamma_2(w) = 0.1\sin(w)^4$, the delays $\rho_k(t) = \exp(t)/(1 + \exp(t))$, $\varrho_k(t) = 1$, and activation functions $f_k(\cdot) = g_k(\cdot) = h_k(\cdot) = \tanh(\cdot), k = 1, 2$. It follows from Assumptions 1–3 that $F_k = G_k = H_k = 1, \rho_2 = 0.25, \varrho_1 = 1, \varrho_2 = 0$. Fig. 1 depicts the trajectories of the state variables $w_1(t, z)$ and $w_2(t, z)$ in system (37), which implies that system (37) is unstable.

Then, design the controllers

$$u_p(t,z) = \sum_{l=1}^{2}\gamma_l(\epsilon(t))\lambda_p^{<l>}w_p(t,z) \tag{38}$$

where the gains are chosen as $\lambda_1^{<1>} = -7.5, \lambda_1^{<2>} = -7.3, \lambda_2^{<1>} = -9.8$, and $\lambda_2^{<2>} = -9.7$. And

$$\begin{aligned}u_p(t,z) &= \sum_{l=1}^{2}\gamma_l(\epsilon(t))\lambda_p^{<l>}(t,z)w_p(t,z)\\ \frac{\partial\lambda_p^{<l>}(t,z)}{\partial t} &= -\zeta_p^{<l>}|w_p(t,z)|^2\end{aligned} \tag{39}$$

where $\zeta_p^{<l>} = 5, p, l = 1, 2$. Then, $\lambda_1(t, z) = \lambda_1^{<l>}(t, z)$ and $\lambda_2(t, z) = \lambda_2^{<l>}(t, z), l = 1, 2$.

Choose $\tau = 2, \xi_p = 1, \acute{\upsilon}_{1pk} = \acute{\upsilon}_{2pk} = \grave{\upsilon}_{1pk} = \grave{\upsilon}_{2pk} = \acute{\varpi}_{1pk} = \acute{\varpi}_{2pk} = \grave{\varpi}_{1pk} = \grave{\varpi}_{2pk} = \acute{\kappa}_{1pk} = \acute{\kappa}_{2pk} = \grave{\kappa}_{1pk} = \grave{\kappa}_{2pk} = 0.5, p, k = 1, 2$, it is easy to check that the conditions of Corollary 1 hold. Then, the results of Corollary 1 show that system (37) is globally exponentially stabilizable via the fuzzy controller (38). Also the results of Theorem 2 show that the global stabilization result for system (37) under controller (39). The trajectories of states variables $w_1(t, z)$ and $w_2(t, z)$ under controllers (38) and (39) are shown in Figs. 2 and 3, respectively. From Figs. 2 and 3, we can see that under controller (38) or (39), the states in system (37) converge to zero as time goes to infinity. Finally, the trajectories of control gains $\lambda_1(t, z)$ and $\lambda_2(t, z)$ in adaptive controller (39) are given in Fig. 4.

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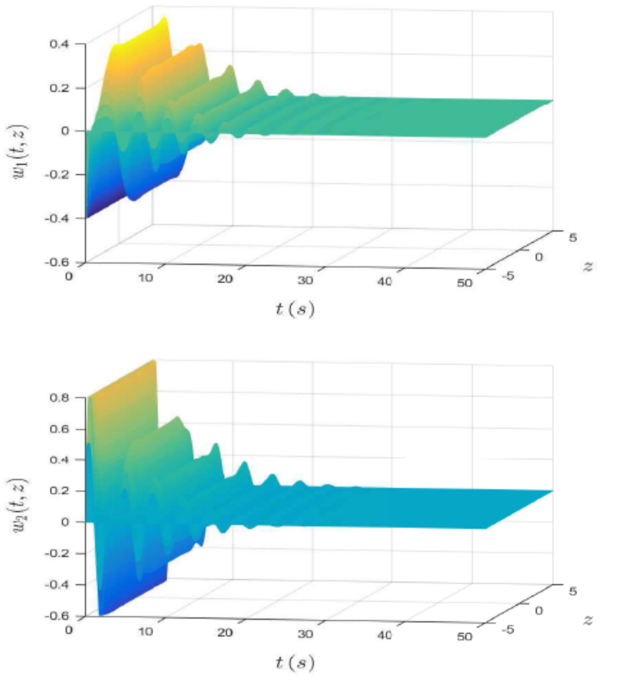

Fig. 3. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (37) under controller (39).

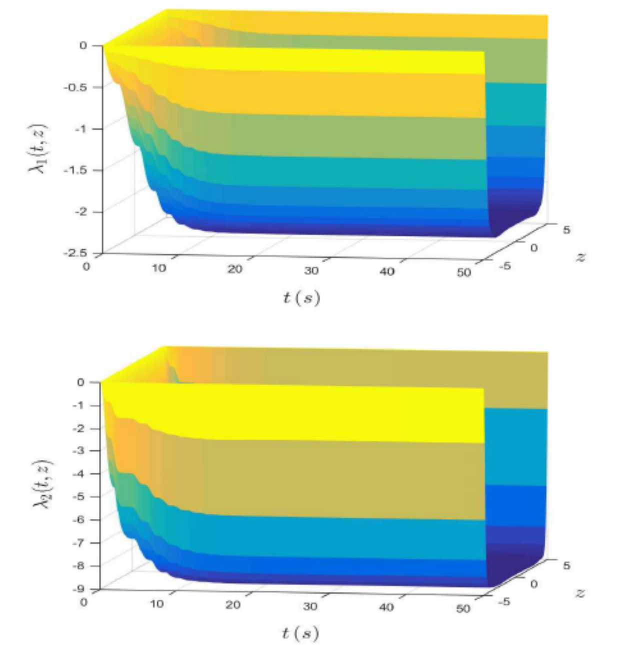

Fig. 4. Trajectories of control gains $\lambda_1(t, z)$ and $\lambda_2(t, z)$ in adaptive controller (39).

Example 2: Consider the 2-D FMRDNNs with discrete delays

$$\begin{aligned}\frac{\partial w_p(t,z)}{\partial t} &= \sum_{l=1}^{2} \gamma_l(\epsilon(t))\Bigg[\varepsilon_p \frac{\partial^2 w_p(t,z)}{\partial z^2} - d_p^{<l>} w_p(t,z) \\ &\quad + \sum_{k=1}^{2} a_{pk}\big(w_p(t,z)\big) f_k(w_k(t,z)) \\ &\quad + \sum_{k=1}^{2} b_{pk}\big(w_p(t,z)\big) g_k(w_k(t-\rho_k(t),z))\Bigg] \end{aligned} \quad (40)$$

where $p, l = 1, 2$, $\varepsilon_1 = \varepsilon_2 = 0.1$, $z \in \Delta = [-5, 5]$, $d_1^{<1>} = 1$, $d_1^{<2>} = 1.2$, $d_2^{<1>} = 1$, $d_2^{<2>} = 0.9$, $\chi_1 = 0.5$, $\chi_2 = 3$, $a_{11}^{+} = 2$, $a_{11}^{-} = 1.8$, $a_{12}^{+} = -0.1$, $a_{12}^{-} = -0.2$, $a_{21}^{+} = -5$, $a_{21}^{-} = -4.9$, $a_{22}^{+} = 3$, $a_{22}^{-} = 2.5$, $b_{11}^{+} = -1.5$, $b_{11}^{-} = -1.2$, $b_{12}^{+} = -0.1$, $b_{12}^{-} = -0.08$, $b_{21}^{+} = -0.2$, $b_{21}^{-} = -0.19$, $b_{22}^{+} = -2.5$, $b_{22}^{-} = -2.4$, $\gamma_1(w) = 1 - 0.1\sin(w)^2$, $\gamma_2(w) = 0.1\sin(w)^2$, the delay $\rho_k(t) = \exp(t)/(1+\exp(t))$, and activation functions $f_k(\cdot) = g_k(\cdot) = \tanh(\cdot)$, $k = 1, 2$. It follows from Assumptions 1–3 that $F_k = G_k = 1$, $\rho_2 = 0.25$, and $\varrho_1 = 1$. Fig. 5 depicts the trajectories of the state variables $w_1(t, z)$ and $w_2(t, z)$ in system (40), which implies that system (40) is unstable.

Then, design the controllers

$$u_p(t,z) = \sum_{l=1}^{2} \gamma_l(\epsilon(t)) \lambda_p^{<l>} w_p(t,z) \quad (41)$$

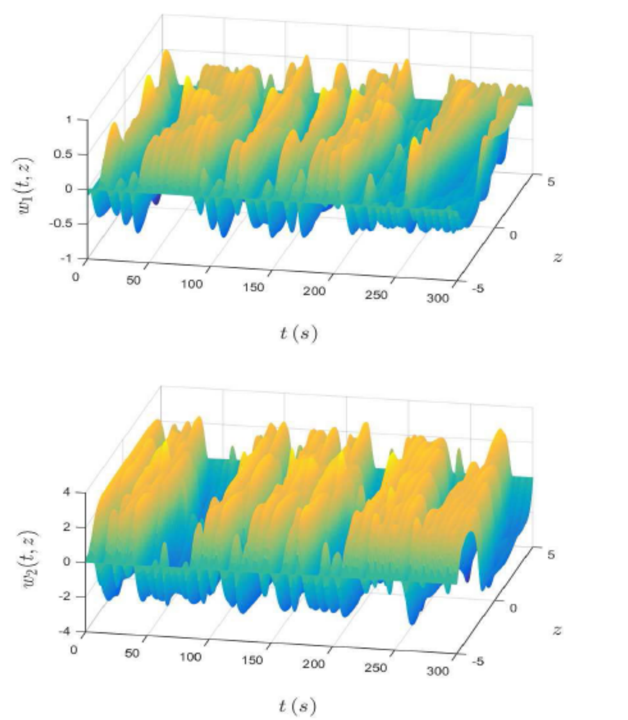

Fig. 5. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (40).

where the gains are chosen as $\lambda_1^{<1>} = -5.6$, $\lambda_1^{<2>} = -5.4$, $\lambda_2^{<1>} = -7.7$, and $\lambda_2^{<2>} = -7.8$. And

$$\begin{aligned} u_p(t,z) &= \sum_{l=1}^{2} \gamma_l(\epsilon(t)) \lambda_p^{<l>}(t,z) w_p(t,z), \\ \frac{\partial \lambda_p^{<l>}(t,z)}{\partial t} &= -\zeta_p^{<l>} |w_p(t,z)|^2 \end{aligned} \quad (42)$$

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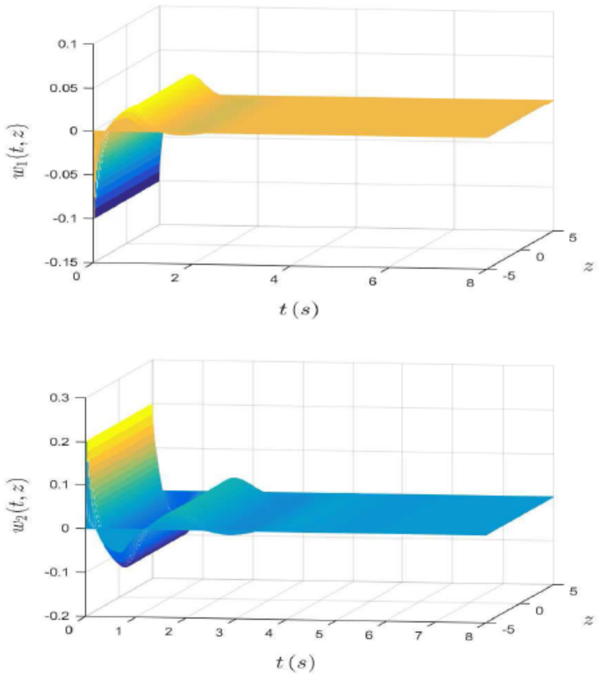

Fig. 6. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (40) under controller (41).

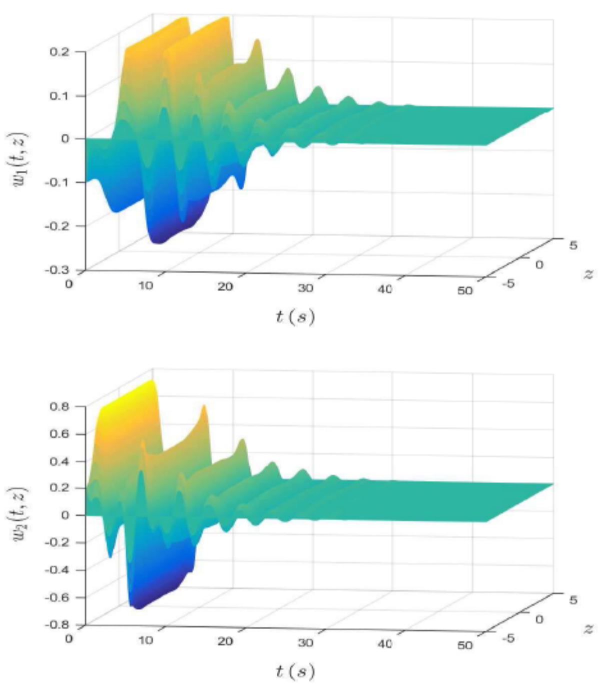

Fig. 7. Trajectories of states $w_1(t, z)$ and $w_2(t, z)$ in system (40) under controller (42).

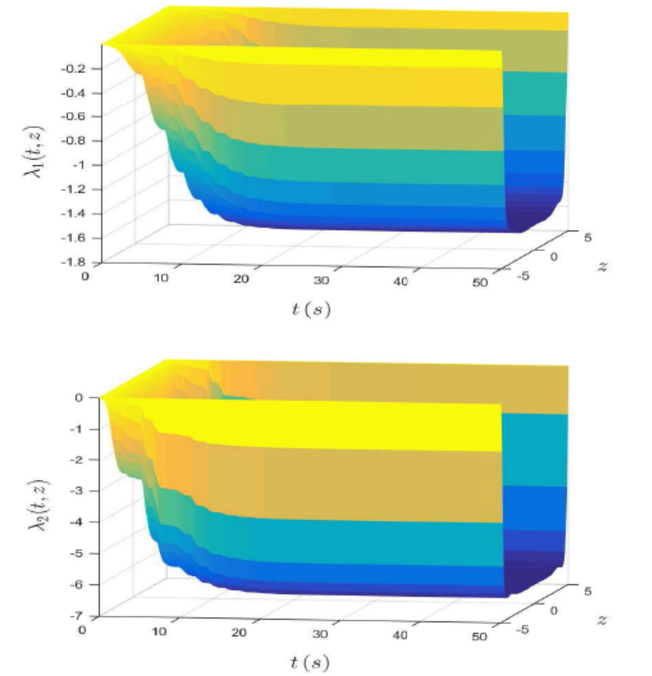

Fig. 8. Trajectories of control gains $\lambda_1(t, z)$ and $\lambda_2(t, z)$ in adaptive controller (42).

where $\zeta_p^{<l>} = 5, p, l = 1, 2$. Then, $\lambda_1(t, z) = \lambda_1^{<l>}(t, z)$ and $\lambda_2(t, z) = \lambda_2^{<l>}(t, z), l = 1, 2$.

Choose $\tau = 2, \xi_p = 1, \acute{\upsilon}_{1pk} = \acute{\upsilon}_{2pk} = \grave{\upsilon}_{1pk} = \grave{\upsilon}_{2pk} = \acute{\varpi}_{1pk} = \acute{\varpi}_{2pk} = \grave{\varpi}_{1pk} = \grave{\varpi}_{2pk} = 0.5, p, k = 1, 2$, it is easy to check that the conditions of Corollary 2 hold. Then, the results of Corollary 2 show that system (40) is globally exponentially stabilizable via the fuzzy controller (41). Also the results of Theorem 2 show that the global stabilization result for system (40) under controller (42). The trajectories of states variables $w_1(t, z)$ and $w_2(t, z)$ under controllers (41) and (42) are shown in Figs. 6 and 7, respectively. From Figs. 6 and 7, we can see that under controller (41) or (42), the states in system (40) converge to zero as time goes to infinity. Finally, the trajectories of control gains $\lambda_1(t, z)$ and $\lambda_2(t, z)$ in adaptive controller (42) are given in Fig. 8.

V. Conclusion

The global stabilizability problem has been discussed for FMRDNNs by adopting the fuzzy set theory, Lyapunov stability theory, and Green formula. Under some inequality techniques, and designed fuzzy controllers, several easily verified criteria have been derived. It is noted that the obtained results are general and include some existing ones as special cases. Finally, two examples have been carried out to show the effectiveness of the presented results.

Since intermittent feedback control methods can save control cost, future work will focus on the stabilization and synchronization problems of FMRDNNs via intermittent control. Moreover, the finite-time or fixed-time stabilization and synchronization problems of FMRDNNs with stochastic disturbances will also be considered in the future.

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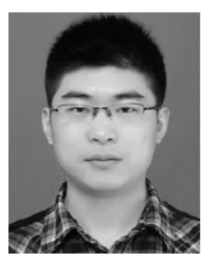

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