Decentralized Event-Triggered Control for a Class of Nonlinear-Interconnected Systems Using Reinforcement Learning

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***Abstract*—In this article, we propose a novel decentralized event-triggered control (ETC) scheme for a class of continuous-time nonlinear systems with matched interconnections. The present interconnected systems differ from most of the existing interconnected plants in that their equilibrium points are no longer assumed to be zero. Initially, we establish a theorem to indicate that the decentralized ETC law for the overall system can be represented by an array of optimal ETC laws for nominal subsystems. Then, to obtain these optimal ETC laws, we develop a reinforcement learning (RL)-based method to solve the Hamilton–Jacobi–Bellman equations arising in the discounted-cost optimal ETC problems of the nominal subsystems. Meanwhile, we only use critic networks to implement the RL-based approach and tune the critic network weight vectors by using the gradient descent method and the concurrent learning technique together. With the proposed weight vectors tuning rule, we are able to not only relax the persistence of the excitation condition but also ensure the critic network weight vectors to be uniformly ultimately bounded. Moreover, by utilizing the Lyapunov method, we prove that the obtained decentralized ETC law can force the entire system to be stable in the sense of uniform ultimate boundedness. Finally, we validate the proposed decentralized ETC strategy through simulations of the nonlinear-interconnected systems derived from two inverted pendulums connected via a spring.**

***Index Terms*—Adaptive dynamic programming (ADP), discounted cost, event-triggered control (ETC), interconnected systems, reinforcement learning (RL).**

I. Introduction

In the control community, the decentralized adaptive control of interconnected systems has been a hot topic over the past several decades [1]–[3]. This is mainly because interconnections have been the common characteristics in many real-world complex systems, such as ecological systems, transportation systems, and computer network systems. Generally, it is difficult to design the stabilizing controllers for the interconnected systems using one-shot approaches [4]. To address this issue, the decentralized control method was proposed. The decentralized control approach differs from the one-shot method in that it first partitions the control problem of the overall system into an array of subproblems which are able to be solved independently. Then, the solutions of subproblems (i.e., independent controllers) all together constitute the decentralized controller, which makes the entire system stable. Moreover, the implementation of the decentralized control algorithm only uses the knowledge of local subsystems rather than the information of the overall system.

The past decades have witnessed many techniques or methods applied to derive the decentralized control, such as the backstepping method [5], the optimal control approach [6], and the fuzzy technique [7]. In this article, we will develop the decentralized control strategy from an optimal control perspective. The early study applying the optimal control theory to design decentralized controllers for nonlinear-interconnected systems could be tracked to Saberi's work [8]. It was proved in [8] that the decentralized controller for the overall system could be derived through solving a set of optimal control problems of independent nonlinear subsystems. Nevertheless, the bottleneck of solving nonlinear optimal control problems is that one often needs to solve the Hamilton–Jacobi–Bellman equations (HJBEs), which generally do not exist in the closed-form solutions. To overcome the bottleneck, adaptive dynamic programming (ADP) [9] and reinforcement learning (RL) [10] were introduced, which aimed at obtaining the numerical solutions of HJBEs. The two names, namely, ADP and RL, are often interchangeable because they have nearly the same characteristics when applied to solve the optimal control problems. In the past decades, ADP and RL have been widely exploited. Various approaches were reported in this field, such as goal representation ADP [11]; local value iteration ADP [12]; policy iteration ADP [13], [14]; robust ADP [15]; online RL [16], [17]; off-policy RL [18], [19]; and integral RL [20], [21].

Though plenty of ADP and RL methods have been successfully applied to obtain numerical solutions of HJBEs, most of them are implemented in the time-triggering mechanism (i.e., the controllers are updated *periodically*). As stated in [22], the time-triggering mechanism generally had a low efficiency in using restricted resources, such as the computation bandwidths

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and the electric power. To address this problem, the event-triggered control (ETC) approaches were proposed [23], [24]. A typical feature of many existing ETC methods is that they update the controllers *aperiodically*. Due to this property, the ETC approaches can save the aforementioned limited resources. Thus, the ETC methods are widely used in real applications, such as islanded microgrids [25] and offshore platforms [26]. In recent years, many event-triggered ADP and RL approaches have been suggested to design adaptive controllers for nonlinear systems. In [27], an RL-based optimal ETC scheme was proposed for continuous-time nonlinear systems. After that, in [28], an ADP-based optimal ETC strategy was developed for partially unknown constrained-input nonlinear systems. Both [27] and [28] employed an actor–critic structure to implement the optimal ETC algorithms. To simplify the actor–critic structure, a single critic network was presented in [29] to obtain the robust ETC of uncertain nonlinear systems. Different from the work of [29], the robust ETC of continuous-time nonlinear systems was derived in [30] by using the H_∞ control theory and the concurrent learning technique together. Later, by using a similar structure as [29] and [30], an ADP-based robust optimal ETC strategy was suggested in [31] for unknown constrained-input nonlinear systems. Recently, in [32], an RL-based distributed approximate optimal ETC scheme was proposed for continuous-time nonlinear-interconnected systems. More recently, in [33], a decentralized ETC policy was developed for nonlinear systems with mismatched interconnections via the combination of the experience replay technique and adaptive critic designs. (*Note:* According to [34], adaptive critic designs were the synonyms for ADP and RL.)

However, a precondition of applying the aforementioned ADP and RL approaches (including the time-triggered and event-triggered ADP and RL methods) is that the equilibrium points of the controlled systems should be zero. In engineering applications, there exist many nonlinear dynamical systems whose equilibrium points are nonzero. Under this circumstance, the aforementioned ADP and RL methods cannot be directly utilized to derive ETC of such systems. For the sake of using these ADP and RL methods, one often has to move the equilibrium points to zero through coordinate transformations. Thus, one needs to acquire the equilibrium points of the controlled systems beforehand. Nevertheless, it is challenging to obtain the equilibrium points of nonlinear systems beforehand, especially for nonlinear-interconnected systems. Therefore, a question to be asked: if the equilibrium points of nonlinear-interconnected systems are nonzero, can we present ADP and RL approaches to obtain the decentralized ETC of such systems *without* requiring coordinate transformations? This motivates this article.

In this article, a novel decentralized ETC scheme is developed for continuous-time nonlinear systems with matched interconnections. The present interconnected systems differ from most of the existing interconnected plants in that their equilibrium points are no more assumed to be zero. Initially, a theorem is established to indicate that the decentralized ETC law for the overall system consists of an array of optimal ETC laws for nominal subsystems. Then, in order to obtain these optimal ETC laws, an RL-based method is developed to solve the HJBEs arising in the discounted-cost optimal ETC problems of nominal subsystems. The implementation of the RL-based approach only uses critic networks. Meanwhile, the critic network weight vectors are tuned by using the gradient descent method and the concurrent learning technique together. With the proposed weight tuning rule, the persistence of excitation condition is relaxed and the critic network weight vectors are uniformly ultimately bounded. Moreover, by using the Lyapunov method, it is proved that the obtained decentralized ETC law forces the entire system to be stable in the sense of uniform ultimate boundedness (UUB).

The novelties of this article are three points.

1) In comparison with [27]–[33], this article removes the restrictive condition that the equilibrium points of nonlinear systems should be zero. Therefore, the present decentralized ETC law is applicable for more general nonlinear plants, especially for those nonlinear-interconnected systems with *nonzero* equilibrium points.
2) Though both this article and [30] employ the concurrent learning technique, an important difference between them is that, in this article, the decentralized ETC is derived via solving an H_2 optimal ETC problem rather than the H_∞ optimal ETC problem. Hence, the decentralized ETC method can avoid the challenge arising in solving the H_∞ optimal ETC problem. (*Note:* According to [35], solving the H_∞ optimal control problems must judge the existence of saddle points beforehand, which is a big challenge.)
3) This article extends the work of [33] to study the decentralized ETC problem of nonlinear systems with *matched* interconnections as well as *nonzero* equilibrium points. Apart from the significant difference stated in 1), another remarkable difference between this article and [33] lies in that this article no longer needs to introduce the auxiliary control, let alone to let it satisfy a restrictive inequality. (*Note:* In [33], the auxiliary control is required to be less than the square root of the term $x_i^\mathsf{T} Q_i x_i$, which is often hard to be directly verified.)

It is worth emphasizing here that due to the introduction of a discount factor into the cost function (i.e., the discounted cost) for each nominal subsystem, the present decentralized ETC method can remove the requirement that the equilibrium point of interconnected systems is zero. Recently, such a discounted cost has been utilized to investigate the optimal *tracking* control (including the time-triggered and event-triggered tracking control) problems of nonlinear systems [36]–[38]. However, there are few studies on developing ETC methods to solve *regulation* problems of nonlinear systems, especially, the regulation problems of nonlinear-interconnected systems with nonzero equilibrium points. This also motivates this article.

Notation: $\mathbb{R}$ denotes the set of all real numbers. $\mathbb{R}^{n_i}$ and $\mathbb{R}^{n_i \times m_i}$ denote the spaces of all real n_i-vectors and all $n_i \times m_i$ real matrices, respectively. Ω_i is a compact subset of $\mathbb{R}^{n_i}$. T is the transpose symbol. "$\triangleq$" means "equal by definition." When $\bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{n_i}]^\mathsf{T} \in \mathbb{R}^{n_i}$, its norm is defined as

$\|\bar{x}\| = \sqrt{\sum_{i=1}^{n_i} |\bar{x}_i|^2}$. When $A \in \mathbb{R}^{n_i \times m_i}$, its norm is defined as $\|A| = \sqrt{\text{tr}(AA^\mathsf{T})}$ with $\text{tr}(AA^\mathsf{T})$ denoting the trace of AA^T.

II. Problem Statement and Preliminaries

A. Problem Statement

We consider the continuous-time nonlinear system with matched interconnections, which consists of N subsystems are given by

$$\begin{aligned}\dot{x}_i(t) &= f_i(x_i(t)) + g_i(x_i(t))(u_i(t) + \omega_i(x(t)))\\ x_{i0} &= x_i(0), i = 1, 2, \dots, N\end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the measurable state vector of the ith subsystem with the initial state x_{i0}, $x = [x_1^\mathsf{T}, x_2^\mathsf{T}, \dots, x_N^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^n$ $\left(n = \sum_{i=1}^N n_i\right)$ is the whole state, $u_i \in \mathbb{R}^{m_i}$ is the control vector of the ith subsystem, $f_i\colon \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $g_i\colon \mathbb{R}^{n_i} \to \mathbb{R}^{n_i \times m_i}$, and $\omega_i\colon \mathbb{R}^n \to \mathbb{R}^{m_i}$ are the known smooth mappings, and $g_i(x_i)\omega_i(x) \in \mathbb{R}^{n_i}$ is the interconnected term.

Assumption 1: For each $i \in \mathbb{I} = \{1, 2, \dots, N\}$, $f_i(0) \neq 0$, that is, $x_i = 0$ is not the equilibrium point of the ith subsystem when $u_i(t) = 0$ and $\omega_i(x(t)) = 0$. In addition, the ith subsystem described as in (1) is controllable.

Assumption 2: For each $i \in \mathbb{I} = \{1, 2, \dots, N\}$, $f_i(x_i)$ and $g_i(x_i)$ have the Lipschitz property on Ω_i and satisfy:

1) $\|f_i(x_i)\| \leq K_{f_i}\|x_i\| + b_{f_i}$, where $K_{f_i} > 0$ is the Lipschitz constant and $b_{f_i} > 0$ is a known constant;
2) $\|g_i(x_i)\| \leq b_{g_i}$, where $b_{g_i} > 0$ is a known constant.

Remark 1: In general, the Lipschitz continuity of $f_i(x_i)$ yields that $\|f_i(x_i)\| \leq K_f\|x_i\|$ (see [28]). However, due to $f_i(0) \neq 0$ (see Assumption 1), we have to let $\|f_i(x_i)\| \leq K_f\|x_i\| + b_{f_i}$ in Assumption 2. Likewise, we can let $g_i(x_i)$ satisfy an inequality like $\|g_i(x_i)\| \leq K_{g_i}\|x_i\| + c_{g_i}$ with $K_{g_i} > 0$ and $c_{g_i} > 0$ be the known constants. Since x_i belongs to the compact set Ω_i, we can conclude that $\|x_i\|$ is upper bounded. Thus, for simplifying the discussion, we let $g_i(x_i)$ be bounded by a constant, that is, $\|g_i(x_i)\| \leq b_{g_i}$ in Assumption 2. This feature is in accordance with the assumption given in [29]–[31].

Assumption 3: For each $i \in \mathbb{I} = \{1, 2, \dots, N\}$, the vector function $\omega_i(x) \in \mathbb{R}^{m_i}$ is bounded as

$$\|\omega_i(x)\| \leq \sum_{j=1}^N a_{ij}P_{ij}(x_j) \tag{2}$$

where $a_{ij} \geq 0$, $j = 1, 2, \dots, N$, are constants and $P_{ij}(x_j) \in \mathbb{R}$, $j = 1, 2, \dots, N$, are positive-definite functions. Furthermore, $\omega_i(0) = 0$ and $P_{ij}(0) = 0$, $i, j = 1, 2, \dots, N$.

Let

$$P_i(x_i) = \max\{P_{1i}(x_i), P_{2i}(x_i), \dots, P_{Ni}(x_i)\}. \tag{3}$$

Then, (2) can be further expressed as

$$\|\omega_i(x)\| \leq \sum_{j=1}^N b_{ij}P_j(x_j), \quad i = 1, 2, \dots, N \tag{4}$$

with $b_{ij} \geq a_{ij}P_{ij}(x_j)/P_j(x_j)$, $j = 1, 2, \dots, N$, being the nonnegative constants.

Remark 2: In real-world systems, there exist interconnected nonlinear systems possessing the feature of system (1) and satisfying Assumption 3. A typical example is the interconnected systems derived from two inverted pendulums connected via a spring (see [1]). As for more detailed analyses, one can refer to Section VI.

The goal of this article is to design an approximate state-feedback decentralized controller for the interconnected system (1), subject to Assumptions 1–3, such that the entire closed-loop system is stable in the sense of UUB. Nonetheless, it is generally difficult to directly design such a decentralized controller. Inspired by the works of [8] and [39], we will divide the decentralized stabilization problem into N optimal control problems of nominal subsystems corresponding to the interconnected system (1).

B. HJBE for ith Nominal Subsystem

For the ith subsystem given as in (1), the nominal system (i.e., the ith nominal subsystem) is

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i. \tag{5}$$

Associated with (5), an infinite-horizon cost function with a discount factor is introduced and written in the form

$$V_i^{u_i}(x_i(t)) = \int_t^\infty e^{-\alpha_i(\tau - t)}\mathcal{R}_i(x_i(\tau), u_i(\tau))\mathrm{d}\tau \tag{6}$$

where $\alpha_i > 0$ is the discount factor, and

$$\mathcal{R}_i(x_i, u_i) = \eta_i P_i^2(x_i) + x_i^\mathsf{T} Q_i x_i + \|u_i\|^2 \tag{7}$$

with $\eta_i > 0$ being the adjustable parameter, $P_i(x_i)$ being defined as (3), $Q_i \in \mathbb{R}^{n_i \times n_i}$ being the positive-definite matrix, and $\|u_i\|^2 = u_i^\mathsf{T} u_i$.

Remark 3: The term $e^{-\alpha_i(\tau - t)}$ $(\tau \geq t)$ in (6) aims at ensuring the cost function $V_i^{u_i}(x_i(t))$ to be convergent. If there is no $e^{-\alpha_i(\tau - t)}$ (i.e., $\alpha_i = 0$), then the cost function (6) will be divergent (or unbounded). This is due to the fact that the equilibrium point of system (5) is nonzero (see Assumption 1).

The cost function, denoted by $V_i^*(x_i)$, is called the optimal cost, that is,

$$V_i^*(x_i) = \min_{u_i \in \mathscr{A}(\Omega_i)} V_i^{u_i}(x_i) \tag{8}$$

where $\mathscr{A}(\Omega_i)$ denotes the set of all admissible control policies defined on Ω_i.

According to [9], $V_i^*(x_i)$ can be obtained by solving the HJBE [*note:* $V_i^*(0) = 0$]

$$\min_{u_i \in \mathscr{A}(\Omega_i)} H\left(x_i, \nabla V_i^*(x_i), u_i\right) = 0 \tag{9}$$

where $H(x_i, \nabla V_i^*(x_i), u_i)$ is the Hamiltonian for $\nabla V_i^*(x_i)$ (i.e., $\partial V_i^*(x_i)/\partial x_i$) and u_i, and its expression is

$$\begin{aligned}H\left(x_i, \nabla V_i^*(x_i), u_i\right) = &\left(\nabla V_i^*(x_i)\right)^\mathsf{T}(f_i(x_i) + g_i(x_i)u_i)\\ &- \alpha_i V_i^*(x_i) + \eta_i P_i^2(x_i)\\ &+ x_i^\mathsf{T} Q_i x_i + \|u_i\|^2.\end{aligned} \tag{10}$$

The optimal control, denoted by $u_i^*(x_i)$, is derived as

$$u_i^*(x_i) = \arg \min_{u_i \in \mathscr{A}(\Omega_i)} H\big(x_i, \nabla V_i^*(x_i), u_i\big) = -\frac{1}{2} g_i^{\mathsf{T}}(x_i) \nabla V_i^*(x_i). \quad (11)$$

Inserting (11) into (9), we can restate the HJBE as

$$\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} f_i(x_i) - \alpha_i V_i^*(x_i) + \eta_i P_i^2(x_i) + x_i^{\mathsf{T}} Q_i x_i - \left\| \frac{1}{2} g_i^{\mathsf{T}}(x_i) \nabla V_i^*(x_i) \right\|^2 = 0 \quad (12)$$

with $V_i^*(0) = 0$.

In general, one can derive the decentralized control law for the interconnected system (1) by solving an array of HJBEs given as in (12) (see [6], [40]). However, as illustrated in [6] and [40], the proposed decentralized control laws were implemented in the time-triggering mechanism. As stated in Section I, the time-triggered control laws often bring about low efficiencies in utilizing the limited resources. Meanwhile, the computational load associated with deriving the time-triggered control policies is heavy. To overcome the two deficiencies, we aim at developing a decentralized ETC strategy for the interconnected system (1).

III. Decentralized ETC Strategy

First, we discuss the stability of the interconnected system (1) in an event-triggering mechanism (ETM). The ETM shares the same spirits as [27]–[33]. In this part, we prove that the decentralized ETC of the interconnected system (1) can be derived by solving an array of event-triggered HJBEs (ET-HJBEs). Then, we solve these ET-HJBEs through the critic-only structure.

A. Decentralized Stabilization in the ETM

Let $\{t_k\}_{k=0}^{\infty}$ (*note:* $t_k < t_{k+1}, k \in \mathbb{N}$) be the sequence of triggering instants. For the ith nominal subsystem, we denote the sampled state at the triggering instant t_k ($k \in \mathbb{N}$) as

$$\bar{x}_{i,k} = x_i(t_k).$$

To describe the error between the sampled state $\bar{x}_{i,k}$ and the current state $x_i(t)$, we define an error function as follows:

$$e_{i,k}(t) = \bar{x}_{i,k} - x_i(t) \quad \forall t \in [t_k, t_{k+1}). \quad (13)$$

Remark 4: Strictly speaking, for the ith nominal subsystem, the sequence of triggering instants should be denoted as $\{t_k^i\}_{k=0}^{\infty}$ (*note:* $t_k^i < t_{k+1}^i$), which is in essence in the same spirits as [17] and [41]. In order not to result in confusions of symbols, we write the sequence $\{t_k^i\}_{k=0}^{\infty}$ as $\{t_k\}_{k=0}^{\infty}$ without mentioning the index i. Likewise, $e_{i,k}(t)$ in (13) ought to be expressed as $e_{i,k}(t) = \bar{x}_{i,k} - x_i(t)$ $\forall t \in [t_k^i, t_{k+1}^i)$. Due to the same reason mentioned above, we present the definition of $e_{i,k}(t)$ as (13). In general, the N subsystems' states are triggered in an *asynchronous* way. The later simulation results have verified this fact (see Figs. 7 and 9 in Section VI).

If an event is triggered at the time instant $t = t_k$, then the error function given in (13) satisfies $e_{i,k}(t_k) = 0$. On the other hand, if letting the state feedback control law be executed at the set of sampled states $\{\bar{x}_{i,k}\}_{k=0}^{\infty}$, then we will obtain a sequence of ETC laws, that is, $\{u_i(\bar{x}_{i,k})\}_{k=0}^{\infty}$. Apparently, the sequence $\{u_i(\bar{x}_{i,k})\}_{k=0}^{\infty}$ consists of discrete-time control signals $u_i(\bar{x}_{i,1}), u_i(\bar{x}_{i,2}), \ldots, u_i(\bar{x}_{i,\infty})$. In order to obtain a continuous-time control signal, one often resorts to the zero-order hold technique [22]. Letting the zero-order hold technique be applied to each control policy $u_i(\bar{x}_{i,k})$, we can generate a continuous-time input signal as follows:

$$\mu_i\big(\bar{x}_{i,k}, t\big) = u_i\big(\bar{x}_{i,k}\big) = u_i(x_i(t_k)) \quad \forall t \in \big[t_k, t_{k+1}\big).$$

Based on the above described ETM, we can derive from (11) that the optimal ETC policy for the ith nominal subsystem (5) with the corresponding discounted cost function (6) is [*note:* $\forall t \in [t_k, t_{k+1})$]

$$\mu_i^*\big(\bar{x}_{i,k}, t\big) = u_i^*\big(\bar{x}_{i,k}\big) = -\frac{1}{2} g_i^{\mathsf{T}}\big(\bar{x}_{i,k}\big) \nabla V_i^*\big(\bar{x}_{i,k}\big) \quad (14)$$

where $\nabla V_i^*(\bar{x}_{i,k}) = (\partial V_i^*(x_i)/\partial x_i)|_{x_i = \bar{x}_{i,k}}$.

Letting u_i in (9) be replaced with $\mu_i^*(\bar{x}_{i,k}, t)$ given as (14), we can obtain the ET-HJBE as follows:

$$\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} f_i(x_i) - \alpha_i V_i^*(x_i) + \eta_i P_i^2(x_i) + x_i^{\mathsf{T}} Q_i x_i - \frac{1}{2}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i) g_i^{\mathsf{T}}\big(\bar{x}_{i,k}\big) \nabla V_i^*\big(\bar{x}_{i,k}\big) + \left\| \frac{1}{2} g_i^{\mathsf{T}}\big(\bar{x}_{i,k}\big) \nabla V_i^*\big(\bar{x}_{i,k}\big) \right\|^2 = 0 \quad (15)$$

with $V_i^*(0) = 0$.

Before continuing our discussion, we make an assumption which was used in [27], [30], and [42].

Assumption 4: $u_i^*(x_i)$ given in (11) is Lipschitz continuous on Ω_i, that is, there exists a Lipschitz constant $K_{u_i^*} > 0$ such that, for every $x_i, \bar{x}_{i,k} \in \Omega_i$, the following inequality holds:

$$\big\| u_i^*(x_i) - u_i^*\big(\bar{x}_{i,k}\big) \big\| \le K_{u_i^*} \big\| x_i - \bar{x}_{i,k} \big\| = K_{u_i^*} \big\| e_{i,k} \big\|.$$

Remark 5: Using (14), Assumption 4 implies

$$\big\| u_i^*(x_i) - \mu_i^*\big(\bar{x}_{i,k}\big) \big\| \le K_{u_i^*} \big\| e_{i,k} \big\|$$

where $\mu_i^*(\bar{x}_{i,k})$ stands for $\mu_i^*(\bar{x}_{i,k}, t)$. For brevity, in subsequent discussion, we write $\mu_i^*(\bar{x}_{i,k}, t)$ as $\mu_i^*(\bar{x}_{i,k})$ without mentioning $t \in [t_k, t_{k+1})$.

Theorem 1: Consider N nominal subsystems formulated as (5) with the corresponding discounted cost functions presented as (6). If Assumptions 1–4 hold, then we can find N positive constants η_i^*, $i = 1, 2, \ldots, N$, such that, for each $\eta_i \ge \eta_i^*$, the N optimal ETC laws $\mu_i^*(\bar{x}_{i,1}), \mu_i^*(\bar{x}_{i,2}), \ldots, \mu_i^*(\bar{x}_{i,N})$ together can force the interconnected system (1) to be stable in the sense of UUB with the following triggering condition:

$$\big\| e_{i,k} \big\|^2 \le \frac{\big(1 - \rho_i^2\big) \lambda_{\min}(Q_i)}{\beta_i K_{u_i^*}^2} \|x_i\|^2 \triangleq \big\| e_{i,T} \big\|^2 \quad (16)$$

where $\rho_i \in (0, 1)$ and $\beta_i \in (0, +\infty)$ are adjustable parameters, $\lambda_{\min}(Q_i)$ is the minimum eigenvalue of Q_i, and $\|e_{i,T}\|$ is the triggering threshold.

Proof: See Appendix A. ■

Theorem 1 shows that the decentralized ETC law for the interconnected system (1) is able to be obtained via finding the N optimal ETC laws $\mu_i^*(\bar{x}_{i,1}), \mu_i^*(\bar{x}_{i,2}), \ldots, \mu_i^*(\bar{x}_{i,N})$. To this end, we solve N ET-HJBEs given as in (15).

B. Solving ET-HJBEs via Critic-Only Structure

According to [43, Theorem. 3.1], $V_i^*(x_i)$ in (8) can be represented via a neural network over Ω_i as follows:

$$V_i^*(x_i) = W_{c_i}^{\mathsf{T}} \sigma_{c_i}(x_i) + \varepsilon_{c_i}(x_i) \tag{17}$$

where $W_{c_i} \in \mathbb{R}^{\tilde{n}_i}$ is the ideal weight vector to be determined, $\sigma_{c_i}(x_i) = [\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\tilde{n}_i}}(x_i)]^{\mathsf{T}} \in \mathbb{R}^{\tilde{n}_i}$ is the continuously differentiable vector activation function [*note:* for every $x_i \neq 0$, $\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\tilde{n}_i}}(x_i)$ are linearly independent; $\sigma_{c_{is}}(0) = 0$, $s = 1, 2, \ldots, \tilde{n}_i$], $\tilde{n}_i \in \mathbb{N}$ is the number of neurons, and $\varepsilon_{c_i}(x_i) \in \mathbb{R}$ is the function reconstruction error. As shown in [44], $\varepsilon_{c_i}(x_i) \to 0$ when $\tilde{n}_i \to \infty$. In other words, $\varepsilon_{c_i}(x_i)$ can be made small via selecting sufficiently large $\tilde{n}_i$.

By using (17), we can obtain the derivative of $V_i^*(x_i)$ at the sampled state $\bar{x}_{i,k}$ as

$$\nabla V_i^*(\bar{x}_{i,k}) = \nabla \sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k}) W_{c_i} + \nabla \varepsilon_{c_i}(\bar{x}_{i,k})$$

where $\nabla \sigma_{c_i}(\bar{x}_{i,k}) = (\partial \sigma_{c_i}(x_i)/\partial x_i)|_{x_i=\bar{x}_{i,k}}$ with $\nabla \sigma_{c_i}(0) = 0$, and $\nabla \varepsilon_{c_i}(\bar{x}_{i,k}) = (\partial \varepsilon_{c_i}(x_i)/\partial x_i)|_{x_i=\bar{x}_{i,k}}$. Thus, we can restate $\mu_i^*(\bar{x}_{i,k})$ in (14) as [*note:* $\forall t \in [t_k, t_{k+1})$]

$$\mu_i^*(\bar{x}_{i,k}) = -\frac{1}{2} g_i^{\mathsf{T}}(\bar{x}_{i,k}) \nabla \sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k}) W_{c_i} + \varepsilon_{\mu_i^*}(\bar{x}_{i,k}) \tag{18}$$

where $\varepsilon_{\mu_i^*}(\bar{x}_{i,k}) = -(1/2) g_i^{\mathsf{T}}(\bar{x}_{i,k}) \nabla \varepsilon_{c_i}(\bar{x}_{i,k})$. Note that W_{c_i} in (18) is often unavailable. Thus, we use the estimated weight vector $\hat{W}_{c_i}$ to replace W_{c_i}. Specifically, we use the critic network to approximate $V_i^*(x_i)$ in (8) as follows:

$$\hat{V}_i(x_i) = \hat{W}_{c_i}^{\mathsf{T}} \sigma_{c_i}(x_i). \tag{19}$$

Using (19), we can formulate the estimated value of $\mu_i^*(\bar{x}_{i,k})$ as [*note:* $\forall t \in [t_k, t_{k+1})$]

$$\hat{\mu}_i(\bar{x}_{i,k}) = -\frac{1}{2} g_i^{\mathsf{T}}(\bar{x}_{i,k}) \nabla \sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k}) \hat{W}_{c_i}. \tag{20}$$

Replacing $V_i^*(x_i)$ and u_i in (10) with aforementioned $\hat{V}_i(x_i)$ and $\hat{\mu}_i(\bar{x}_{i,k})$, respectively, we obtain the approximate Hamiltonian as

$$\begin{aligned}\hat{H}\left(x_i, \nabla \hat{V}_i(x_i), \hat{\mu}_i(\bar{x}_{i,k})\right) = \ & \hat{W}_{c_i}^{\mathsf{T}} \phi_i + \eta_i P_i^2(x_i) + x_i^{\mathsf{T}} Q_i x_i \\ & + \left\| \hat{\mu}_i(\bar{x}_{i,k}) \right\|^2 \end{aligned} \tag{21}$$

where

$$\phi_i = \nabla \sigma_{c_i}(x_i) \big(f_i(x_i) + g_i(x_i) \hat{\mu}_i(\bar{x}_{i,k}) \big) - \alpha_i \sigma_{c_i}(x_i). \tag{22}$$

On the other hand, as pointed out by [27] and [45], $\mu_i^*(\bar{x}_{i,k})$ is actually the discretized value of $\mu_i^*(x_i)$ at the triggering instant t_k. Thus, (9) implies $H(x_i, \nabla V_i^*(x_i), \mu_i^*(\bar{x}_{i,k})) = 0$. Then, by using (21), we can define the error arising in the approximating Hamiltonian as

$$\begin{aligned} e_{c_i} &= \hat{H}\left(x_i, \nabla \hat{V}_i(x_i), \hat{\mu}_i(\bar{x}_{i,k})\right) - H\big(x_i, \nabla V_i^*(x_i), \mu_i^*(\bar{x}_{i,k})\big) \\ &= \hat{W}_{c_i}^{\mathsf{T}} \phi_i + \eta_i P_i^2(x_i) + x_i^{\mathsf{T}} Q_i x_i + \left\| \hat{\mu}_i(\bar{x}_{i,k}) \right\|^2. \end{aligned} \tag{23}$$

For the purpose of making $\hat{\mu}_i(\bar{x}_{i,k}) \to \mu_i^*(\bar{x}_{i,k})$, we need to force $e_{c_i} \to 0$. That is, we should keep e_{c_i} small enough. To this end, one usually tunes $\hat{W}_{c_i}$ to minimize the target function $E_i = (1/2) e_{c_i}^{\mathsf{T}} e_{c_i}$. In order to make a high efficiency in utilizing the historical state data and motivated by the works of [46]–[48], we change the objective function E_i to be

$$E_i = \underbrace{\frac{1}{2} e_{c_i}^{\mathsf{T}} e_{c_i}}_{E_{c_i}} + \sum_{s=1}^{l_0} \underbrace{\frac{1}{2} e_{c_{i,s}}^{\mathsf{T}} e_{c_{i,s}}}_{E_{c_{i,s}}} \tag{24}$$

where e_{c_i} is given in (23), $s \in \{1, 2, \ldots, l_0\}$ is the index of the historical state $x_i(t_s)$, $t_s \in [t_k, t_{k+1})$, l_0 is the number of the historical state (*note:* $l_0 > \tilde{n}_i$ with $\tilde{n}_i$ denoting the aforementioned number of neurons), and $e_{c_{i,s}}$ is the value of e_{c_i} at the historical state $x_i(t_s)$, that is,

$$\begin{aligned} e_{c_{i,s}} = e_{c_i}(x_i(t_s)) = \ & \hat{W}_{c_i}^{\mathsf{T}} \phi_{i,s} + \eta_i P_i^2(x_i(t_s)) \\ & + \ x_i^{\mathsf{T}}(t_s) Q_i x_i(t_s) + \left\| \hat{\mu}_i(\bar{x}_{i,k}) \right\|^2 \end{aligned}$$

where

$$\begin{aligned} \phi_{i,s} = \ & \nabla \sigma_{c_i}(x_i(t_s)) \big(f_i(x_i(t_s)) + g_i(x_i(t_s)) \hat{\mu}_i(\bar{x}_{i,k}) \big) \\ & - \alpha_i \sigma_{c_i}(x_i(t_s)). \end{aligned} \tag{25}$$

Applying the gradient descent approach to E_i [*note:* $E_i = E_{c_i} + \sum_{s=1}^{l_0} E_{c_{i,s}}$ in (24)] and selecting the two different normalization terms $(1+\phi_i^{\mathsf{T}}\phi_i)^{-2}$ and $(1+\phi_{i,s}^{\mathsf{T}}\phi_{i,s})^{-2}$, we can update the estimated weight vector $\hat{W}_{c_i}$ via [*note:* $\forall t \in [t_k, t_{k+1})$]

$$\begin{aligned} \dot{\hat{W}}_{c_i} &= -\frac{\ell_{c_i}}{\left(1+\phi_i^{\mathsf{T}}\phi_i\right)^2} \frac{\partial E_{c_i}}{\partial \hat{W}_{c_i}} - \sum_{s=1}^{l_0} \frac{\ell_{c_i}}{\left(1+\phi_{i,s}^{\mathsf{T}}\phi_{i,s}\right)^2} \frac{\partial E_{c_{i,s}}}{\partial \hat{W}_{c_i}} \\ &= -\frac{\ell_{c_i}\phi_i}{\left(1+\phi_i^{\mathsf{T}}\phi_i\right)^2} e_{c_i} - \sum_{s=1}^{l_0} \frac{\ell_{c_i}\phi_{i,s}}{\left(1+\phi_{i,s}^{\mathsf{T}}\phi_{i,s}\right)^2} e_{c_{i,s}} \end{aligned} \tag{26}$$

where $\ell_{c_i} > 0$ is an adjustable parameter, and ϕ_i and $\phi_{i,s}$ are defined as (22) and (25), respectively.

Denote

$$\psi_i = \phi_i \Big/ \left(1 + \phi_i^{\mathsf{T}} \phi_i\right) \quad \text{and} \quad \psi_{i,s} = \phi_{i,s} \Big/ \left(1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}\right).$$

Let the weight estimation error be $\tilde{W}_{c_i} = W_{c_i} - \hat{W}_{c_i}$. Then, from (26), we can obtain [*note:* $t \in [t_k, t_{k+1})$]

$$\begin{aligned} \dot{\tilde{W}}_{c_i} = & -\ell_{c_i} \left(\psi_i \psi_i^{\mathsf{T}} + \sum_{s=1}^{l_0} \psi_{i,s} \psi_{i,s}^{\mathsf{T}} \right) \tilde{W}_{c_i} \\ & + \frac{\ell_{c_i} \psi_i}{1 + \phi_i^{\mathsf{T}} \phi_i} \varepsilon_{H_i} + \sum_{s=1}^{l_0} \frac{\ell_{c_i} \psi_{i,s}}{1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}} \varepsilon_{H_{i,s}} \end{aligned} \tag{27}$$

where ε_{H_i} and $\varepsilon_{H_{i,s}}$ are the residual errors formulated as [27]

$$\begin{aligned} \varepsilon_{H_i} &= -\nabla \varepsilon_{c_i}^{\mathsf{T}}(x_i) \big(f_i(x_i) + g_i(x_i) \hat{\mu}_i(\bar{x}_{i,k}) \big) + \alpha_i \varepsilon_{c_i}(x_i) \\ \varepsilon_{H_{i,s}} &= -\nabla \varepsilon_{c_i}^{\mathsf{T}}(x_i(t_s)) \big(f_i(x_i(t_s)) + g_i(x_i(t_s)) \hat{\mu}_i(\bar{x}_{i,k}) \big) \\ & \quad + \alpha_i \varepsilon_{c_i}(x_i(t_s)). \end{aligned}$$

Remark 6: According to [46]–[48], the summation term in (27) can force $\tilde{W}_{c_i}$ to converge to a small compact set without requiring the persistence of excitation condition only if the historical state dataset

$$\left\{ \sigma_{c_i}(x_i(t_1)), \ldots, \sigma_{c_i}\big(x_i(t_{\tilde{n}_i})\big), \ldots, \sigma_{c_i}\big(x_i(t_{l_0})\big) \right\} \ (l_0 > \tilde{n}_i)$$

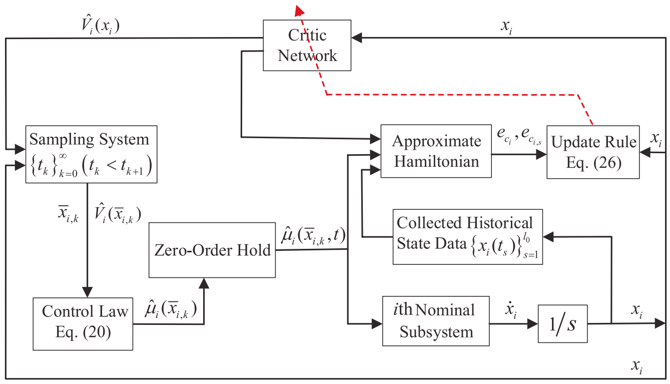

Fig. 1. Block diagram of the proposed ETC scheme for the ith nominal subsystem.

satisfies

$$\operatorname{rank}\big[\sigma_{c_i}(x_i(t_1)), \sigma_{c_i}(x_i(t_2)), \ldots, \sigma_{c_i}\big(x_i(t_{l_0})\big)\big] = \tilde{n}_i. \tag{28}$$

In this article, we also need the collected historical state dataset to satisfy (28). Obviously, the validity of (28) can be guaranteed by choosing the large number of the historical state, that is, l_0. In addition, it can be seen from (26) that both the current state data and the historical state data [see the first term and the summation term in (26)] are utilized to make $\hat{W}_{c_i}$ converge to the ideal weight vector W_{c_i} within a small compact set (or rather, to make the weight estimation error $\tilde{W}_{c_i}$ converge to a small compact set). Chowdhary [46] first coined this technique called the concurrent learning. A synonym for the concurrent learning technique was the experience replay method [49], [50]. In this article, we call it the concurrent learning, which is in accordance with [46].

To summarize the above analyses, we provide a block diagram to illustrate the proposed ETC scheme for the ith nominal subsystem (see Fig. 1).

IV. Stability Analysis

We first present an augmented hybrid system comprised of the ith closed-loop nominal subsystem and the dynamics of $\tilde{W}_{c_i}$ in (27). That is, letting $z_i = [x_i^{\mathsf{T}}, \bar{x}_{i,k}^{\mathsf{T}}, \tilde{W}_{c_i}^{\mathsf{T}}]^{\mathsf{T}}$, we have the following.

1) *Continuous Dynamics:* [*Note*: $z_i \triangleq z_i(t)$, $\forall t \in [t_k, t_{k+1})$]

$$\dot{z}_i = \begin{bmatrix} f_i(x_i) - \frac{1}{2} g_i(x_i) g_i^{\mathsf{T}}(\bar{x}_{i,k}) \nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k}) \hat{W}_{c_i} \\ 0 \\ -\ell_{c_i} \Phi(\psi_i, \psi_{i,s}) \tilde{W}_{c_i} + \Sigma(\varepsilon_{H_i}, \varepsilon_{H_{i,s}}) \end{bmatrix} \tag{29}$$

where

$$\Phi(\psi_i, \psi_{i,s}) = \psi_i \psi_i^{\mathsf{T}} + \sum_{s=1}^{l_0} \psi_{i,s} \psi_{i,s}^{\mathsf{T}} \tag{30}$$

$$\Sigma(\varepsilon_{H_i}, \varepsilon_{H_{i,s}}) = \frac{\ell_{c_i} \psi_i \varepsilon_{H_i}}{1 + \phi_i^{\mathsf{T}} \phi_i} + \sum_{s=1}^{l_0} \frac{\ell_{c_i} \psi_{i,s} \varepsilon_{H_{i,s}}}{1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}}.$$

2) *Discrete Dynamics:* (*Note*: $t = t_{k+1}$, $k \in \mathbb{N}$)

$$z_i(t^+) = z_i(t) + \begin{bmatrix} 0 \\ \bar{x}_{i,k} - x_i(t) \\ 0 \end{bmatrix} \tag{31}$$

where $z_i(t^+) = \lim_{\varsigma \to 0^+} z_i(t+\varsigma)$ with $\varsigma \in (0, t_{k+1} - t_k)$.

Before studying the stabilities of (29) and (31), we impose an assumption which was utilized in [29], [33], and [51].

Assumption 5: For every $x_i \in \Omega_i$, there exists a constant $b_{\sigma_{c_i}} > 0$ such that $\|\nabla\sigma_{c_i}(x_i)\| \le b_{\sigma_{c_i}}$. Meanwhile, for every $x_i \in \Omega_i$, there exist constants $b_{\varepsilon_{\mu_i^*}} > 0$ and $b_{\varepsilon_{H_i}} > 0$ such that $\|\varepsilon_{\mu_i^*}(x_i)\| \le b_{\varepsilon_{\mu_i^*}}$ and $\|\varepsilon_{H_i}\| \le b_{\varepsilon_{H_i}}$.

Theorem 2: Consider the ith nominal subsystem (5) with associated ET-HJBE (15). Let Assumptions 1–5 be satisfied and provide an initial admissible control for the ith nominal subsystem (5). Meanwhile, take the ith ETC law as (20) and update the critic network weight by using (26). Then, the ith closed-loop nominal subsystem (5) and the weight estimation error $\tilde{W}_{c_i}$ are stable in the sense of UUB if the triggering condition is constructed as

$$\|e_{i,k}\|^2 \le \frac{(1-\rho_i^2)\lambda_{\min}(Q_i)}{(1+\gamma_i)K_{u_i^*}^2} \|x_i\|^2 \triangleq \|\bar{e}_{i,T}\|^2 \tag{32}$$

where $\rho_i \in (0, 1)$, $\gamma_i \in (0, +\infty)$, and $\|\bar{e}_{i,T}\|$ is the triggering threshold, and as long as the following inequality holds:

$$2\ell_{c_i} \lambda_{\min}\big(\Phi(\psi_i, \psi_{i,s})\big) - (1 + 1/\gamma_i)^2 b_{g_i}^2 b_{\sigma_{c_i}}^2 > 0 \tag{33}$$

with $\lambda_{\min}(\Phi(\psi_i, \psi_{i,s}))$ being the minimum eigenvalue of $\Phi(\psi_i, \psi_{i,s})$ given in (30).

Proof: See Appendix B. ■

Remark 7: In (33), the term $\lambda_{\min}(\Phi(\psi_i, \psi_{i,s}))$ is positive under condition (28). Now, we prove this fact. Similar to the process of [52, Lemma 2], we can derive

$$\begin{aligned} &\operatorname{rank}\big[\psi_i(x_i(t_1)), \psi_i(x_i(t_2)), \ldots, \psi_i(x_i(t_{l_0}))\big] \\ &\quad = \operatorname{rank}\big[\sigma_{c_i}(x_i(t_1)), \sigma_{c_i}(x_i(t_2)), \ldots, \sigma_{c_i}\big(x_i(t_{l_0})\big)\big] = \tilde{n}_i. \end{aligned}$$

Let $\xi_i = [\psi_i(x_i(t_1)), \psi_i(x_i(t_2)), \ldots, \psi_i(x_i(t_{l_0}))]$. Then, by using the matrix theory [53, Chapter. 0.4], we have $\operatorname{rank} \xi_i \xi_i^{\mathsf{T}} = \operatorname{rank} \xi_i$. Thus, $\operatorname{rank} \xi_i \xi_i^{\mathsf{T}} = \tilde{n}_i$, that is,

$$\operatorname{rank} \sum_{s=1}^{l_0} \psi_{i,s} \psi_{i,s}^{\mathsf{T}} = \tilde{n}_i.$$

Hence, $\sum_{s=1}^{l_0} \psi_{i,s} \psi_{i,s}^{\mathsf{T}} \in \mathbb{R}^{\tilde{n}_i \times \tilde{n}_i}$ is positive definite. Note that $\psi_i \psi_i^{\mathsf{T}}$ is a positive semidefinite matrix. Therefore, we obtain that $\Phi(\psi_i, \psi_{i,s}) \in \mathbb{R}^{\tilde{n}_i \times \tilde{n}_i}$ given in (30) is positive definite. Then, $\lambda_{\min}(\Phi(\psi_i, \psi_{i,s})) > 0$ holds.

V. Lower Bound of the Minimal Intersample Time

When designing the event-triggered controllers, one has to guarantee the minimal intersample time to be positive. Now, we prove this fact. First, we provide an assumption associated with $\nabla\sigma_{c_i}(x_i)$. The assumption was used in [28] and [29].

Assumption 6: For every $i \in \mathbb{I} = \{1, 2, \ldots, N\}$, $\nabla\sigma_{c_i}(x_i)$ satisfies the Lipschitz property on Ω_i. To be specific, for every $x_i', x_i'' \in \Omega_i$, there exists a Lipschitz constant $K_{\sigma_{c_i}} > 0$ making the following inequality hold:

$$\|\nabla\sigma_{c_i}(x_i') - \nabla\sigma_{c_i}(x_i'')\| \le K_{\sigma_{c_i}} \|x_i' - x_i''\|.$$

Theorem 3: Consider the ith nominal subsystem (5) with the ETC $\hat{\mu}_i(\bar{x}_{i,k})$ proposed as in (20). Let Assumptions 2 and 6 hold and let the triggering condition be given as (32). Then, the minimal intersample time, denoted by $(\Delta t_k)_{\min}$ (*note:* $\Delta t_k = t_{k+1} - t_k$, $k \in \mathbb{N}$), satisfies the following inequality:

$$(\Delta t_k)_{\min} \geq \frac{1}{\pounds_i} \ln(1 + \bar{\varrho}_{\min}) > 0 \tag{34}$$

where $\pounds_i$ and $\bar{\varrho}_{\min}$ are positive constants to be given in later (38) and (44), respectively.

Proof: Under Assumption 2, the ith nominal subsystem (5) with the ETC law $\hat{\mu}_i(\bar{x}_{i,k})$ yields

$$\begin{aligned}\|\dot{x}_i\| &= \left\|f_i(x_i) + g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k})\right\| \\ &\leq K_{f_i}\|x_i\| + b_{g_i}\left\|\hat{\mu}_i(\bar{x}_{i,k})\right\| + b_{f_i}.\end{aligned} \tag{35}$$

Meanwhile, by using Assumptions 2 and 6, we are able to obtain from (20) that

$$\left\|\hat{\mu}_i(\bar{x}_{i,k})\right\| \leq \frac{1}{2} b_{g_i} K_{\sigma_{c_i}} \left\|\bar{x}_{i,k}\right\| \left\|\hat{W}_{c_i}\right\|. \tag{36}$$

Combining (35) with (36), we have

$$\|\dot{x}_i\| \leq K_{f_i}\|x_i\| + \frac{1}{2} b_{g_i}^2 K_{\sigma_{c_i}} \left\|\hat{W}_{c_i}\right\| \left\|\bar{x}_{i,k}\right\| + b_{f_i}. \tag{37}$$

According to Theorem 2, $\tilde{W}_{c_i}$ is UUB. Note that the ideal weight W_{c_i} is typically bounded. We thus obtain that $\hat{W}_{c_i}$ is bounded (*note:* $\hat{W}_{c_i} = W_{c_i} - \tilde{W}_{c_i}$). Here, we write $\|\hat{W}_{c_i}\| \leq \delta_{\hat{W}_{c_i}}$ with $\delta_{\hat{W}_{c_i}} > 0$ being the known constant. Let

$$\pounds_i = \max\left\{K_{f_i}, \frac{1}{2} b_{g_i}^2 K_{\sigma_{c_i}} \delta_{\hat{W}_{c_i}}\right\}. \tag{38}$$

Then, (37) can be further written as

$$\|\dot{x}_i\| \leq \pounds_i\|x_i\| + \pounds_i\left\|\bar{x}_{i,k}\right\| + b_{f_i}. \tag{39}$$

According to (13), it follows that $x_i(t) = \bar{x}_{i,k} - e_{i,k}(t)$ and $\dot{x}_i(t) = -\dot{e}_{i,k}(t)$. We thus derive from (39) that

$$\left\|\dot{e}_{i,k}\right\| \leq \pounds_i\left\|e_{i,k}\right\| + 2\pounds_i\left\|\bar{x}_{i,k}\right\| + b_{f_i}. \tag{40}$$

Note that at the triggering instant t_k, it follows $e_{i,k}(t_k) = 0$. Then, according to the comparison lemma [54, Lemma 3.4], we can find that the solution of (40) satisfies [*note:* $\forall t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$]

$$\left\|e_{i,k}\right\| \leq \frac{2\pounds_i\left\|\bar{x}_{i,k}\right\| + b_{f_i}}{\pounds_i}\left(e^{\pounds_i(t-t_k)} - 1\right). \tag{41}$$

After doing some computations, we obtain from (41) that

$$\Delta t_k = t_{k+1} - t_k \geq \frac{1}{\pounds_i} \ln(1 + \varrho_k), \quad k \in \mathbb{N} \tag{42}$$

where

$$\varrho_k = \frac{\pounds_i\left\|\bar{x}_{i,k} - x_i\left(t_{k+1}^-\right)\right\|}{2\pounds_i\left\|\bar{x}_{i,k}\right\| + b_{f_i}} > 0, \quad k \in \mathbb{N} \tag{43}$$

with $x_i(t_{k+1}^-) = \lim_{\varsigma \to 0^+} x_i(t_{k+1} - \varsigma)$.

Denote the minimum value of ϱ_k for all $k \in \mathbb{N}$ as

$$\bar{\varrho}_{\min} = \min_{k \in \mathbb{N}}\{\varrho_k\}. \tag{44}$$

According to (43) and (44), we have $\bar{\varrho}_{\min} > 0$. Then, taking the minimum value on both sides of (42), we obtain

$$(\Delta t_k)_{\min} \geq \frac{1}{\pounds_i} \ln(1 + \bar{\varrho}_{\min}) > 0.$$

That is, (34) holds. ■

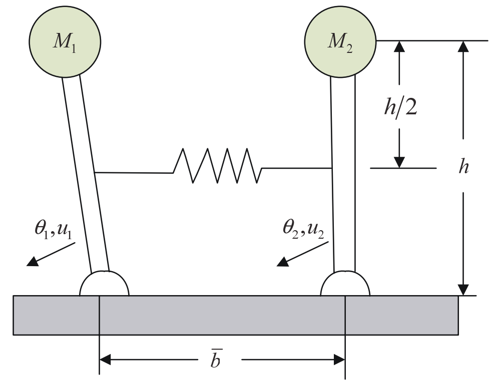

Fig. 2. Two inverted pendulums connected via a spring.

TABLE I
PARAMETERS USED IN THE TWO INVERTED PENDULUMS

Parameter	Meaning	Value
M_1	mass of the first pendulum end	2 (kg)
M_2	mass of the second pendulum end	2.5 (kg)
J_1	moment of inertia (the first pendulum)	0.5 (kg)
J_2	moment of inertia (the second pendulum)	0.625 (kg)
$\bar{g}$	gravitational acceleration	9.81 (m/s^2)
h	the pendulum height	0.5 (m)
k_0	spring constant	100 (N/m)
$\bar{l}$	natural length of the spring	0.5 (m)
$\bar{b}$	distance between the pendulum hinges	0.4 (m)

Remark 8: As pointed out by [22], Zeno behavior occurs only when the minimal intersample time is zero. Theorem 3 indicates that the lower bound of the minimal intersample time is positive. Thus, Zeno behavior is avoided under the condition given in Theorem 3.

VI. SIMULATION STUDY

To validate the established theoretical results, we consider the two inverted pendulums connected via a spring proposed in [1]. The structure of the two inverted pendulums is displayed as Fig. 2. Meanwhile, the motion of the two inverted pendulums can be described via a state-space mode as

$$\begin{aligned}\dot{x}_{11} &= x_{12}, \dot{x}_{12} = \left(\frac{M_1\bar{g}h}{J_1} - \frac{k_0h^2}{4J_1}\right)\sin(x_{11}) + \frac{k_0h}{2J_1}\left(\bar{l} - \bar{b}\right) \\ &\quad + \frac{u_1}{J_1} + \frac{k_0h^2}{4J_1}\sin(x_{21}) \\ \dot{x}_{21} &= x_{22} \\ \dot{x}_{22} &= \left(\frac{M_2\bar{g}h}{J_2} - \frac{k_0h^2}{4J_2}\right)\sin(x_{21}) - \frac{k_0h}{2J_2}\left(\bar{l} - \bar{b}\right) \\ &\quad + \frac{u_2}{J_2} + \frac{k_0h^2}{4J_2}\sin(x_{12})\end{aligned} \tag{45}$$

with $\theta_1 = x_{11}$ and $\theta_2 = x_{21}$ denoting the angular displacements of the pendulums from vertical. The meanings and values of the parameters used in the two inverted pendulums are provided in Table I.

Let $x_i = [x_{i1}, x_{i2}]^{\mathsf{T}} \in \mathbb{R}^2$, $i = 1, 2$. Then, we can restate

$$f_i(x_i) = \begin{bmatrix} x_{i2} \\ \bar{\iota}_i \sin(x_{i1}) + (-1)^{i+1} k_0 h \big(\bar{l} - \bar{b}\big)/(2J_i) \end{bmatrix} \quad (46)$$

where $\bar{\iota}_i = M_i \bar{g} h / J_i - k_0 h^2/(4J_i)$, $i = 1, 2$. As shown in Table I, there is $\bar{l} \neq \bar{b}$. Then, we can see from (46) that $f_i(0) \neq 0$, $i = 1, 2$, which satisfy Assumption 1. Therefore, the equilibrium point of system (45) is nonzero. Note that $\|\sin(x_{i1})\| \leq \|x_{i1}\|$ for $x_{i1} \in \mathbb{R}$, $i = 1, 2$. Then, it can be found that $f_i(x_i)$, $i = 1, 2$, given in (46) satisfy Assumption 2. Denoting $g_i(x_i) = [0, 1/J_i]$, $i = 1, 2$, we can find that $\|g_i(x_i)\| \leq 1/J_i$, $i = 1, 2$, which satisfy Assumption 2. Let $\omega_1(x) = (k_0 h^2/4) \sin(x_{21})$. Then, according to Table I, we have $\|\omega_1(x)\| \leq 6.25|x_{21}| \leq 6.25\|x_2\|$. Likewise, letting $\omega_2(x) = (k_0 h^2/4) \sin(x_{12})$, we obtain $\|\omega_2(x)\| \leq 6.25|x_{12}| \leq 6.25\|x_1\|$. Thus, to satisfy the inequality (4) (or rather, Assumption 3), we can select $P_1(x_1) = \|x_1\|$, $P_2(x_2) = \|x_2\|$, and design the corresponding parameters as follows: $b_{11} = 0$, $b_{12} = 6.25$, $b_{21} = 6.25$, and $b_{22} = 0$. In addition, the initial state vector of interconnected system (45) is $x_0 = [0.5, -0.5, 1, -1]^{\mathsf{T}}$.

By using (5), we are able to obtain the nominal subsystems 1 and 2 for the interconnected system (45). According to (6), we can separately present the discounted cost function for nominal subsystems 1 and 2 as

$$V_1^{u_1}(x_1) = \int_t^{\infty} e^{-\alpha_1(\tau - t)} \Big(\eta_1 \|x_1\|^2 + x_1^{\mathsf{T}} Q_1 x_1 + u_1^2\Big) \mathrm{d}\tau$$
$$V_2^{u_2}(x_2) = \int_t^{\infty} e^{-\alpha_2(\tau - t)} \Big(\eta_2 \|x_2\|^2 + x_2^{\mathsf{T}} Q_2 x_2 + u_2^2\Big) \mathrm{d}\tau.$$

To make the matrix $\tilde{\mathcal{A}}$ in (56) positive definite, we choose $\eta_1 = 40$ and $\eta_2 = 40$. At the same time, we let $\alpha_1 = 0.85$, $\alpha_2 = 6.9$, and $Q_1 = Q_2 = 2.85 I_2$ with I_2 denoting the 2×2 identity matrix. As indicated in Theorem 1, we need to solve the ET-HJBEs related to nominal subsystems 1 and 2 [such as (15)] for obtaining the decentralized ETC of system (45). To this end, we use the critic network (19) to approximately solve the two ET-HJBEs. The vector activation functions $\sigma_{c_i}(x_i)$, $i = 1, 2$, are, respectively, given in the form (*note:* $\tilde{n}_1 = 3$ and $\tilde{n}_2 = 3$):

$$\sigma_{c_1}(x_1) = \Big[x_{11}^2, x_{11}x_{12}, x_{12}^2\Big]^{\mathsf{T}}$$
$$\sigma_{c_2}(x_2) = \Big[x_{21}^2, x_{21}x_{22}, x_{22}^2\Big]^{\mathsf{T}}.$$

The weight parameters associated with $\sigma_{c_1}(x_1)$ and $\sigma_{c_2}(x_2)$ are denoted as $\hat{W}_{c_1} = [\hat{W}_{c_{11}}, \hat{W}_{c_{12}}, \hat{W}_{c_{13}}]^{\mathsf{T}}$ and $\hat{W}_{c_2} = [\hat{W}_{c_{21}}, \hat{W}_{c_{22}}, \hat{W}_{c_{23}}]^{\mathsf{T}}$, respectively. The parameters used in (26) and (32) are designed as follows: $\ell_{c_i} = 0.8$, $\rho_i = 0.3$, $\gamma_i = 3$, and $K_{u_i^*} = 4.5$, where $i = 1, 2$.

Remark 9: Generally, there is no direct method to verify Assumption 4. To make Assumption 4 hold, a promising method is to select sufficiently large Lipschitz constant $K_{u_i^*}$. However, choosing large $K_{u_i^*}$ will make the triggering threshold $\|e_{i,T}\|$ in (16) very small. Therefore, there is a dilemma to select the Lipschitz constant $K_{u_i^*}$. In this example, we determine $K_{u_i^*}$ via computer simulations. We find that selecting $K_{u_i^*} = 4.5$ can lead to satisfactory results.

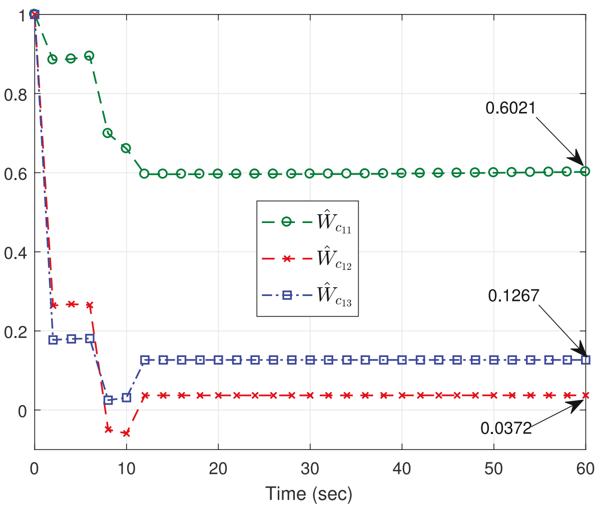

Fig. 3. Performance of the weight vector $\hat{W}_{c_1} = [\hat{W}_{c_{11}}, \hat{W}_{c_{12}}, \hat{W}_{c_{13}}]^{\mathsf{T}}$.

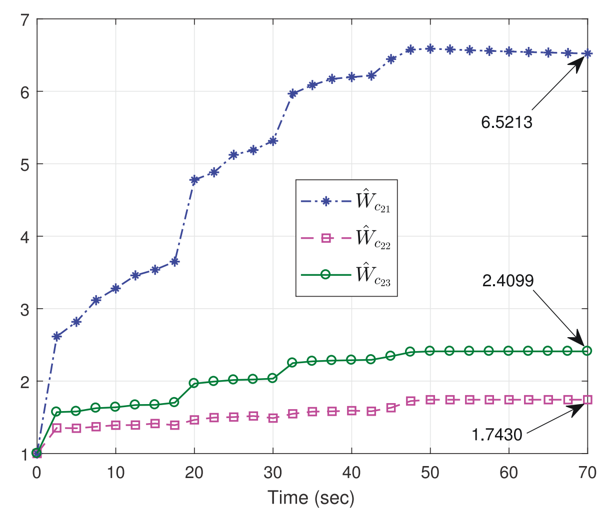

Fig. 4. Performance of the weight vector $\hat{W}_{c_2} = [\hat{W}_{c_{21}}, \hat{W}_{c_{22}}, \hat{W}_{c_{23}}]^{\mathsf{T}}$.

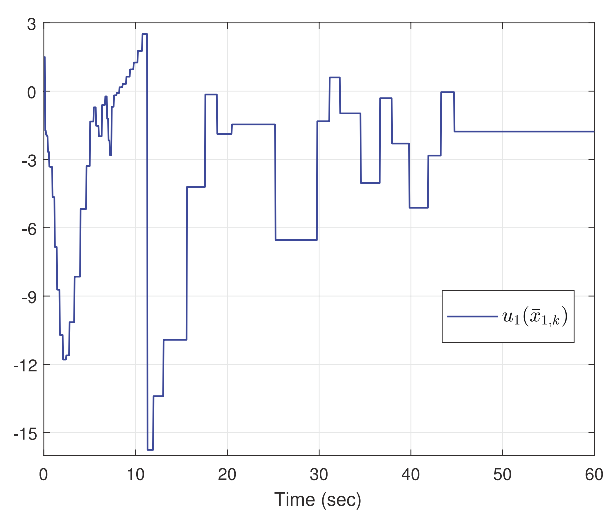

Fig. 5. ETC $u_1(\bar{x}_{1,k})$ for the nominal subsystem 1.

The experimental study is performed by using the MATLAB (R2017a) software package. Meanwhile, the computer simulation results are displayed in Figs. 3–11. Figs. 3 and 4 show

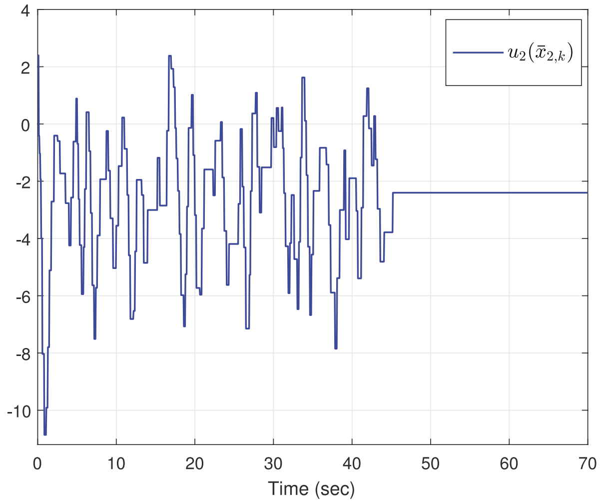

Fig. 6. ETC $u_2(\bar{x}_{2,k})$ for the nominal subsystem 2.

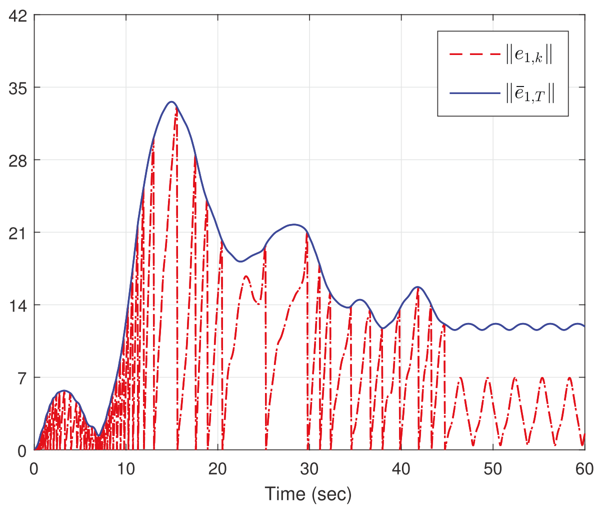

Fig. 7. Norm of the state error function $e_{1,k}$ (i.e., $\|e_{1,k}\|$) and the event-triggering threshold $\|\bar{e}_{1,T}\|$.

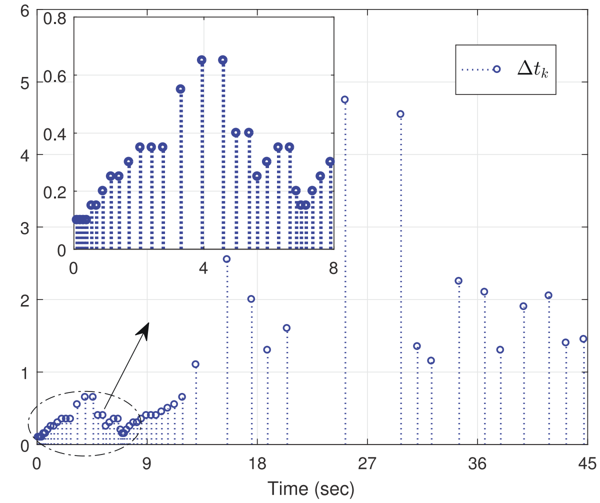

Fig. 8. Intersampling time Δt_k (*note:* $\Delta t_k = t_{k+1} - t_k$).

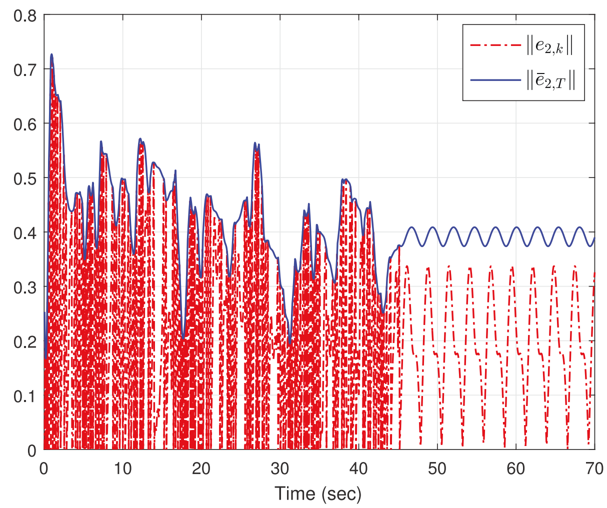

Fig. 9. Norm of the state error function $e_{2,k}$ (i.e., $\|e_{2,k}\|$) and the event-triggering threshold $\|\bar{e}_{2,T}\|$.

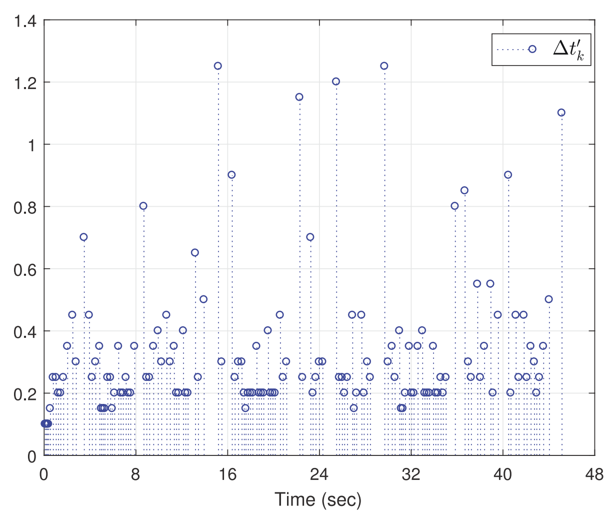

Fig. 10. Intersampling time $\Delta t'_k$.

the performance of weight vectors $\hat{W}_{c_1}$ and $\hat{W}_{c_2}$ used in the critic networks, which aim at approximating the discounted cost functions associated with nominal subsystems 1 and 2. It can be observed from Figs. 3 and 4 that $\hat{W}_{c_1}$ converges to $\hat{W}_{c_1}^{\text{final}} = [0.6021, 0.0372, 0.1267]^{\mathsf{T}}$ after the first 40 s, and $\hat{W}_{c_2}$ converges to $\hat{W}_{c_2}^{\text{final}} = [6.5213, 1.743, 2.4099]^{\mathsf{T}}$ after the first 50 s. Figs. 5 and 6 illustrate the ETC $u_1(\bar{x}_{1,k})$ for the nominal subsystem 1 and the ETC $u_2(\bar{x}_{2,k})$ for the nominal subsystem 2, respectively. Fig. 7 displays the norm of the state error function $e_{1,k}$ (i.e., $\|e_{1,k}\|$) and the event-triggering threshold $\|\bar{e}_{1,T}\|$ when considering the nominal subsystem 1. Meanwhile, Fig. 8 indicates the intersampling time Δt_k (*note:* $\Delta t_k = t_{k+1} - t_k$). It should be noted here that, as shown in Fig. 7, the event is no longer triggered after the first 45 s [*note:* when $\|e_{1,k}\| \leq \|\bar{e}_{1,T}\|$, the triggering condition (32) is not violated]. Thus, after 45 s, there is no intersampling time. Hence, in Fig. 8, we only present the intersampling time during the first 45 s. Likewise, when considering the nominal subsystem 2, we use Fig. 9 to depict the norm of the state error function $e_{2,k}$ (i.e., $\|e_{2,k}\|$) and the event-triggering threshold $\|\bar{e}_{2,T}\|$. At the same time, we present Fig. 10 to illustrate the intersampling time $\Delta t'_k$ (*note:* $\Delta t'_k$ is defined similar

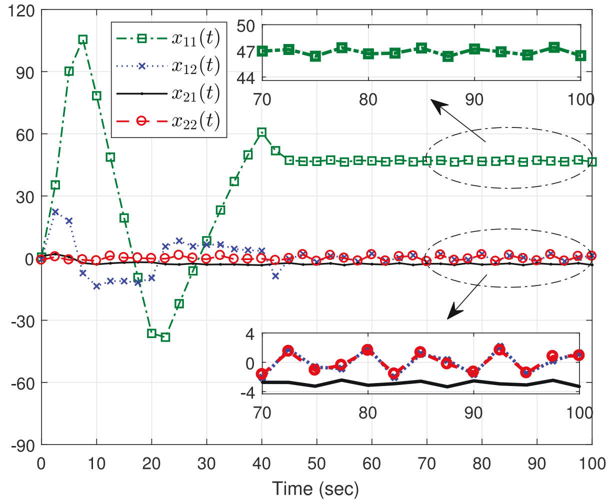

Fig. 11. Whole state vector $x(t) = [x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t)]^{\mathsf{T}}$.

to Δt_k). It should also be mentioned here that, as displayed in Fig. 9, the event is no longer triggered after the first 48 s. Therefore, in Fig. 10, we only need to present the intersampling time during the first 48 s. Observing Figs. 8 and 10, we have that $\min\{\Delta t_k, \Delta t'_k\} = 0.1$ s. Accordingly, we can keep the Zeno behavior from happening. Furthermore, from Fig. 8 (or Fig. 10), we can see that there are 53 (or 135) state samples. This indicates that only 53 (or 135) state samples are utilized to implement the present ETC strategy. While implementing the related time-triggered control scheme, we have to use 1200 (or 1400) state samples. Therefore, the controller updates can be reduced up to 95.58% (or 90.36%). In this sense, the proposed ETC strategy significantly decreases the computational burden. On the other hand, inserting the converged weight vectors $\hat{W}_{c_1}^{\text{final}}$ and $\hat{W}_{c_2}^{\text{final}}$ into (20), we can obtain the approximate optimal ETC policies for nominal subsystems 1 and 2, respectively. Then, according to Theorem 1, we derive the decentralized ETC of the interconnected system (45) by putting these obtained approximate optimal ETC policies together. Under the derived decentralized ETC, the closed-loop interconnected system (45) turns out to be stable in the sense of UUB (see Fig. 11).

VII. Conclusion

We have developed a novel RL-based decentralized ETC scheme for continuous-time nonlinear systems subject to matched interconnections. The present decentralized ETC scheme not only can be implemented without the persistence of excitation condition but also can be directly applied to those nonlinear-interconnected systems without zero equilibrium points. The interconnected terms are required to satisfy the matched condition when designing the decentralized event-triggered controller. Indeed, this is a restrictive condition. In engineering applications, the knowledge of the interconnected terms of large-scale nonlinear systems is often unavailable, which results in the difficulty in judging whether the interconnected terms satisfy the matched condition or not. Hence, how to extend the present control approach to solve decentralized ETC problems of nonlinear systems with unknown interconnections is our consecutive work.

On the other hand, when proposing the decentralized ETC scheme, one often has the problem with data congestions, which is a challenge. Recently, Ding *et al.* [55], [56] presented an effective approach based on the actor–critic structure to design a neural-network-based output-feedback controller for stochastic nonlinear systems subject to data congestions. Hence, how to develop an RL-based decentralized ETC strategy for stochastic nonlinear-interconnected systems with data congestions is also a topic in our future work. More recently, the distributed control method was introduced to handle control problems of industrial cyber-physical systems [57]. Thus, whether the present control method can be extended to tackle decentralized ETC problems of industrial cyber-physical systems is another topic in our future study.

Appendix A
Proof of Theorem 1

We take the Lyapunov function candidate in the form

$$\mathcal{L}(x) = \sum_{i=1}^{N} V_i^*(x_i) \tag{47}$$

where $V_i^*(x_i)$, $i = 1, 2, \ldots, N$ are defined as (8). According to expressions (6) and (7), we can deduct from (8) that, for each $i \in \mathbb{I} = \{1, 2, \ldots, N\}$, $V_i^*(x_i) > 0, \forall x_i \neq 0$ and $V_i^*(x_i) = 0 \Leftrightarrow x_i = 0$. Thus, $V_i^*(x_i)$, $i = 1, 2, \ldots, N$, satisfy the definition of positive-definite functions. Then, we can conclude that $\mathcal{L}(x)$ in (47) is a positive-definite function.

Differentiating $\mathcal{L}(x)$ with respect to t (i.e., $\mathrm{d}\mathcal{L}(x(t))/\mathrm{d}t$) and using the N state trajectories $\dot{x}_i = f_i(x_i) + g_i(x_i)(\mu_i^*(\bar{x}_{i,k}) + \omega_i(x))$, $i = 1, 2, \ldots, N$, we have

$$\begin{aligned}\dot{\mathcal{L}}(x) = \sum_{i=1}^{N}\Big\{&\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i) + g_i(x_i)u_i^*(x_i)\big)\\ &+ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i)\big(\mu_i^*(\bar{x}_{i,k}) - u_i^*(x_i)\big)\\ &+ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i)\omega_i(x)\Big\}.\end{aligned} \tag{48}$$

On the other hand, we can derive from (10) and (11) that

$$\begin{cases}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i) + g_i(x_i)u_i^*\big)\\ = \alpha_i V_i^*(x_i) - \eta_i P_i^2(x_i) - x_i^{\mathsf{T}} Q_i x_i - \big\|u_i^*(x_i)\big\|^2\\ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}} g_i(x_i) = -2\big(u_i^*(x_i)\big)^{\mathsf{T}}.\end{cases} \tag{49}$$

Inserting (49) into (48), we have

$$\begin{aligned}\dot{\mathcal{L}}(x) = \sum_{i=1}^{N}\Bigg\{&-\eta_i P_i^2(x_i) - x_i^{\mathsf{T}} Q_i x_i - \big\|u_i^*(x_i)\big\|^2\\ &\underbrace{-2\big(u_i^*(x_i)\big)^{\mathsf{T}}\big(\mu_i^*(\bar{x}_{i,k}) - u_i^*(x_i)\big)}_{\pi_1}\\ &\underbrace{-2\big(u_i^*(x_i)\big)^{\mathsf{T}}\omega_i(x)}_{\pi_2} + \alpha_i V_i^*(x_i)\Bigg\}.\end{aligned} \tag{50}$$

Note that $2\tilde{c}^{\mathsf{T}}\tilde{d} \le \tilde{c}^{\mathsf{T}}\tilde{c}/\beta_i+\beta_i\tilde{d}^{\mathsf{T}}\tilde{d}$ (*note:* $\beta_i > 0$ is an adjustable constant) holds for arbitrary vectors $\tilde{c}$ and $\tilde{d}$ with appropriate dimensions. By using Remark 5, we shall find that π_1 in (50) satisfies

$$\begin{aligned}\pi_1 &\le \left\|u_i^*(x_i)\right\|^2/\beta_i + \beta_i\left\|\mu_i^*(\bar{x}_{i,k}) - u_i^*(x_i)\right\|^2\\ &\le \left\|u_i^*(x_i)\right\|^2/\beta_i + \beta_i K_{u_i^*}^2\left\|e_{i,k}\right\|^2. \end{aligned}\tag{51}$$

Applying Cauchy's inequality $\tilde{c}^{\mathsf{T}}\tilde{d} \le \|\tilde{c}\|\|\tilde{d}\|$ (*note:* $\tilde{c}$ and $\tilde{d}$ are vectors with appropriate dimensions) to π_2 in (50) and using (4), we have

$$\pi_2 \le 2\left\|u_i^*(x_i)\right\|\|\omega_i(x)\| \le 2\left\|u_i^*(x_i)\right\|\sum_{j=1}^{N} b_{ij}P_j(x_j). \tag{52}$$

By using inequalities (51) and (52) as well as the fact that $\lambda_{\min}(Q_i)\|x_i\|^2 \le x_i^{\mathsf{T}}Q_i x_i$, we can obtain from (50) that

$$\begin{aligned}\dot{\mathcal{L}}(x) \le &-\sum_{i=1}^{N}\Big(\rho_i^2\lambda_{\min}(Q_i)\|x_i\|^2 - \Upsilon_i\big(V_i^*, u_i^*(x_i)\big)\Big)\\ &-\sum_{i=1}^{N}\Big(\Big(1-\rho_i^2\Big)\lambda_{\min}(Q_i)\|x_i\|^2 - \beta_i K_{u_i^*}^2\|e_{i,k}\|^2\Big)\\ &-\sum_{i=1}^{N}\Bigg\{\eta_i P_i^2(x_i) + \left\|u_i^*(x_i)\right\|^2\\ &\qquad -2\left\|u_i^*(x_i)\right\|\sum_{j=1}^{N} b_{ij}P_j(x_j)\Bigg\} \end{aligned}\tag{53}$$

where

$$\Upsilon_i\big(V_i^*, u_i^*(x_i)\big) = \alpha_i V_i^*(x_i) + \|u_i^*(x_i)\|^2/\beta_i. \tag{54}$$

Noticing that $u_i^*(x_i)$ is an admissible control policy, that is, $u_i^*(x_i) \in \mathscr{A}(\Omega_i)$, we can obtain that for every $x_i \in \Omega_i$, $u_i^*(x_i)$ and $V_i^*(x_i)$ are bounded [58]. Thus, $\Upsilon_i(V_i^*, u_i^*(x_i))$ is bounded. We denote $\|\Upsilon_i(V_i^*, u_i^*(x_i))\| \le \epsilon_{M_i}$ with $\epsilon_{M_i} > 0$ being the constant. Meanwhile, we let

$$\begin{aligned}\tilde{\boldsymbol{\eta}} &= \operatorname{diag}\{\eta_1, \eta_2, \ldots, \eta_N\}\\ \tilde{\mathbf{1}} &= \operatorname{diag}\{1_1, 1_2, \ldots, 1_N\} \quad (1_i = 1,\ i \in \mathbb{I})\\ y(x) &= \big[-P_1(x_1), -P_2(x_2), \ldots, -P_N(x_N)\\ &\qquad \left\|u_1^*(x_1)\right\|, \left\|u_2^*(x_2)\right\|, \ldots, \left\|u_N^*(x_N)\right\|\big]^{\mathsf{T}}. \end{aligned}$$

Then, by using (16), we can see that (53) yields

$$\dot{\mathcal{L}}(x) \le -\sum_{i=1}^{N}\Big(\rho_i^2\lambda_{\min}(Q_i)\|x_i\|^2 - \epsilon_{M_i}\Big) - y^{\mathsf{T}}(x)\tilde{\mathcal{A}}y(x) \tag{55}$$

where $\tilde{\mathcal{A}}$ is given in the form

$$\tilde{\mathcal{A}} = \begin{bmatrix}\tilde{\boldsymbol{\eta}} & B^{\mathsf{T}}\\ B & \tilde{\mathbf{1}}\end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N}\\ \vdots & \vdots & \ddots & \vdots\\ b_{N1} & b_{N2} & \cdots & b_{NN}\end{bmatrix}. \tag{56}$$

It can be seen from (56) that $\tilde{\boldsymbol{\eta}}$ (i.e., $\operatorname{diag}\{\eta_1, \eta_2, \ldots, \eta_N\}$) lies along the principal diagonal of the block matrix $\tilde{\mathcal{A}} \in \mathbb{R}^{2N\times 2N}$. Accordingly, $\tilde{\mathcal{A}}$ can be made positive definite by choosing appropriate η_i, $i = 1, 2, \ldots, N$. In other words, we can find N positive constants η_i^*, $i = 1, 2, \ldots, N$, such that $\eta_i \ge \eta_i^*$ makes $-y^{\mathsf{T}}(x)\tilde{\mathcal{A}}y(x) < 0$ valid. Then, (55) further yields

$$\dot{\mathcal{L}}(x) \le -\sum_{i=1}^{N}\Big(\rho_i^2\lambda_{\min}(Q_i)\|x_i\|^2 - \epsilon_{M_i}\Big). \tag{57}$$

Thus, (57) implies $\dot{\mathcal{L}}(x) < 0$ only if, for every $i \in \mathbb{I} = \{1, 2, \ldots, N\}$, the subsystem state x_i is out of the set

$$\mathcal{D}_{x_i} = \left\{x_i\colon \|x_i\| \le \frac{1}{\rho_i}\sqrt{\epsilon_{M_i}/\lambda_{\min}(Q_i)}\right\}.$$

According to the Lyapunov extension theorem [59], this proves that the state of the interconnected system (1) is uniformly ultimately bounded. Specifically, with the N optimal ETC laws $\mu_i^*(\bar{x}_{i,1}), \mu_i^*(\bar{x}_{i,2}), \ldots, \mu_i^*(\bar{x}_{i,N})$ together, the UUB stability of the interconnected system (1) is guaranteed. Moreover, for each $i \in \mathbb{I}$, the ultimate bound of x_i is $\sqrt{\epsilon_{M_i}/\lambda_{\min}(Q_i)}/\rho_i$. This completes the proof.

Appendix B
Proof of Theorem 2

We take the Lyapunov function candidate in the form

$$L_1(t) = V_i^*(\bar{x}_{i,k}) + \underbrace{V_i^*(x_i(t))}_{L_{11}(t)} + \underbrace{(1/2)\tilde{W}_{c_i}^{\mathsf{T}}\tilde{W}_{c_i}}_{L_{12}(t)}. \tag{58}$$

As aforementioned, the ith closed-loop augmented system consists of two parts: 1) continuous dynamical system (29) and 2) discrete dynamical system (31). Thus, the stability of the ith closed-loop augmented system will be studied from following two situations.

Situation I: Events are not triggered, that is, $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. Then, we have $\mathrm{d}V_i^*(\bar{x}_{i,k})/\mathrm{d}t = 0$. Taking the time derivative of $L_{11}(t)$ in (58) and using the solution of the differential equation $\dot{x}_i = f_i(x_i) + g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k})$, we have

$$\begin{aligned}\dot{L}_{11}(t) &= \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i) + g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k})\big)\\ &= \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i) + g_i(x_i)u_i^*(x_i)\big)\\ &\quad + \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\big(\hat{\mu}_i(\bar{x}_{i,k}) - u_i^*(x_i)\big). \end{aligned}\tag{59}$$

On the other hand, according to (11) and (12), it follows that:

$$\begin{cases}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}f_i(x_i) = \alpha_i V_i^*(x_i) - \eta_i P_i^2(x_i)\\ \qquad\qquad -x_i^{\mathsf{T}}Q_i x_i + \left\|u_i^*(x_i)\right\|^2\\ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i) = -2\big(u_i^*(x_i)\big)^{\mathsf{T}}. \end{cases}\tag{60}$$

Inserting (60) into (59), we have

$$\begin{aligned}\dot{L}_{11}(t) = &\ \alpha_i V_i^*(x_i) - \eta_i P_i^2(x_i) - x_i^{\mathsf{T}}Q_i x_i\\ &- \left\|\hat{\mu}_i(\bar{x}_{i,k})\right\|^2 + \underbrace{\left\|u_i^*(x_i) - \hat{\mu}_i(\bar{x}_{i,k})\right\|^2}_{\zeta}. \end{aligned}\tag{61}$$

Note that the following inequality holds:

$$\left\|\tilde{c} + \tilde{d}\right\|^2 \le (1 + 1/\gamma_i)\|\tilde{c}\|^2 + (1 + \gamma_i)\left\|\tilde{d}\right\|^2$$

with $\gamma_i > 0$, $i = 1, 2, \ldots, N$, being the constants, $\tilde{c}$ and $\tilde{d}$ being the vectors with suitable dimensions. Then, by using

Assumptions 2, 4, and 5 as well as (18) and (20), we can derive from $\zeta \in \mathbb{R}$ in (61) that

$$\begin{aligned}\zeta &= \left\| \left(u_i^*(x_i) - \mu_i^*(\bar{x}_{i,k})\right) + \left(\mu_i^*(\bar{x}_{i,k}) - \hat{\mu}_i(\bar{x}_{i,k})\right) \right\|^2 \\ &\le (1+1/\gamma_i) \left\| \mu_i^*(\bar{x}_{i,k}) - \hat{\mu}_i(\bar{x}_{i,k}) \right\|^2 \\ &\quad + (1+\gamma_i) \left\| u_i^*(x_i) - \mu_i^*(\bar{x}_{i,k}) \right\|^2 \\ &\le (1+1/\gamma_i) \left\| -\frac{1}{2} g_i^{\mathsf{T}}(\bar{x}_{i,k}) \nabla \sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k}) \tilde{W}_{c_i} + \varepsilon_{\mu_i^*}(\bar{x}_{i,k}) \right\|^2 \\ &\quad + (1+\gamma_i) K_{u_i^*}^2 \left\| e_{i,k} \right\|^2 \\ &\le \frac{(1+1/\gamma_i)^2}{4} b_{g_i}^2 b_{\sigma_{c_i}}^2 \left\| \tilde{W}_{c_i} \right\|^2 + b_{\varepsilon_{\mu_i^*}}^2 (1+\gamma_i)^2/\gamma_i \\ &\quad + (1+\gamma_i) K_{u_i^*}^2 \left\| e_{i,k} \right\|^2. \end{aligned} \tag{62}$$

Noting that $\eta_i P_i^2(x_i) \ge 0$ and $\|\hat{\mu}_i(\bar{x}_{i,k})\|^2 \ge 0$ and using (62), we can derive from (61) that

$$\begin{aligned}\dot{L}_{11}(t) \le &- x_i^{\mathsf{T}} Q_i x_i + \alpha_i V_i^*(x_i) + b_{\varepsilon_{\mu_i^*}}^2 (1+\gamma_i)^2/\gamma_i \\ &+ (1+\gamma_i) K_{u_i^*}^2 \left\| e_{i,k} \right\|^2 \\ &+ \frac{(1+1/\gamma_i)^2}{4} b_{g_i}^2 b_{\sigma_{c_i}}^2 \left\| \tilde{W}_{c_i} \right\|^2. \end{aligned} \tag{63}$$

According to the definition of $\Upsilon_i(V_i^*, u_i^*(x_i))$ in (54), it follows $\|\alpha_i V_i^*(x_i)\| \le \|\Sigma_i(V_i^*, u_i^*(x_i))\| \le \epsilon_{M_i}$. In addition, $\lambda_{\min}(Q_i)\|x_i\|^2 \le x_i^{\mathsf{T}} Q_i x_i$. Thus, we further develop (63) as

$$\begin{aligned}\dot{L}_{11}(t) \le &- \lambda_{\min}(Q_i)\|x_i\|^2 + (1+\gamma_i) K_{u_i^*}^2 \left\| e_{i,k} \right\|^2 \\ &+ \frac{(1+1/\gamma_i)^2}{4} b_{g_i}^2 b_{\sigma_{c_i}}^2 \left\| \tilde{W}_{c_i} \right\|^2 + \hbar_i \end{aligned} \tag{64}$$

where $\hbar_i = \epsilon_{M_i} + b_{\varepsilon_{\mu_i^*}}^2 (1+\gamma_i)^2/\gamma_i$.

Differentiating $L_{12}(t)$ in (58) with respect to t and utilizing (27), we obtain

$$\begin{aligned}\dot{L}_{12}(t) = &- \ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \Phi(\psi_i, \psi_{i,s}) \tilde{W}_{c_i} \\ &+ \frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_i \varepsilon_{H_i}}{1 + \phi_i^{\mathsf{T}} \phi_i} + \sum_{s=1}^{l_0} \frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_{i,s} \varepsilon_{H_{i,s}}}{1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}} \end{aligned} \tag{65}$$

with $\Phi(\psi_i, \psi_{i,s})$ being defined as (30).

Applying the inequality $\tilde{c}^{\mathsf{T}} \tilde{d} \le \tilde{c}^{\mathsf{T}} \tilde{c}/2 + \tilde{d}^{\mathsf{T}} \tilde{d}/2$ (*note:* $\tilde{c}$ and $\tilde{d}$ are vectors with suitable dimensions) to the second term in (65) and noting that $1/(1 + \phi_i^{\mathsf{T}} \phi_i) \le 1$, we have

$$\begin{aligned}\frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_i \varepsilon_{H_i}}{1 + \phi_i^{\mathsf{T}} \phi_i} &\le \frac{\ell_{c_i}}{1 + \phi_i^{\mathsf{T}} \phi_i} \left(\frac{1}{2} \tilde{W}_{c_i}^{\mathsf{T}} \psi_i \psi_i^{\mathsf{T}} \tilde{W}_{c_i} + \frac{1}{2} \varepsilon_{H_i}^{\mathsf{T}} \varepsilon_{H_i} \right) \\ &\le \frac{\ell_{c_i}}{2} \tilde{W}_{c_i}^{\mathsf{T}} \psi_i \psi_i^{\mathsf{T}} \tilde{W}_{c_i} + \frac{\ell_{c_i}}{2} \varepsilon_{H_i}^{\mathsf{T}} \varepsilon_{H_i}. \end{aligned} \tag{66}$$

Similarly, we obtain

$$\begin{aligned}\sum_{s=1}^{l_0} \frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_{i,s} \varepsilon_{H_{i,s}}}{1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}} \le &\frac{\ell_{c_i}}{2} \tilde{W}_{c_i}^{\mathsf{T}} \left(\sum_{s=1}^{l_0} \psi_{i,s} \psi_{i,s}^{\mathsf{T}} \right) \tilde{W}_{c_i} \\ &+ \frac{\ell_{c_i}}{2} \sum_{s=1}^{l_0} \varepsilon_{H_{i,s}}^{\mathsf{T}} \varepsilon_{H_{i,s}}. \end{aligned} \tag{67}$$

Combining (66) with (67) and using Assumption 5, we have

$$\begin{aligned}&\frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_i \varepsilon_{H_i}}{1 + \phi_i^{\mathsf{T}} \phi_i} + \sum_{s=1}^{l_0} \frac{\ell_{c_i} \tilde{W}_{c_i}^{\mathsf{T}} \psi_{i,s} \varepsilon_{H_{i,s}}}{1 + \phi_{i,s}^{\mathsf{T}} \phi_{i,s}} \\ &\qquad \le \frac{\ell_{c_i}}{2} \tilde{W}_{c_i}^{\mathsf{T}} \Phi(\psi_i, \psi_{i,s}) \tilde{W}_{c_i} + \frac{\ell_{c_i}(l_0+1)}{2} b_{\varepsilon_{H_i}}^2. \end{aligned}$$

Thus, we can obtain from (65) that

$$\begin{aligned}\dot{L}_{12}(t) \le &-\frac{\ell_{c_i}}{2} \lambda_{\min}\left(\Phi(\psi_i, \psi_{i,s})\right) \left\| \tilde{W}_{c_i} \right\|^2 \\ &+ \frac{\ell_{c_i}(l_0+1)}{2} b_{\varepsilon_{H_i}}^2. \end{aligned} \tag{68}$$

Using (64) and (68), the derivative of $L_1(t)$ in (58) satisfies

$$\begin{aligned}\dot{L}_1(t) \le &-\rho_i^2 \lambda_{\min}(Q_i)\|x_i\|^2 - \left(1 - \rho_i^2\right) \lambda_{\min}(Q_i)\|x_i\|^2 \\ &+ (1+\gamma_i) K_{u_i^*}^2 \|e_{i,k}\|^2 + \hbar_i + \frac{\ell_{c_i}(l_0+1)}{2} b_{\varepsilon_{H_i}}^2 \\ &- \frac{1}{4} \Big(2\ell_{c_i} \lambda_{\min}\left(\Phi(\psi_i, \psi_{i,s})\right) \\ &\qquad - (1+1/\gamma_i)^2 b_{g_i}^2 b_{\sigma_{c_i}}^2 \Big) \left\| \tilde{W}_{c_i} \right\|^2 \end{aligned} \tag{69}$$

with $\hbar_i$ being given as in (64).

Therefore, if letting (32) and (33) be valid, then (69) implies $\dot{L}_1(t) < 0$ provided that we are able to make $x_i \notin \Omega_{x_i}$ or $\tilde{W}_{c_i} \notin \Omega_{\tilde{W}_{c_i}}$ with Ω_{x_i} and $\Omega_{\tilde{W}_{c_i}}$, respectively, defined as

$$\Omega_{x_i} = \left\{ x_i \colon \|x_i\| \le \frac{1}{\rho_i} \sqrt{\frac{\ell_{c_i}(l_0+1) b_{\varepsilon_{H_i}}^2 + 2\hbar_i}{2\lambda_{\min}(Q_i)}} \right\}$$

$$\Omega_{\tilde{W}_{c_i}} = \left\{ \tilde{W}_{c_i} \colon \left\| \tilde{W}_{c_i} \right\| \le \sqrt{\frac{\ell_{c_i}(l_0+1) b_{\varepsilon_{H_i}}^2 + 2\hbar_i}{\ell_{c_i} \lambda_{\min}\left(\Phi(\psi_i, \psi_{i,s})\right) - \frac{\varpi_i}{2}}} \right\}$$

where $\varpi_i = b_{g_i}^2 b_{\sigma_{c_i}}^2 (1+1/\gamma_i)^2$.

This verifies that the UUB stability of the ith subsystem state x_i and the weight estimation error $\tilde{W}_{c_i}$ based on the Lyapunov extension theorem [59]. In addition, the ultimate bound of x_i (or $\tilde{W}_{c_i}$) is the same as the bound of Ω_{x_i} (or $\Omega_{\tilde{W}_{c_i}}$).

Situation II: Events are triggered, that is, $t = t_k$, $k \in \mathbb{N}$. In this situation, we take the difference of the Lyapunov function candidate described as in (58) into account, that is,

$$\Delta L_1(t_k) = V_i^*(\bar{x}_{i,k+1}) - V_i^*(\bar{x}_{i,k}) + \Delta \Pi_i \tag{70}$$

where

$$\begin{aligned}\Delta \Pi_i = &V_i^*\left(x_i(t_k^+)\right) - V_i^*(x_i(t_k)) \\ &+ \frac{1}{2} \tilde{W}_{c_i}^{\mathsf{T}}(t_k^+) \tilde{W}_{c_i}(t_k^+) - \frac{1}{2} \tilde{W}_{c_i}^{\mathsf{T}}(t_k) \tilde{W}_{c_i}(t_k) \end{aligned} \tag{71}$$

where $x_i(t_k^+) = \lim_{\varsigma \to 0^+} x_i(t_k + \varsigma)$ and $\tilde{W}_{c_i}(t_k^+) = \lim_{\varsigma \to 0^+} \tilde{W}_{c_i}(t_k + \varsigma)$ with $\varsigma \in (0, t_{k+1} - t_k)$.

As proved in *Situation I*, if either $x_i \notin \Omega_{x_i}$ or $\tilde{W}_{c_i} \notin \Omega_{\tilde{W}_{c_i}}$ holds, then we have $\dot{L}_1(t) < 0$ for all $t \in [t_k, t_{k+1})$. Specifically, for all $t \in [t_k, t_{k+1})$, it follows $\dot{L}_2(t) < 0$ [*note:* $L_2(t) = L_{11}(t) + L_{12}(t)$ with $L_{11}(t)$ and $L_{12}(t)$ being defined as (58)]. This indicates that $L_2(t)$ is strictly monotonically decreasing over the interval $[t_k, t_{k+1})$. Noting that $t_k < t_k + \varsigma$ for all $\varsigma \in (0, t_{k+1} - t_k)$, we thus have

$$L_2(t_k) > L_2(t_k + \varsigma) \quad \forall \varsigma \in (0, t_{k+1} - t_k). \tag{72}$$

Taking the right limit with respect to ς on both sides of (72) (i.e., $\varsigma \to 0^+$) and according to the property of the limit [60, Ch. 4], we have

$$L_2(t_k) \geq \lim_{\varsigma \to 0^+} L_2(t_k + \varsigma) = L_2\big(t_k^+\big). \tag{73}$$

From (73), we obtain

$$\begin{aligned} &V_i^*(x_i(t_k)) + \frac{1}{2}\tilde{W}_{c_i}^{\mathsf{T}}(t_k)\tilde{W}_{c_i}(t_k) \\ &\quad \geq V_i^*\big(x_i\big(t_k^+\big)\big) + \frac{1}{2}\tilde{W}_{c_i}^{\mathsf{T}}\big(t_k^+\big)\tilde{W}_{c_i}\big(t_k^+\big). \end{aligned}$$

Hence, $\Delta\Pi_i$ defined as (71) satisfies $\Delta\Pi_i \leq 0$. On the other hand, since $x_i(t)$ is UUB in *Situation I*, we can conclude

$$V_i^*\big(\bar{x}_{i,k+1}\big) \leq V_i^*\big(\bar{x}_{i,k}\big).$$

Thus, if $x_i \notin \Omega_{x_i}$ or $\tilde{W}_{c_i} \notin \Omega_{\tilde{W}_{c_i}}$ holds, then $\Delta L_1(t_k)$ in (70) satisfies $\Delta L_1(t_k) < 0$. This demonstrates that x_i and $\tilde{W}_{c_i}$ are uniformly ultimately bounded through the Lyapunov extension theorem [59]. In addition, the ultimate bound of x_i (or $\tilde{W}_{c_i}$) is the same as the bound of Ω_{x_i} (or $\Omega_{\tilde{W}_{c_i}}$). This completes the proof.

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