Team-Triggered Practical Fixed-Time Consensus of Double-Integrator Agents With Uncertain Disturbance

Jian Liu, *Graduate Student Member, IEEE*, Yao Yu, Haibo He, *Fellow, IEEE*, and Changyin Sun, *Senior Member, IEEE*

***Abstract*—This article addresses the team-triggered fixed-time consensus problems for a class of double-integrator agents subject to uncertain disturbance. Compared with the finite-time results, the convergence time of the fixed-time results is independent of the initial conditions. Furthermore, a novel team-triggered control (TTC) strategy is presented. This control strategy incorporates the event-triggered control (ETC) and self-triggered control (STC). The ETC and STC are proposed to achieve the fixed-time consensus of second-order multiagent systems (MASs), and no Zeno behavior occurs. The TTC scheme, derived by combining the ETC scheme and the STC scheme, is able to relax the requirement of continuous communication and thus lowering the energy consumption of communication while ensuring the performance of the system. The effectiveness of the proposed algorithms is validated by numerical simulations.**

***Index Terms*—Fixed-time consensus, multiagent systems (MASs), self-triggered control (STC), team-triggered control (TTC).**

I. Introduction

OVER the recent years, the fixed-time consensus control has received increasing attention in multiagent systems (MASs) [1], [2], including first-order [3], second-order [4], and high-order MASs [5]. Besides, the disturbances [6], chained-form dynamics [7], and nonlinear dynamics [8] are all considered. Moreover, different from the finite-time consensus results [9]–[11], the fixed-time results are more practical when the initial conditions are sufficiently large or unknown.

However, in the aforementioned studies, continuous update of controllers is required. Furthermore, due to the limited resources of the embedded processors [12], event-triggered control (ETC) strategies were developed for MASs [13]. The event-triggered consensus was reached for single-integrator agents in [14]. Furthermore, the input saturation was discussed in [15]. Subsequently, the event-triggered consensus results were extended to double-integrator agents in [16]. For the general linear MASs, the consensus problems were solved in [17]. However, there exists a problem that each agent requires monitoring its neighbor's states continuously. To amend the drawback, the self-triggered control (STC) was also presented to avoid continuous communication in [14], [17], and [18], and another research line for the event-triggered function design was proposed in [19]–[21], where continuous communication was avoided. In [20], both the ETC and STC were investigated for a class of identical linear MASs. In [21], the convex optimization problems were addressed via ETC under the balanced directed graphs. Moreover, the adaptive mechanism was considered in the ETC in [22].

Considering the convergence rate with ETC, the finite-time control of sensor networks was obtained in [23] and the finite/fixed-time consensus of first-order MASs was obtained in [24]–[28]. Moreover, the finite-time consensus of general linear MASs was reached in [29]. In [24]–[26], the finite-time consensus results of linear MASs were obtained via ETC. Then, the results were extended to nonlinear disturbed MASs in [27]. In [28], the event-triggered fixed-time consensus of leader–follower MASs was achieved. In [24], [28], and [29], the Zeno behavior cannot be excluded. Moreover, the ETC of the above works requires to monitor its neighboring states continuously. Later, improved ETC frameworks were developed for the finite-time consensus [30], [31] and the fixed-time consensus [32]–[34], and continuous communication was not required. The finite-time consensus results of first-order MASs were obtained in [30], and the results were extended to the fixed-time consensus with nonlinear uncertainties in [32] and [33]. Nevertheless, the existing event-triggered fixed-time consensus results are mainly based on first-order MASs. In [34], the fixed-time consensus was achieved via ETC with continuous communication and the finite-time consensus

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Jian Liu is with the School of Automation, Southeast University, Nanjing 210096, China, and also with the Key Laboratory of Measurement and Control of Complex System of Engineering, Ministry of Education, Southeast University, Nanjing 210096, China (e-mail: bkliujian@163.com).

Yao Yu is with the Key Laboratory of Knowledge Automation for Industrial Processes of Ministry of Education, School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China (e-mail: yuyao@ustb.edu.cn).

Haibo He is with the Department of Electrical, Computer and Biomedical Engineering, University of Rhode Island, Kingston, RI 02881 USA (e-mail: he@ele.uri.edu).

Changyin Sun is with the School of Automation, Southeast University, Nanjing 210096, China (e-mail: cysun@seu.edu.cn).

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was reached via STC. To avoid continuous monitoring, the authors assumed that the leader's input is zero in [35].

More recently, a hybrid-triggered sampling control was proposed in [36]–[38], which incorporates the time-triggered sampling control and the ETC, and the communication strategy is switched between the event-triggered sampling and the time-triggered sampling. The hybrid-triggered control was presented in [36] for networked systems, and the stochastic cyberattacks were considered in [37]. Later, the quantized stabilization problem was solved for fuzzy systems via hybrid-triggered control in [38]. For the MASs, there are also some hybrid event-time-driven consensus results [39]–[41], and the event detections are performed discontinuously. The hybrid event-time-driven consensus problems were addressed in [39] and [40], and the time delay was considered in [39]. Then, the corresponding results were extended to double-integrator networks in [41]. Moreover, a team-triggered control (TTC) framework was presented in [42], which incorporates the ETC and the STC, and the communication strategy is switched between the event-triggered sampling and the self-triggered sampling. Hence, the energy consumption of communication can be reduced due to the existence of self-triggered sampling, and the system performance can be ensured.

Motivated by these existing studies, this article investigates the problem of the fixed-time consensus for second-order MASs with uncertain disturbance via TTC and STC. The contributions are listed as follows. First, a new control framework is proposed based on the backstepping design method and a hyperbolic tangent function to avoid the communication loop problem in [4] and [6] and the Zeno behavior in [24], [28], and [29]. Second, different from the existing event-triggered finite-time results [24]–[26], [29] and fixed-time results [28], [34] with continuous communication, our team-triggered and self-triggered fixed-time approaches not only reduce the energy consumption of communication but also ensure system performance. Third, the discontinuous problem of the sign function existing in [24], [26], [27], [30], [31], [34], and [35] is avoided. Fourth, compared with the existing event-triggered finite/fixed-time consensus [24]–[35], we obtain the fixed-time consensus of double-integrator agents via STC, and a new TTC strategy is proposed.

The remainder of this article is outlined as follows. In Section II, some basic notations are introduced. The corresponding results are presented in Section III. Section IV shows an example. The conclusions and future outlook are given in Section V.

II. Preliminaries

A. Algebraic Graph Theory

Consider an undirected graph $\mathcal{G} = (N, \mathcal{E})$, where N is the set of nodes and $\mathcal{E}$ is the set of edges. The adjacency matrix is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with

$$a_{ij}\begin{cases} > 0, & \text{if } (j,i) \in \mathcal{E} \\ = 0, & \text{otherwise.}\end{cases}$$

The degree matrix is $\mathcal{D} = \text{diag}[d_1, \ldots, d_N]$ with $d_i = \sum_{j=1, j\neq i}^{N} a_{ij}$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. In addition, the graph $\mathcal{G}$ is connected if a path exists between any two nodes.

B. Definition and Lemma

Assume that the origin is an equilibrium point of the following system:

$$\begin{cases} \dot{x}(t) = f(t, x(t)) \\ x(0) = x_0 \end{cases} \tag{1}$$

where $f(t, x(t)) : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is an unknown nonlinear function.

Definition 1 [43]: The origin of (1) is globally finite-time stable if it is asymptotically stable and there exists a settling time $T > 0$, such that the solution $x(t, x_0)$ can reach the origin in $T(x_0)$. If $\exists T_{\max} > 0$, such that $T \leq T_{\max}$ for any initial conditions, it is fixed-time stable.

Lemma 1 [44]: For an undirected connected graph $\mathcal{G}$, the eigenvalues of $\mathcal{L}$ are $0, \lambda_2, \ldots, \lambda_N$ and satisfy $0 < \lambda_2 \leq \ldots \leq \lambda_N$. Furthermore, if $\mathbf{1}^T x = 0$ with $x = [x_1, x_2, \ldots, x_N]^T$, then $x^T \mathcal{L} x \geq \lambda_2 x^T x$.

Lemma 2 [43], [45]: If there exists a Lyapunov function $\mathcal{V}(x(t))$ satisfying

$$\dot{\mathcal{V}}(x(t)) \leq -c_1 \mathcal{V}^{\rho}(x(t)) - c_2 \mathcal{V}^{\omega}(x(t)) + \iota$$

with $c_1 > 0, c_2 > 0, \iota > 0, \rho \in (0, 1)$, and $\omega \in (1, \infty)$, the origin of system (1) is practical fixed-time stable. Moreover, the residual set of the solution is

$$\left\{\lim_{t\to T} x(t) | \mathcal{V}(x(t)) \leq \min\left\{c_1^{-\frac{1}{\rho}}\left(\frac{\iota}{1-\theta}\right)^{\frac{1}{\rho}}, c_2^{-\frac{1}{\omega}}\left(\frac{\iota}{1-\theta}\right)^{\frac{1}{\omega}}\right\}\right\}$$

where $\theta \in (0, 1)$ is a scalar. The settling time T satisfies

$$T \leq T_{\max} := \frac{1}{c_1\theta(1-\rho)} + \frac{1}{c_2\theta(\omega - 1)}.$$

Furthermore, if $\iota = 0$, the origin of system (1) is fixed-time stable, which is definited in Definition 1, and $T_{\max}$ will be rewritten as $(1/[c_1(1-\rho)]) + (1/[c_2(\omega-1)])$.

Lemma 3 [4]: Let $\zeta_1, \zeta_2, \ldots, \zeta_N \geq 0$. Then

$$\sum_{i=1}^{N} \zeta_i^{\rho} \geq \left(\sum_{i=1}^{N} \zeta_i\right)^{\rho}, \quad 0 < \rho \leq 1$$

$$N^{1-\omega}\left(\sum_{i=1}^{N}\zeta_i\right)^{\omega} \leq \sum_{i=1}^{N}\zeta_i^{\omega} \leq \left(\sum_{i=1}^{N}\zeta_i\right)^{\omega}, \quad 1 < \omega \leq \infty.$$

Lemma 4 [46]: For any $y \in \mathbb{R}$, we have

$$0 \leq |y| - y\tanh(\vartheta y) \leq \frac{\kappa}{\vartheta}$$

where $\vartheta \gg 1$ and $\kappa = 0.2785$.

C. Background

The second-order MASs have N agents, and the dynamics of agent i is given as

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= u_i(t) + \Delta_i(x_i(t), v_i(t), t) \end{aligned} \tag{2}$$

where $x_i(t)$, $v_i(t)$, and $u_i(t)$ are the position, velocity, and control input of agent i, respectively, $\Delta_i(x_i(t), v_i(t), t)$ is the uncertain disturbance.

Assumption 1: The uncertain disturbance of agent i is bounded by a non-negative constant $\bar{\Delta}$ as follows:

$$|\Delta_i(x_i(t), v_i(t), t)| \leq \bar{\Delta}. \tag{3}$$

Assumption 2: The communication graph is a connected undirected graph.

Definition 2: The practical fixed-time consensus is that there exist sufficiently small positive constants δ_1 and δ_2, and a value T such that $|x_i(t) - x_j(t)| \leq \delta_1$ and $|v_i(t) - v_j(t)| \leq \delta_2$ when $t \geq T$. Moreover, $\exists T_{\max} > 0$, s.t., $T \leq T_{\max}$ for arbitrary initial conditions.

III. Main Results

In this section, a new TTC strategy is presented. The ETC scheme is developed first for second-order disturbed MASs with continuous communication. Then, the TTC scheme, derived by combining the ETC scheme and the STC scheme, is able to relax the requirement of continuous communication. Moreover, in order to make a comparison, we also present an STC scheme.

A. Event-Triggered Fixed-Time Consensus

The design of the algorithm has the following two steps. First, based on the backstepping design method, the virtual velocity $\hat{\upsilon}_i(t)$ is constructed to make agents reach an agreement within a fixed time. Second, $u_i(t)$ is constructed to ensure true velocity $v_i(t)$ can track virtual velocity $\hat{\upsilon}_i(t)$ within a fixed time. The fixed-time consensus results without event-triggered sampling have been presented in [47]. Herein, the specific design ideas are as follows.

First, the virtual velocity $\hat{\upsilon}_i(t)$ is designed as

$$\hat{\upsilon}_i(t) = -\alpha_1 \chi_i^{\beta}(t) - \alpha_2 \tanh(\vartheta \chi_i(t)) + \varsigma \tag{4}$$

where $\chi_i(t)$ is that

$$\chi_i(t) = \sum_{j=1}^{N} a_{ij}\big(x_i(t) - x_j(t)\big) \tag{5}$$

and α_1, α_2, and ς are positive constants, ϑ has been introduced in Lemma 4 and can be chosen as required, and $\beta \in (2, \infty)$ is the ratio of positive odd numbers.

Then, define the error $e_i(t)$ as follows:

$$\begin{aligned} e_i(t) &= v_i(t) - \hat{\upsilon}_i(t) \\ &= v_i(t) + \alpha_1 \chi_i^{\beta}(t) + \alpha_2 \tanh(\vartheta \chi_i(t)) - \varsigma \end{aligned} \tag{6}$$

subsequently, we design the controller $u_i(t)$ to guarantee the true velocity $v_i(t)$ can track the virtual velocity $\hat{\upsilon}_i(t)$ within a fixed time, which is constructed as follows:

$$\begin{aligned} u_i(t) = &-\alpha_1 \beta \chi_i^{\beta-1}\Big(t_{k_i}^i\Big) z_i\Big(t_{k_i}^i\Big) \\ &- \alpha_2 \vartheta \Big(1 - \tanh^2\Big(\vartheta \chi_i\Big(t_{k_i}^i\Big)\Big)\Big) z_i\Big(t_{k_i}^i\Big) \\ &- \alpha_3 e_i^{\gamma}\Big(t_{k_i}^i\Big) - \alpha_4 \tanh\Big(\vartheta e_i\Big(t_{k_i}^i\Big)\Big), \quad t \in \Big[t_{k_i}^i, \quad t_{k_i+1}^i\Big) \end{aligned} \tag{7}$$

where $z_i(t)$ is that

$$z_i(t) = \sum_{j=1}^{N} a_{ij}\big(v_i(t) - v_j(t)\big) \tag{8}$$

and α_3 and α_4 are positive constants, $\gamma \in (1, \infty)$ is the ratio of positive odd numbers, and $t_{k_i}^i$ ($t_{k_i+1}^i$) is the latest (next) triggering time of agent i.

The measurement error is designed as

$$\begin{aligned} E_i(t) = &\, \alpha_3 e_i^{\gamma}\Big(t_{k_i}^i\Big) + \alpha_4 \tanh\Big(\vartheta e_i\Big(t_{k_i}^i\Big)\Big) \\ &+ \alpha_1 \beta \chi_i^{\beta-1}\Big(t_{k_i}^i\Big) z_i\Big(t_{k_i}^i\Big) \\ &+ \alpha_2 \vartheta \Big(1 - \tanh^2\Big(\vartheta \chi_i\Big(t_{k_i}^i\Big)\Big)\Big) z_i\Big(t_{k_i}^i\Big) \\ &- \alpha_3 e_i^{\gamma}(t) - \alpha_4 \tanh(\vartheta e_i(t)) - \alpha_1 \beta \chi_i^{\beta-1}(t) z_i(t) \\ &- \alpha_2 \vartheta \Big(1 - \tanh^2(\vartheta \chi_i(t))\Big) z_i(t). \end{aligned} \tag{9}$$

The triggering function of agent i is

$$\varphi_i(t) = |E_i(t)| - \eta \alpha_3 \big|e_i^{\gamma}(t)\big| - \eta \alpha_4 \tag{10}$$

where $\eta \in (0, 1)$ can be chosen as required. Thus

$$t_{k_i+1}^i = \inf\Big\{t > t_{k_i}^i | \varphi_i(t) \geq 0\Big\} \tag{11}$$

and the controller of agent i is updated at its own sampling time sequence $t_0^i, t_1^i, \ldots$

Theorem 1: For the MASs (2) with uncertain disturbance, if the following inequalities hold:

$$\begin{aligned} \alpha_2 &> \varsigma + \Gamma \\ \alpha_4(1 - \eta) &> \bar{\Delta} \end{aligned} \tag{12}$$

with

$$\Gamma = \min\left\{\frac{N\alpha_4 \kappa}{\vartheta(1-\theta)\big(\alpha_4(1-\eta) - \bar{\Delta}\big)},\ N^{\frac{1}{2}}\left(\frac{\alpha_4 \kappa}{\alpha_3 \vartheta (1-\theta)(1-\eta)}\right)^{\frac{1}{\gamma+1}}\right\}$$

the practical fixed-time consensus is obtained under the controller (7) and the triggering function (10).

Proof: First, we will show that the true velocity $v_i(t)$ can track the virtual velocity $\hat{\upsilon}_i(t)$ within a fixed time, and we have

$$\begin{aligned} \dot{e}_i(t) &= u_i(t) + \Delta_i(x_i(t), v_i(t), t) + \alpha_1 \beta \chi_i^{\beta-1}(t) z_i(t) \\ &\quad + \alpha_2 \vartheta \Big(1 - \tanh^2(\vartheta \chi_i(t))\Big) z_i(t) \\ &= -\big(E_i(t) + \alpha_3 e_i^{\gamma}(t) + \alpha_4 \tanh(\vartheta e_i(t))\big) \\ &\quad + \Delta_i(x_i(t), v_i(t), t). \end{aligned} \tag{13}$$

Construct the Lyapunov candidate function

$$V_1(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^2(t). \tag{14}$$

In the following, the concept of Filippov solutions should be applied [48], [49]. In [2], [25], [30], and [32], the similar results based on Filippov solutions were given.

According to the triggering condition and Lemmas 3 and 4, the derivative of (14) can be written as

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{N} e_i(t)\dot{e}_i(t)\\ &\le \sum_{i=1}^{N} |e_i(t)|\big(\eta\alpha_3\big|e_i^{\gamma}(t)\big| + \eta\alpha_4\big) - \alpha_3\sum_{i=1}^{N} e_i^{\gamma+1}(t)\\ &\quad - \alpha_4\sum_{i=1}^{N}|e_i(t)| + \bar{\Delta}\sum_{i=1}^{N}|e_i(t)| + \frac{N\alpha_4\kappa}{\vartheta}\\ &\le -\alpha_3(1-\eta)\sum_{i=1}^{N}|e_i(t)|^{\gamma+1}\\ &\quad - \big(\alpha_4(1-\eta)-\bar{\Delta}\big)\sum_{i=1}^{N}|e_i(t)| + \frac{N\alpha_4\kappa}{\vartheta}\\ &\le -\alpha_3(1-\eta)N^{\frac{1-\gamma}{2}}\left(\sum_{i=1}^{N} e_i^2(t)\right)^{\frac{\gamma+1}{2}}\\ &\quad - \big(\alpha_4(1-\eta)-\bar{\Delta}\big)\left(\sum_{i=1}^{N} e_i^2(t)\right)^{\frac{1}{2}} + \frac{N\alpha_4\kappa}{\vartheta}\\ &\le -\alpha_3(1-\eta)N^{\frac{1-\gamma}{2}}(2V_1(t))^{\frac{\gamma+1}{2}}\\ &\quad - \big(\alpha_4(1-\eta)-\bar{\Delta}\big)(2V_1(t))^{\frac{1}{2}} + \frac{N\alpha_4\kappa}{\vartheta}. \end{aligned} \tag{15}$$

According to Lemma 2, $e_i(t)$ converges into the region

$$\begin{aligned}|e_i(t)| &\le \|e(t)\| \le \Gamma\\ &= \min\left\{\frac{N\alpha_4\kappa}{\vartheta(1-\theta)\big(\alpha_4(1-\eta)-\bar{\Delta}\big)}\right.\\ &\qquad\left. N^{\frac{1}{2}}\left(\frac{\alpha_4\kappa}{\alpha_3\vartheta(1-\theta)(1-\eta)}\right)^{\frac{1}{\gamma+1}}\right\}\end{aligned}$$

in a settling time T_1, which satisfies that

$$\begin{aligned}T_1 \le \bar{T}_1 &= \frac{1}{c_1\theta(1-\rho)} + \frac{1}{c_2\theta(\omega-1)}\\ &= \frac{\sqrt{2}}{\theta\big(\alpha_4(1-\eta)-\bar{\Delta}\big)}\\ &\quad + \frac{2}{\alpha_3\theta(1-\eta)(\gamma-1)N^{\frac{1-\gamma}{2}}2^{\frac{\gamma+1}{2}}}.\end{aligned} \tag{16}$$

It is noteworthy that when $|e_i(t)| \le \Gamma$, one has that $v_i(t) = -\alpha_1\chi_i^{\beta}(t) - \alpha_2\tanh(\vartheta\chi_i(t)) + \varsigma + e_i(t)$.

Construct the Lyapunov candidate function

$$V_2(t) = \frac{1}{2}x^T(t)\mathcal{L}x(t).$$

Based on Lemmas 2 and 3, we obtain

$$\begin{aligned}\dot{V}_2(t) &= x^T(t)\mathcal{L}\dot{x}(t)\\ &= \sum_{i=1}^{N}\chi_i(t)v_i(t)\\ &\le -\alpha_1\sum_{i=1}^{N}\chi_i^{\beta+1}(t) - \alpha_2\sum_{i=1}^{N}|\chi_i(t)|\\ &\quad + \varsigma\sum_{i=1}^{N}|\chi_i(t)| + \sum_{i=1}^{N}|\chi_i(t)||e_i(t)| + \frac{N\alpha_2\kappa}{\vartheta}\\ &\le -\alpha_1 N^{\frac{1-\beta}{2}}\left(\sum_{i=1}^{N}\chi_i^2(t)\right)^{\frac{\beta+1}{2}}\\ &\quad - (\alpha_2-\varsigma-\Gamma)\left(\sum_{i=1}^{N}\chi_i^2(t)\right)^{\frac{1}{2}} + \frac{N\alpha_2\kappa}{\vartheta}\\ &\le -\alpha_1 N^{\frac{1-\beta}{2}}(2\lambda_2 V_2(t))^{\frac{\beta+1}{2}}\\ &\quad - (\alpha_2-\varsigma-\Gamma)(2\lambda_2 V_2(t))^{\frac{1}{2}} + \frac{N\alpha_2\kappa}{\vartheta}\end{aligned} \tag{17}$$

where λ_2 denotes the second smallest eigenvalue of $\mathcal{L}$. According to Lemma 2, the residual set can be calculated as

$$\begin{aligned}\Bigg\{\lim_{t\to T_1+T_2} x(t)|V_2(t) \le \min\Bigg\{&(2\lambda_2)^{-1}\left(\frac{N\alpha_2\kappa}{\vartheta(1-\theta)(\alpha_2-\varsigma-\Gamma)}\right)^2,\\ &\frac{N}{2\lambda_2}\left(\frac{\alpha_2\kappa}{\alpha_1\vartheta(1-\theta)}\right)^{\frac{2}{\beta+1}}\Bigg\}\Bigg\}\end{aligned} \tag{18}$$

and T_2 satisfies

$$\begin{aligned}T_2 \le \bar{T}_2 &= \frac{1}{c_1\theta(1-\rho)} + \frac{1}{c_2\theta(\omega-1)}\\ &= \frac{\sqrt{2}}{(\alpha_2-\varsigma-\Gamma)\theta\sqrt{\lambda_2}}\\ &\quad + \frac{2}{\alpha_1(\beta-1)N^{\frac{1-\beta}{2}}\theta(2\lambda_2)^{\frac{\beta+1}{2}}}.\end{aligned} \tag{19}$$

Therefore, the practical consensus will be achieved, and the total

$$\begin{aligned}T = T_1 + T_2 &\le \frac{\sqrt{2}}{\theta\big(\alpha_4(1-\eta)-\bar{\Delta}\big)}\\ &\quad + \frac{2}{\alpha_3\theta(1-\eta)(\gamma-1)N^{\frac{1-\gamma}{2}}2^{\frac{\gamma+1}{2}}}\\ &\quad + \frac{\sqrt{2}}{(\alpha_2-\varsigma-\Gamma)\theta\sqrt{\lambda_2}} + \frac{2}{\alpha_1(\beta-1)N^{\frac{1-\beta}{2}}\theta(2\lambda_2)^{\frac{\beta+1}{2}}}.\end{aligned}$$

Moreover, one has $v_i(t) = \hat{\upsilon}_i(t) + e_i(t) \le |-\alpha_1\chi_i^{\beta}(t) - \alpha_2\tanh(\vartheta\chi_i(t)) + \varsigma| + \Gamma$. Hence, $v_i(t)$ enters into a small neighborhood of ς. In addition, we can choose the suitable $v_i(t)$ in practical applications because ς and Γ can be chosen as required.

Global Stability Analysis: First, when $t \in [0, T_1]$, according to (4) and (6), $e_i(t)$ and $\hat{\upsilon}_i(t)$ are bounded. From (2), (4), and (6), we have

$$\dot{x}_i(t) = e_i(t) - \alpha_1\chi_i^{\beta}(t) - \alpha_2\tanh(\vartheta\chi_i(t)) + \varsigma. \tag{20}$$

Because $e_i(t)$ is bounded when $t \in [0, T_1]$, $x_i(t)$ is bounded by the input-to-state stable property [50]. Moreover, $v_i(t)$ is bounded based on the definition of $v_i(t)$. Then, when $|e_i(t)| \le \Gamma$, the practical consensus is obtained in a settling time T_2. Hence, the practical consensus is obtained in a settling time T. ■

Theorem 2: For the MASs (2) with uncertain disturbance, the interevent interval is lower bounded, and thus no Zeno behavior occurs.

Proof: From the definition of $\chi_i(t)$, one has that $\sum_{i=1}^{N}\chi_i^2(t) = x^T(t)\mathcal{L}^2x(t)$. Based on Lemma 1, we have the following results:

$$\lambda_N x^T(t)\mathcal{L}x(t) \geq \sum_{i=1}^{N} \chi_i^2(t) \geq \lambda_2 x^T(t)\mathcal{L}x(t). \tag{21}$$

Hence, we can obtain $|\chi_i(t)| < \|\chi(t)\| \leq \sqrt{2\lambda_N V_2(0)}$ with $V_2(0) = (1/2)x^T(0)\mathcal{L}x(0)$.

According to the definition of $z_i(t)$, we can also obtain

$$\begin{aligned}|z_i(t)| &= \left|\sum_{j=1}^{N} a_{ij}\big(v_i(t) - v_j(t)\big)\right| \\ &\leq \left|\sum_{j=1}^{N} a_{ij}\big(e_i(t) - e_j(t)\big)\right| + 2\alpha_2 \sum_{j=1}^{N} a_{ij} \\ &\quad + \alpha_1 \left|\sum_{j=1}^{N} a_{ij}\Big(-\chi_i^{\beta}(t) + \chi_j^{\beta}(t)\Big)\right| \\ &\leq \|e(t)\|_1 + (l_{ii} - 1)\|e(t)\|_2 + 2\alpha_1 l_{ii}\|\chi(t)\|_2^{\beta} + 2\alpha_2 l_{ii} \\ &\leq \varpi_{i1} \end{aligned} \tag{22}$$

where $\varpi_{i1} = (l_{ii} - 1 + N^{(1/2)})(2V_1(0))^{(1/2)} + 2\alpha_2 l_{ii} + 2\alpha_1 l_{ii}(2\lambda_N V_2(0))^{(\beta/2)}$, and λ_N denotes the largest eigenvalue of $\mathcal{L}$.

According to (9), we obtain

$$\begin{aligned} D^+|E_i(t)| &\leq \big|\dot{E}_i(t)\big| \\ &= \Big|\big(-\alpha_3 e_i^{\gamma}(t) - \alpha_4 \tanh(\vartheta e_i(t))\big)' \\ &\quad - \Big(\alpha_1\beta\chi_i^{\beta-1}(t)z_i(t)\Big)' \\ &\quad - \Big(\alpha_2\vartheta\Big(1 - \tanh^2(\vartheta\chi_i(t))\Big)z_i(t)\Big)'\Big| \\ &\leq \Big|-\Big(\alpha_3\gamma e_i^{\gamma-1}(t) + \alpha_4\vartheta\Big(1 - \tanh^2(\vartheta e_i(t))\Big)\Big)\dot{e}_i(t) \\ &\quad - \Big(\alpha_1\beta\chi_i^{\beta-1}(t) + \alpha_2\vartheta\Big(1 - \tanh^2(\vartheta\chi_i(t))\Big)\Big)\dot{z}_i(t) \\ &\quad - z_i(t)\dot{\chi}_i(t)\Big(\alpha_1\beta(\beta-1)\chi_i^{\beta-2}(t) \\ &\qquad - 2\alpha_2\vartheta^2\tanh(\vartheta\chi_i(t)) \\ &\qquad \times\Big(1 - \tanh^2(\vartheta\chi_i(t))\Big)\Big)\Big| \\ &\leq \varpi_2|\dot{e}_i(t)| + \varpi_3|\dot{z}_i(t)| + \varpi_4 z_i^2(t) \\ &\leq \varpi_2\big(|u_i(t)| + \bar{\Delta}\big) + \varpi_2\Big|\alpha_1\beta(2\lambda_N V_2(0))^{\frac{\beta-1}{2}} + \alpha_2\vartheta\Big| \\ &\quad \times |z_i(t)| + \varpi_3|\dot{z}_i(t)| + \varpi_4 z_i^2(t) \\ &\leq \varpi_2\big(|u_i(t)| + \bar{\Delta}\big) + \varpi_2\varpi_3|z_i(t)| + \varpi_4\Big|z_i^2(t)\Big| \\ &\quad + \varpi_3\left(\left|\sum_{j=1}^{N} a_{ij}\big(u_i(t) - u_j(t)\big)\right| + 2l_{ii}\bar{\Delta}\right) \\ &\leq \varpi_2\Big(\Big|u_i\Big(t_{k_i}^i\Big)\Big| + \bar{\Delta}\Big) + \varpi_{i1}\varpi_2\varpi_3 + \varpi_{i1}^2\varpi_4 \\ &\quad + \varpi_3\left(\left|\sum_{j=1}^{N} l_{ij}u_j\Big(t_{k_{j(t)}}^j\Big)\right| + 2l_{ii}\bar{\Delta}\right) \\ &\leq \psi\Big(t_{k_i}^i, t_{k_{j(t)}}^j\Big) \end{aligned} \tag{23}$$

where $\varpi_2 = |\alpha_3\gamma(2V_1(0))^{[(\gamma-1)/2]} + \alpha_4\vartheta|$, $\varpi_3 = |\alpha_1\beta(2\lambda_N V_2(0))^{[(\beta-1)/2]} + \alpha_2\vartheta|$, $\varpi_4 = |(\alpha_1\beta(\beta - 1)(2\lambda_N V_2(0))^{[(\beta-2)/2]} + 2\alpha_2\vartheta^2)|$, $\psi(t_{k_i}^i, t_{k_{j(t)}}^j) = \varpi_2(|u_i(t_{k_i}^i)| + \bar{\Delta}) + \varpi_3(|\sum_{j=1}^{N} l_{ij}u_j(t_{k_{j(t)}}^j)| + 2l_{ii}\bar{\Delta}) + \varpi_{i1}\varpi_2\varpi_3 + \varpi_{i1}^2\varpi_4$, and $\psi(t_{k_i}^i, t_{k_{j(t)}}^j)$ has a maximum value $\bar{\psi}$. Moreover, $t_{k_{j(t)}}^j$ denotes the latest triggering time of agent j, and D^+ is the right derivative. Since $E_i(t_{k_i}^i) = 0$, based on (23), we have

$$\begin{aligned}|E_i(t)| &\leq \int_{t_{k_i}^i}^{t} \big|\dot{E}_i(s)\big| ds \\ &\leq \int_{t_{k_i}^i}^{t} \psi\Big(t_{k_i}^i, t_{k_{j(t)}}^j\Big) ds. \end{aligned} \tag{24}$$

From the triggering function (10) and (24), one has

$$\begin{aligned}\Big|E_i\Big(t_{k_i+1}^i\Big)\Big| &= \eta\alpha_3\Big|e_i^{\gamma}\Big(t_{k_i+1}^i\Big)\Big| + \eta\alpha_4 \\ &\leq \int_{t_{k_i}^i}^{t_{k_i+1}^i} \psi\Big(t_{k_i}^i, t_{k_{j(t)}}^j\Big) ds \end{aligned} \tag{25}$$

which yields $t_{k_i+1}^i - t_{k_i}^i \geq ([\eta\alpha_4]/[\psi(t_{k_i}^i, t_{k_{j(t)}}^j)]) \geq (\eta\alpha_4/\bar{\psi}) > 0$, this completes the proof. ■

Remark 1: The residual set (18) and equality (25) indicate that the convergence region and interevent interval are determined by ϑ. Besides, $\psi(t_{k_i}^i, t_{k_{j(t)}}^j)$ and ϑ are positively correlated as shown in the equations after (23). Therefore, with the expansion of ϑ, the convergence region and the interevent interval will narrow down. Moreover, for different actual systems, we can select different ϑ to obtain better control properties. Hence, the parameter ϑ can be chosen according to the specific situation.

Remark 2: The previous studies [24]–[28], [30], [32], [33] have discussed the finite/fixed-time consensus problems of first-order MASs via ETC. Herein, we consider the second-order MASs.

Remark 3: For the ETC, continuous communication is required to monitor the triggering condition. In the following, in order to save limited resources, we will present a TTC scheme and an STC scheme, such that communication frequency is reduced with the system performance ensured.

B. Team-Triggered Fixed-Time Consensus

First, define

$$\Xi_i(t) = \alpha_1\beta\chi_i^{\beta-1}(t)z_i(t) + \alpha_2\vartheta\Big(1 - \tanh^2(\vartheta\chi_i(t))\Big)z_i(t). \tag{26}$$

Then, define $Y_i(t) = |E_i(t)| + [\eta/(1+\eta)]|\Xi(t)|$, similar to (23), one has

$$\begin{aligned} D^+Y_i(t) &\leq \big|\dot{E}_i(t)\big| + \frac{\eta}{1+\eta}\big|\dot{\Xi}(t)\big| \\ &\leq \varpi_2|\dot{e}_i(t)| + \frac{(1+2\eta)\varpi_3}{1+\eta}|\dot{z}_i(t)| + \frac{(1+2\eta)\varpi_4}{1+\eta}z_i^2(t) \\ &\leq \frac{(1+2\eta)\varpi_3}{1+\eta}\left(|\sum_{j=1}^{N} a_{ij}\big(u_i(t) - u_j(t)\big)| + 2l_{ii}\bar{\Delta}\right) \\ &\quad + \varpi_2\big(|u_i(t)| + \bar{\Delta}\big) \\ &\quad + \varpi_2\varpi_3|z_i(t)| + \frac{(1+2\eta)\varpi_4}{1+\eta}\Big|z_i^2(t)\Big| \end{aligned}$$

$$\leq \varpi_2\left(\left|u_i\left(t_{k_i}^i\right)\right| + \bar{\Delta}\right) + \varpi_{i1}\varpi_2\varpi_3 + \frac{(1+2\eta)\varpi_{i1}^2\varpi_4}{1+\eta} + \frac{(1+2\eta)\varpi_3}{1+\eta}\left(\left|\sum_{j=1}^N l_{ij}u_j\left(t_{k_{j(t)}}^j\right)\right| + 2l_{ii}\bar{\Delta}\right) \leq \tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right) \quad (27)$$

where

$$\tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right) = \varpi_2\left(\left|u_i\left(t_{k_i}^i\right)\right| + \bar{\Delta}\right) + \varpi_{i1}\varpi_2\varpi_3 + \frac{(1+2\eta)\varpi_{i1}^2\varpi_4}{1+\eta} + \frac{(1+2\eta)\varpi_3}{1+\eta} \times \left(\left|\sum_{j=1}^N l_{ij}u_j\left(t_{k_{j(t)}}^j\right)\right| + 2l_{ii}\bar{\Delta}\right).$$

The team-triggered function of agent i is that

$$\Psi_i(t) = \begin{cases} \varphi_i(t), & \text{if } \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right| + \sigma \geq \bar{\chi}\left(t_{k_i}^i\right) \\ \int_{t_{k_i}^i}^t \tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds + \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right| - \bar{\chi}\left(t_{k_i}^i\right), & \text{otherwise} \end{cases}$$

where $\bar{\chi}(t_{k_i}^i) = [\eta/(1+\eta)]|\alpha_3 e_i^\gamma(t_{k_i}^i) + \alpha_4 \tanh(\vartheta e_i(t_{k_i}^i)) + \Xi_i(t_{k_i}^i)|$, σ can be chosen as required (a very small positive constant). Thus

$$t_{k_i+1}^i = \inf\left\{t > t_{k_i}^i | \Psi_i(t) \geq 0\right\}. \quad (28)$$

When $[\eta/(1+\eta)]|\Xi_i(t_{k_i}^i)| + \sigma \geq \bar{\chi}(t_{k_i}^i)$, it is the event-triggered condition in Theorem 1. When $[\eta/(1+\eta)]|\Xi_i(t_{k_i}^i)| + \sigma < \bar{\chi}(t_{k_i}^i)$, it is the self-triggered condition that $t_{k_i+1}^i$ occurs at most τ_i time units after $t_{k_i}^i$ for agent i, where

$$\tau_i = \frac{\bar{\chi}\left(t_{k_i}^i\right) - \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right|}{\tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)} \geq \frac{\sigma}{\tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)}.$$

Hence, no Zeno behavior occurs because $\tilde{\psi}(t_{k_i}^i, t_{k_{j(t)}}^j)$ is upper bounded. For all $t \in [t_{k_i}^i, t_{k_i}^i + \tau_i]$, if there is an event triggered in one of its neighbors, update $\tilde{\psi}(t_{k_i}^i, t_{k_{j(t)}}^j)$ with the latest states. Otherwise, agent i waits until $t_{k_i+1}^i$. It can be shown that the team-triggered condition has the same effect as the single event-triggering condition, which can guarantee that $\varphi_i(t) \leq 0$. The proof is presented in the following.

When $[\eta/(1+\eta)]|\Xi_i(t_{k_i}^i)| + \sigma < \bar{\chi}(t_{k_i}^i)$. The triggering function enforces

$$\int_{t_{k_i}^i}^t \tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds + \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right| \leq \bar{\chi}\left(t_{k_i}^i\right). \quad (29)$$

Similar to (24), we obtain

$$|E_i(t)| + \frac{\eta}{1+\eta}|\Xi(t)| \leq \int_{t_{k_i}^i}^t \tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds + \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right| \leq \bar{\chi}\left(t_{k_i}^i\right). \quad (30)$$

According to (9) and (26), we have $|E_i(t)| + |\Xi(t)| + |\alpha_3 e_i^\gamma(t) + \alpha_4 \tanh(\vartheta e_i(t))| \geq |\alpha_3 e_i^\gamma(t_{k_i}^i) + \alpha_4 \tanh(\vartheta e_i(t_{k_i}^i)) + \Xi(t_{k_i}^i)| = [(1+\eta)/\eta]\bar{\chi}(t_{k_i}^i)$. Together with (30), we have

$$|E_i(t)| \leq \frac{\eta}{1+\eta}\left(|E_i(t)| + \left|\alpha_3 e_i^\gamma(t)\right| + |\alpha_4 \tanh(\vartheta e_i(t))|\right). \quad (31)$$

Inequality (31) implies that $|E_i(t)| \leq \eta\alpha_3|e_i^\gamma(t)| + \eta\alpha_4$. Together with the case $[\eta/(1+\eta)]|\Xi_i(t_{k_i}^i)| + \sigma \geq \bar{\chi}(t_{k_i}^i)$, we obtain that the triggering condition based on $\Psi_i(t)$, which can guarantee $\varphi_i(t) \leq 0$. The proof is completed.

Under the team-triggered rule and previous analysis, we obtain the following results based on Theorem 1.

Theorem 3: For the MASs (2) with uncertain disturbance, if the following inequalities hold:

$$\alpha_2 > \varsigma + \Gamma$$
$$\alpha_4(1-\eta) > \bar{\Delta} \quad (32)$$

the practical fixed-time consensus is reached with the controller (7) via the TTC, and the settling time $T = T_1 + T_2$.

Remark 4: For $\int_{t_{k_i}^i}^t \tilde{\psi}(t_{k_i}^i, t_{k_{j(t)}}^j)ds + [\eta/(1+\eta)]|\Xi_i(t_{k_i}^i)| - \bar{\chi}(t_{k_i}^i)$, we only use $\tilde{\psi}(t_{k_i}^i, t_{k_{j(t)}}^j)$, $\Xi(t_{k_i}^i)$, and $e(t_{k_i}^i)$ to determine $t_{k_i+1}^i$, which means that continuous communication is avoided in the case of the STC. Hence, for the TTC, the energy consumption of communication is reduced.

Remark 5: Under the TTC strategy, there is a tradeoff between the convergence rate, communication cost, and triggering times, which will be shown in the following and the simulation results.

C. Self-Triggered Fixed-Time Consensus

In order to make a comparison, we also present a STC scheme.

Consider the following condition:

$$\int_{t_{k_i}^i}^t \psi\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds \leq \eta\alpha_4 \quad (33)$$

which can guarantee that $\varphi_i(t) \leq 0$. The detailed proof is shown as follows.

Based on inequality (24), we have

$$|E_i(t)| \leq \int_{t_{k_i}^i}^t \psi\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds. \quad (34)$$

Substituting (34) into (33), we have

$$|E_i(t)| \leq \eta\alpha_4 \leq \eta\alpha_3\left|e_i^\gamma(t)\right| + \eta\alpha_4. \quad (35)$$

Hence, the proof is completed. ■

Define

$$\Phi_i\left(t_{k_i}^i, t_{k_{j(t)}}^j\right) = \begin{cases} \int_{t_{k_i}^i}^t \psi\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds - \eta\alpha_4, & \text{if } \frac{\eta}{1+\eta}\left|\Xi_i\left(t_{k_i}^i\right)\right| + \sigma \geq \bar{\chi}\left(t_{k_i}^i\right) \\ \int_{t_{k_i}^i}^t \tilde{\psi}\left(t_{k_i}^i, t_{k_{j(t)}}^j\right)ds + \frac{\eta}{1+\eta}|\Xi_i\left(t_{k_i}^i\right)| - \bar{\chi}\left(t_{k_i}^i\right), & \text{otherwise.} \end{cases}$$

Thus, the following self-triggered rule is given.

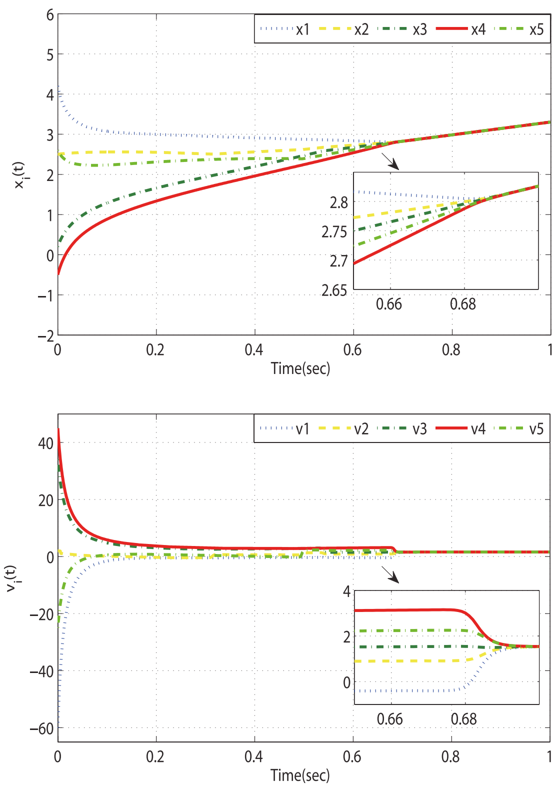

Fig. 1. States evolutions of five agents under the ETC.

Self-Triggered Rule: For each agent i, if exists a positive constant ξ_i such that

$$\xi_i = \begin{cases} \frac{\eta\alpha_4}{\psi\left(t^i_{k_i}, t^j_{k_{j(t)}}\right)}, & \text{if } \frac{\eta}{1+\eta}\left|\Xi_i\left(t^i_{k_i}\right)\right| + \sigma \geq \bar{\chi}\left(t^i_{k_i}\right) \\ \frac{\bar{\chi}\left(t^i_{k_i}\right) - \frac{\eta}{1+\eta}\left|\Xi_i\left(t^i_{k_i}\right)\right|}{\tilde{\psi}\left(t^i_{k_i}, t^j_{k_{j(t)}}\right)}, & \text{otherwise.} \end{cases}$$

Then, the next triggering time $t^i_{k_i+1}$ occurs at most ξ_i time units after $t^i_{k_i}$, that is, $t^i_{k_i+1} \leq t^i_{k_i} + \xi_i$. For all $t \in [t^i_{k_i}, t^i_{k_i} + \xi_i]$, if one of its neighbors triggered, update $\psi(t^i_{k_i}, t^j_{k_{j(t)}})$ with the latest states.

Similar to the team-triggered rule [the triggering condition based on $\Psi_i(t)$], one has that the self-triggered condition [the triggering condition based on $\Phi_i(t^i_{k_i}, t^j_{k_{j(t)}})$] has the same effect as the single event-triggering condition, which can also guarantee that $\varphi_i(t) \leq 0$. Hence, the following results are obtained as Theorem 3.

Theorem 4: For the MASs (2) with uncertain disturbance, if the following inequalities hold:

$$\begin{aligned} \alpha_2 &> \varsigma + \Gamma \\ \alpha_4(1-\eta) &> \bar{\Delta} \end{aligned} \tag{36}$$

the consensus is obtained with the controller (7) via the STC, and $T = T_1 + T_2$. Based on the self-triggering rule, we have

$$\xi_i \geq \min\left\{ \frac{\eta\alpha_4}{\psi\left(t^i_{k_i}, t^j_{k_{j(t)}}\right)}, \frac{\sigma}{\tilde{\psi}\left(t^i_{k_i}, t^j_{k_{j(t)}}\right)} \right\} > 0.$$

Hence, no Zeno behavior occurs.

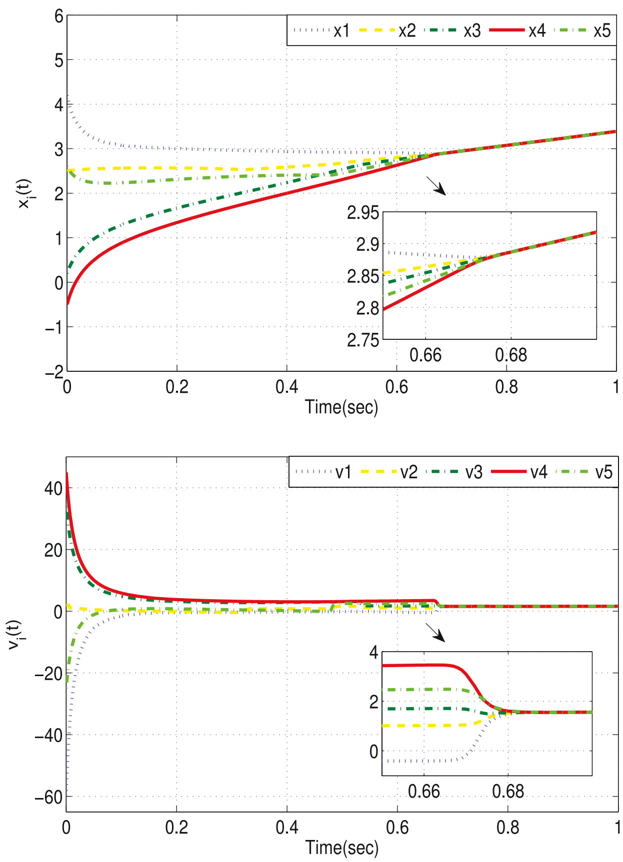

Fig. 2. States evolutions of five agents under the TTC.

Remark 6: In [29] and [31], the finite-time event-triggered consensus problems of general linear MASs were tackled. Moreover, in [34], the fixed-time consensus was achieved via ETC and can only ensure that the position information is consistent. However, continuous communication is required in [29] and [34]. Guo and Chen [35] assumed that the leader's input is zero under intermittent communication. In this article, we propose the TTC and STC to reduce the energy consumption, and ς can be chosen as required, implying that we can choose the suitable $v_i(t)$ in practical applications. Moreover, the uncertain disturbance was considered.

Remark 7: For the STC, $\Phi_i(t^i_{k_i}, t^j_{k_{j(t)}})$ is completely independent of real-time status values. Hence, compared with the TTC, the STC can avoid continuous monitoring completely. However, more triggering times may be required.

Remark 8: The STC needs numerous triggering times and calculations in order to guarantee a fast convergence rate. It is important for us to make a tradeoff between communication cost and triggering times. Herein, we proposed three different triggering mechanisms, and we can choose the appropriate triggering mechanism as required.

IV. Simulation Results

Consider a connected network topology of five agents with the following Laplacian matrix:

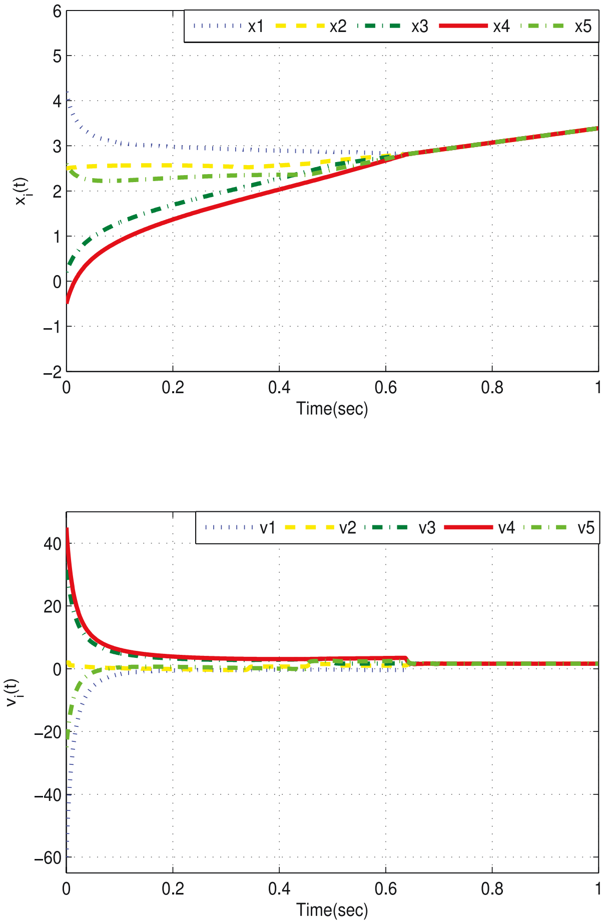

Fig. 3. States evolutions of five agents under the STC.

$$\mathcal{L} = \begin{bmatrix} 2.5 & -1.5 & -1 & 0 & 0 \\ -1.5 & 3.5 & -1 & 0 & -1 \\ -1 & -1 & 3.6 & -1.6 & 0 \\ 0 & 0 & -1.6 & 3.1 & -1.5 \\ 0 & -1 & 0 & -1.5 & 2.5 \end{bmatrix}.$$

Moreover, the dynamics of agent i is

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= u_i(t) + 0.6\sin(v_i(t)) + 0.8\cos(x_i(t)) \end{aligned} \tag{37}$$

and the uncertain disturbance satisfies Assumption 1 with $\bar{\Delta} = 1.4$. λ_2 and λ_N are 1.7 and 5.82. Assume that $x(0) = [4.2 \quad 2.5 \quad 0.2 \quad -0.5 \quad 2.6]^T$ and $v(0) = [-60 \quad 2 \quad 34 \quad 45 \quad -25]^T$.

Under the controller (7), we set $\alpha_1 = 0.5$, $\alpha_2 = 2$, $\alpha_3 = 2.5$, $\alpha_4 = 3$, $\alpha = 13/5$, $\gamma = 7/5$, $\eta = 0.5$, $\vartheta = 80$, $\varsigma = 1.6$, $\theta = 0.2$, and $\sigma = 0.001$. The parameters satisfy the inequality (12). Fig. 1 shows the state evolutions of five agents under the ETC, and $v_i(t)$ converges into a small neighborhood of ς.

In order to relax the requirement of continuous communication, the team-triggered function and the self-triggered function are proposed to replace the event-triggered function. Fig. 2 shows the state evolutions of five agents under the TTC, and $v_i(t)$ converges into a small neighborhood of ς. Fig. 3 shows the state evolutions of five agents under the STC.

TABLE I
TRIGGERING TIMES UNDER THE ETC, TTC, AND STC

	ETC	TTC	STC
Agent 1	143	230	704
Agent 2	205	529	764
Agent 3	152	581	764
Agent 4	135	583	753
Agent 5	178	397	688

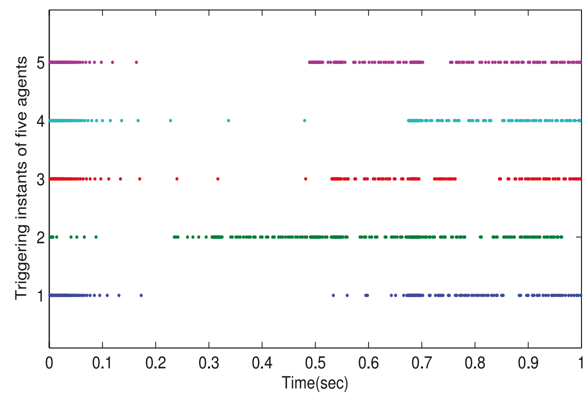

Fig. 4. Triggering instants of five agents under the ETC.

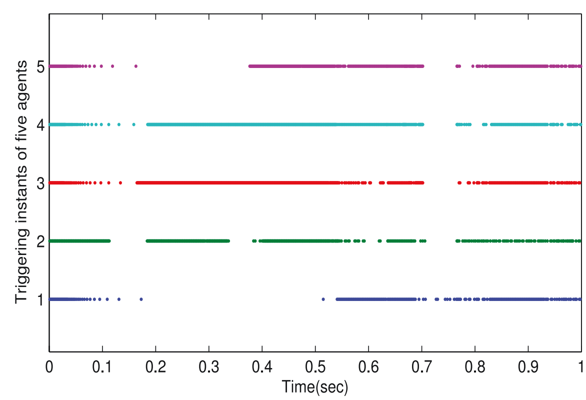

Fig. 5. Triggering instants of five agents under the TTC.

Comparing Fig. 1 with Fig. 2, we find that the systems have a faster convergence rate under the TTC even though the communication is intermittent. The faster convergence is likely due to increasing numbers of triggering. Moreover, the STC has the fastest convergence.

Figs. 4–6 show the corresponding triggering instants. The triggering times under the ETC, the STC, and the TTC are listed in Table I. In the simulations, the sampling interval is chosen as 0.001 s. From Table I, we can find that the STC needs the most triggering times. Because the communication is intermittent in STC and the continuous communication is required in our ETC, the STC needs more triggering times to offset the defects from discontinuous communication. According to Tables I and II and Figs. 1–3, we can find that TTC and STC can ensure the system performance with intermittent communication at the cost of more triggering times.

Table II shows that the self-triggered case is abundant in the TTC, which means that energy consumption is reduced

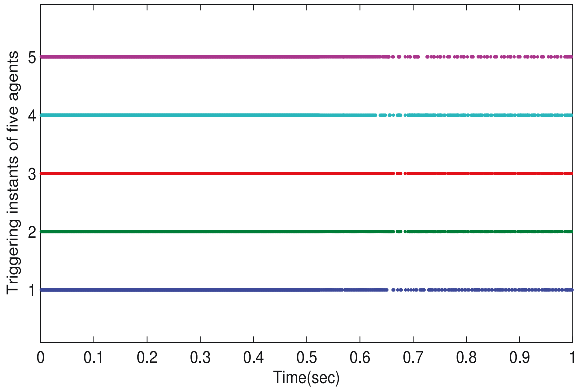

Fig. 6. Triggering instants of five agents under the STC.

TABLE II
TRIGGERING TIMES UNDER THE TTC

	Total	Event-triggered	Self-triggered
Agent 1	230	75	155
Agent 2	529	68	461
Agent 3	581	98	483
Agent 4	583	74	509
Agent 5	397	92	305

substantially. Furthermore, compared with the ETC or the STC (Table I), the TTC not only leads to a tradeoff between the convergence rate, communication cost, and triggering times but also ensures the system performance.

V. CONCLUSION

The fixed-time consensus problem of second-order MASs with uncertain disturbance is considered via TTC. First, an ETC is proposed, and no Zeno behavior occurs. Then, the TTC and STC are presented, where the energy consumption is reduced substantially and the system performance is guaranteed. Moreover, in the simulation results, the TTC has a tradeoff between the convergence rate, communication cost, triggering times, and calculated amount. The effectiveness of the proposed algorithms is demonstrated by numerical simulations.

In this article, the results are based on the undirected topology, and the directed topology is more practical. Moreover, we only consider the linear MASs with uncertain disturbance. In addition, future work will focus on the team/self-triggered fixed-time consensus problem for nonlinear MASs under directed topology.

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Jian Liu (Graduate Student Member, IEEE) received the B.S. and Ph.D. degrees from the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, China, in 2015 and 2020, respectively.

He is currently a Postdoctoral Fellow with the School of Automation, Southeast University, Nanjing, China. From 2017 to 2018, he was a joint training student with the Department of Mathematics, Dartmouth College, Hanover, NH, USA. His current research interests include multiagent systems, nonlinear control, and event-triggered control.

Yao Yu received the B.S. degree from the Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, China, in 2004, and the M.S. and Ph.D. degrees from the Department of Automation, Tsinghua University, Beijing, China, in 2010.

She was a Postdoctoral Fellow with Tsinghua University. She is currently an Associate Professor with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing. Her current research interests include nonlinear control, robust control, and time-delay systems.

Haibo He (Fellow, IEEE) received the B.S. and M.S. degrees in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1999 and 2002, respectively, and the Ph.D. degree in electrical engineering from Ohio University, Athens, OH, USA, in 2006.

He is currently the Robert Haas Endowed Chair Professor with the Department of Electrical, Computer, and Biomedical Engineering, University of Rhode Island, South Kingstown, RI, USA.

Dr. He received the IEEE International Conference on Communications Best Paper Award in 2014, the IEEE CIS Outstanding Early Career Award in 2014, and the National Science Foundation CAREER Award in 2011. He was the General Chair of the IEEE Symposium Series on Computational Intelligence in 2014. He is currently the Editor-in-Chief of the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS.

Changyin Sun (Senior Member, IEEE) received the B.S. degree from the College of Mathematics, Sichuan University, Chengdu, China, in 1996, and the M.S. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2001 and 2003, respectively.

He is a Distinguished Professor with the School of Automation, Southeast University. His current research interests include intelligent control, flight control, pattern recognition, and optimal theory.

Prof. Sun is an Associate Editor of the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS.