

Optimal Investment in Prevention and Recovery for Mitigating Epidemic Risks

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The worldwide healthcare and economic crisis caused by the COVID-19 pandemic highlights the need for a deeper understanding of investing in the mitigation of epidemic risks. To address this, we built a mathematical model to optimize investments into two types of measures for mitigating the risks of epidemic propagation: prevention/containment measures and treatment/recovery measures. The new model explicitly accounts for the characteristics of networks of individuals, as a critical element of epidemic propagation. Subsequent analysis shows that, to combat an epidemic that can cause significant negative impact, optimal investment in either category increases with a higher level of connectivity and intrinsic loss, but it is limited to a fraction of that total potential loss. However, when a fixed and limited mitigation investment is to be apportioned among the two types of measures, the optimal proportion of investment for prevention and containment increases when the investment limit goes up, and when the network connectivity decreases. Our results are consistent with existing studies and can be used to properly interpret what happened in past pandemics as well as to shed light on future and ongoing events such as COVID-19.

KEY WORDS: COVID-19; epidemic risk mitigation; optimal investment; prevention and containment; treatment and recovery

1. INTRODUCTION

Within a few short months, COVID-19 has ravaged the world, resulting in millions of people infected and, among them, deaths in the hundreds of thousands. Countries and regions tried many different measures to mitigate and control the disease with varying levels of success—while some, most notably

New Zealand, Taiwan, and Vietnam, showed significant success, others instead suffered badly. The devastating social and economic impacts of the pandemic crisis in these harder hit countries highlight the importance of investing in the containment and treatment of contagious diseases. However, finding the optimal combination of investments in epidemic prevention and treatment is a complex problem, due to such factors as the uncertainty associated with a novel disease, the nonlinear nature of an epidemic, the nonadditive effects of interventions, and budgetary limitations (Alistar, Long, Brandeau, & Beck, 2014; Coşgun & Büyüktaktın, 2018). The ongoing impacts of COVID-19, along with worldwide trends of increasing globalization, population mobility, and interconnectivity, emphasize the need for critical understanding of how to invest in mitigating the risks

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of future epidemics (Saker, Lee, Cannito, Gilmore, & Campbell-Lendrum, 2004).

Understanding how an epidemic spreads is vital to containing it. For over a century, researchers have used mathematical models to examine how communicable diseases propagate through a population. The susceptible-infected-removed (SIR) model, the classical compartmental model in mathematical epidemiology developed by Kermack and McKendrick (1927), divides the population into three classes (compartments): individuals who have not yet been infected (susceptible), those who are currently infected, and those who are now immune or have died. Since then, a number of modifications and improvements have been made to the original SIR model, motivated by the growing number of outbreaks in recent years such as the 2002–2004 SARS (severe acute respiratory syndrome) outbreak and the 2009 H1N1 pandemic. Brauer (2017) provides a comprehensive review of the available models.

One of the important additions to the basic SIR model, among others, is modeling the transmission of infection as a stochastic process, which captures the fact that in most outbreaks a few infected individuals usually spread the infection to many people, while most other infected individuals either do not spread it or only spread it to a few others (Brauer, 2008; Riley *et al.*, 2003). This highlights the importance of modeling the connections between individuals in the context of epidemic propagation. Network analysis, because of its ability to capture the realistic settings of population structure, has received significant attention for modeling epidemic propagation during the past decades.

In a population network, individuals are represented by vertices (nodes) and connections/contacts between individuals are given by edges. An infection thus can be transmitted from one vertex to another through edges. One of the most common network types in the literature is a random graph, where nodes are added to the network by randomly connecting to any other existing nodes with a uniform probability (Barabási & Albert, 1999). Random graph networks have been used to model epidemic propagation in different settings; for example, SIR epidemics by Volz (2008) and Miller (2011) and the susceptible-infectious-susceptible spread by Parshani, Carmi, and Havlin (2010) and Shang (2012).

Despite their wide popularity, however, random graph models are unlikely to represent population networks in reality. Instead, “popular” individuals

(i.e., those with already large number of connections) are more likely to receive more connections, and those with few connections are likely to remain less connected. This “preferential attachment,” where a new node is likely to connect to the nodes with an already large number of connections, will result in a network composed of a majority of nodes that have limited connections and a small number of “hubs” with large number of links. Such a network topology, first proposed by physicists Barabási and Albert (1999), results in a “scale-free network” and has been successfully used to simulate numerous industry networks such as the Internet (Kim & Altmann, 2012) and the power grid (Chassin & Posse, 2005), as well as social networks such as journal citations (McGuigan, 2018) and academic collaborations (Dorogovtsev & Mendes, 2002). In particular, studies in human sexual contact (Liljeros, Edling, Nunes Amaral, Stanley, & Åberg, 2001; Schneeberger *et al.*, 2004) and the existence of “supercarriers” in epidemics (Brauer, 2008; Riley *et al.*, 2003) like COVID-19 (Hamner *et al.*, 2020; Jones & Maxouris, 2020; Kwon, 2020; Stieg, 2020) show that this network topology applies well in the case of epidemic propagation. Therefore, in this study, we adopt the structure of a scale-free network, where network growth is based on preferential attachment, to evaluate investments for mitigating epidemic risks.

Epidemic propagation models have been used extensively in evaluating the effectiveness of different types of interventions and investments. According to the World Health Organization (WHO), epidemics happen in four phases: emergence, localized transmission, amplification, and reduced transmission (WHO, 2018). Governments can intervene and mitigate the impact by preventing further propagations of epidemics (during the emergence and localized transmission phases) and/or reducing propagation through treatment and recovery (T & R; during the amplification and reduced transmission phases). However, finding an optimal set of interventions with budgetary constraints and different community structures is challenging.

Several studies have used compartmental models to study the optimal investment allocation problem (Mylius, Hagenaars, Lagnér, & Wallinga, 2008; Ren, Ordóñez, & Wu, 2013). Among them, Brandeau, Zaric, and Richter (2003) considered optimal resource allocation in a basic susceptible and infected epidemic model in several distinct populations. Alistar *et al.* (2014) studied the optimal prevention

and treatment resources allocation in HIV epidemic control, using a susceptible-infected-treated model, to minimize the reproduction number R_0 . However, these compartmental models do not consider the effect of the network structure of the population, a significant factor of disease transmission.

Another stream of research focuses on the evaluation of a limited set of strategies to counter epidemics in different settings using network analysis and/or simulation (Zhang, Zhong, Gao, & Li, 2018). Among such efforts, Fu, Small, Walker, and Zhang (2008) evaluated epidemic thresholds using network analysis with infectivity and immunization and found that the targeted immunization scheme is more effective than the proportional scheme. Their results show that without an effective vaccine or treatment, lockdown of infected sections of the population is the best option to contain the epidemic. Siettos, Anastassopoulou, Russo, Grigoras, and Mylonakis (2016) used a small-world network model with an agent-based simulation to evaluate several policies for the containment of the Ebola virus disease. More recently, Nicolaides, Avraam, Cueto-Felgueroso, González, and Juanes (2020) used a human mobility model and simulation to study the effectiveness of hand-hygiene, recommended by WHO, in mitigating flu-like virus transmission through the air transportation networks. They showed that increasing hand-washing rate in influential locations can reduce the risk of pandemic by around 40%.

The COVID-19 crisis highlights the need to understand the measures that can be most effective against an epidemic, given the size of the impact, characteristics of epidemic transmission, the (network) structure of a population, and the budgetary and economic constraints. The losses are staggering: the pandemic and the associated fiscal actions and lockdowns have resulted in \$11.7 trillion, or close to 12% of global GDP, of negative economic impacts as of September 2020 (International Monetary Fund, 2020), and Cutler and Summers (2020) estimated \$16 trillion of loss in the United States alone if the disease runs its course. It is critical for decision-makers in governments and other organization to respond with measures that save lives, support vulnerable people and businesses, minimize the fallout on economic activity, and speed up the recovery (Carlsson-Szlezak, Swartz, & Reeves, 2020; Nagarajan, 2020). The current study, therefore, sets out to identify optimal prevention and treatment investments for mitigating epidemic propagation through a population network. In support of this, we introduce

a new objective function to minimize risk due to an epidemic and adopt the preferential-attachment network to model the population network for the purpose of disease transmission. The investment functions that we incorporate are in general forms and can represent different types of prevention and treatment strategies. The proposed model thus allows for comparing the effectiveness of different kinds of investments in mitigating the risk of epidemic. The results of the study, consistent with studies of prior epidemic events, provide a decision framework for policymakers facing a pending pandemic.

2. RESEARCH MODEL

We model the epidemic propagation based on individual-to-individual transmission within a “network of contact,” which can be an organization, a community, a city, a metropolitan area, a country, or something even larger. Such a network of contact is assumed to be closed at the time of an examination; that is, all individuals can have contact with others in the same network but not with those outside of the network. The disease propagates throughout the network via individual-to-individual transmissions, and those who contract it may be asymptomatic, need rest before recovery, require treatment to recover, or succumb to the disease. The total loss, L , due to an epidemic includes all possible consequences due to individuals contracting the disease, such as the cost of care, lost productivity, loss of life, etc. Adopting a commonly used risk measure (Aven, 2010; Boholm, 2019), we calculate the risk Z of an epidemic as the product of the probability p of individuals contracting the disease and the resulting loss L :

$$Z = pL \quad (1)$$

At the network level, Equation (1) captures the sum of the risks of the entire population in the network. We use L_0 and p_0 to denote the *intrinsic* loss and infection probability observed in an epidemic without any intervention or mitigation, and $Z_0 = p_0 L_0$ is thus the associated intrinsic risk of epidemic propagation in the network. Because the objective of any intervention or mitigation is to reduce the negative impact of an epidemic, Z_0 , p_0 , and L_0 represent the maximal risk, infection probability, and potential loss of an epidemic, respectively.

To examine how an epidemic can be propagated throughout the network (i.e., how individuals in this network can get infected, or the behavior of p), it is necessary to understand the structure of

the network of contacts. Every individual has contact with a certain number of other individuals, and the combination of individuals (“nodes” or “vertices”) and contacts (“links” or “edges”) form the network. As discussed earlier, the prevalent model for such a network is based on “preferential attachment”—individuals with an already large number of connections are more likely to receive more connections, and those with fewer connections are likely to remain less connected—and this results in the “scale-free network” topology, composed of the majority of nodes with limited connections and a small number of “hubs” with large number of links. To find p , the probability of a node in the network being infected by the epidemic, we adopt the derivation of the epidemic spread in such a network (see Appendix 1 for details):

$$p = \beta\mu^{\frac{1}{\lambda}}. \quad (2)$$

Eq. (2) implies that epidemic spreading is determined by three factors: μ , the degree of connectivity of the node in the network; λ , the likelihood that the node may be exposed to the disease from its connections; and β , the susceptibility that the individual may be infected by the disease when exposed to it. To focus on overall network behavior in the discussion below, the same (average) value is used for each parameter across all nodes in the given network. In addition, all three factors are normalized to the interval between 0 and 1.

We consider two general categories of risk mitigation measures—prevention/containment and treatment/recovery. Prevention and containment (P&C) measures aim at reducing the rate at which individuals contract and/or spread the disease, while T&R lessen the impact of the disease on those who contract it. In other words, to mitigate the epidemic risk of the network, one can make an investment of S_p in P&C—to reduce p , the probability of transmission—or an investment of S_L in T&R—to reduce L , the potential loss as a result of the disease. Thus, $p = p(S_p)$ and $L = L(S_L)$. (When no investment is made, $S_p = S_L = 0$, and we have $p_0 = p(S_p = 0)$ and $L_0 = L(S_L = 0)$ as the intrinsic infection probability and intrinsic loss, respectively, of the epidemic propagation in the network). The benefit achieved with mitigation investments is then

$$W = (Z_0 - Z) - (S_p + S_L) \quad (3)$$

and the task of finding an optimal investment is to maximize W , which itself is a function of p (thus β , μ , and λ) and L . In what follows, we attempt to

analyze the optimal investments made by a rational decisionmaker (or by rational decisionmakers collectively) in P&C versus in T&R to mitigate the risk of epidemic propagation in a network of individuals.

3. MODELING MITIGATION INVESTMENT

3.1. Investing in P&C

We first examine the case when an investment is only made in P&C (i.e., $S_p > 0$ and $S_L = 0$). Note that the P&C investment S_p affects (i.e., reduces) the disease exposure rate λ in Eq. (2) but not the susceptibility β , which reflects the nature of the disease, or the connectivity μ , which is determined by the topology of the network. As such, we can rewrite Eq. (2) as follows (see Appendix 1 for details):

$$p = \beta\mu^{1+\varepsilon_p S_p}, \quad (4)$$

where ε_p is a parameter describing the effectiveness of investment S_p . And therefore,

$$R = \beta L_0 \mu^{1+\varepsilon_p S_p}, \quad (5)$$

$$W = \beta\mu L_0 - \beta L_0 \mu^{1+\varepsilon_p S_p} - S_p, \quad (6)$$

where we use L_0 since $S_L = 0$. Before finding the optimal solution for the P&C investment, it is necessary to verify the boundary conditions of Eq. (6). First, the initial condition has to hold that the benefit increases when the very first investment is made (otherwise it makes no sense to even invest in such measures). In other words,

$$\left. \frac{\partial W}{\partial S_p} \right|_{S_p=0} \geq 0. \quad (7)$$

Inserting Eq. (6) into Eq. (7) and rearranging the terms, we have

$$\beta\varepsilon_p L_0 \mu (-\ln \mu) \geq 1. \quad (8)$$

This initial condition Eq. (8) will be revisited later. Additionally, in order to find the maximum W , it is necessary that the second derivative of W with respect to S_p is negative. To check, we note that

$$\frac{\partial^2 W}{\partial S_p^2} = -\beta L_0 \left(\frac{\varepsilon_p}{\mu} \mu^{1+\varepsilon_p S_p} + (\varepsilon_p \ln \mu)^2 \mu^{1+\varepsilon_p S_p} \right) \quad (9)$$

is indeed negative, as the terms inside the parenthesis are both positive. Therefore, W will yield its maximum when we optimize S_p by setting:

$$\frac{\partial W}{\partial S_p} = -\beta L_0 \varepsilon_p (\ln \mu) \mu^{1+\varepsilon_p S_p} - 1 = 0. \quad (10)$$

Rearranging the terms and solving for S_p^* , the optimal investment in P&C, we have

$$S_p^* = \left(\frac{1}{\varepsilon_p (-\ln \mu)} \right) \ln(\beta \varepsilon_p L_0 \mu (-\ln \mu)). \quad (11)$$

Since S_p^* is always positive, the argument in the logarithm has to be greater than 1. This yields the same equation as the boundary condition Eq. (8). Rearranging the terms, we get

$$L_0 \geq \frac{1}{\beta \varepsilon_p \mu (-\ln \mu)} \equiv \bar{L}, \quad (12)$$

Therefore, we have the following proposition.

Proposition 1. *Investment in P&C only makes sense when the intrinsic loss of the epidemic without mitigation is larger than a critical amount $\bar{L}$.*

This proposition outlines the importance of carefully investigating the nature of an epidemic (i.e., β) and the network characteristics (i.e., μ) to avoid “overinvestment,” because not all epidemics are worth protecting against. Unless the intrinsic loss of epidemic is greater than a critical amount $\bar{L}$, one is better off not investing in P&C at all. Note that $\bar{L}$ is a decreasing function of the connectivity μ . In other words, the more connected the network is, the lower the threshold is for making P&C investment.

We examine first the behavior of optimal P&C investment in relationship with the intrinsic loss of epidemic L_0 . To do so, we differentiate S_p^* in (11) with respect to L_0 :

$$\frac{\partial S_p^*}{\partial L_0} = \frac{1}{\varepsilon_p L_0 (-\ln \mu)} > 0 \quad (13)$$

and

$$\frac{\partial^2 S_p^*}{\partial L_0^2} = \frac{-1}{\varepsilon_p L_0^2 (-\ln \mu)} < 0. \quad (14)$$

Therefore, we have the following proposition.

Proposition 2. *The optimal investment in P&C is a strictly increasing concave function of the intrinsic loss of the epidemic without mitigation.*

It is not hard to see why Proposition 2 holds in practice. When the intrinsic loss of the epidemic is high, one would likely attempt a higher level of risk mitigation with investment in P&C. However, the pace of increase in investment cannot keep up with the increasing level of potential intrinsic loss. Fig. 1 illustrates how S_p^* varies with respect to L_0 .

Next, we examine the relationship between the optimal P&C investment and the connectivity, μ .

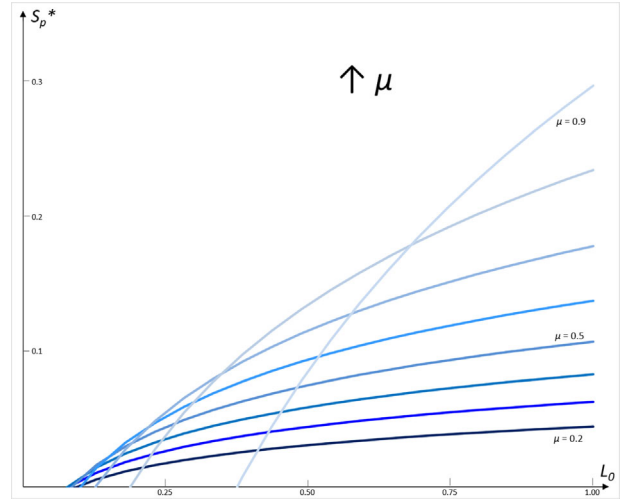


Fig 1. Optimal prevention and containment investment with respect to potential loss (S_p^* and L_0 scaled to some maximum loss value).

Taking the derivative with respect to μ in Eq. (11) and rearranging the terms, we get

$$\frac{\partial S_p^*}{\partial \mu} = \frac{\ln(\beta \varepsilon_p L_0 (-\ln \mu)) - 1}{\varepsilon_p (\ln \mu)^2}. \quad (15)$$

An examination of this result shows that when L_0 is sufficiently large (greater than or equal to $e(\beta \varepsilon_p (-\ln \mu))^{-1}$), Eq. (15) is always positive, and S_p^* increases with μ . However, for a small L_0 value, the numerator of Eq. (15) turns negative for high values of μ and eventually drives S_p^* to 0 when μ is large enough (Fig. 2). Therefore, we also have the following proposition.

Proposition 3. *The optimal P&C investment increases with the average connectivity of the network when the intrinsic loss of the epidemic, in the absence of mitigation, is high. However, if this intrinsic loss is small, it is better to stop investing in P&C when the connectivity becomes too high.*

Proposition 3 implies that, when the potential epidemic intrinsic loss is high, one should invest more in P&C when the individuals in the network are more connected (thus more exposed to possible transmission). This effectively means that the highly connected individuals (the “hubs”) should invest more in prevention than the sparsely connected nodes, according to this proposition. Assuming, however, that the potential loss faced by each individual node is similar for all nodes, then it is likely that the hubs will instead invest less than the optimal amount for their

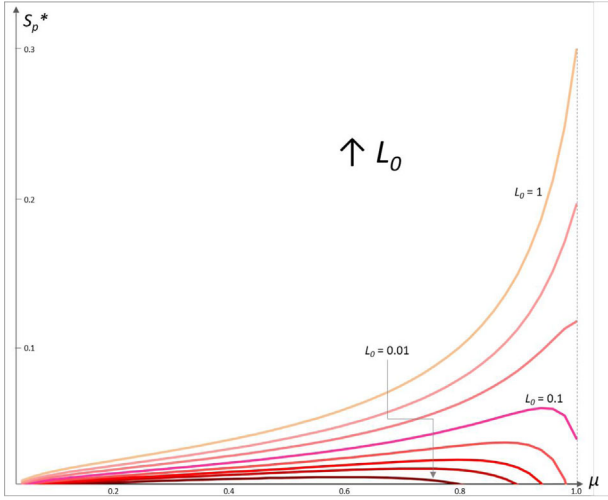


Fig 2. Optimal prevention and containment investment with respect to connectivity (S_p^* and L_0 scaled to some maximum loss value).

level of connectivity, thus jeopardizing the safety of the whole network. This moral hazard issue will be further discussed in Section 4.2.

Finally, we examine the property of S_p^* by rewriting Eq. (11) as

$$S_p^* = \frac{Z_0 \ln c}{c}, \quad (16)$$

where $Z_0 = \beta \mu L_0$, and $c \equiv Z_0 \varepsilon_p (-\ln \mu)$. To find the maximum of S_p^* , we differentiate Eq. (16) with respect to c and set it to 0:

$$\frac{\partial S_p^*}{\partial c} = \frac{Z_0 (1 - \ln c)}{c^2} = 0. \quad (17)$$

(Eq. (16) does yield a maximum of S_p^* since $\frac{\partial^2 S_p^*}{\partial c^2} < 0$.) So, when $1 - \ln c = 0$, or $c = e$ ($= 2.718...$, the exponential constant), we have the maximum of S_p^* .

$$S_p^* \leq \frac{Z_0 \ln e}{e} = \frac{Z_0}{e} \quad (18)$$

Since $e^{-1} \approx 0.368$, we have the following proposition.

Proposition 4. *The optimal P&C investment to mitigate epidemic propagation risks will never exceed $0.368 \cdot Z_0$, the intrinsic epidemic risk without any mitigation measures.*

Proposition 4 places an upper limit on the optimal P&C investment. In other words, any investment higher than $0.368 \cdot Z_0$ is deemed unjustifiable. Note

that this upper limit applies to *all* values of potential intrinsic loss L_0 and connectivity μ , since it is derived independently of and in parallel with Propositions 2 and 3. This important result will be further discussed in Section 5.

3.2. Investing in T&R

Instead of focusing on P&C, one can instead invest in T&R to mitigate the risk of epidemic propagation. The T&R investment will reduce the potential loss, L , due to the spread of the epidemic, although the chance of getting infected, p , stays constant.

Studies have attempted to identify the proper form for describing the impact of investing in reducing the total loss due to a disaster or catastrophic event. Among them, Mackenzie and Zobel (2016) propose four different deterministic reduction functions and find that the logarithmic allocation function yields the best fit with data from actual disaster recovery. In this study, therefore, we adopt this best match function to describe the effect of reducing the potential loss of epidemic L by making T&R investment S_L as follows:

$$L = L_0 (1 - k \ln(1 + \varepsilon_L S_L)), \quad (19)$$

where ε_L is a parameter describing the effectiveness of investment S_L , and $k \in (0, 1)$ is a normalization such that $k \ln(1 + \varepsilon_L S_L)$ is less than 1 (Mackenzie & Zobel, 2016). Inserting Eq. (19) into Eq. (3), we have

$$W = \beta \mu L_0 - \beta \mu L_0 (1 - k \ln(1 + \varepsilon_L S_L)) - S_L = \beta \mu k L_0 \ln(1 + \varepsilon_L S_L) - S_L. \quad (20)$$

Similar to the P&C case, we want to ensure that the initial condition holds such that the benefit increases when the very first investment is made. In other words,

$$\left. \frac{\partial W}{\partial S_L} \right|_{S_L=0} \geq 0. \quad (21)$$

Applying this condition to Eq. (20) and rearranging the terms, we get

$$L_0 \geq \frac{1}{\beta \mu k \varepsilon_L} \equiv L_{\star} \quad (22)$$

In other words, one will not invest in T&R unless the intrinsic loss is higher than a critical value $L_{\star}$. To look for the optimal investment in T&R, we dif-

ferentiate Eq. (20) with respect to S_L and set it to 0:

$$\frac{\partial W}{\partial S_L} = \frac{\beta\mu k \varepsilon_L L_0}{1 + \varepsilon_L S_L} - 1 = 0. \quad (23)$$

Note that this yields the maximum W since $\frac{\partial^2 W}{\partial S_L^2} < 0$. Solving for S_L , we have the optimal T&R investment S_L^* as:

$$S_L^* = \frac{\beta\mu k \varepsilon_L L_0 - 1}{\varepsilon_L}. \quad (24)$$

We first note that S_L^* is a linear function of both L_0 and μ . That is, the optimal T&R investment increases linearly with both intrinsic loss and connectivity. To further investigate the property of S_L^* , we rewrite Eq. (24) using Eq. (22) as follows:

$$S_L^* = \beta\mu k (L_0 - \bar{L}_c). \quad (25)$$

Since all β , μ , and k are between 0 and 1, the product $\beta\mu k \in (0, 1)$, and we have the following proposition.

Proposition 5. *Investment in T&R should only be made if the potential loss of an epidemic without mitigation is larger than a critical value $\bar{L}_c$. In addition, the optimal T&R investment is a fraction of the difference between the intrinsic loss and the critical loss value $\bar{L}_c$.*

Proposition 5 provides important guidance for investing in T&R. First of all, only those epidemics with the intrinsic loss higher than some critical value are worth T&R investments. This amount, $\bar{L}_c$ as noted in Eq. (25), is inversely proportional to the connectivity of nodes in the network. When the intrinsic loss of the epidemic is deemed higher than $\bar{L}_c$, the optimal investment in T&R is still only a fraction of the difference between the intrinsic loss and $\bar{L}_c$. It is thus worthwhile to carefully examine all the relevant factors—connectivity in the network, effectiveness of the T&R effort, and the likelihood of any members of the network catching the disease—to determine the best level of recovery and treatment investment.

3.3. Allocation of Risk Mitigation Investments Among Prevention/Containment and Treatment/Recovery

To mitigate epidemic risks, one may decide to invest simultaneously in both P&C and T&R. In reality, there is always a limit to which mitigation investments can be made. Such a limit not only includes any

formal budgets to spend on curbing epidemics (as direct investments) but also the “acceptable” economic impact caused by mitigation policies. In this section, we examine the optimal *allocation* to these two investments in order to maximize the benefits due to such a limit.

We assume that the limit of total mitigation investment due to budgetary and economic constraints is S , which will be split between P&C investment and T&R investment, that is, $S = S_p + S_L$. We use $s_p = S_p/S$ and $s_l = S_L/S$ to denote the proportions of investment allocated to P&C and T&R, respectively. Then, the total benefit W can be expressed as (with $s_l = 1 - s_p$):

$$W = \beta\mu L_0 - \beta L_0 (1 - k \ln(1 + \varepsilon_L S(1 - s_p))) \mu^{1 + \varepsilon_p s_p S} - S. \quad (26)$$

To find the optimal allocation ratio s_p (and thus s_l), we take the derivative of W with respect to s_p and set it to 0. After collecting terms, we have:

$$-\ln\mu + \ln\mu \ln(1 + \varepsilon_L S(1 - s_p)) - \frac{1}{1 + \varepsilon_L S(1 - s_p)} = 0. \quad (27)$$

Note that L_0 is not in Eq. (27). That is, s_p (and thus s_l) does not depend on L_0 . This is a somewhat surprising result, as one might expect to see impact of intrinsic loss on the allocation of total mitigation investment limited by budgetary and economic constraints between these two investments. But it is not the case here.

A careful examination of Eq. (27) reveals that no closed-form solution of s_p is possible. To investigate how the allocation varies with key parameters such as the total investment limit S and the connectivity μ , we can use the implicit function technique to derive the relationships without knowing the closed-form solution of s_p . Taking the derivative of s_p with respect to μ using this technique and collecting terms, we get

$$\frac{\partial s_p}{\partial \mu} = \frac{(1 + \varepsilon_L S(1 - s_p)) (\ln(1 + \varepsilon_L S(1 - s_p)) - 1)}{\varepsilon_L \mu S ((1 + \varepsilon_L S(1 - s_p)) \ln\mu + 1)}. \quad (28)$$

Examining the sign of Eq. (28), we note that when $|\ln\mu|$ is larger than 1 (i.e., $n > 1$), $\frac{\partial s_p}{\partial \mu}$ is always negative. Therefore, we have the following proposition (see Appendix 2 for proof).

Proposition 6. *With a fixed limit of total mitigation investment due to budgetary and economic constraints, the intrinsic loss of the epidemic does not affect the*

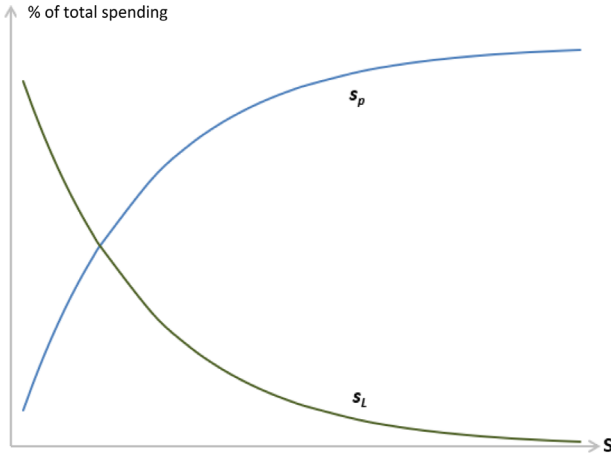


Fig 3. Allocation ratio of total spending versus investment limit.

allocation between prevention/containment and treatment/recovery. In addition, if an average individual in the network connects to more than one other member, the more connections it has, the less portion of the limit should be allocated to P&C investment.

Since it is more than likely that an average individual would have contact with more than one other individual in the same network, Proposition 6 states that the allocation to P&C investment given a fixed limit of total mitigation investment will decline with increasing connectedness. This result seems reasonable, considering that reducing the likelihood of transmission becomes more difficult and costly when there are a large number of connections to defend against, and investing in T&R can be a more effective approach when P&C is hard to achieve.

We now examine how allocation varies with a fixed limit on the total mitigation investment. To do so, we take the derivative of s_p with respect to S . After manipulation, we arrive at the following relationships:

$$s_p \propto 1 - \frac{1}{S}; \quad s_L \propto \frac{1}{S}. \quad (29)$$

These relationships, shown in Fig. 3, give the following proposition (the proof is in Appendix 3).

Proposition 7. *When the limit of total mitigation investment due to budgetary and economic constraints is small, one should focus on T&R investment. As the constraint relaxes, a higher portion should be allocated to P&C, while allocation to T&R decreases at a rate inversely proportional to the increasing limit of mitigation investment.*

Proposition 7 states an important principle in allocating investment to mitigate epidemic among P&C and T&R. When the total investment limit is small, it would be difficult and ineffective to defend against the likelihood of transmission via all the connections, and it is better to focus on reducing the impact by investing in T&R programs. The higher the limit is, however, the more realistic and economical it gets to effectively target P&C. And when the ability to invest in mitigation is very high, it would make sense to focus on containing the epidemic and preventing transmission altogether to reduce the number of individuals infected (thus lowering the overall loss).

4. DISCUSSION

4.1. Interpretation of Modeling Results

We reiterate the assumptions of several key factors in this study. The intrinsic loss L_0 is the total (economic) loss associated with an epidemic in a population network (city, country, community, etc.) if the disease spreads *naturally* (i.e., without any mitigation), including such items as loss of human lives, cost to the healthcare system to test and treat the infected, productivity lost due to infection (patients, healthcare providers, etc.), and opportunity cost due to providers' (such as hospitals) inability to treat other patients. Although some of these impacts may be difficult to measure or subject to ethical debate (such as the use of statistical value of human lives), they are real and assumed to be quantifiable in this study. As an example, Cutler and Summers (2020) estimated the total loss due to COVID-19 at more than \$16 trillion in the United States. On the other hand, mitigation investments S_p and S_L are made to prevent the spread or treat the disease. Some investments are direct, such as those for vaccines, therapeutics, and hospital capacity. There are also indirect investments incurred due to the consequences of mitigation measures. For instance, social distancing policy requires closing or limiting the operations of certain businesses, resulting in economic impact not directly paid for initially. It is with this understanding that we now discuss the propositions developed in this study.

Proposition 1 states that investment in P&C only makes sense when the intrinsic loss of the epidemic is larger than a critical amount L_c . In other words, when the intrinsic loss is potentially low enough, it

is advisable to let the disease run its course. Interestingly, the critical value $\underline{L}$ is a decreasing function of μ (see Eq. (15)), implying that a highly connected community has a lower threshold for prevention investment. This makes sense because a disease is much more likely to become an epidemic in highly connected urban hubs (like New York City) or densely populated countries (like Taiwan), presenting in a high intrinsic loss that necessitates mitigation measures. This is also consistent with Proposition 2, which states that the optimal investment in P&C is a strictly increasing concave function of the intrinsic loss of the epidemic. So to prevent and contain a transmittable disease, it is necessary to spend more when the intrinsic loss is higher, but the spending increase cannot keep pace with the potential loss as it becomes very high (see also the discussion of Proposition 4).

Proposition 3 states that when the intrinsic loss is high, the optimal investment in P&C increases as the network connectivity increases. The SARS epidemic, the first pandemic of the 21st century when it appeared in 2002, spread through 26 countries, infecting 8,098 people and resulting in 774 deaths.¹ Its potential negative impact (as a result of the high fatality rate and difficulty of treatment) prompted countries such as Taiwan, a highly connected island nation of over 23 million people (density of over 1,700 per square mile), to invest significantly in P&C infrastructure to coordinate their public health and medical services across the country for future epidemics, consistent with Proposition 3. This investment has been tested and modified through the pandemics of H1N1 (2009–2010) and Ebola (2014–2016) (Kao, Ko, Guo, Chen, & Chou, 2017) and is proven to be effective in dealing with the current case of COVID-19 (more in Section 4.2).

On the other hand, Proposition 3 states that when the intrinsic loss is small, it may be advisable to not invest in P&C in a highly connected community. Prevention of a transmittable disease is always difficult in densely populated or highly connected areas; when the disease is less potent or lethal, it may be better to allow the infection to spread instead of trying to contain it. For instance, before the varicella vaccine was developed for chickenpox, people in some communities intentionally exposed themselves in the hope that they would achieve immunity. Such an approach for less severe diseases is consistent with Proposition 3.

Proposition 4 states that the optimal P&C investment to mitigate epidemic propagation risks should not exceed $0.368 \cdot Z_0$ ($Z_0 = \beta\mu L_0$, the intrinsic epidemic risk without any mitigation measures). Since both β and μ are less than 1, this upper bound for the optimal investment in P&C is a fraction of (and can be significantly lower than) the intrinsic loss L_0 of an epidemic. Interestingly, Courtemanche, Garuccio, Le, Pinkston, and Yelowitz (2020) indicate that based on evidence from the 1918 influenza pandemic, social distancing measures would reduce the average contact rate by 38%, a number very close to the factor (as the inverse of e , the Euler's number or the natural exponential constant) in this proposition. This relationship between this “limit” of prevention effectiveness and the upper bound for optimal prevention investment is worth further exploration in future research.

Proposition 5 addresses investments in T&R. It states that such investments are called for only in those epidemics with potential loss higher than some critical value $\underline{L}$. This may be the case for seasonal flu, especially when the flu vaccine proves to be effective, in that there is little need for specific investment targeting specific treatment for such a disease. This proposition also states that the optimal T&R investment is a fraction of the difference between the intrinsic loss and $\underline{L}$. In a Dutch study, Lugnér and Postma (2009) estimate that the cost-effectiveness cut-off point for investment in stockpiling a combination of antiviral drugs for treatment is an 11% risk of a pandemic during a 30-year period (9% for a single antiviral drug treatment), or 29% (23% for a single antiviral drug treatment) when production loss offsets are not included. (The authors note that there is no official threshold below which a cost-effectiveness ratio is considered acceptable in the Netherlands, possibly due to the ethical difficulty in measuring the value of human lives.) Balicer, Huerta, Davidovitch, and Grotto (2005) evaluate multiple strategies of treatment for the entire population, or those at heightened risk, in Israel and find that investing in antiviral stockpiling is always cost-effective as long as the estimated annual pandemic risk remains greater than one in 80 years. Both cases are consistent with Proposition 5.

Propositions 6 and 7 deal with the optimal allocation between both P&C and T&R measures when there is a limit to total investment due to budgetary and economic constraints. Proposition 6 states that the allocation between prevention efforts and treatment efforts is not dependent on potential intrinsic

¹ https://www.who.int/csr/sars/country/table2004_04_21/en/

loss. Although surprising, this result appears to be consistent with the concept of risk tolerance, where one responds to varying levels of risk by spending more or less on mitigation instead of changing the mitigation strategies. Proposition 6 also states that, in a more connected network, a greater proportion of the limited investment amount should be allocated to treatment and less to prevention. Preventing or containing the spreading of an epidemic in a highly connected network can be difficult and costly, and with a limited investment capability it may be more effective to focus on treatment. Wang, Li, and Guo (2012) discuss the allocation of a fixed budget between vaccines (prevention) and antivirals (treatment) based on the cost and efficacy of the two options. They find that if both options are 100% effective, a higher investment in vaccines is the obvious option. But even in this case, they allocate a certain amount to antivirals to address any possibility of $R_0 > 1$. As the infection rate increases, with greater connectivity, there should be more investment allocated to treatment and less to prevention. This allocation is further supported in cases when there is no availability or low efficacy of prevention measures (such as vaccines against a virus).

Finally, Proposition 7 states that if the investment is very limited, it is better to focus primarily on treatment. This can be seen in the typical antiviral stockpiles that many countries maintain, especially when the source of threats is not specifically known in advance. It is also the usual strategy to prepare for responding to potential bio-terrorist attacks. The proposition also states that as the investment limit increases, a higher proportion should be allocated to P&C. After the SARS and H1N1 epidemics, many Asian nations opted to invest significantly over the past two decades to combat future epidemics, and a large proportion was allocated to prevention, consistent with Proposition 7. Xue and Zeng (2019) detail investments by China in a variety of prevention measures including strengthening national emergency response teams, building institutional capacity for national monitoring and epidemiological training, growing vaccine production capacity, and developing international and regional collaborations. It is expected that as countries begin to see epidemics as a national biosecurity threat based on the experience of COVID-19, governments are likely to invest significantly in combating future pandemics and a large portion of such spending would be applied toward P&C.

4.2. Analysis of COVID-19 Pandemic

Faced with the COVID-19 pandemic, countries and communities have rushed to mitigate the risks via a plethora of strategies (see Table I), with varying degree of success. In this section, we examine how the modeling results developed in this study can inform this current case and provide guidelines for the future.

Walker et al. (2020) estimate that in the absence of any interventions, COVID-19 would have resulted in 7 billion infections and 40 million deaths (at $R_0 = 3$) globally in 2020. More recently, Cutler and Summers (2020) put the total loss in the United States due to COVID-19 at more than \$16 trillion. Such a potential impact is so enormous that it would be likely exceed any conceivable critical threshold for intervention (Proposition 1). Perhaps the most natural reaction to an epidemic breakout is prevention, particularly when investment limit is not an issue (Propositions 2 and 7). Although almost all nations spent heavily in prevention when facing a pandemic as serious as COVID-19, the key difference appears to be not only the amount and but the type and pace of mitigation efforts. As of fall 2020, several countries and regions, most notably Taiwan, South Korea, and Vietnam, have successfully contained COVID-19 in spite of the proximity (both geographically and socially) to the disease epicenter, China. These countries all have densely populated cities and regions, resulting in high connectivity and thus requiring massive investment in prevention (Proposition 3). To be able to afford such a large investment (primarily in isolation, tracing, and tracking systems), it must be made preemptively and over time. For instance, Kao et al. (2017) discuss the extensive and continued investment in the development of the Communicable Disease Control Medical Network (CDCMN), a collaboration of the public health system and the medical system that Taiwan established in 2003 following the SARS outbreak. CDCMN was successfully activated during the H1N1 influenza (2009–2010) and the Ebola outbreak (2014–2016) and has been effective in addressing the COVID-19 pandemic (Duff-Brown, 2020; Wang, Ng, & Brook, 2020).

For countries and regions that do not have mitigation measures in place at the time of an outbreak, the most effective prevention measure is social distancing or even complete lockdown (Chu et al., 2020; Flaxman et al., 2020; Hsiang et al., 2020). Although

Table I. Examples of Mitigation Measures amid COVID-19

	Prevention/Containment	Treatment/Recovery
Short term	• Social distancing and lockdown • Universal mask requirement • Quarantine and isolation • Tracing and tracking	• Herd immunity through natural transmission • Scaling up disease-targeted healthcare capacity
Long term	• Public health education • Vaccine development; herd immunity through vaccination	• Developing therapeutics

the “intrinsic cost” of social distancing—that is, to separate people from one another—may appear to be low, the economic impact of executing such lockdowns, as reflected in business closures, layoffs and furloughs, and overall reduced economic output, can be extremely high (Proposition 2), requiring billions or even trillions of dollars in government support (Cassim, Handjiski, Schubert, & Zouaoui, 2020; Humphries, Neilson, & Ulysea, 2020). In densely populated areas without preventive alert systems, like New York City, prevention is almost impossible without virtually unlimited monetary support (Proposition 3 and Fig. 2). In this case, the investment can perhaps be better made to boost treatment (such as increasing hospital capacity) until the spread is under control (Proposition 7). In general, although prevention may be desirable, highly connected communities or countries with no little prior investment in disease containment should allocate heavily to treatment to reduce the impact of the disease, because prevention may turn out to be too expensive (Proposition 6).

In the case of a very limited ability to invest, Proposition 7 recommends focusing more on treatment than prevention, and one such option is to develop herd immunity through natural transmission. Herd immunity occurs when most individuals of a population are immune to an infectious disease and thereby will not spread it, indirectly protecting those who are not immune (D’Souza & Dowdy, 2020). During the COVID-19 pandemic, Sweden, with its broadly liberal society and decentralized healthcare system, has taken on a strategy to protect the elderly and the fragile and to avoid overloading hospitals, while keeping the economy open with minimum curtailment of people’s movements and by relying on the individual judgment of people to behave appropriately. Such an approach was justified by low population density and high percentage of single dwelling (therefore low connectivity, cf. Proposi-

tion 6), as well as the relative healthy population, which has an effect of lowering the cost of treatment (Leatherby & McCann, 2020). The result, however, was controversial: although there were signs of success as of summer 2020 (Erdbrink, 2020), Sweden’s economy has generally experienced slumps similar to those of other European nations that enforced stricter social distancing policies (Lindeberg, 2020), and, by fall 2020, Sweden maintains three to five times the number of infections and deaths per million population compared to its Scandinavian neighbors of Denmark, Finland, and Norway. Such an approach is certainly even less advisable in densely populated areas (high connectivity) or less healthy populations (high cost of treatment). It is also important to note that the ethical issue of putting a price tag on human lives makes such a decision difficult, even when it is supported by economic and risk analysis.

Although the model discussed above uses network-wide averaged parameters, it is also interesting to examine the mitigation measures at the individual level. In a population network that exhibits preferential attachment characteristics, the optimal investment in prevention would be disproportionately high for the few “hubs” with high connectivity compared to the average individuals in an epidemic with high intrinsic loss (Proposition 3). To be effective in slowing down the transmission of COVID-19, it is necessary and even paramount to shut down those hubs (often small businesses such as bars and gyms), but such closings cost them disproportionately more compared to individuals staying at home. This creates an inherent moral hazard: the hubs may not have the incentive to spend much more than the vast majority of the nodes for the good of the whole network, and it is natural for some to attempt to open despite the shutdown order to reduce their outsized burden from the prevention measures. It is, therefore, recommended that decisionmakers identify and subsidize those businesses and individuals that act as hubs

in order to effectively enforce prevention mitigation measures such as social distancing.

Finally, it is worth noting that Proposition 7 is about the optimal allocation between prevention and treatment, given the size of the budgetary and economic constraints; it does not provide an either-or choice. Based on the modeling results, allocating all resources to prevention or treatment only happens when limit of investment is infinite or infinitesimal, neither of which is possible in reality. For instance, although treatment is highly favored in the case of very limited investment capability (Proposition 7) and high connectivity (Proposition 6), low-cost prevention measures can still be pursued (such as policies adopted in African countries during COVID-19 pandemic; see, for instance, Massinga Loembé *et al.*, 2020). On the other hand, no matter how much effort one puts into preventing an epidemic from spreading, it is always necessary to invest in effective treatment of the disease in order to minimize the impact on the population in the network. This has also been verified in the case of COVID-19, where stockpiling personal protection equipment for healthcare workers, a modest measure to ensure treatment availability, proved to be critical in mitigating risks of the epidemic.

5. CONCLUSION

In this article, we build a mathematical model to optimize the investments in mitigating the risks of epidemic propagation throughout a network of individuals by treating the prevention/containment and treatment/recovery measures separately. We find that when investment concentrates on one category of measures (i.e., prevention or treatment only), given an epidemic that may cause large enough potential loss, then optimal investment increases with connectivity and potential loss but is limited to only a fraction of loss amount. When the total investment is limited by budgetary and economic constraints, however, the proportion allocated to prevention increases when the total investment limit goes up and when the network connectivity is lower. We further show that these results are consistent with previous studies and can be used to properly interpret what happened in past pandemics as well as to shed light on future and ongoing events such as COVID-19.

As with all analytical research, assumptions that are made to keep the models manageable always result in a simplified representation of reality. One such key assumption is the use of average transmission pa-

rameters for all nodes. In reality, transmission rates can be different for each node and they can be dynamic over time. We also assume that investments are made at a single point in time and that their effects are instantaneous, whereas in reality they can be made over time and have delayed impacts. Our model is deterministic in nature at a snapshot in time. In other words, we assume that one can obtain (or estimate) the values of all the parameters at a particular point in time in order to derive a steady state solution to inform the decisionmakers of their investment options. In the future, we can extend the current work to include probabilistic models for input parameters such as the exposure (μ) and effectiveness of investment (ε), in order to take into account the uncertainties of those parameters. Finally, we adopt the expected loss (i.e., the probability of an event multiplied by its consequences) to quantify the risk of a pandemic because the objective of our model is to minimize the total expected loss of a pandemic. Although such a definition is supported by and frequently adopted in the literature, we do acknowledge that, as highlighted by Aven (2010, 2019), using expected loss in its statistical term may not be informative when comparing risks of different scenarios. In such cases, uncertainties beyond probabilities should be considered in measuring risk. Future studies that relax these and other assumptions, as well as provide empirical verification of the modeling results, can extend the applicability and generalizability of this line of research.

This study makes several theoretical and practical contributions. Instead of using epidemiological parameters within the objective function, our economics-based model optimizes the risk-reducing performance of mitigation investments. Additionally, our model explicitly accounts for network topology, a critical element of epidemic propagation, and derives critical relationships between optimal investments and key network characteristics. Overall, our results provide both a solid theoretical foundation and practical guidance to decisionmakers in determining proper actions to take to mitigate the risks of an epidemic such as COVID-19.

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APPENDIX 1

Derivation of p

To find p , the probability of a node in a network being infected by the epidemic, we follow the derivation of the SIR-based epidemic spread in a scale-free network (Chang & Young, 2005; Pastor-Satorras & Vespignani, 2001). Let $P_k(t)$ denote the relative density of infected nodes with k connections—that is, the probability that a node with k connections is infected—at time t . The mean field rate equation gives

$$\frac{\partial P_k(t)}{\partial t} = -P_k(t) + \lambda k [1 - P_k(t)] \Theta(\lambda), \quad (\text{A.1})$$

where $\Theta(\lambda)$ is the probability that any given connection points to an infected node, as a function of λ , the epidemic spreading rate (Pastor-Satorras & Vespignani, 2001). Solving for P_k in a steady state (i.e., $\partial P_k(t)/\partial t = 0$), one gets

$$P_k = \frac{k\lambda\Theta(\lambda)}{1 + k\lambda\Theta(\lambda)}. \quad (\text{A.2})$$

Note that $\Theta(\lambda)$ can be expressed in the lowest order of λ and n , the average number of node connections (Chang & Young, 2005):

$$\Theta(\lambda) = \frac{e^{-\lambda n}}{\lambda n}. \quad (\text{A.3})$$

Substituting Eq. (A.3) into Eq. (A.2) and averaging P_k over k , one gets the average probability that a disease would spread to a random node (Pastor-Satorras & Vespignani, 2001):

$$p = \beta e^{-\frac{1}{\lambda n}} = \beta \mu^{\frac{1}{n}}, \quad (\text{A.4})$$

where β represents the probability that any individual may be infected when exposed to the disease, a factor determined by the nature of the epidemic but exogenous to the network, and $\mu = e^{-1/n}$ represents the connectivity of the node ($\mu \in [0, 1]$, $\mu = 0$ when $n = 0$, and $\mu = 1$ when $n \rightarrow \infty$).

The effect of the P&C investment S_p is in the reduction of the epidemic spreading rate λ in Eq. (A.4). As such, λ and S_p satisfy certain boundary conditions. We know that, without any investment, a disease would be spread freely to any other nodes; in other words, $\lambda = 1$ when $S_p = 0$. (Note that although the epidemic propagates “freely” without P&C investment, the rate of individuals contracting the disease is determined by the susceptibility β in Eq. (A.4).) But, any finite investments, no matter how large, would

never be able to block propagation completely, that is, $\lambda \rightarrow 0$ only when $S_p \rightarrow \infty$. Without loss of generality, the relationship between P&C investment and spreading rate can be expressed as $\lambda \equiv \frac{1}{1 + \varepsilon_p S_p}$ to satisfy the above boundary conditions, where ε_p is a parameter describing the effectiveness of investment S_p . Therefore, we have

$$p = \beta \mu^{1 + \varepsilon_p S_p} \quad (\text{A.5})$$

APPENDIX 2

Proof of Proposition 6

The first part of the proposition that the allocation is independent of the intrinsic potential loss is trivial, since L_0 is not in Eq. (27), the equation for s_p .

To prove the second part of this proposition, we first have to derive Eq. (28). To facilitate the implicit function manipulation, we make the following assignments:

$$\varphi \equiv 1 + \varepsilon_L S(1 - s_p). \quad (\text{A.6})$$

$$F \equiv (-\ln \mu) + \ln \mu \ln \varphi - \frac{1}{\varphi}. \quad (\text{A.7})$$

Since F in Eq. (A.7) is a function of both s_p and μ , we can use the implicit function rule to find the derivative of s_p with respect to μ as follows:

$$\frac{\partial s_p}{\partial \mu} = - \frac{\frac{\partial F}{\partial \mu}}{\frac{\partial F}{\partial s_p}} = - \frac{\frac{\partial F}{\partial \mu}}{\frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial s_p}}. \quad (\text{A.8})$$

Inserting Eq. (A.6) and Eq. (A.7) into Eq. (A.8) and making the simplification that $\varepsilon_L = \varepsilon_p$, we find

$$\frac{\partial s_p}{\partial \mu} = \frac{\varphi \ln \varphi}{\varepsilon_L \mu S (\varphi \ln \mu + 1)} = \frac{(1 + \varepsilon_L S(1 - s_p)) (\ln(1 + \varepsilon_L S(1 - s_p)) - 1)}{\varepsilon_L \mu S ((1 + \varepsilon_L S(1 - s_p)) \ln \mu + 1)} \quad (\text{A.9})$$

Since the numerator of Eq. (A.9) is always positive, the sign is determined by the denominator, or, to be exact, the sign of the term $1 - (1 + \varepsilon_L S(1 - s_p)) \ln \frac{1}{\mu}$. (We rewire the logarithmic term to make it positive, since $\ln \mu$ is always negative.) Because $1 + \varepsilon_L S(1 - s_p)$ is always greater than 1, the term is negative when $\ln \frac{1}{\mu} > 1$, or $e^{-1} > \mu = e^{-\frac{1}{n}}$, where n is the number of connections. Therefore, when $n > 1$, $\frac{\partial s_p}{\partial \mu}$ is always negative, and s_p , the allocation to P&C, decreases with increasing connectivity μ . ■

APPENDIX 3

Proof of Proposition 7

We proceed in the same fashion as the proof for Proposition 6. Since F in Eq. (A.7) is a function of both s_p and S , we can use the implicit function rule to find the derivative of s_p with respect to S as follows:

$$\frac{\partial s_p}{\partial S} = - \frac{\frac{\partial F}{\partial S}}{\frac{\partial F}{\partial s_p}} = - \frac{\frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial S}}{\frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial s_p}} = - \frac{\frac{\partial \varphi}{\partial S}}{\frac{\partial \varphi}{\partial s_p}}. \quad (\text{A.10})$$

Note that from Eq. (A.6),

$$\frac{\partial \varphi}{\partial s_p} = -\varepsilon_L S; \quad \frac{\partial \varphi}{\partial S} = \varepsilon_L (1 - s_p). \quad (\text{A.11})$$

Therefore, we have the differential equation:

$$\frac{\partial s_p}{\partial S} - \frac{1 - s_p}{S} = 0. \quad (\text{A.12})$$

Solving for the differential equation, we find the relationship of s_p as a linear function $1 - \frac{1}{S}$, or

$$s_p \propto 1 - \frac{1}{S}. \quad (\text{A.13})$$

Proceeding in the same fashion, we can also find that

$$s_L \propto \frac{1}{S}. \quad (\text{A.14})$$

■

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