Quantifying Hierarchical Indicators of Water Distribution Network Structure and Identifying their Relationships with Surrogate Indicators of Hydraulic Performance

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ABSTRACT

Enhancing the performance of water distribution networks (WDN) on a day-to-day basis, or under extreme disturbances is an utmost priority for utilities. Previous research has characterized the structure of WDN in the pipe-junction or segment-valve representation to gain insight on various aspects of their performance; however, the research on characterizing WDN structure in a hierarchical representation and its relationship with performance is lacking. Two key properties of WDNs are loops and pipe diameters that are organized in a hierarchical way. Novel indicators have been created to quantify the network hierarchy related to these key properties in other spatial flow distribution networks: loop nestedness and pipe diameter gradation along flow paths. The goal of this study is to adopt such indicators to characterize the hierarchy of WDNs and evaluate its relationship with WDN performance. This study applies a hierarchical decomposition process to model the relationships among loops as a tree network for quantifying loop nestedness. Flow paths of monotonically increasing and decreasing pipe diameters are traced to quantify pipe diameter gradation. Statistical distributions are approximated for these two indicators. Then, relationships between these network hierarchy indicators and two performance indicators (measuring path redundancy and power surplus) are identified. For 15 benchmark networks, this study finds the statistical

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distributions representing loop nestedness and pipe diameter gradation closely follow a power-law. Results suggest gradual pipe diameter gradation along flow paths and high loop nestedness increase WDN path redundancy, and gradual pipe diameter gradation increases WDN power surplus. The study demonstrates that the hierarchical analysis of WDNs can significantly supplement traditional topological analyses in explaining WDN performance.

Keywords: water distribution networks, complex network analysis, systems analysis, redundancy, hierarchy

INTRODUCTION

Water distribution networks (WDNs) are lifelines of the global urban fabric. They transport water to end users via a network of pipes, valves, pumps and water sources. At the same time, to maintain their performance, water distribution networks face increasing stress (e.g., climate related extreme weather, dwindling budgets, increased demand, aging) (ASCE, 2017; Di Nardo et al., 2017; Evans et al., 2018; Gheisi et al., 2016; Blaha and Gaewski, 2007; Diao et al., 2016; Butler et al., 2017; Abdel-Mottaleb et al., 2019; Pagani et al., 2020; Giustolisi, 2020). WDN performance is defined as the extent to which amount, pressure, and quality of delivered water are met under various scenarios (e.g., power outages, flooding) (Gheisi et al., 2016; Dziedzic and Karney, 2015; Skipworth, 2002; Farmani and Butler, 2014; Pagano et al., 2019; Ostfeld et al., 2002) and can be measured for both the component-level and network-wide level (Diao et al., 2016; Pagano et al., 2019). Network-wide indicators of performance are useful to utilities, especially in comparing design scenarios and identifying failure scenarios that cause the greatest loss in service (Diao et al., 2016; Pagano et al., 2019). Thus, many indicators have and continue to be defined, aiming to evaluate network-wide performance, such as reliability, resilience, robustness, redundancy, and flexibility. Among them, reliability is a widely used aggregate indicator and can be quantified using the probabilities of component failure, probabilities of demand exceedance, and system redundancies inherent in the WDN layout– all of which can be obtained from network analysis, hydraulic simulations, and/or real monitoring data (Goulter, 1987; Walski, 1987; Ostfeld et al., 2002; Farmani et al., 2005; Giustolisi, 2020). Because probabilities of failure and demand exceedance are not easily available (Ostfeld, 2005), entropy and the Todini index are two commonly used surrogate indicators for reliability (Farmani et al., 2005; Raad et al., 2010; Reca et al., 2008; Dziedzic and Karney, 2014; Ulusoy et al., 2018; Santonastaso et al., 2018). Entropy is a measure of flow path redundancy. The Todini index is a measure of power surplus (which has also been referred to as energy redundancy in the literature, see Ulusoy et al. (2018)). Entropy and the Todini index are just two of many available reliability performance indicators (Farmani et al., 2005; Ulusoy et al., 2018). Despite their limitations at capturing the totality of water distribution network performance or being indirectly related to reliability, the literature has generally found that as these two indicators increase, so does the reliability of a water distribution network and that the two indicators each capture different aspects of WDN performance (they are by no means a complete representation of hydraulic reliability or performance in general) (Farmani et al., 2005; Raad et al., 2010; Reca et al., 2008; Dziedzic and Karney, 2014; Liu et al., 2017a).

Various factors impact the performance of WDN: internal factors such as WDN structure and water hammer, and external factors such as natural disturbances and limited resources (Pagano et al., 2019; Abdel-Mottaleb and Zhang, 2019). WDN structure refers to the configuration of components-how they relate to each other. WDN structure can be represented in various ways, including pipe-junction topology, segment-valve topology, hydraulic (or logical), and hierarchical. Pipe-junction topology is what is represented by most distribution network models for hydraulic simulations, where nodes represent junctions and edges represent pipes (i.e., the actual connections in space). Segment-valve topology represents segments as nodes and valves as edges (Zischg et al., 2017, 2019; Walski, 1993, 1994). Abdel-Mottaleb et al. (2019) and Abdel-Mottaleb and Zhang (2019) investigated the so-called logical structure where nodes represent WDN components (i.e., pipes, junctions) and edges represent the logical (hydraulic) influence the components have on each other. The various representations of WDN structure are often analyzed via network science techniques (or graph theory). The application of graph theory in WDN has been centered around pipe-junction topology including characterization, connectivity analyses, sectorization, and attempts to derive surrogate performance indicators (Jacobs and Goulter, 1988; Ostfeld, 2005;

Yazdani and Jeffrey, 2011, 2012; Ormsbee and Kessler, 1990; Hernandez et al., 2016; Giustolisi et al., 2017; Giudicianni et al., 2018; Giustolisi et al., 2019; Herrera et al., 2016; Meng et al., 2018; Pagano et al., 2019; Giudicianni et al., 2020; Balekelayi and Tesfamariam, 2019; Torres et al., 2017) with a focus on quantifying network connectivity (Giustolisi, 2020). Though pipe-junction topology is useful for hydraulic simulation and correlates with hydraulic behavior (Giustolisi, 2020; Walski, 1993), characterization of structure other than the pipe-junction representation is also important to provide insights on WDN performance.

Given that water distribution systems are looped to reduce dissipation and create alternate flow paths in case of breaks (Dziedzic and Karney, 2014), there is a need to consider loops in network structure analysis. Previous research has developed or adopted measures considering loops in a WDN, such as the ratio between numbers of existing to maximal loops in a pipe-junction representation (Yazdani and Jeffrey, 2012). Such measures, however, do not distinguish between "layouts with the same number of loops", and account for the organization of the loops (Singh and Fiorentino, 1992). Hernandez et al. (2016) attempted to distinguish WDN layout based on loops and classified WDNs as branched, grid or loopy, yet it remains qualitative and does not distinguish between two different branch, grid or loopy networks. Previous research has also considered pipe diameters in analyzing pipe-junction topology, such as evaluating the uniformity of diameters of pipes meeting at junctions or along the loops of WDNs (Prasad and Park, 2004; Creaco et al., 2016). While providing insight, the uniformity of pipe diameters along individual loops or junctions does not provide a holistic network-wide characterization because it does not trace flow paths in their entirety. These measures (e.g., number of loops, uniformity of pipe diameters along loops) fail to represent the organization of loops and pipe diameters (i.e., how the loops and pipe diameters in a WDN are arranged and organized) (Katifori and Magnasco, 2012; Barthelemy, 2018). The organization of both loops and pipe diameters along flow paths has been represented hierarchically for many spatial flow distribution networks. Mathematically representing and subsequently quantifying the hierarchy, in particular of loops and pipe diameters, is a gap in the WDN literature that this study addresses. This study introduces loop nestedness and pipe diameter gradation along flow paths to characterize the hierarchical representation of network structure. Loop nestedness captures how loops are arranged relative to each other (e.g., which smaller loops are contained within larger loops, how the number of loops changes with loop size). Pipe diameter gradation along paths quantifies how gradually or abruptly diameters change along the flow paths of a WDN.

Much less research has been conducted to relate WDN structure to performance indicators than to apply network theory to characterize network structure. Studies relating WDN structure to various performance indicators are skewed towards pipe-junction topology (Torres et al., 2017; Meng et al., 2018; Ulusoy et al., 2018; Giustolisi et al., 2019; Giudicianni et al., 2020; Pagano et al., 2019; Balekelayi and Tesfamariam, 2019). Torres et al. (2017) and Pagano et al. (2019) found that though network analysis of pipe-junction topology provides insight on global WDN behavior and complements physics-based hydraulic simulation, it is severely limited in explaining hydraulic performance impacted by other factors. Limitations such as representing water sources in the same manner as demand nodes are identified as a drawback to relating pipe-junction topology to performance-based indicators (Meng et al., 2018; Giustolisi et al., 2019). Though segment-valve topology accounts for water sources, the information obtained is focused on valve placement rather than pipe layout (Liu et al., 2017b; Giustolisi et al., 2019; Abdel-Mottaleb and Walski, 2020). As the hierarchical representation of WDNs has not yet been thoroughly characterized, there is not a single study relating hierarchical measures of WDNs to hydraulic performance indicators. However, the hierarchical representations of other spatial flow distribution networks have been characterized and shown to provide additional insights beyond traditional connectivity/topology measures, such as classification (e.g., distinguishing between different species) and correlation with the redundancy and robustness to damage. (Papadopoulos et al., 2018; Ronellenfitsch and Katifori, 2017; Gavrilchenko and Katifori, 2018).

Building on previous research applying graph theory to WDNs and the research analyzing the hierarchy of spatial flow distribution networks, this study proposes a methodology for characterizing and quantifying hierarchy of loops and pipe diameters in WDNs. This study also evaluates how the two studied hierarchical indicators, loop nestedness and pipe diameter gradation along flow paths, relate to two commonly used surrogate indicators of performance based on hydraulic simulations.

METHODOLOGY

The hierarchy of spatial flow distribution networks from various domains has been characterized using loop nestedness and edge diameter gradation along paths. These two indicators are quantified by delineating the hierarchy, of both loops and edges (e.g., how smaller loops or edges are connected to larger ones). In each domain for which these indicators have been quantified, edges represent a different component (e.g., blood vessels, plant leaf veins). In this study, edge diameter refers to pipe diameter. Pipe diameter gradation is measured along entire flow paths. Loop nestedness is obtained after constructing a decomposition tree where nodes represent loops. If a loop is directly conained in a larger loop, nodes representing the two loops are connected by an edge. After quantifying and characterizing the hierarchy of WDNs, loop nestedness and pipe diameter gradation along flow paths are put into physical context of the WDNs. Then, their respective contribution to water distribution network performance is evaluated.

Benchmark water distribution networks are tested for illustration and reproducibility. The networks span a large parameter space, from small number of nodes and edges, to large numbers of nodes and edges; from low values of cyclicity or loops to high cyclicity values; and single to multiple sources and/or pumps. Similar to Santonastaso et al. (2018), the networks span the space of low entropy and Todini index values to high values of entropy and Todini index, and both synthetic and real WDN. The WDN are analyzed in their junction-pipe layout as opposed to the segment-valve representation that has been shown to be more realistic regarding isolation (Walski, 1993, 1994; Liu et al., 2017b). Figures of the tested WDNs and a table summarizing their properties are included in the supplementary information.

Decomposition Process

This study adopts the hierarchical loop decomposition algorithm presented in Katifori and Magnasco (2012) to obtain a decomposition tree, representing the hierarchy of loops in the network. The decomposition tree is the network model used to represent hierarchical organization of loops (i.e., representing loops contained within loops). This algorithm is part of the *nesting* python package (Ronellenfitsch et al., 2015). Prior to inputting the WDN model for decomposition, *networkx* and *WNTR*, open source python packages, are used to convert the *.inp* file for each network into a *networkx* graph object. The algorithm begins with the pruning of all nodes connected to a single edge in the water network (i.e., all junctions connected to a single pipe), only the part of the network with all edges (i.e., pipes) participating in loops remains. Then, the pipes are ordered based on their diameter, and the pipe with the smallest diameter is identified. In Figure 1, the pipe with the smallest diameter is e1. If there are pipes with the same diameter, the values are randomly perturbed infinitesimally such that they are no longer the same (a procedure that was found not to impact the results). Then the pipe with the smallest diameter is removed from the network. When the pipe is part of two loops, its removal leads to the merging of the two loops into a single larger loop. In Figure 1, when e1 is removed, loops 3 and 4 merge into loop 2. The ordering of pipes and merging of loops is repeated iteratively until every pipe is removed from the network and all the loops have been merged into the largest, or most exterior, loop of the network. In Figure 1, after e2 is removed loops 2 and 5 merge, and loop 1 is identified as the largest, most exterior loop.

In Figure 1, the resulting tree contains a single subtree. Terminal nodes of the tree are defined as the nodes for which there is no subtree; in Figure 1, the terminal nodes correspond to Nodes 5, 4, and 3. That means loops 5, 4, and 3 in the original network do not contain other loops. The subtree degree of a node is determined by the total number of terminal nodes contained in that subtree. Node 1 in Figure 2 contains three terminal nodes, and thus has a subtree degree of 3. Likewise, Node 2 contains two terminal nodes: Nodes 3 and 4. By definition, the subtree degree of terminal nodes is 0 as terminal nodes do not contain subtrees. The Nesting python package (Ronellenfitsch et al., 2015) is used to construct the decomposition tree and calculate measures representing properties of the tree. A limitation of using this algorithm is that it only allows for planar graphs (i.e pipes intersect only at their endpoints) as input. WDNs are often near-planar (Barthelemy, 2018) and the WDNs in this study are all planar.

Quantification of Hierarchy: Loops

Meshedness using Pipe-Junction Representation

One of the reasons we seek to quantify loop hierarchy, is to investigate how much more it can explain an often-used performance indicator (entropy) of water distribution networks in comparison with the commonly used measure of loops, meshedness. Meshedness has been shown to be a good indicator of water distribution network redundancy (Yazdani and Jeffrey, 2011) because it is the ratio of the existing number of loops to the maximal potential number of loops for the same number of junctions while maintaining planarity. Meshedness (for planar networks) is calculated using the following equation for the tested water distribution networks:

$$r = \frac{m - n + 1}{2n - 5} \tag{1}$$

Where m is the number of edges (i.e., pipes), and n is the number of nodes (i.e., junctions).

Nestedness using Decomposed Tree

Then, the water distribution networks are analyzed for their loop hierarchy using the tree network from the decomposition process. Only the part of the network with the highest number of connected loops is represented by the decomposition tree. If a water distribution networks is fully looped, this entails the entire network is included such as Modena; for other WDNs such as D-Town, only a fraction of the network is selected (see supplementary information). The hierarchical organization of smaller loops within larger loops can be quantified on two different levels: as a distribution of measures for each individual node in a given tree, and also as an aggregate measure over the entire tree.

Nodal Measures For the nodal level, there are two measures that have been previously developed: the nesting ratio, and the partition asymmetry. For each node j in the tree, there are two branches, r and s: branch r is the branch with a larger number of terminal nodes than branch s. The number of terminal nodes contained in branch r is referred to as r_j, and the number of terminal nodes contained in branch s is referred to as s_j. For each node j in the nesting tree, the nesting ratio is calculated as shown in Equation 2:

$$q_j = s_j / r_j \tag{2}$$

Where $r_j \geq s_j$ (so that q is a fraction less than 1) are the numbers of terminal nodes in branches r and s.

The partition asymmetry, also measured for each node in the graph, (referred to in the rest of the paper as asymmetry) has previously been introduced by Modes et al. (2016) and Van Pelt et al. (1992). It is calculated as shown in Equation 3 for a node j:

$$a(j) = \frac{r_j - s_j}{r_j + s_j - 1} \tag{3}$$

From Equations 2 and 3, it is clear that the nesting ratios and asymmetries calculated for a given tree would be inversely proportional, meaning high asymmetry values correspond to lower nesting ratios (i.e., less hierarchically nested loops).

Network-Wide Measure The aggregate tree-level measure has been developed and called the nesting number by Ronellenfitsch et al. (2015). The nesting number is an average of the nesting ratio distribution for a given tree and it decreases as the nestedness of loop hierarchy decreases. The nesting number is defined as a weighted average, shown in Equation 4:

$$i = \sum_j w_j q_j, where \sum_j w_j = 1 \tag{4}$$

Both unweighted and degree weighted nesting number can be calculated: unweighted nesting number i_u, with $w_j = 1 \forall j$, and degree-weighted nesting number i_w, with weight, w, proportional to the subtree degree of a given node, j ($w_j \propto \delta_j - 1 = r_j + s_j - 1$, where δ_j is the subtree degree). A high value nesting number ($i_{u,w}$) qualitatively represents graphs that are highly nested.

In addition to the nesting number, from the obtained decomposition tree, this study evaluates the relationships among different measures such as loop subtree degree versus loop area, subtree degree versus asymmetry, and subtree degree versus mean pipe radii to further characterize the loops of WDNs in the context of physical properties. The loop area is calculated as the physical-spatial area (in square meters), converted to a directly proportional measure of "square pixels". Mean pipe radii of a loop is the average pipe radius (in meters) of all of the pipes forming a given loop.

Quantification of Hierarchy: Pipe Diameter Gradation

Another measure of the network hierarchy quantifies the gradual change from larger diameter to smaller diameter pipes along flow paths, and Ronellenfitsch et al. (2015) and Modes et al. (2016) termed it Topological Length. Similar to asymmetry, the value is calculated for many segments of the network but can be evaluated as a network-wide measure by characterizing its distribution. Topological length is relevant to water distribution, because generally, abrupt fluctuations destabilize the system (Creaco et al., 2016). Due to the obtained type of distributions (power law), we define γ, as the power law exponent of a given topological length distribution for each water distribution network (Equation 5). The exponent γ allows capturing the power law distribution of topological length without bias as the mean would (see Faloutsos et al. (1999)).

$$P(L_e = l) \propto l^{-\gamma} \tag{5}$$

Where the frequency of a topological length , L_e, is a function of the topological length of the pipe raised to a power $-\gamma$. An exponent (γ) of larger magnitude indicates that pipes in a given water distribution network have a higher likelihood of having high topological length (see complementary empirical cumulative distribution plot in the supporting information) (Newman, 2005; Kunegis and Preusse, 2012). In other words, it is more likely to have paths with gradual change in pipe diameters, rather than abrupt change. The exponent γ is more accurate at capturing the behavior of the distribution then a mean or median because of the heavy tail (Faloutsos et al., 1999). The calculation procedure is described in detail in Modes et al. (2016) but in brief here: Starting from an initial pipe $e_1 \equiv < i_1, j_1 >$ between junctions i_1 and j_1, we identify all the pipes e that are adjacent to it (share the node i_1 or j_1) and have diameter smaller than or equal to the diameter of pipe e_1. We choose the pipe with the maximum diameter from the set e, which is now e_2, and add it to the trail, which now becomes (e_1, e_2) . The process is repeated for e_2 (identify all links that are adjacent to e_2 with diameter smaller than e_2 and choose the maximum) and iterate. The algorithm terminates when the set of pipes that have diameter smaller than that of the last link e_k in the trail is empty. The length of the trail associated with edge e1 is $l(e_1) = k$. The process is iterative, starting from every pipe of the network, in this way associating a trail length $l(e)$ with every pipe e.

Simulation-Based Performance Indicators

Hydraulic Simulation

The selected performance indicators require hydraulic modeling of the networks. Different kinds of hydraulic simulations can be conducted to calculate various WDN performance indicators. In this study, pressure driven, extended period simulations were run for the range of demands for 24-hour duration (when demands are available). Pressure driven analysis was calculated using the WNTR simulator within python (see Klise et al. (2017)). After conducting hydraulic simulations using the WNTR package within python, both the entropy and the Todini index are calculated for the water distribution networks using the WNTR package within python (Klise et al., 2017). Only 9 of the 15 networks are tested using the Todini index due to software limitations (i.e., a lack of convergence in solving equations within reasonable time), but from the 9 available data points, statistical validity is still established. For entropy, the data provided in Santonastaso et al. (2018) is used to confirm obtained values.

Entropy: Path Redundancy

The entropy surrogate indicator assumes that greatest uniformity between supply paths to all nodes minimizes expected shortfall in case of a pipe breakdown (Tanyimboh and Templeman, 1993; Farmani et al., 2005). Flows through pipes are obtained by simulating either a single demand scenario or the average of an extended period simulation. This study uses the entropy gap, $\triangle S$, meaning two simulations are conducted, one for the network with its current structure, and one for the maximal path redundancy structure (i.e., structure with greatest supply path uniformity). Santonastaso et al. (2018) proposed a measure of path redundancy, $\triangle S$, which is akin to normalizing the entropy of a given network by taking the entropy and dividing it by the maximum possible entropy given a network. By using $\triangle S$, we can compare a given network's reliability to its ideal path redundancy. The equations for S, and $\triangle S$ are given in Equations 6 and 7 , as shown in Santonastaso et al. (2018):

$$S = -\sum_{i=1}^{NS} \frac{Q_i}{T} ln(\frac{Q_i}{T}) - \frac{1}{T}\sum_{j=1}^{NN} T_j [\frac{Q_j}{T_j} ln(\frac{Q_j}{T_j}) + \sum_{ji \in N_j} \frac{q_{ji}}{T_j} ln(\frac{q_{ji}}{T_j})] \tag{6}$$

The first term is the entropy of supply nodes and the second is the entropy of demand nodes; NS is the number of supply nodes; T is the total supplied flow rate; NN is the number of demand nodes; Q_i represents the inflow at the i-th source node; T_j is the total flow rate reaching the j-th demand node; Q_j is the water demand at the j-th demand node; q_{ij} is the flow rate in the pipe connecting node j with surrounding node i; and N_j is the number of pipes carrying water from the j-th demand node towards neighboring nodes. Similar to Santonastaso et al. (2018) , the maximization of S, MS, is computed by the procedure proposed in Yassin-Kassab et al. (1999) and $\triangle S$ is calculated as follows in Equation 7.

$$\triangle S = 1 - \frac{S}{MS} \tag{7}$$

Todini Index: Power Surplus

The Todini index, or resilience index, is found by simulating either a single demand scenario of a network or the average of extended period simulation. The simulation results provide the flows from reservoirs to nodes, the available head at each reservoir and demand node, and the power introduced by pumps in the system (Dziedzic and Karney, 2014). Again, though this measure provides surplus power available to be dissipated in the network in case of failure, it neglects how the system actually performs or recovers after a failure (Farmani et al., 2005). The Todini index evaluates excess pressure head available at junctions, and is calculated as shown in Equation 8 (Todini, 2000; Dziedzic and Karney, 2015):

$$RI = \frac{\sum_{j=1}^{n} q_j (ha_j - hr_j)}{(\sum_{r=1}^{R} Q_r H_r + \sum_{b=1}^{B} P_b) - \sum_{j=1}^{n} q_j hr_j} \tag{8}$$

where n =number of demand nodes; q_j = demand at node j; ha_j = head available at node j; hr_j = minimum head required to meet constraints at node j; R = number of reservoirs; Q_r = flow being supplied to the system by reservoir r; H_r = head at reservoir r; P_b = power introduced in the system by pump b; and B = number of pumps. The Todini index is intended to compare different designs for the same network. As it does not always fall between [0,1], and the comparison with hierarchical indicators is on a pipe-basis, the index is size-normalized by the number of pipes in each network prior to conducting the regression analyses explained in the Results.

Identifying Relationships Between Hierarchical Metrics and Performance Indicators

After calculating both the entropy and the Todini index for the water distribution networks, linear multi-regression analyses with both the nesting numbers (i) and topological length exponents (γ) are conducted, using the *scipy* (Virtanen et al., 2020) and *seaborn* (Michael et al., 2018) packages in python, to evaluate relationships between WDN hierarchy and hydraulic performance using R^2 and standard error (SE) .

RESULTS AND DISCUSSION

Characterization of Hierarchy

This study characterizes the distribution of subtree degrees of nesting tree nodes for each network. Though subtree degrees are used as input for calculating loop nestedness measures, it is beneficial to understand the type of distribution subtree degrees follow to gain insight on the organization of loops. For the most part, subtree degree distributions for all water distribution networks are significantly approximated by a power-law distribution, $p \ll 0.05$. Only very small networks, such as TLN are not considered, because they do not have enough data to estimate a distribution at all (e.g., only 2 points). The distribution for Net6 is shown in Figure 2. The linear relationship on the log-log scale of Figure 2b indicates power-law behavior. See supplementary information for the remaining graphs. We also characterize the distribution of topological lengths for each network. For the most part, they are all significantly appproximated by a power-law distribution. After confirming their significant power-law distributions, with ($p \ll 0.05$), using methods described in Clauset et al. (2007), we store the power law exponent, γ, characterizing each network's distribution. Again, only very small networks, such as TLN are not considered, because they do not have enough data (only two data points). The distribution for Net6 is shown in Figure 3. The linear relationship on the log-log scale of Figure 3b indicates power-law behavior. Individual figures for a given network are shown in supplementary information. These findings are interesting because node-based topological measures accounting for nodal degree and other centralities of water distribution networks often follow poisson distribution rather than a power-law ((Giustolisi et al., 2017) among others). If water distribution network component connectivity follows a poisson distribution, that implies the networks are generally robust to targeted and random modes of failure (i.e., criticality and vulnerability is relatively randomly distributed among components). Whereas a power-law distribution indicates that there are few components that are especially critical or vulnerable, such that their failure may be catastrophic for the given network. This study suggests, from the approximate power-law distributions, that perhaps water distribution networks do not always follow a poisson distribution with respect to some key properties (e.g., loops), and thus are not necessarily immune from targeted modes of failure. These findings are consistent with the observation of power-law type of hierarchical behavior that is observed in other urban infrastructure networks (Yang et al., 2017; Krueger et al., 2017; Klinkhamer et al., 2019) and previous research on water distribution networks (Abdel-Mottaleb and Zhang, 2019).

Physical Context

Subtree Degree versus Loop Area and Asymmetry

This study also examined relationships between subtree degree of loops and the corresponding loop area and asymmetry. As shown in Figure 4(a, c, e, g, and i), higher subtree degree nodes correspond to loops with larger areas (i.e., smaller loops have smaller subtree degree). At the same time, there are generally more data points (i.e., nodes or subtrees) concentrated in the smaller loop area and smaller subtree degree space of the plots shown in Figure 4. This indicates that water distribution networks generally have more smaller loops than larger loops. As shown in Figure 4(b, d, f, h, and j), for many networks, the higher the subtree degree, the higher the asymmetry of a given node. However, this observation is not consistent for all of the water distribution networks. For many of the tested networks, there is a decrease in asymmetry near the mid-range of subtree degree. Given that the subtree degree of a loop is closely related to its physical area, it seems that loops of mid-range area are more hierarchically nested than larger area loops. The reason for this may be that larger loop redundancies are more expensive and less feasible than adding redundancies (and thus more nestedness) to smaller loops. This suggests a tradeoff between adding less expensive redundancies (at the smaller loop level) and adding redundancies that will serve more of the population (i.e., larger distribution mains). There also seems to be a larger spread of asymmetry values for smaller subtree degrees and thus smaller area loops may contain either highly evenly or unevenly distributed redundancies. However, path redundancies are not solely quantified using the asymmetry, nesting ratio and nesting numbers. There are other factors interplaying with these measures of nestedness, such as source location and pipe diameters.

Source Location and Subtree Degree versus Mean Pipe Radii of a Loop

Though previous research has quantified shape (e.g., grid, branch, loop) of water distribution networks, such as Hernandez et al. (2016), there remains a gap of accounting for water sources in research on water distribution network structure (Giustolisi et al., 2017; Meng et al., 2018; Giustolisi et al., 2019). Location of the water source influences whether larger pipes are on the outer, larger loops, or whether larger pipes are within smaller internally nested loops. Hence this study further examined the relationship between degree and mean pipe radii; differences in the relationship between these two properties of the decomposition tree may be attributable to demand allocation and source(s) placement. Not all studied water distribution networks have the same relationship between subtree degree (and consequently, loop area) and mean pipe radii of that subtree (i.e., loop). In Figure 5, five plots of subtree degree versus mean pipe radii are shown (see supplementary information for remaining plots).

From Figure 5e and f, Fossolo's larger diameter pipes are not part of the largest loop, whereas they are for Net3 (5c and d) and Modena (5a and b). Because Fossolo has a single water source, pipe diameters are not as uniform throughout the network as they are for multiple source networks (e.g., Net3 and Modena). Another observation is that larger pipes are on the periphery of the Modena, Net3 and D-Town networks, whereas for Net6, the largest diameter pipe is not even in the largest looped part selected for analysis. Similarly, the degree versus asymmetry distributions for Modena and Net3 are much closer in distribution shape than they are to Fossolo and Net6 (even low degree loops in Fossolo and Net6 show high asymmetry, whereas only the highest degree loops show highest asymmetry in Net3 and Modena). This observation is confirmed upon calculating the Kolmogorov-Smirnov statistic between the asymmetry distributions of the networks, following the method in Ronellenfitsch et al. (2015) (see supplementary information). It is interesting that for these five networks, the nesting number, shown in Table 1, is higher for networks with the highest mean pipe radii on the periphery of the largest looped part. In addition, both Net3 and Modena have multiple sources, whereas Fossolo has a single water source that no doubt influences pipe size along the gradient of larger area to smaller area loops within the network (i.e., the location of the source(s) of water, and number of sources influences the distribution of pipe diameters relative to the networks' loops). Multiple storage tanks in a network, depending on their placements relative to the demands, can result in increased entropy or pathway redundancy, as the pipe diameters can be smaller than they otherwise would have to be if there weren't as many source redundancies (Chin et al., 2000; Walski, 2000). These observations open the question of how physical network components (e.g., number and location of sources) influence the hierarchy, and consequently performance, to manage network maintenance and operations.

Relationships Between Hierarchy and Performance

There is a significant correlation between the network hierarchy and simulation-based performance indicators. For entropy, in the normalized form ($\triangle S$), smaller values (i.e., closer to zero) indicate a network is closer to its "ideal" or maximal path redundancy. First, a regression analysis is conducted between the baseline topological measure of meshedness, r, and $\triangle S$. The significant R^2 value of 0.3 indicates that there must be other factors influencing the path redundancy in addition to the ratio of existing to maximal potential number of loops. Meshedness alone is limited at capturing path redundancy in water distribution networks. Though meshedness has been shown to be a robust measure of path redundancy (Yazdani and Jeffrey, 2011), it is limited in explaining variability observed in the gap between actual and optimal path redundancy ($R^2 = 0.3$). However, when a multi-regression is run with the nesting number, the R^2 increases to 0.63. The R^2 further increases to 0.83 when the pipe diameter gradation is included. When the pipe diameter gradation is included, the number of samples decreases to 14 because two of the networks do not have enough data points to calculate γ (still, the results are significant with greater than 95 percent confidence, p$\leq$0.0003). Regardless, this indicates that, not only the pipe-junction topology of a WDN impact its flow path redundancy, but so does its loop and pipe diameter hierarchy.

Networks with higher nesting number values (i.e., more nestedness and loop symmetry) tend to have a significantly lower gap between their actual and maximum entropy values, $\triangle S$ (Figure 6a). The pipe diameter gradation measure (γ) also has the same effect on $\triangle S$, but stronger (Figure 6b). However, when evaluating their impact on the Todini index (excess pressure head at junctions, or energy redundancy), the nesting number and pipe gradation displayed different relationships. The nesting number of a network did not have as significant of a relationship with the Todini index (Figure 6c), but higher values of γ, pipe diameter gradation, increased the Todini index (Figure 6d). Higher values of the exponent, γ, indicate more gradual changes in diameter in the network. Pipe diameter gradation along flow paths, γ, explained the variability in the Todini index (R^2=0.856, n=9, p$\leq$0.001, $SE = 0.010$) more than that observed for entropy (R^2= 0.66, n=15, p$\leq$0.005, $SE = 0.053$). However, when meshedness, loop nestedness and pipe diameter gradation are accounted for, the correlation significantly increases (R^2=0.83, n=15, with p$\leq$0.001, $SE = 0.039$). This supports the hypothesis that hierarchy of loops and pipe diameters influences the two tested performance indicators (i.e., is an integral part of WDN structure). However, actual performance for WDNs depends not only on their pipe-junction or hierarchical representations, but also on operational, design, and dynamic conditions that are not considered in this work (but can be included in future studies).

Increasing loop nestedness has leverage on path redundancy, whereas more gradual pipe diameter change can simultaneously enhance path redundancy and power surplus (or energy redundancy).

This suggests that WDN design optimization can be improved by including decision variables related to pipe diameter gradation along flow paths. Previous studies have focused on optimizing the design of WDN loops or sizing pipes without considering diameter gradation along paths ((Todini, 2000; Creaco et al., 2016; Dziedzic and Karney, 2015) among others). Though Creaco et al. (2016) took it further by considering diameter gradation of loops, it does not extend to flow paths. When diameter gradation is accounted for even just within loops, Creaco et al. (2016) found additional solutions to WDN design optimization problems. Instead of solely focusing on adding loops or maintaining diameter uniformity of loops to enhance WDN redundancy and subsequent reliability, effort should be made to increase the pipe diameter gradation along paths throughout a network for increased leverage.

CONCLUSION

We characterize WDN hierarchy, showing that loop nestedness (i.e., subtree degree) and pipe diameter gradation along flow paths (i.e., topological length), are approximated by power-law distributions. This indicates that WDNs are more vulnerable at the loop level than at the junction level. With respect to network design, differences in the relationships between location of larger pipes and nestedness, and loop area and nestedness were observed based on water source location, suggesting that hierarchical indicators capture more information regarding water source location than pipe-junction topology. This study also found that the hierarchy of WDNs, as quantified by loop nestedness and pipe diameter gradation along flow paths, explained variability of simulation-based performance indicators (specifically flow path redundancy and power surplus). Quantifying and characterizing the different representations of WDN structure is a necessary step before applying structural measures in network optimization, failure analysis, and design/scenario comparisons.

There are however several limitations to the study that can be addressed in future work. The nesting number, or measure of loop nestedness, used in this work captures the largest looped part, and not the entirety of the network. This limitation can be addressed in future work by modifying the measure to account for more loopy parts. This study analyzes the network as a single snapshot (i.e., static not dynamic as it actually is). The hierarchy of components may likely change as water distribution infrastructure co-evolves with cities. Real WDNs must also be analyzed to confirm the identified relationships before hierarchical indicators are adopted to estimate surrogate performance indicators. Nonetheless, analyzing the hierarchical representation of WDNs adds much more insight on their structure that contributes to the hydraulic performance than only analyzing their pipe-junction topology.

DATA AVAILABILITY STATEMENT

- All data, models, or code generated or used during the study are available from the corresponding author by request.

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List of Tables

TABLE 1. Measurements of r (meshedness), size-normalized RI (Todini Index, energy redundancy), $\triangle S$ (entropy or path redundancy gap), γ (topological length distribution exponent), and i (nesting number) for WDNs

Network	r	RI (size normalized)	$\triangle S$	γ	i
Anytown	0.422	–	0.121	0.621	0.365
Net1	0.176	0.0900	0.106	1.128	0.813
Net2	0.075	0.0125	0.127	0.618	0.700
Net3	0.121	0.0053	0.144	0.634	0.486
Net6	0.080	0.0001	0.203	0.537	0.164
Fossolo	0.318	0.0236	0.128	0.772	0.386
Modena	0.085	0.0008	0.162	0.570	0.531
Pescara	0.201	–	0.139	0.617	0.534
Richmond	0.049	–	0.181	0.428	0.649
D-Town	0.065	–	0.169	0.632	0.762
TRN/Gessler	0.315	–	0.108	0.825	0.435
ZJ	0.228	0.0000	0.147	0.573	0.791
Rural	0.126	0.0018	0.167	0.428	0.688
Jilin	0.137	0.0000	0.122	0.613	0.829
TLN	0.222	–	0.098	–	0.700