

# 1 Evaluating Segment and Valve Importance and Vulnerability

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## 3 ABSTRACT

4 Because consideration of segments and valves is essential for evaluating the reliability and  
 5 resilience of water distribution networks (WDNs) when shutdowns are required, a quick method  
 6 of identifying critical and vulnerable segments and valves would benefit utilities. While the  
 7 importance and vulnerability of segments can best be evaluated by extensive hydraulic analysis,  
 8 hydraulic analyses can be time consuming. It can also be challenging to visualize the segments  
 9 of a water distribution network and their associated valves. To address these limitations, this  
 10 study develops a method based on graph theory to identify important and vulnerable segments  
 11 without hydraulic calculations. The method generates a matrix that represents how reachable water  
 12 sources are from segments when a given segment must be isolated, while distinguishing between  
 13 continuous water sources and ephemeral storage. This study also applies measures from graph  
 14 theory to determine the number of valves to operate to isolate a segment and provides a rigorous  
 15 proof to support the intuitive equation. A method to visualize the connectivity of segments with  
 16 the graph theory measures is demonstrated. The developed methods are applied to multiple valving  
 17 scenarios of a case study and two real water distribution networks. Correlations between graph-  
 18 theory based measures derived from the segment-valve topology and hydraulic simulation-based  
 19 criticality are higher than in previous studies that apply graph theory to the pipe-junction topology  
 20 of WDNs ( $r \geq 0.6$ ). Results indicate that the developed methods can be used by utilities as a  
 21 preliminary screening to eliminate the need for some hydraulic simulations. These findings are  
 22 expected to provide decision-support for utilities.

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23      **Keywords:** water distribution networks, complex network analysis, graph theory, segments,  
24      isolation valves, vulnerability, criticality

25      **INTRODUCTION**

26      Resilience of Water Distribution Networks (WDN) is the ability of the WDN to maintain  
27      adequate function in face of diverse and unanticipated failures and to bounce back (Vugrin et al.,  
28      2010; Diao et al., 2016; Klise et al., 2017; Meng et al., 2018; He and Yuan, 2019). As such,  
29      enhancing the resilience of WDNs continues to be an important, open research topic (Diao et al.,  
30      2016; Butler et al., 2017; Abdel-Mottaleb et al., 2019; Pagani et al., 2020). Many municipalities  
31      struggle with pipe breaks as systems age or morph in unexpected ways. Because there may be  
32      insufficient isolation valves in place to isolate each pipe in the system, when a pipe breaks, needing  
33      repair or replacement, it is often more than one pipe that must be isolated for repairs. This  
34      exacerbates the effects failures have on customers and is a situation that utilities seek to avoid  
35      (Walski, 1993; Liu et al., 2017; Zischg et al., 2019; Giustolisi, 2020).

36      Much of the literature focuses on identifying critical pipes in WDN or evaluating resilience  
37      by assuming individual pipes can always be isolated in the system (Shuang et al., 2014; Abdel-  
38      Mottaleb et al., 2019; He and Yuan, 2019; Balekelayi and Tesfamariam, 2019). However, it is  
39      the valves located along pipes that isolate failures, limiting widespread impacts due to the failure  
40      (Giustolisi, 2020). In evaluating WDN reliability, a common but faulty assumption has been that a  
41      single pipe can be isolated in the network. More often than not, there are not valves at each end of  
42      every pipe in the network. This means that a shutdown cannot always be isolated to a single pipe.  
43      A WDN segment is a portion of the system that can be isolated by valves; segments often contain  
44      more than one pipe (Walski, 1993; Giustolisi and Savic, 2010). Thus, a crucial step in enhancing  
45      reliability and resilience of WDNs is to evaluate the vulnerabilities and criticality of isolation valve  
46      systems in place. Isolation valves are critical when their operability is important for maintaining  
47      the performance of a given WDN.

48      Figure 1 shows a typical pipe, P-1, in a hydraulic model connecting nodes J-1 and J-2. Account-  
49      ing for isolation valves, the pipe is part of three distribution segments S-101, S-102 and S-103.

50 Segments are bounded by isolation valves or other control devices. Segments may be made up of  
51 node elements and parts of pipe elements from a hydraulic model. Since there is no way to close a  
52 node, there is not a one-to-one relationship between pipe elements and segments.

53 Both graph theory and the emerging field of complex network analysis (CNA) are powerful  
54 tools to analyze networks and have been applied to WDN over the years for varying purposes,  
55 including evaluation of various aspects of reliability and resilience (Agathokleous et al., 2017;  
56 Hwang and Lansey, 2017; Ulusoy et al., 2018; Abdel-Mottaleb and Zhang, 2019; Yazdani and  
57 Jeffrey, 2012a,b; Torres et al., 2017; Giustolisi et al., 2017, 2019; Zeng et al., 2017; Balekelayi  
58 and Tesfamariam, 2019; Pagano et al., 2019). CNA builds on graph theory to focus more on the  
59 relationship between structure and function of a graph and the complex system (e.g., a WDN) it  
60 represents (Zweig et al., 2016). CNA provides tools to analyze more complex networks such as  
61 networks with multiple edges connecting nodes, weighted edges and nodes, among other things.  
62 However, both graph theory and CNA have often been applied to what is called the pipe-junction  
63 topology or the primal graph, where nodes represent demand junctions or other point features and  
64 edges represent pipes (Zischg et al., 2019). This representation of WDN is the most common,  
65 being necessary for hydraulic analyses and an accurate spatial representation of the system in maps  
66 (Walski, 1993). Further, this is most commonly visualized topology of WDNs, as there have  
67 been computer programs automating it and most discussions have been pipe-centered. Thus, some  
68 methods for identifying critical WDN components have been focused on this representation.

69 The number and location of valves has generally received very little attention in the research  
70 literature compared with other aspects of distribution design and analysis with some exceptions  
71 (Walski, 1994; Deb et al., 2006; Walski, 2011). This includes the application of CNA. While CNA  
72 has increasingly been applied to the pipe-junction representation of WDNs, limitations include  
73 that the pipe-junction topology is not representing the operational reality in event of pipe failures  
74 and their associated shutdowns because valves are not considered. For example, a cut set, which  
75 by definition has to do with isolation of portions of a network, is not accurate if applied to the  
76 pipe-junction topology because an edge that represents a pipe may not actually have valves on either

77 end. However, if applied to the segment-valve representation could in fact represent the collection  
78 of valves that can result in excessive unintended isolation. Graph theory and CNA measures have  
79 hardly been applied to the dual, segment-valve representation where nodes represent segments and  
80 edges represent valves connecting them (Walski, 1993; Zischg et al., 2019; Giustolisi, 2020).

81 Using a segment-valve representation facilitates inspection and evaluation of potential isolation  
82 scenarios that can face a system to allow repair when system elements fail (Jun and Loganathan,  
83 2007). This representation was first explained and visualized in (Walski, 1993), where the au-  
84 thor differentiated between the Bouchart and Goulter (1991) definition of segments. The first  
85 commercial software to utilize segment-valve topology was WaterGEMS (Bentley Systems, 2020)  
86 which introduced its segmentation and criticality tool in the early 2000s. The first application to  
87 a full-scale system was Walski et al. (2008). Since then, there has been considerable research  
88 on identifying WDN segments, analyzing shutdowns with different valving scenarios and more  
89 recently, isolation valve system design.

90 There are two major research areas with respect to WDN segments and valving: analysis and  
91 planning (Jun and Loganathan, 2007). Planning refers to identifying optimal isolation valve sys-  
92 tems (IVS) (e.g., Jun (2005); Alvisi et al. (2011); Choi et al. (2018); Giustolisi (2020)), DMA  
93 configuration (e.g., Santonastaso et al. (2019); Creaco and Haidar (2019)). Analysis involves iden-  
94 tifying and evaluating impact of unintended isolations (e.g., Deb et al. (2006); Jun and Loganathan  
95 (2007); Creaco et al. (2012)), comparing different valving scenarios (e.g., Liu et al. (2017); Zischg  
96 et al. (2019); Atashi et al. (2020)), identifying optimal near-real time response strategies (e.g.,  
97 Mahmoud et al. (2018)), and improving hydraulic modeling (e.g., Vasilic et al. (2018)). Zischg  
98 et al. (2019) and Jun and Loganathan (2007) focused on obtaining insight from the structure, or  
99 physical configuration of segments and valves, in lieu of hydraulic simulations.

100 However, there remain gaps to be addressed regarding connectivity analyses of WDNs using  
101 graph theory. As Meng et al. (2018) and Giustolisi et al. (2019) note, water sources are often not  
102 accounted for in studies that use connectivity analysis to evaluate reliability and resilience of WDNs  
103 (e.g., Yazdani and Jeffrey (2012a)). The concept of reachability (emphasizing the connectivity to

water sources) has been introduced in previous research, but previously has referred to the pipe-junction representation. While Dziedzic and Karney (2014) used a connectivity matrix to evaluate WDN component connectivity to water sources, the matrix did not account for segments or different types of water sources, such as distribution tanks and reservoirs. Similarly, to evaluate WDN reliability, Wagner et al. (1988) quantified the reachability of water sources to demand junctions rather than to segments and did not distinguish between different water sources. Though the matrices developed by Jun and Loganathan (2007) are based on the segment-valve representation, they also do not account for the differences between remaining connected to water sources, such as reservoirs and more ephemeral storage, such as tanks. While the matrices allow tracing unintended isolations, they do not provide a prioritization scheme, or ranking, of segments and they do not measure the aggregate vulnerability of segments due to potential unintended isolations.

Additionally, the developed segment-valve diagrams would be useful if automated such that it is feasible for utilities to readily use them for mapping large system valving. While Zischg et al. (2019) apply CNA to the segment-valve topology, their methodology provides more global network-wide insight than it does local, component-level (see Pagano et al. (2019) for distinction between local and global graph theory or CNA-based measures). Similarly, Atashi et al. (2020) apply global resilience analysis to the segment-valve topology that provides global network-wide insight. Such a methodology, while practical to compare among isolation valves in different systems, does not provide details on which segments are problematic.

Further, traditional techniques of evaluating topological reliability and quantifying connectivity that have been applied to the pipe-junction representation have not been applied to the segment-valve representation (e.g., minimum edge cut sets by Su et al. (1987) and Yang et al. (1996), articulation points by Jacobs and Goulter (1989); Jacobs and Coulter (1991)). However, there are direct operational insights that might be gained by using these metrics from graph theory on the segment-valve representation because of how the graph theory measures physically correspond to this representation of WDNs.

The most accurate way to determine the criticality of each segment in a WDN is to remove

131 each segment from a water distribution system model and conduct a pressure dependent demand,  
132 extended period simulation to identify impacted customers and calculate the demand shortfall.

133 This type of calculation is performed in software such as WaterGEMS (Bentley Systems, 2020).

134 Utilities attempting to analyze this problem manually would have to run many simulations because  
135 there no quick preliminary screening tool to hone in on important segments. Moreover, there is  
136 no tool to quickly identify vulnerable segments to help utilities identify problematic locations of a  
137 given WDN.

138 To address these gaps, the authors expand on previous research (specifically, connectivity  
139 analyses) that applies graph theory to WDNs. The major contribution of this study is in applying  
140 graph theory to the segment-valve topology for preliminary screening to identify problem areas.  
141 The study provides methods for displaying results in a useful way to utilities by visualizing various  
142 measures to help quickly identify important (i.e., critical) and vulnerable segments and also to  
143 help map segments that do not necessitate closing all downstream valves. Additionally, the paper  
144 presents an importance and vulnerability index that account for the difference between continuous  
145 water sources and ephemeral water storage when evaluating the impact of segment isolation. The  
146 paper first presents an improved approach to visualizing the segment-valve topology, followed by  
147 the use of a reachability matrix to determine important and vulnerable segments. Then, a method  
148 to determine the number of valves that must be operated to isolate segments in different cases is  
149 presented. Several case study scenarios are studied to illustrate the use of the topology display and  
150 reachability-based indices for a variety of systems, including some real systems.

## 151 **METHODOLOGY**

### 152 **Development of Segment-Valve Topology Display**

153 Using the graph representation of segment connectivity to visualize the interconnections be-  
154 tween segments is valuable and advantageous to communicate the relationship of valves and  
155 segments (Jun and Loganathan, 2007). Such a segment-valve graph can be constructed by treating  
156 each segment as a node and each valve as an edge connecting the segments. A segment-valve graph  
157 can be considered as a dual topology compared with the usual pipe-junction topology.

158 The information necessary to construct a segment-valve display includes: the list of segments  
159 and their associated pipes, locations of pipes (x,y) coordinates used to place the segment spatially,  
160 and valves connecting the segments. Segments containing water sources are also identified, and  
161 their IDs stored. For this study, WaterGEMS is used to identify segments and their elements. The  
162 segment-valve graph can be constructed using only open source software packages within python.  
163 An empty *networkx* (Hagberg et al., 2008) graph is initialized, and all segments are stored as nodes.

164 In WDNs where there are segments connected to each other by more than one valve, a *multigraph*  
165 object is initialized instead of a graph object to allow for multiple edges between nodes. Similarly,  
166 if a WDN has directional elements (e.g., pumps, PRVs), a *digraph* object (i.e., directed graph) is  
167 initialized to account for direction. When studying a given pressure zone with directional elements,  
168 segments flowing into the zone should be treated as open links while those flowing out of the zone  
169 should be treated as closed to prevent water from moving backward through a directional element.  
170 For each node, (x,y) coordinates are assigned as the centroid of the pipes belonging to the given  
171 segment. The centroid is calculated using the *shapely* package with the *multipoint* and *centroid*  
172 functions (Gillies, 2013). If two segments are connected by a valve, then an edge or link is assigned  
173 between the two nodes in the graph. The exact location of the valve is not needed, and it is assumed  
174 that all valves are operable although the graph can be reconstructed to eliminate inoperable valves.  
175 A schematic of a segment-valve diagram is shown in Figure 2a.

176 To display the topology, the *matplotlib* and *networkx* packages are used. The graph object is  
177 used as input into the *networkx draw* function along with settings for node and edge (i.e. segment  
178 and valve, respectively) sizes and displaying labels and coordinates. The size and color of nodes and  
179 edges can be used to represent different attributes by passing the dictionary object of the attribute  
180 values as input to the *networkx draw* function. For the case study presented in this paper, sample  
181 code is provided in a repository (see Abdel-Mottaleb and Walski (2020b)).

182 **Reachability Matrix**

183 *Determination of Reachability Matrix*

184 Constructing the reachability matrix,  $\mathbf{R}$ , is an integral step of assessing importance and vulner-  
185 ability of segments and was introduced in Abdel-Mottaleb and Walski (2020a). Rows,  $m$ , of the  
186 matrix represent segment closure and columns,  $n$ , represent segment impact. This section illustrates  
187 matrix construction with a small example. First, every segment closure is considered. For each  
188 segment closure, the impact on other segments connectivity to water sources is evaluated. Water  
189 sources are distinguished as continuous or ephemeral. Continuous sources such as large reservoirs,  
190 wells and treatment plants are long lasting and ephemeral sources, such as distribution tanks, are  
191 temporary because they can drain in a few hours if they are not connected to a continuous source.

192 If the closure of a given segment ( $m$ ) causes another segment ( $n$ ) to lose connection with all  
193 water sources, a value of 2 is assigned to  $R_{mn}$  for all isolated segments. If the closure of a given  
194 segment ( $m$ ) causes another segment ( $n$ ) to lose connection with all continuous water sources yet  
195 maintain connection to an ephemeral water source, a value of 1 is assigned to  $R_{mn}$  the closure of  
196 a given segment ( $m$ ) does not cause another segment ( $n$ ) to lose connection with continuous water  
197 sources, a value of 0 is assigned to  $R_{mn}$  is repeated for each segment in the network until the cells  
198 of matrix  $\mathbf{R}$  are populated, as shown in Figure 2 for a small example. In Figure 2, the reservoir is  
199 the continuous source while the Tank is an ephemeral source.

200 Summing the values of row  $m$ , corresponding to a given segment, gives an indication of how  
201 important a segment is because its closure results in many unintended isolations, and this sum is  
202 called the importance index. Summing the values of column  $n$ , corresponding to a given segment,  
203 gives an indication of how vulnerable that segment is to unintended isolation. A high value of the  
204 importance index indicates shutting down this segment will affect many other segments, while a  
205 value of 2 indicates that segment will only impact itself. A high value for the vulnerability index  
206 indicates that a shutdown of several other segments can isolate this segment, while a value of 2  
207 indicates that failures in other segments will not affect it.

208 Calculations based on the reachability matrix can be represented on the segment-valve topology.

209 To compare the indexes of different valving scenarios, the importance and vulnerability indexes  
210 are min-max normalized between 0 and 1. Meaning, for a given network scenario, the minimum  
211 index is subtracted from all the values and then divided by the difference between the maximum  
212 and minimum values of the segments in the network scenario.

213 Different valving scenarios of a given network will also differ in their segments. Because each  
214 scenario would also have a different number of segments, the data (i.e., importance and vulnerability  
215 indexes) measured for each scenario is on a different scale. When the data for each scenario is  
216 re-scaled or min-max normalized between 0 and 1, the different valving scenarios can be compared  
217 with each other. For example, if the spread of the vulnerability index of a scenario is mostly above  
218 0.5 and another scenarios vulnerability index is mostly below 0.5, this provides information relating  
219 how vulnerable a valving scenario is compared with another. The normalization procedure also  
220 allows location-specific comparisons between valving scenarios. Because the scale is normalized,  
221 comparing segments within a given system is also easier as the maximum and minimum values are  
222 known to be 1 and 0, respectively.

223 *Validation by Comparison with WaterGEMS*

224 The reachability matrix importance index is validated with WaterGEMS criticality as follows.  
225 First the reachability matrix is constructed, and the importance index is calculated for each given  
226 segment isolation. The WaterGEMS (Bentley Systems, 2020) criticality tool is also run for the  
227 three valving scenarios of the case study using a 24-hour extended period simulation (pressure  
228 driven) to quantify the system demand shortfall for each segment isolation starting at  $t = 0$  and  
229 lasting through the 24-hour duration.

230 For the case study presented in the following section, the average supplied flow in the network  
231 is  $44L/s$ . The tank has a base elevation of  $49m$ , initial elevation of  $55m$  and maximum elevation  
232 of  $58m$ . The reservoir and pumps are designed to fill the tank during the simulations. The analysis  
233 mode was set to hydraulics only, the reference pressure is  $50m$ , and the threshold pressure is set to  
234 the reference pressure.

235 The Pearson correlation coefficient and linear regression statistics are calculated in python using

236     scipy (Jones et al., 2016), and are used to evaluate the correlations between the reachability matrix  
237     value and the hydraulic simulation-based result.

238     **Identifying Valves that must be Operated for Segment Shutdowns**

239     The location of a segment in the network has operational implications for valve closure when  
240     segments must be isolated. Depending on the structure that a given segment is part of (e.g. loop,  
241     tree, or both) the number of valves that must be closed to isolate the segment differ. Trees refer  
242     to the portions of the network where two or more segments are connected by only one path (i.e.,  
243     free of loops). Additionally, a failed valve may mean that more valves must be closed to isolate a  
244     segment, or it might not have an impact— depending on its location.

245     *Impact of Failed (OPEN) valves*

246     Failed valves Require Extensive Shutdowns In this work, a valve is referred to as "failed" when  
247     it cannot be closed when required, hence the term failed (OPEN). Isolation valves can be considered  
248     to have failed for various reasons including being paved over, cannot be operated for a variety of  
249     reasons, or cannot be located. The more failed valves there are, the more unintended isolations  
250     there will be, and the more valves need to be operated to isolate a given segment. This leads to  
251     the observation that the number of valves that must be operated when the sole valve between two  
252     segments fails can be given by

$$N = N_1 + N_2 - 2 \quad (1)$$

253     for looped areas, where  $N$  is the number of valves to be closed off to isolate a segment in the event  
254     of a broken valve between two adjacent segments, 1 and 2.  $N_1$  is the number of valves to close  
255     segment 1 and  $N_2$  is the number of valves to close segment 2. Graphically, an inoperable valve  
256     between two segments means that the two segments become merged into one. The proof of this  
257     equation is as follows.

258     **Proof** Let a connected graph,  $G$ , with a number of nodes greater than two, represent a water  
259     network, where the nodes represent segments and the edges represent valves. Let  $G$  contain two  
260     adjacent nodes, 1 and 2, representing two adjacent segments, 1 and 2, in a water network. The

262 degree,  $k$ , is the number of edges that are incident on a node (i.e., the number of connections). Let  
 263 the degree of node 1 be denoted by  $k_1$  and the degree of node 2 be denoted by  $k_2$ . Because node 1  
 264 and node 2 are connected to each other,  $k_1$  and  $k_2$  each account for the edge between nodes 1 and  
 265 2. Let a modified graph from  $G$ , denoted by  $G'$ , contain nodes 1 and 2 merged as a super-node,  
 266  $1 + 2$  (see Figure 3). The degree of node  $1 + 2$  in  $G'$ ,  $k_{1+2}$ , is equivalent to  $k_1 + k_2 - 2$ , where  $k_1$   
 267 and  $k_2$  are the degrees of node 1 and node 2 from the original graph,  $G$ . Two is subtracted from  
 268 the total degree because when node 1 and node 2 are merged, they are considered as a single node  
 269 without a connection between them (i.e., there is no longer node 1 or node 2, but only node  $1 + 2$ ).  
 270 Therefore, the number of edges that must be removed from  $G$  to disconnect a subgraph consisting  
 271 of only nodes 1 and 2 is also equal to  $k_1 + k_2 - 2$ . Meaning that to isolate segment 1 or segment 2 in  
 272 a water network where segments 1 and 2 are connected by a broken valve,  $N_1 + N_2 - 2$  valves must  
 273 be operated, where  $N_x$  is the number of valves connected to a given segment  $x$ . This illustrates how  
 274 a failure of the valve between segments 1 and 2 can greatly impact the extent of the outage caused  
 275 by a failure in either segment 1 or 2.

276 There are two exceptions to Equation 1. The first is when either segment 1, segment 2, or both  
 277 are connected to a tree structure of segments that are fed by the loop containing segments 1 and 2.  
 278 The second is when there is more than one valve between the two segments that will be merged as  
 279 a single node in their corresponding graph representation due to a at least one valve failure. In this  
 280 case, the number of valves that must be operated can be generalized to Equation 2:

$$281 \quad N = N_1 + N_2 - 2N_{1,2} \quad (2)$$

282 where  $N$  is the number of valves that must be operated,  $N_1$  is the number of valves connected to  
 283 segment 1,  $N_2$  is the number of valves connected to segment 2, and  $N_{1,2}$  is the number of valves  
 284 connecting segments 1 and 2.

285 Thus far, the segment-valve graph has been referred to as undirected. Considering the segment-  
 286 valve graph as directed, with link direction indicating flow through, observations are made in the

287 following section related to the operation of valves that isolate segments that bridge loops and the  
288 tree structures they feed.

289 *Identifying of Valves Needed to close Tree Segments*

290 A segment can be part of a loop on the upstream side but on the downstream side (the side  
291 that is isolated from any source), it can be the beginning of a tree. The standard practice for such  
292 segments is only closing valves upstream of the segment to isolate it. It is not necessary to operate  
293 downstream valves to isolate a segment. Possible reasons include time constraints, and the location  
294 of valves and traffic disruption. Though uncommon, utilities may opt to close downstream valves.

295 Identifying segments that are part of a loop and have a downstream tree structure can be achieved  
296 by identifying and mapping which valves are connected to loops (called cycles in graph theory)  
297 and which are the beginning of a tree and do not need to be closed. To identify those valves that are  
298 located on the downstream side of a segment, first, articulation points are identified. An articulation  
299 point (also called a cut vertex) is any node (i.e., segment) whose removal disconnects the graph.  
300 Physically, an articulation point is a segment, which when shut down, also isolates downstream  
301 segments beyond itself, if there is not a source downstream. Any segment in a tree or serving as  
302 the root of a tree is an articulation point. In a completely looped system, there are no articulation  
303 points. The *articulation\_points* function within the *networkx* package used to identify articulation  
304 points.

305 A cycle is a path in the network that begins and ends at the same node (in this case, the same  
306 segment) without repeating segments. The cycle basis of a graph is a minimal collection of cycles  
307 such that any cycle in the network can be written as a sum of cycles in the basis (a basis is a set of  
308 elements from which all other elements can be derived as a combination of that set of elements).  
309 In other words, the cycle basis of a given WDN, contains all the segments that are part of loops.  
310 The cycle basis of the graph is identified using the *cycle\_basis* function within *networkx*.

311 Then, to exclude any segments from the articulation points that are not also part of a cycle, the  
312 intersection between the sets of segments in the articulation points and cycle basis is identified and  
313 stored. Thus far, the method does not account for the location of the water sources in the network.

314 With an additional step, we can exclusively identify which valves are connected to loops and which  
315 are the beginning of a tree and do not need to be closed. The subgraphs containing segments with  
316 water sources must be identified and excluded from the intersection of the articulation points and  
317 cycle basis. The *biconnected\_component\_subgraphs* function in networkx is used to identify  
318 subgraphs that contain segments with water sources; that way, only segments with downstream  
319 trees are included and not upstream trees.

## 320 CASE STUDY

321 The WDN used as a case study has previously been studied by Liu et al. (2017). The network has  
322 279 pipes and 188 junctions. The segment-valve topology is obtained for three valving scenarios  
323 for this network ( $N$  valves at each intersection,  $N-1$  valves at each intersection and a scenario with  
324 fewer than  $N-1$  which is referred to as scarce valving). The  $N$  valve rule refers to the most  
325 complete allocation of valves, with valves located at the end of each pipe (i.e., at the junction) and  
326 the  $N-1$  valve rule refers to one less valve than the number of pipes at each junction located at  
327 the given junction. The scarce valving scenario refers to the WDN having fewer valves installed  
328 than according to the  $N-1$  rule (but more than  $N-2$ ).  $N$  valves corresponds to 183 valves  
329 and 157 segments,  $N-1$  valves corresponds to 130 valves and 104 segments, and scarce valves  
330 corresponds to 91 valves and 65 segments. The pipe-junction topology of the  $N$ -valve scenario is  
331 shown in Figure 4. Figure 5 is a display of the segments highlighted in the WaterGEMS software.  
332 Figure 6 is the corresponding segment-valve display in python from which the reachability matrix  
333 is constructed.

## 334 RESULTS AND DISCUSSION

### 335 Reachability Matrix

336 The importance and vulnerability indexes for three different valving scenarios of the case study  
337 are displayed in Figures 7, 8, and 9, where larger and darker nodes indicate higher importance (left)  
338 and vulnerability (right). The axes represent the x and y coordinates of segment centroids and the  
339 legend represents a normalized index for each scenario, where 0 is the minimum value and 1 is the

maximum value. The segment identifiers are labeled in each node. For high resolution maps, refer to the supplementary information. Such figures make it easy to identify important (i.e., critical) and vulnerable segments. It is also possible to compare how important and vulnerable segments are in a given location for different scenarios. For example, the location with the three circled segments (11, 13, 24 in  $N$  and  $N - 1$  and 10, 12, 22 in scarce) increases in vulnerability as the number of valves decreases. On the other hand, the importance of a given segment decreases as the number of valves increase. Due to the reduction of valves and increased size of segments, there are more critical segments. In another example, the location of Segment 23 in Figure 8 becomes more critical than it was in Figure 7 (i.e., has a higher importance index as indicated by the darker color and larger node size). These results are consistent with the findings from Liu et al. (2017) and Atashi et al. (2020) that demonstrated  $N$  valving was more resilient than  $N - 1$  valving for a given network. This method adds to these previous works by allowing spatial comparison of network locations in addition to the comparison of overall global network-wide performance.

### 353 *Validation by Comparison with WaterGEMS*

354 The importance index values significantly correlated with demand shortfall values from WaterGEMS. Figure 10 shows that there is a strong correlation between the importance index and the 355 model-based system demand shortfall for the system, for the three valving scenarios ( $p \leq 0.001$ ). 356 The standard error for  $N$  and  $N - 1$  valving scenarios is 2 percent, and for the scarce valving scenario 357 is 3 percent.

359 Properties of the reachability matrix provide a rough method to quickly evaluate the impacts 360 segment closures or failed valves may have, given a valving scenario of a network. This can help 361 identify areas needing additional study. Alternatively, it can narrow down the number of scenarios 362 to simulate for failure analysis.

### 363 **Impact of Failed (OPEN) valves**

364 Failed valves cause extensive segment shutdowns. The more failed valves there are, the more 365 unintended isolations there will be, and the more valves need to be operated to isolate a given 366 segment. For example, in Figure 11, consider that segment 9 must be isolated but the valve between

367 segment 9 and segment 121 is not operable. Instead of simply closing two valves, between 125  
368 and 9 and 121 and 9, now the valves between 90 and 121 and 6 and 121 must be closed. This also  
369 means that water cannot pass quickly from segment 90 to segment 6. This provides impetus for  
370 identifying valves needed to close tree segments and identifying loops that feed trees.

371 *Identifying Valves Needed to Close Tree Segments*

372 As an example, in Figure 12, if segment 138 must be isolated, the valves between segment  
373 138 and 112 and segment 138 and 31 must be isolated because the water source is to the north of  
374 138. However, valves in the tree downstream of segment 138 do not need to be closed off because  
375 there is not a water source between the segments downstream and 138 itself. This observation  
376 indicates that, in contrast to segments attached only to loops where the sum of the number of valves  
377 determines how many valves must be closed to shut down that segment, when some segments are  
378 the root of a tree, there is no need to shut down valves in the downstream direction. A segment can  
379 be part of a loop on the upstream side but on the downstream side (the side that is isolated from  
380 any source), it can be the beginning of a tree.

381 Interestingly, from the reachability matrix, for segments that are part of loops without being the  
382 beginning of a tree, the importance index values would always be 2. Whereas for segments in loops  
383 that are also the beginning of trees, the importance index values would always be greater than 2.  
384 Segments with importance values of 2 would automatically be eliminated from the set of segments  
385 that are both part of a loop and the beginning of a tree.

386 *Identifying Loops that Feed Trees*

387 For the scarce valving scenario of the case study network shown in Figure 13a, the articulation  
388 points are shown in Figure 13b. While many of the segments in the set of articulation points  
389 are actually important as indicated by their importance index, there are some segments that can  
390 disconnect the graph by only isolating a single segment making them less important (e.g., segment  
391 12 circled in the bottom right of Figure 9b in comparison to segment 60, upstream of it). All  
392 of the segments with valves that do not necessarily need to be shut off are included in the set of  
393 articulation points. But the set of articulation points can also include segments that are not also

394 part of a loop, such as segments 12, 43, 15 and 16 (in Figure 13b). After obtaining the intersection  
395 of the segments in the sets of articulation points and cycle basis, shown in Figures 13c and 13d,  
396 segments that are not part of loops are no longer highlighted.

397 This provides different information than the reachability matrix. Though the location of segment  
398 23 has a high importance index value as shown in Figure 9a and is an articulation point that can  
399 disconnect the WDN, if it must be isolated, all valves (upstream and downstream) must be closed  
400 because it does not have tree segments downstream of it only loops. That is why the location of  
401 segment 23 is not highlighted in Figure 13e, after the location of water sources is accounted for.  
402 This result can help utilities focus on valves that must actually be shut off. As this process is  
403 conducted in python, the programming language used in ArcGISit is easy to integrate in the GIS  
404 systems utilities already use.

## 405 **APPLICATIONS TO REAL SYSTEMS**

406 The proposed methodology has been applied to real WDNs, System 1 and System 2 to maintain  
407 confidentiality. The primary limitation of the reachability matrix methodology occurs when there  
408 are many directional components (e.g., PRVs) because the model assumes flow occurs in both  
409 directions, which occurred with one of the real models tested. However, even without accounting  
410 for directionality, the correlation coefficient between the matrix importance index and system  
411 demand shortfall is comparable to the values obtained by Balekelayi and Tesfamariam (2019)  
412 and Meng et al. (2018). System 1 has two reservoirs, two tanks, six pumps, seven PRVs, three  
413 TCVs, and 1236 isolation valves. The segment-valve topology is displayed in Figure 14a. When  
414 directionality of PRVs was accounted for (i.e., a directed graph) in generating the reachability  
415 matrix, the correlation coefficient between the importance index and WaterGEMS criticality of  
416 segments is higher than that obtained by Balekelayi and Tesfamariams methodology ( $r = 0.6$  as  
417 opposed to 0.37) while using a larger network (System 1). Similarly, the correlation coefficient is  
418 higher than most of the coefficients reported by Meng et al. (2018).

419 When the data from the boundary elements (e.g., directional elements, reservoirs, tanks) is not  
420 included in the analysis, the correlation coefficient becomes more than double ( $r = 0.86$ ). However,

421 as can be seen from the scatterplot in Figure 14b, the matching between the importance index and  
422 system demand shortfall is more accurate for higher values of importance. Utilities already know to  
423 pay attention to the segments with boundary elements and they do not need a preliminary screening  
424 to indicate that these elements are important.

425 System 2 has two reservoirs, four tanks, six pumps, 1730 isolation valves, a PRV, and a PSV. For  
426 this WDN, a correlation coefficient of 0.89 is obtained between the importance index and system  
427 demand shortfall after removing the segments with boundary elements from the analysis. This  
428 suggests that proximity to the boundary elements may also impact how accurately the importance  
429 index of a segment represents its actual importance. More experiments with water distribution  
430 networks containing directional elements must be conducted to discern this.

431 For different valving scenarios of the real networks, the indexes from the matrix were able to  
432 capture that the more valving there is in the network (applied according to rules such as  $N$  and  
433  $N - 1$ ), the less vulnerable and important the most vulnerable and important segments become for a  
434 given network which has been shown before in Liu et al. (2017). See the supplementary information  
435 for box plots from the analysis of variance between valving scenarios of a given network.

436 These results indicate that graph theory measures can in fact be used for preliminary screen-  
437 ing for failure scenario analysis or initial design. As it pertains to the articulation points, and  
438 identification of segments that have valves downstream that do not necessarily need to be oper-  
439 ated, the physical correspondence between the WDNs and the graph theory measures indicates  
440 that the measures can still be used for real systems, when quick screening is needed to facilitate  
441 decision-making.

## 442 **SUMMARY**

443 Using a segment-valve topology is more powerful than a pipe-junction topology when analyzing  
444 the reliability of water distributions systems. This study advanced the state-of-the-art in reliability  
445 analysis based on segments and valves in several areas by developing:

446 A method to quickly display segment and valve topology in a way that makes it easy to  
447 graphically identify and display important and vulnerable segments,

448 A method to quickly screen a network to identify important and vulnerable segments that not  
449 only quantifies these properties but also accounts for whether water sources are continuous sources  
450 with relatively continuous supply vs. those (e.g. tanks) that are ephemeral in nature and can only  
451 serve as a supply for a limited time,

452 A method to identify the number of valves to be operated to isolate a segment depending on  
453 the topology of the system, and A method to identify segments that will be isolated when other  
454 segments are closed.

455 The methods are both theoretically justified and useful for practicing engineers and operators.  
456 Applying CNA to the segment-valve topology demonstrates the impact certain valve failures may  
457 have on segment isolation and helps identify valves that do not need to be closed when segments  
458 must be isolated. While the pipe-junction topology of WDNs is necessary for design and hydraulic  
459 analysis, the segment-valve topology provides insight that may provide decision-support for utilities  
460 attempting to ensure their system is resilient. Future research can expand on the current study  
461 by including population-based metrics and likelihood of failure in the analysis. More WDN  
462 configurations under a wide array of loading conditions can also be tested to more clearly establish  
463 limitations of the methods.

## 464 DATA AVAILABILITY STATEMENT

- 465 • All data, models, or code generated or used during the study are available from the corre-  
466 sponding author by request.
- 467 • Some or all data, models, or code generated or used during the study are available in a  
468 repository or online in accordance with funder data retention policies (<https://github.com/N->  
469 abdel/segment-valve-visualization).

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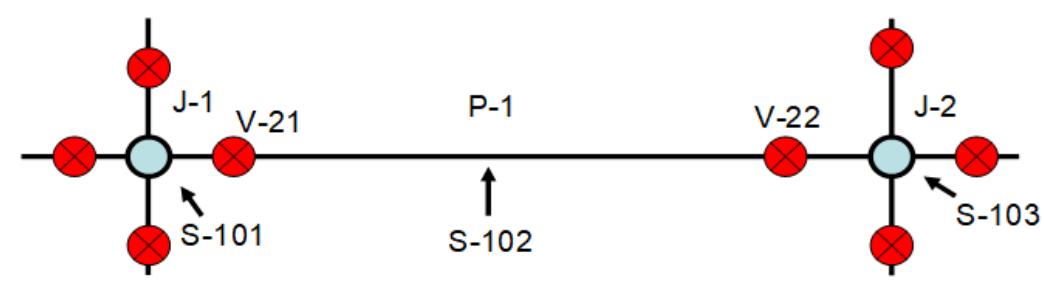
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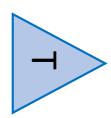
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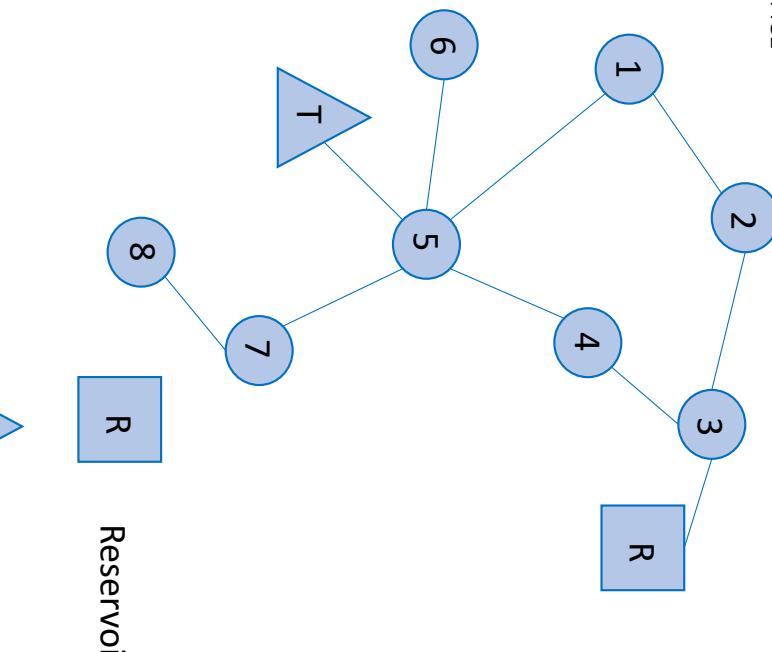
(a)

Isolation  
valve

Segment



Tank

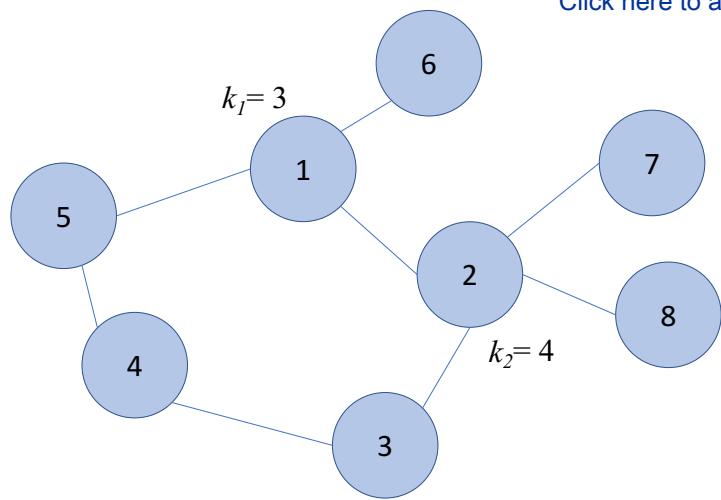


Isolated

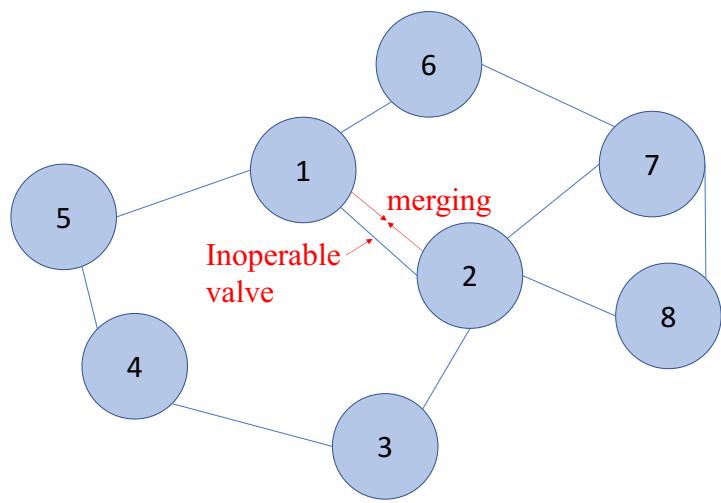
Affected

Segment	1	2	3	4	5	6	7	8	Importance
1	2	0	0	0	0	0	0	0	2
2	0	2	0	0	0	0	0	0	2
3	1	1	2	1	1	1	1	1	9
4	0	0	0	2	0	0	0	0	2
5	0	0	0	0	2	2	2	2	8
6	0	0	0	0	0	2	0	0	2
7	0	0	0	0	0	0	0	2	4
8	0	0	0	0	0	0	0	2	2
Vulnerability	3	3	2	3	3	5	5	7	

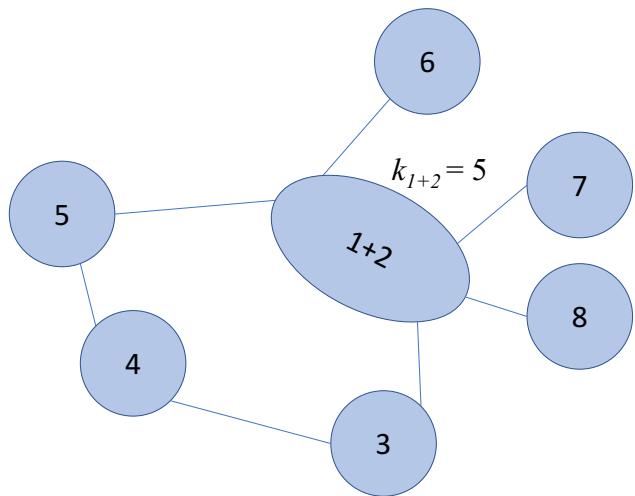
(b)



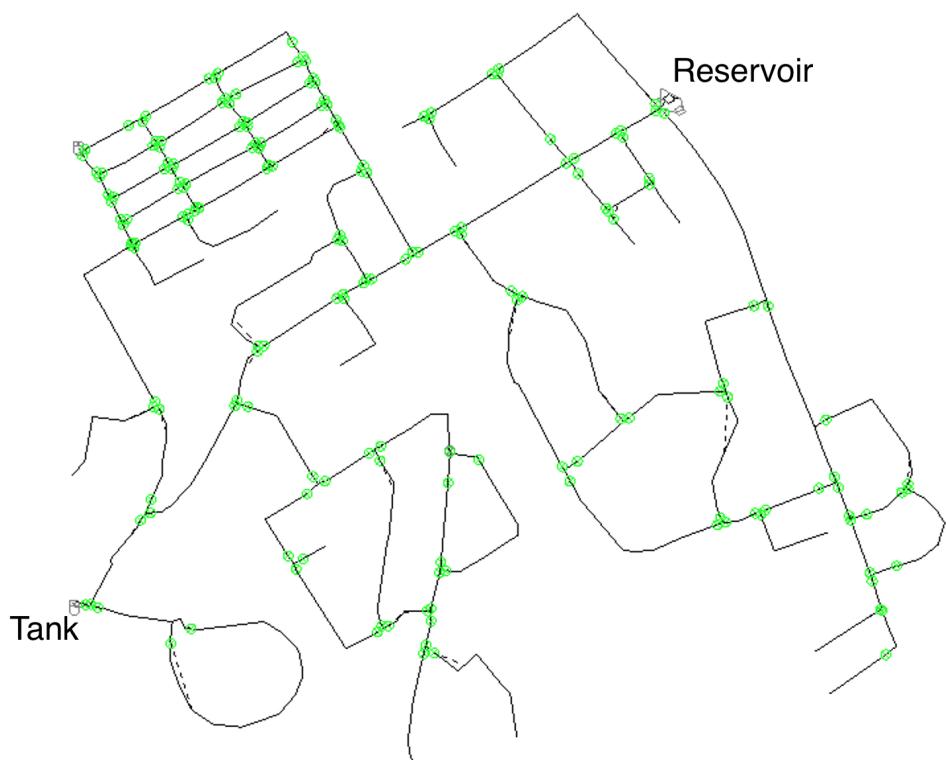
(a)

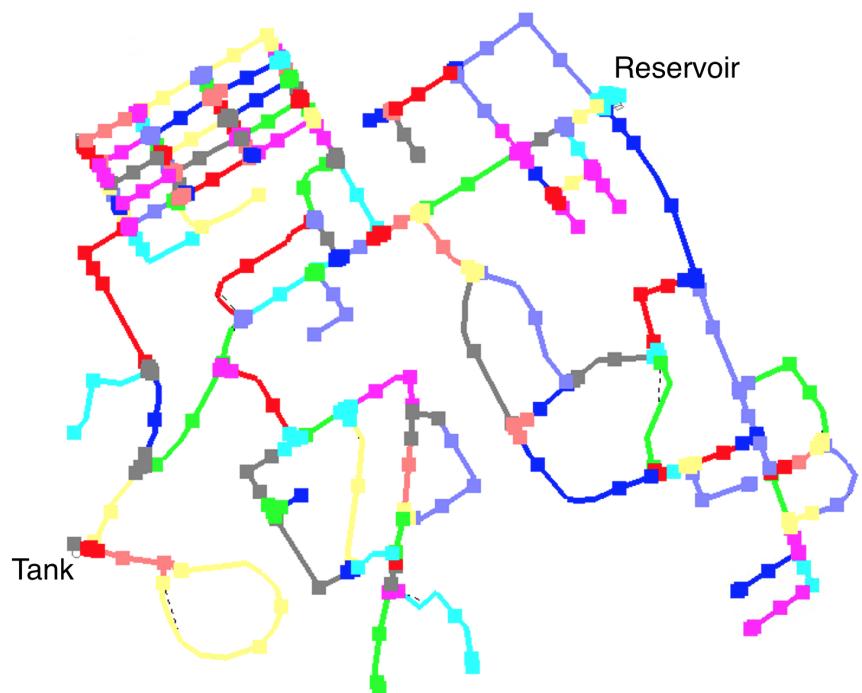


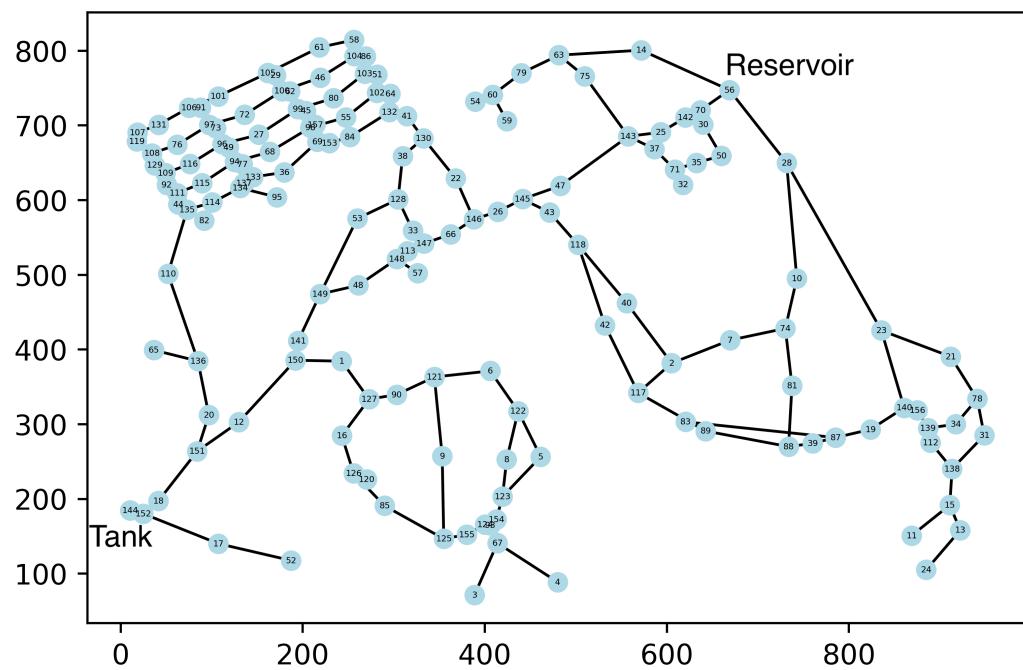
(b)

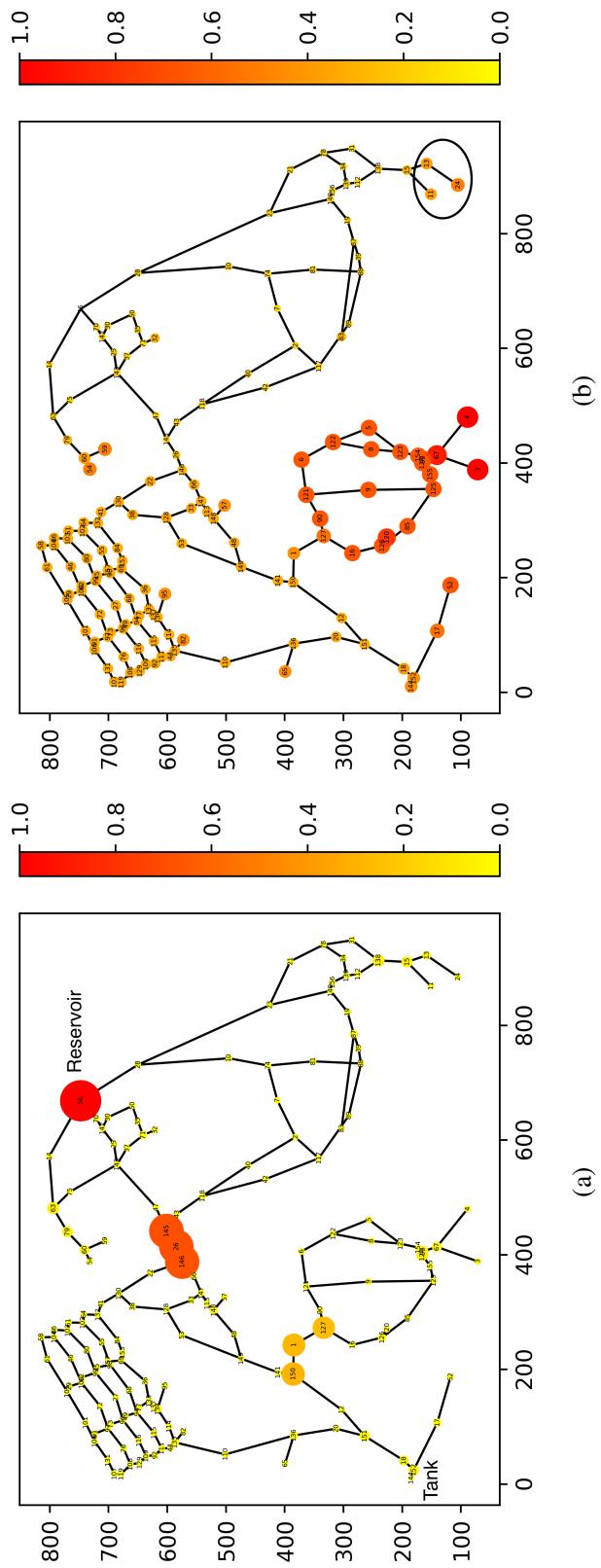


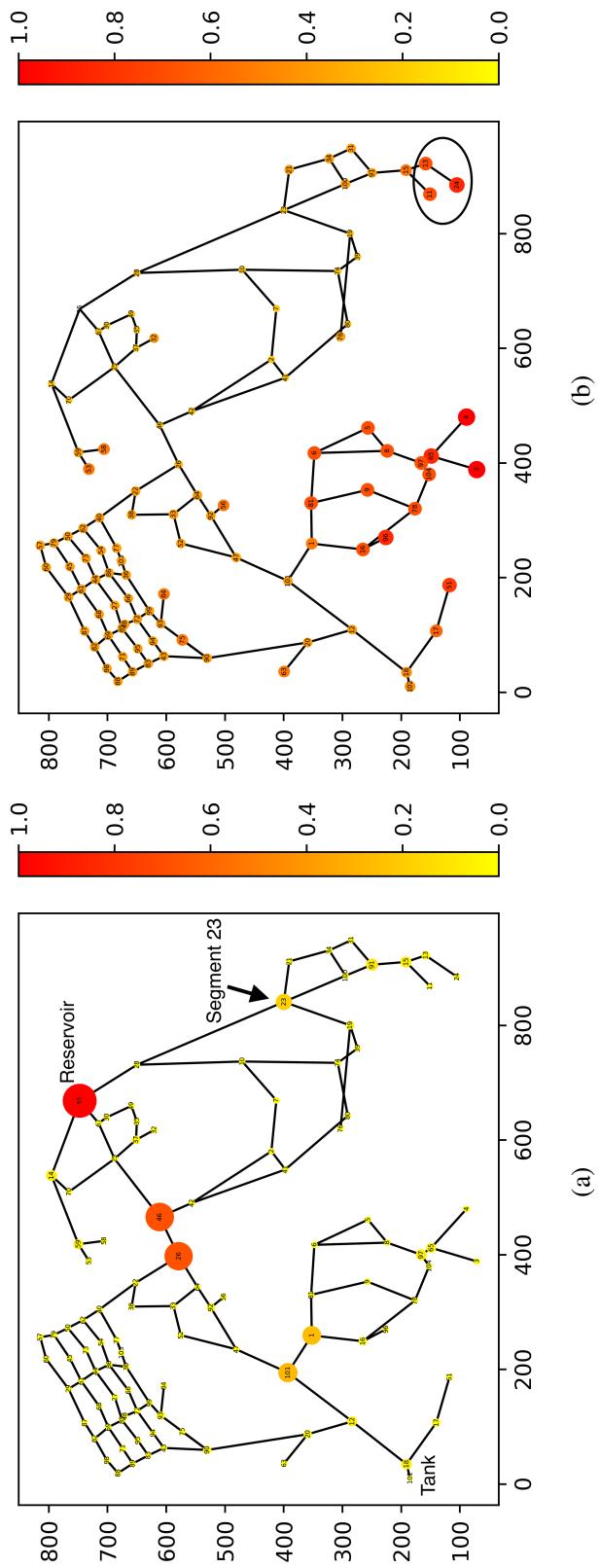
(c)

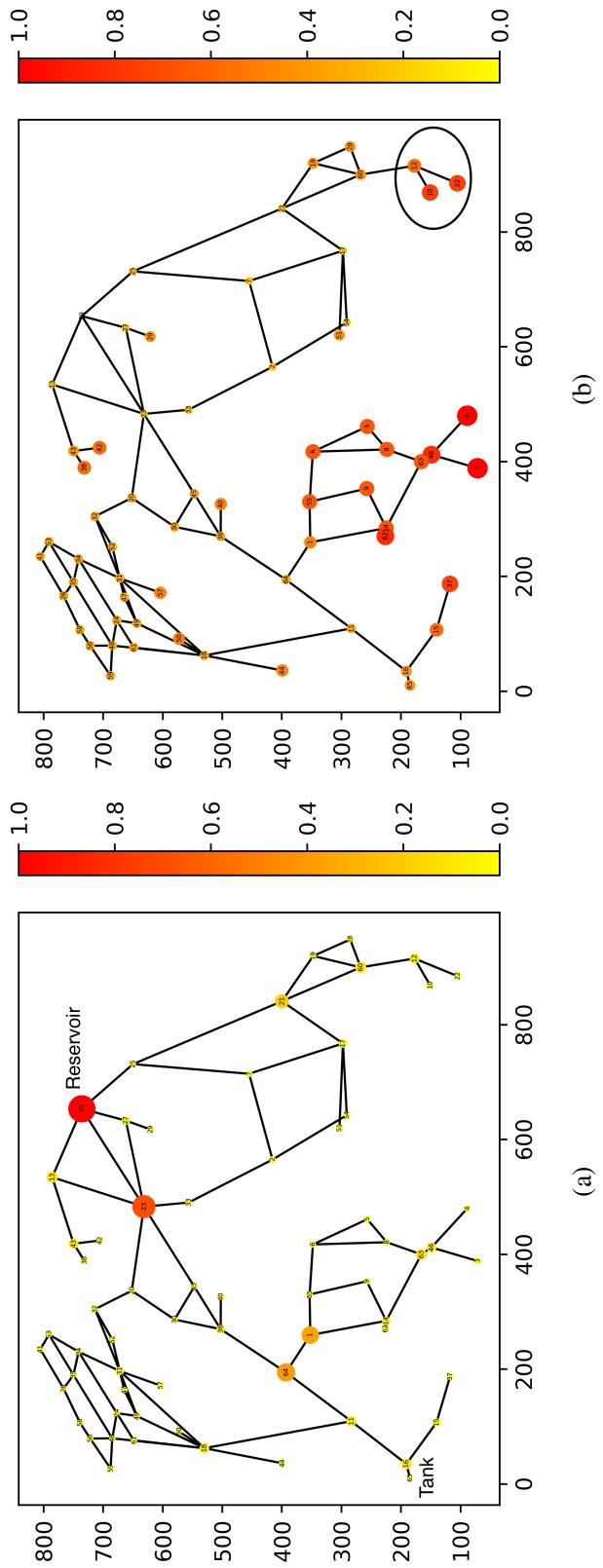


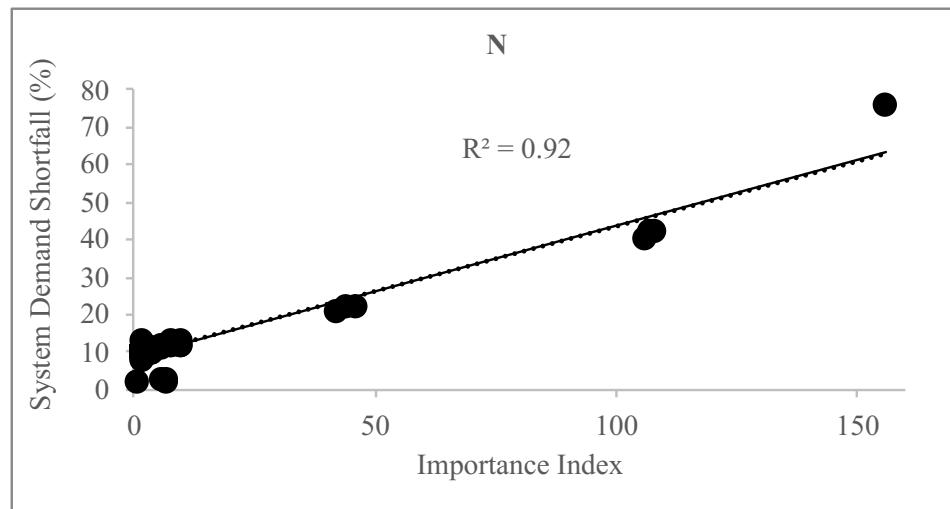




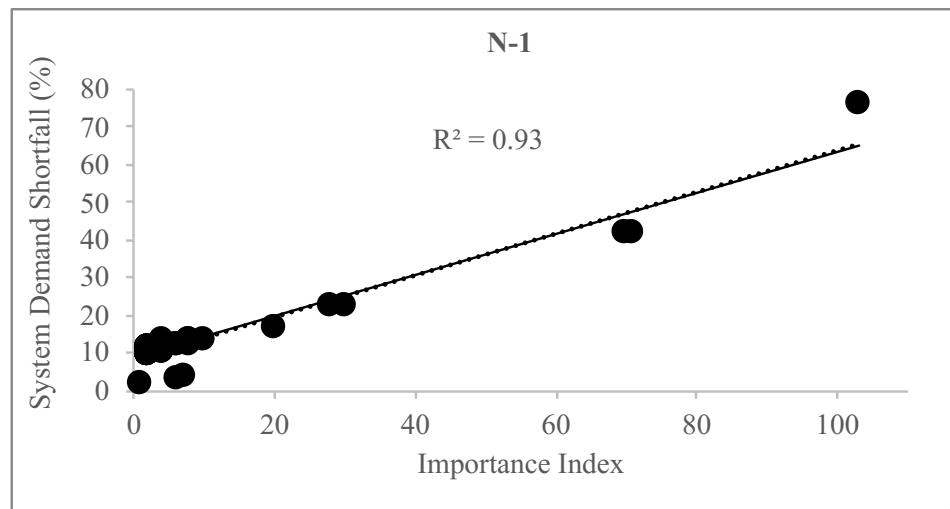




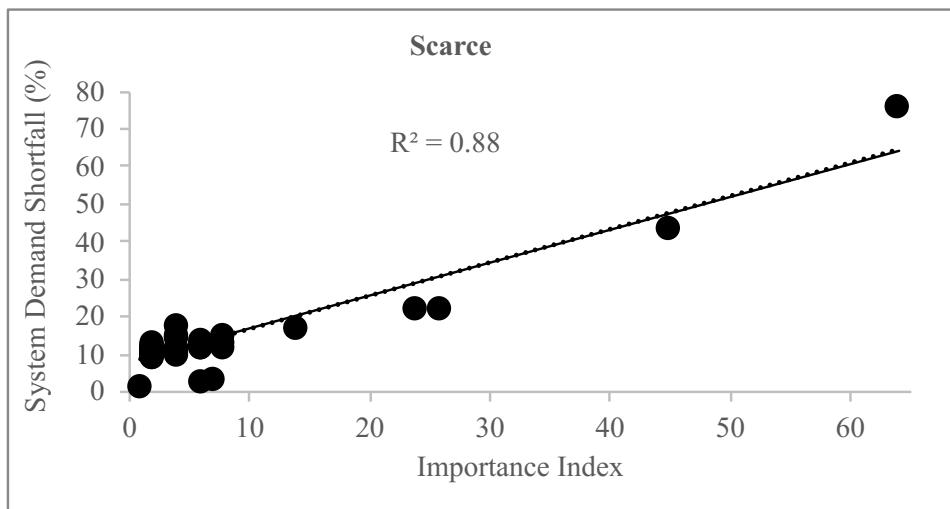




(a)



(b)



(c)

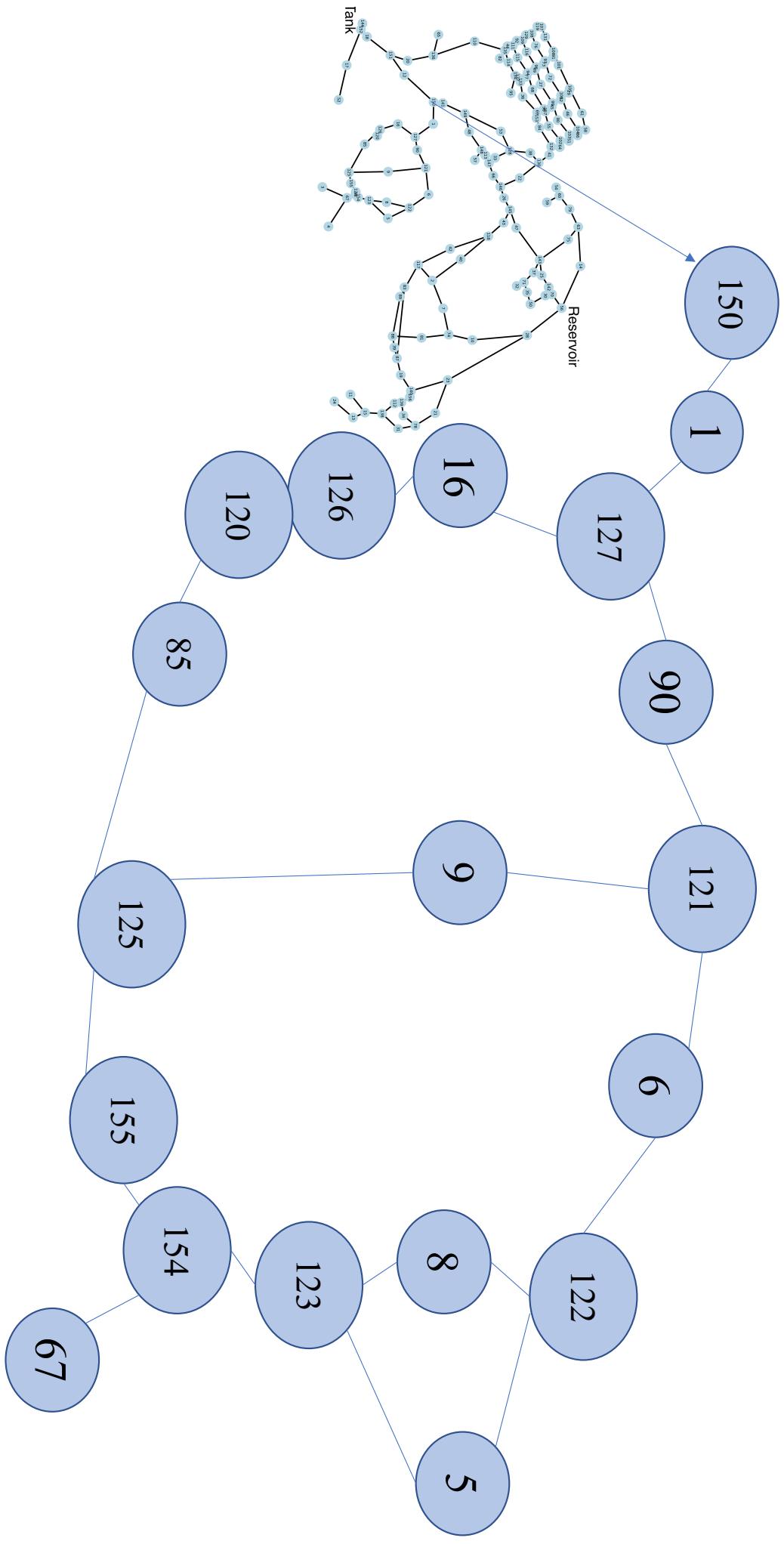
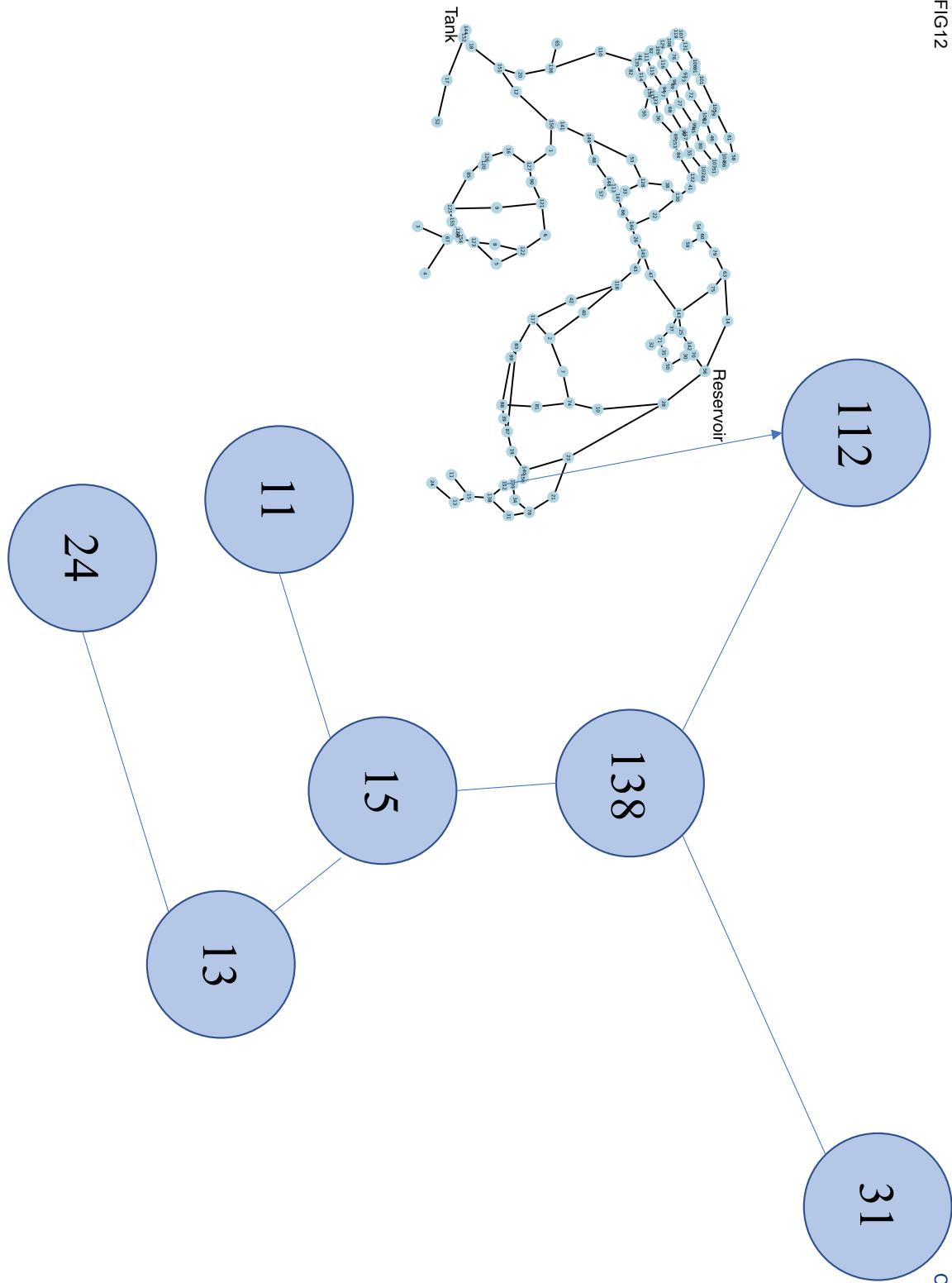


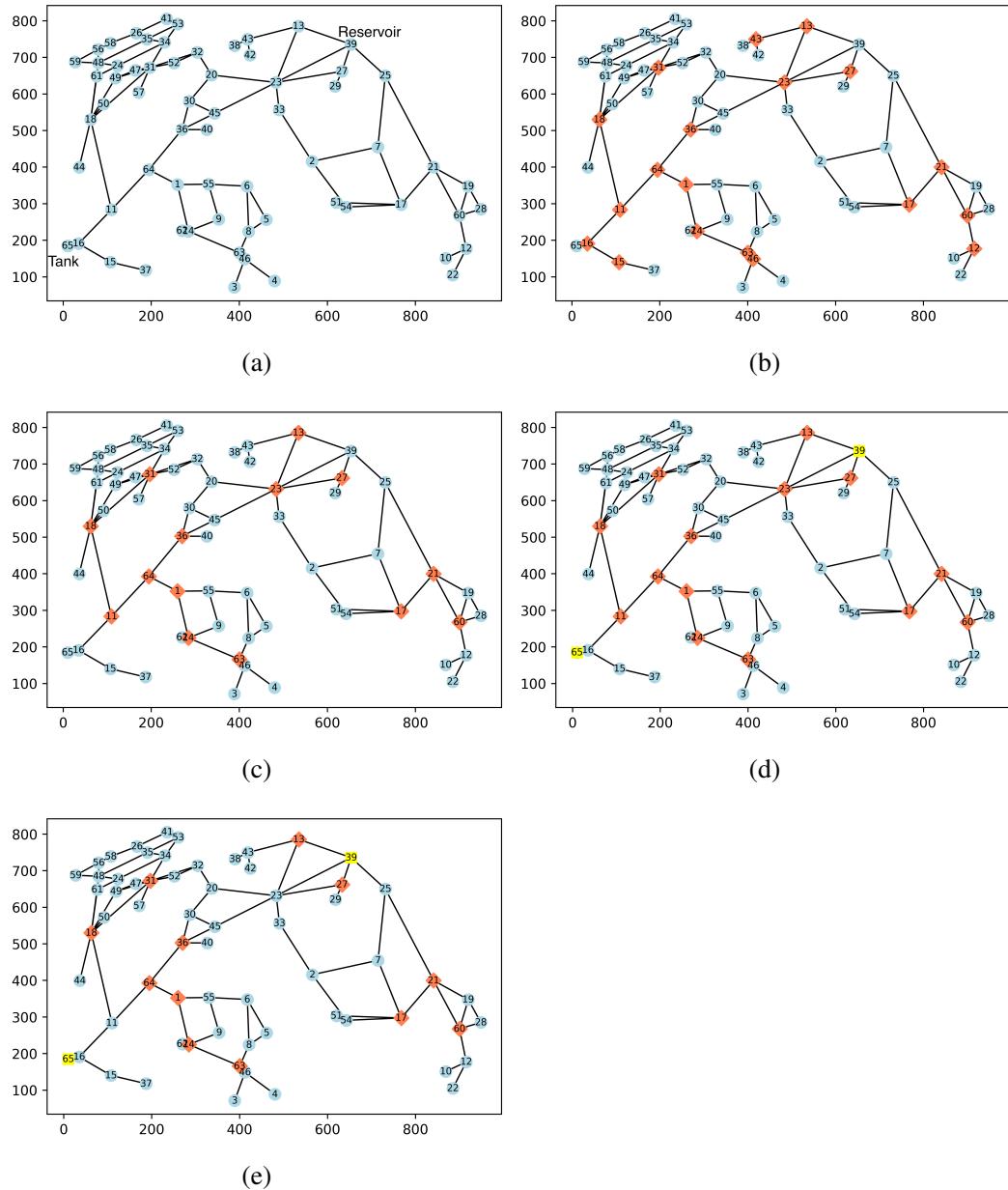
FIG12



Click here to access/download;Figure;FIG12.pdf \*

FIG13

[Click here to access/download;Figure;FIG13.pdf](#) 



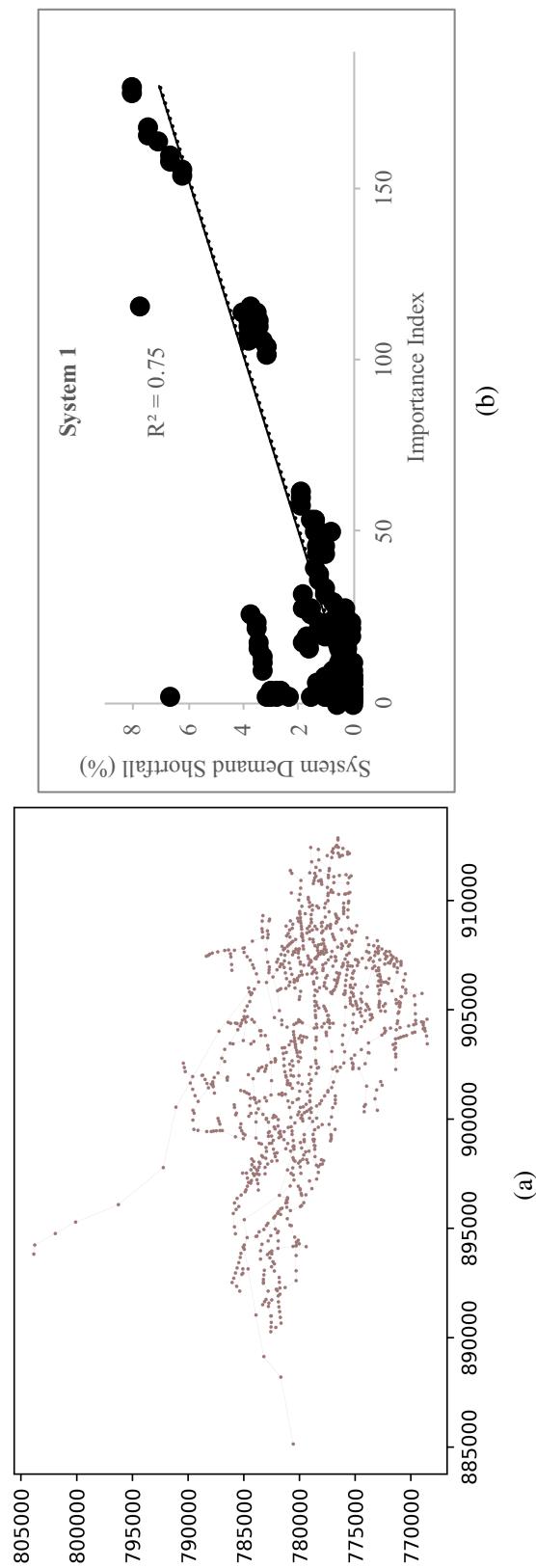


FIG. 1. Illustration of WDN segments, a single pipe can be part of several segments and segments can contain more than one pipe

FIG. 2. (a) Segment-valve topology of a small example (b) Reachability matrix

FIG. 3. Illustration of proof for a small example, showing segments 1 and 2 merging to form a single node in the segment-valve topology

FIG. 4. Pipe-junction topology of N-valve scenario highlighting valve locations

FIG. 5. Segments highlighted on the Pipe-junction topology of N-valve scenario in WaterGEMS

FIG. 6. Segment-valve topology of N valve scenario for the system shown in Figure 4, where segments are nodes and valves are edges

FIG. 7. N valve scenario, where normalized index values are represented in the legend on the right; smaller nodes correspond to lower values and larger nodes correspond to higher values; axes correspond to x,y coordinates of segment centroids (a) Importance (b) Vulnerability

FIG. 8. N-1 valve scenario, where normalized index values are represented in the legend on the right; smaller nodes correspond to lower values and larger nodes correspond to higher values; axes correspond to x,y coordinates of segment centroids (a) Importance, the location of segment 23 increases in criticality from the N valve scenario (b) Vulnerability, the bottom right tree segments increase in vulnerability from the N valve scenario

FIG. 9. Scarce valve scenario, where normalized index values are represented in the legend on the right; smaller nodes correspond to lower values and larger nodes correspond to higher values; axes correspond to x,y coordinates of segment centroids (a) Importance (b) Vulnerability

FIG. 10. Validation of importance index derived from the reachability matrix.

All three valving scenarios' segment importance values significantly correlated ( $p<0.01$ ) with the system demand shortfalls obtained using WaterGEMS

FIG. 11. Zoomed in section of case study network, originating at source segment 150

FIG. 12. Zoomed in section of case study network, originating at source segments 112 and 31

FIG. 13. (a) Segment-valve topology of scarce valving scenario (b) Articulation points denoted with diamond shaped nodes of the scarce valving scenario (c) Intersection of articulation points and segments that are part of the cycle basis denoted with diamond shaped nodes for the scarce valving scenario (d) Intersection of articulation points and segments that are part of the cycle basis, with Reservoir (continuous source) and Tank (ephemeral source) highlighted (Segments 39 and 65, respectively) (e) Segments that are part of loops for which isolation does not necessitate downstream valve closures

FIG. 14. (a) Segment-valve topology of System 1 where the x-axis represents the x-coordinates and the y-axis represents the y-coordinates (b) x-axis represents the importance index for System 1 and the y-axis represents the system demand shortfall, where the correlation coefficient is 0.86 ( $p<0.01$ )