

Drop size distribution broadening mechanisms in a bin microphysics

Eulerian model

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ABSTRACT

In this study, processes that broaden drop size distributions (DSDs) in Eulerian models with two-moment bin microphysics are analyzed. Numerous tests are performed to isolate the effects of different physical mechanisms that broaden DSDs in two- and three-dimensional Weather Research and Forecasting model simulations of an idealized ice-free cumulus cloud. Sensitivity of these effects to modifying horizontal and vertical model grid spacings is also examined. As expected, collision-coalescence is a key process broadening the modeled DSDs. In-cloud droplet activation also contributes substantially to DSD broadening, whereas evaporation has only a minor effect and sedimentation has little effect. Cloud dilution (mixing of cloud-free and cloudy air) also broadens the DSDs considerably, whether or not it is accompanied by evaporation. This mechanism involves the reduction of droplet concentration from dilution along the cloud's lateral edges, leading to locally high supersaturation and enhanced drop growth when this air is subsequently lifted in the updraft. DSD broadening ensues when the DSDs are mixed with those from the cloud core. Decreasing the horizontal and vertical model grid spacings from 100 m to 30 m has limited impact on the DSDs. However, when these physical broadening mechanisms (in-cloud activation, collision-coalescence, dilution, etc.) are turned off, there is a reduction of DSD width by up to ~20-50% when the vertical grid spacing is decreased from 100 m to 30 m, consistent with effects of artificial broadening from vertical numerical diffusion. Nonetheless, this artificial numerical broadening appears to be relatively unimportant overall for DSD broadening when physically based broadening mechanisms in the model are included for this cumulus case.

1. Introduction

Understanding the physical mechanisms responsible for the formation of rain drops in warm clouds remains an open question for the cloud physics community. Adiabatic diffusional growth of droplets in non-turbulent air leads to narrowing of drop size distributions (DSDs), which inhibits collision-coalescence (Pruppacher and Klett 2012). In contrast, observed DSDs are often much wider than DSDs obtained from adiabatic calculations of condensational growth above cloud base (e.g., see Fig. 11 in Jensen et al. 1985). For decades, many explanations have been proposed for bridging this gap between the classical effects of the diffusional and collision-coalescence growth processes (Pruppacher and Klett 2012). These mechanisms usually center on physical processes that broaden DSDs relative to what would occur from only diffusional growth (we will refer to this as DSD “broadening” hereafter). However, this controversial topic continues to be the focus of many modeling and observational studies.

It has been argued that considering aspects such as the solute and curvature effects (Korolev 1995; Yin et al. 2000; Jensen and Nugent 2017; Yang et al. 2018) as well as the radiative cooling (Hartman and Harrington 2005a,b; Lebo et al. 2008) in the condensation calculations are important for explaining observed width of DSDs. On the other hand, several effects have been attributed to turbulence in clouds (Devenish et al. 2012; Grabowski and Wang 2013), such as the generation of larger-scale inhomogeneities that drive the “eddy hopping” mechanism (i.e., DSD broadening by subsequent mixing of droplets that have experienced different condensational growth histories in the clouds) (e.g., Baker et al. 1984; Cooper 1989; Lasher-trapp et al. 2005; Grabowski and Abade 2017; Abade et al. 2018; Dodson and Small Griswold 2019). In addition, by inducing multiscale inhomogeneities in the droplet number concentration (Ayala et al. 2008a; Lu et al. 2010), differences in the horizontal and vertical velocity in a droplet population (Franklin et al.

2005; Pinsky et al. 2006; Dávila and Hunt 2001; Ayala et al. 2008b) and also a reduction in the effect of the aerodynamic interactions between droplets (Pinsky et al. 2007; Wang et al. 2008; Chen et al. 2018), turbulence can enhance the collision rate in clouds and thus broaden DSDs.

The entrainment of dry air and aerosols has also been considered as a source of DSD broadening in clouds. Inhomogeneous mixing of entrained air, where the microphysical response is fast compared to the mixing timescale (in contrast to homogeneous mixing, where the opposite occurs), decreases the droplet concentration (Latham and Reed 1977; Baker and Latham 1979; Baker et al. 1980; Pinsky et al. 2016), favoring the diffusional growth of the remaining droplets when the diluted air ascends. The activation of new droplets from entrained aerosols also broadens the DSD toward the smaller sizes, which can even result in bimodality (Baker et al. 1980; Blyth 1993; Brenguier and Grabowski 1993; Lasher-trapp et al. 2005; Cooper et al. 2013; Hoffmann et al. 2015). However, there is no consensus regarding the role of these processes in the development of observed DSDs (see discussion in Khain et al. 2000). For instance, Khain et al. (2013) suggest that adiabatic processes play a dominant role in DSD evolution and the formation of precipitation from warm processes in deep convective clouds, given observational and modeling evidence showing that the formation of the first raindrops occurs in undilute or mostly undilute cloud cores.

Several modeling studies regarding DSDs evolution in clouds, including Khain et al. (2013), employed two-dimensional (2D) and three-dimensional (3D) Eulerian models with bin microphysics (here by “Eulerian”, we mean using an Eulerian grid discretization both in physical space and mass spectrum space for the microphysical variables). However, Morrison et al. (2018) showed that the numerical diffusion associated with vertical advection in Eulerian models, combined with growth in radius or mass space in bin microphysics schemes, can artificially broaden DSDs. Because ascending/descending air parcels undergo adiabatic cooling/warming from expansion/compression, droplets moving up/down in cloudy parcels simultaneously experience condensation/evaporation.

Mixing associated with numerical diffusion from *vertical* advection (hereinafter, vertical numerical diffusion) does not account for the growth/shrinkage of droplets in ascending/descending air and thus constitutes an unphysical DSD broadening mechanism. According to Morrison et al. (2018), the effects of unphysical DSD broadening from vertical numerical diffusion are more pronounced at lower vertical resolutions. They suggested this numerical broadening may compensate for under-represented horizontal variability and mixing of different droplet populations in typical large-eddy simulation (LES) configurations, or the neglect of other physical mechanisms that could broaden DSDs in real clouds.

In this paper, we investigate how physical DSD broadening processes are represented in an Eulerian bin microphysics model. This is studied using idealized 2D and 3D simulations of an isolated warm (ice-free) cumulus cloud. The overall goal of this study is to investigate the degree to which physical broadening mechanisms contribute to DSD evolution in the model. The contributions of various processes to DSD broadening and evolution are identified with various sensitivity tests, including evaporation, in-cloud droplet activation, collision-coalescence, turbulent mixing, and cloud dilution from entrainment and mixing with cloud-free air. We follow the approach of “mechanism denial” experiments, whereby specific processes are turned off individually or in combination to understand their influence. We also examine the role of model grid spacing on these contributions. In this way, we examine possible effects of artificial DSD broadening from vertical numerical diffusion, given the strong sensitivity of this broadening to the model vertical resolution.

2. Model description

In this study, we focus on DSD evolution simulated by the Weather and Research Forecasting (WRF) model (Skamarock et al. 2008) with a two-moment bin microphysics scheme.

As initial conditions, we use vertical profiles of potential temperature and water vapor mixing ratio from an atmospheric sounding launched at 1730Z on 11 September 2014 from Manacapuru, Brazil (Holdridge et al. 2014) (Fig. 1), as part of the Observations and Modeling of the Green Ocean Amazon (GoAmazon2014/5) Experiment (Martin et al. 2016). This allows us to compare the model output with the in-cloud measurements obtained by the coordinated ACRIDICON-CHUVA Experiment (Wendisch et al. 2016; Machado et al. 2014). A spheroid-shaped (4-km horizontal- and 1.5-km vertical-axis length) warm bubble is applied to force convective initiation. The potential temperature (θ) perturbation is +3 K at the center of the bubble and decreases according to a cosine function toward its edges.

Subgrid-scale turbulent mixing is included in the model by evaluating the diffusion terms in physical space using a 1.5-order closure for the turbulent kinetic energy equation (Skamarock et al. 2008). Additionally, to enhance resolved turbulent motions, some simulations (e.g., TURB in Table 1) apply small random perturbations (± 0.05 K) to the initial θ field. This configuration rapidly (within the first ~ 5 min) produces turbulent-like motion and a $-5/3$ slope of the kinetic energy spectrum (Peters et al. 2019, see Fig. 3 therein).

In both 2D and 3D simulations, we use a 20-km horizontal domain with open lateral boundary conditions. The top of the model vertical grid is located at 10 km height, with a 3-km damping layer, where the vertical grid-spacing is coarser than at lower levels for a more efficient use of computational resources. We analyze the simulations before the cloud reaches the bottom of the damping layer.

In the text, we refer to the model resolution in a “(vertical grid spacing) $\times$ (horizontal grid spacing)” format, where the grid spacings are listed in meters; however, the value of the vertical grid spacing specified in this way represents only an approximate value as the vertical grid spacing slightly decreases with height below the damping layer. Four resolution configurations are employed:

100×100, 30×30, 30×100 and 100×30. A 1-s time step is used for the 100×100 simulations. For the simulations with a grid spacing of 30 m in at least one spatial dimension, the time step was set to 0.25 s.

We use the Tel Aviv University (TAU) size-bin-resolved microphysics, which solves for two moments of the DSD in each of the bins thus limiting numerical diffusion across bins from growth processes (Tzivion et al. 1987, 1989; Feingold et al. 1988; Stevens et al. 1996a). In this version of the TAU microphysics, the DSD is divided into 35 mass-doubling bins with radii ranging between 1.56 μm and 5080 μm , approximately. The method of moments is used to compute mass and number mixing ratios in each size bin resulting from collision-coalescence (Tzivion et al. 1987; Feingold et al. 1988). Turbulent enhancement of collisions is not considered in the calculations of collision-coalescence here. To calculate the diffusional growth and evaporation of droplets, the top-hat method of moments from Stevens et al. (1996a) is employed. Solute and curvature effects on cloud droplet growth are neglected. Aerosols are represented by a constant bulk number concentration set to 200 cm^{-3} in our simulations. The aerosols are assumed to follow a log-normal size distribution, with a geometric mean radius of 0.08 μm and geometric standard deviation of 1.9 μm . The hygroscopicity¹ of the aerosols (κ) is 0.1, based on previous studies of aerosol properties over the Amazon (Gunthe et al. 2009; Martin et al. 2010; Pöhlker et al. 2016). In this version of the TAU scheme, new droplets are activated whenever the number mixing ratio of activated cloud condensation nuclei (CCN) exceeds the number mixing ratio of existing cloud droplets (N_d). Then, the change in N_d due to activation $\left(\frac{dN_d}{dt}\right)_{act}$, is

$$\left(\frac{dN_d}{dt}\right)_{act} = \begin{cases} \frac{(N_a - N_d)}{\Delta t} & N_a \geq N_d \\ 0 & N_a < N_d \end{cases}. \quad (1)$$

¹The hygroscopicity parameter here corresponds to the B parameter in Hudson and Da (1996), which is a particular case of the κ parameter in Petters and Kreidenweis (2007).

where Δt is the model time step. N_a represents the number mixing ratio of aerosols with radii larger than a critical radius (r_c), for a given temperature (T) and supersaturation (S). According to Köhler theory:

$$r_c = \sqrt[3]{\frac{4A^3}{27S^2\kappa}} \quad \text{with} \quad A = \frac{2\sigma M_w}{RT\rho_w}, \quad (2)$$

where σ is the surface tension of the solution/air interface, M_w is the molecular weight of water, R is the universal gas constant and ρ_w is the density of liquid water. All newly activated droplets are placed in the smallest size bin (droplet mass between 1.6×10^{-11} g and 3.2×10^{-11} g, approximately); thus, we do not explore the effects of large aerosol and GCCN.

Note that, by not explicitly representing aerosol scavenging, this method can overestimate the activation rate in the model (Hernández Pardo et al. 2019) compared to a real cloud with the same background aerosol field. Thus, this simplified approach can lead to an artificial enhancement of droplet activation when used in combination with N_d sink processes. For instance, if the rate of collision-coalescence reduces N_d below the diagnosed N_a then activation will occur, which can broaden the DSDs. Here, we test different idealized configurations to understand the effects of activation on DSD broadening (see tests ACT and CBACT described in Table 1).

3. Droplets in an Eulerian-model rising thermal

To characterize the modeled cloud microphysical evolution, we use N_d , the mean droplet diameter (D_m), the DSD standard deviation (σ), the difference between the sizes of the 99th and 50th percentiles of the DSD (ΔD_{99}), and the collision-coalescence rate (CCrate) computed as the total (summed across bins) number mixing ratio of droplets collected per second. For all subsequent analysis and plots, cloudy points are defined as those with $N_d \geq 1 \text{ mg}^{-1}$ (this threshold defines our “cloud mask”). Figure 2 illustrates the distribution of these variables in vertical cross-sections located at the center of the 3D-modeled cloud at different times. For complementing the

analysis, Fig. 3 shows the distributions of supersaturation and the droplet activation and condensation/evaporation rates. The latter indicates the rate of change of the mean droplet size at a given point due to diffusion of water vapor to/from the cloud droplets.

Overall, the values of N_d ($\sim 200 - 250 \text{ mg}^{-1}$) and D_m ($\sim 10 - 30 \text{ }\mu\text{m}$) shown in Fig. 2 are reasonably consistent with the observations of clouds over the Amazon forest obtained during the same day of the sounding employed to initialize the model (Table 2 in Cecchini et al. 2017). However, relatively large values of N_d ($> 250 \text{ mg}^{-1}$) as well as small values of D_m ($< 20 \text{ }\mu\text{m}$) are found near the top of the modeled cloud, a feature that is accentuated at later times. Figure 3 shows that the more intense activation events are located near the boundaries of the cloud, especially near its top. Given that N_a represents an effective upper limit to N_d , the fact that cloud top values of N_d exceed N_a are suggestive of numerical errors²; monotonicity is not preserved for integral quantities of the DSD when bin variables are advected individually even when using a monotonicity-preserving advection scheme (Ovtchinnikov and Easter 2009).

Figure 3 also shows that the activation rate increases in the upper part of the cloud core, especially after 10 min, coinciding with the onset of collision-coalescence (Fig. 2). This occurs because collision-coalescence reduces N_d , which increases the phase relaxation timescale, resulting in an increase in the supersaturation particularly in the updraft core where there is strong vertical motion consistent with Hall (1980). The decrease of N_d and consequent increase in supersaturation from collision-coalescence, such that N_d falls below the diagnosed number of activated CCN, leads to activation of new droplets. This effect causes a strong reduction of D_m near cloud top. It also manifests in the maxima of σ and ΔD_{99} in the upper part of the cloud seen in Fig. 2c,d; the DSDs are broadened toward smaller and larger sizes from activation and collision-coalescence,

²Note that, while the aerosol number concentration is constant and equal to 200 cm^{-3} in these simulations, N_a (kg^{-1}) varies, mainly with height, according to variations in the density of the air.

respectively. Large supersaturations in these regions of collision-coalescence and strong upward motion also lead to large condensation growth rates (Fig. 3c).

Figure 4 illustrates the cloud water mixing ratio and wind field for vertical cross-sections of the 3D simulation, as well as the analogous 2D simulation. The thermal-like structure of the updraft (Levine 1959; Turner 1963) is seen in both the 3D and 2D simulations. A primary spherical vortex-like circulation (“toroidal circulation” hereafter) drives inflow near cloud base and outflow near cloud top, as well as the entrainment of environmental air through the lateral edges of the cloud, most evident at a height approximately one-half the cloud depth (e.g., seen in Fig. 4a3,b3 at approximately 3-4 km height). Secondary toroidal circulations that develop near cloud top (e.g., seen in Fig. 4a3 at approximately 5.5 km height) also favor the intrusion of dry air into the cloud.

Because the equivalent potential temperature (θ_e) is conserved for moist adiabatic processes, it serves as a proxy for the degree of mixing with environmental air and dilution that cloud air parcels undergo as they rise from cloud base (i.e., smaller θ_e implies greater dilution because mid-tropospheric θ_e is smaller than θ_e at cloud base). The $\theta_e = 350$ -K isoline is represented by black contours in Fig. 4. Note that changes in the area encompassed between the $N_d = 1\text{-mg}^{-1}$ and $\theta_e = 350$ -K isolines in Fig. 4 reflect the increase in cloud dilution with time. Dilution is discussed further in the context of an analysis of θ_e in section 4c.

Changes induced in the model dynamics by reducing the number of spatial dimensions from three to two lead to some differences between the simulations. Firstly, as a consequence of its weaker updraft, the cloud develops later in 2D than in 3D. This is consistent with previous studies that compared the updraft strength in 2D and 3D simulations, for the same environmental and initial conditions (e.g., Wilhelmson 1974; Tao et al. 1987; Phillips and Donner 2006; Zeng et al. 2008). The studies of Morrison (2016a,b) showed that differences in vertical velocity between 3D and 2D arise directly from the differences in the continuity equation for each geometry. Figure

4 also shows that, from the initial time of the cloud development, the toroidal circulation of the thermal is stronger in the 2D simulation and, consequently, the cloud appears to entrain more than in 3D, as indicated by the shape of the cloud area encompassed by the $\theta_e = 350$ -K isoline. A stronger toroidal circulation in 2D than 3D is consistent with greater vertical pressure gradient forcing (Morrison 2016a,b) and regions of divergence near cloud top and convergence near cloud base in 2D.

Despite the dynamical differences between the 2D and 3D simulations, their cloud microphysical and macrophysical features are similar (Fig. 5). The largest differences occur in the position of the eddies and lobes near cloud top, but the DSD characteristics inside these cloud lobes are similar in 2D and 3D. Figure 6 shows the DSDs at different heights at the center of the domain (x_o) and 1 km to the left ($x_o - 1$ km), from the 2D and 3D simulations. DSD broadening with height is evident, including the development of bimodalities. Overall, the DSD evolution is similar in the 2D and 3D simulations. Note that because of differences in the location of the cloud lobes along the cloud top, the broadest DSDs are located at x_o in the 3D simulation, and in $x_o - 1$ km in the 2D simulation. Similarity between DSDs in the 3D and 2D simulations is further illustrated in Fig. 7. It shows that the mean profiles of N_d , D_m , σ , ΔD_{99} , and CCrate, analyzed separately for high- and low-dilution cloud regions (separated by the 350-K θ_e isotherm), are similar in 2D and 3D. Differences in the height of the cloud lead to some quantitative differences between the 2D and 3D profiles, mainly toward cloud top, but this does not affect conclusions regarding the vertical distributions of the DSD characteristics.

Given the qualitative consistency between the results obtained with the 3D and 2D configurations of the model, specifically those regarding the microphysical characteristics of the cloud, we employ the latter to explore mechanisms of DSD broadening in the WRF-TAU framework. Being computationally cheaper and easier to analyze and interpret results, the 2D configuration is used

to perform numerous sensitivity tests varying the resolution and turning off the representation of various physical processes that affect the DSDs. Having fewer degrees of freedom, 2D simulations can respond differently to a given change in the model configuration or forcing than 3D simulations. For instance, Wang and Sobel (2011) found that precipitation increases more rapidly in 2D than in 3D as sea surface temperature is increased. However, we expect the sensitivities to have the correct sign, although their magnitude might be somewhat different in 2D than 3D.

4. Role of DSD broadening mechanisms at different model resolutions

In this section, we analyze the role of different mechanisms in broadening the DSDs at two different grid resolutions in the 2D version of the model. There are several potentially important processes for DSD evolution in the simulations. These include collision-coalescence, sedimentation, evaporation, resolved “turbulent-like” transport³, and in-cloud droplet activation. We analyze different sensitivity tests including or neglecting the effects of these processes. Table 1 summarizes the sensitivity tests discussed in this section. Note that here “in-cloud droplet activation” refers specifically to the activation that occurs above either the cloud base or the 2000-m height model level. For each time step and each point in the horizontal grid, the cloud base is defined as a 120-m layer starting at the first model level, from bottom to top, where activation occurs (provided that its height does not exceed 2000 m).

In all of the simulations, transport and mixing of model fields occurs by resolved (grid-scale) transport and parameterized sub-grid scale mixing, with contributions from numerical diffusion associated with the advection calculations. However, the resolved flow is much more turbulent in the TURB and CTRL cases, in which random initial θ perturbations are applied, than in other

³Because these simulations are 2D, “turbulent-like” may be a more appropriate way to describe the smaller-scale eddies in them; 3D turbulence of course has notably different properties from 2D. However, hereafter we will refer to these motions simply as “turbulence” for brevity.

simulations. Therefore, we study the effects of resolved turbulent transport on DSD evolution by comparing simulations with and without θ perturbations, keeping in mind that the resolved flow also affects the subgrid mixing directly by impacting spatial gradients of model fields. By “effects of turbulence”, we specifically mean the effects via resolved turbulent transport and mixing of model fields, not considering the effects of turbulence on collision-coalescence, small-scale fluctuations of supersaturation, droplet clustering, etc. For simplicity, we will refer to the simulations without initial random θ perturbations as “laminar”. In effect, the spinup of turbulent motion in the laminar simulations is greatly delayed so that the flow remains quasi-laminar over the entire simulation time.

To retain dynamical consistency between the varying- N_d and the fixed- N_d tests (ND100 in Table 1), we neglect the weight of condensate in the calculation of buoyancy for the tests shown in this section. Although there remain some cloud structure and dynamical differences among the simulations, particularly when the model grid spacing is altered, these differences are generally small. We note that the “piggybacking” methodology (e.g., Grabowski 2014) could be used to separate the impacts of dynamical and microphysical changes clearly, but we leave such an effort to future work.

Figure 8 shows the microphysical properties of the cloud at $t = 16$ min for the 100×100 and 30×30 simulations, where all the processes mentioned in Table 1 are considered (CTRL simulations). This configuration is similar to those discussed in the previous section except that the weight of the droplets is neglected in the buoyancy calculation here. This has the effect of increasing the vertical velocity and increasing the physical size of the cloud.

Decreasing the grid spacing from 100 m to 30 m, in the CTRL simulations, allows the model to better represent the fine-scale structure of the cloud, especially the sub-cloud-scale turbulent-like eddies. However, there is generally little impact on the cloud microphysical and DSD structure

overall, seen by comparing the top and bottom panels in Fig. 8, with one important exception: both σ and ΔD_{99} are significantly smaller in the high-resolution simulation compared to the low-resolution simulation below ~ 2.5 km. This is a region less affected by entrainment, evaporation, and collision-coalescence. As we will show later, this feature is seen in all simulations and is mainly associated with changes in vertical rather than horizontal model grid spacing. Narrowing of the DSDs with decreasing vertical grid spacing is discussed further in section 4d.

a. Effects of collision-coalescence, evaporation, and turbulent transport

Collision-coalescence plays a dominant role in the evolution of the DSDs in our simulations. This is demonstrated by the CCS test, in which collision-coalescence and sedimentation are included but evaporation is neglected and the flow is laminar (note that additional simulations indicate little impact from also turning off sedimentation). Figure 9 shows vertical cross-sections of model fields from the CCS simulations at high and low model resolutions, in the same format as Fig. 8. With laminar flow, the cloud is nearly symmetric, especially at the lower resolution. Otherwise note the similarity between the distributions of DSD parameters in the CCS simulations and those in the CTRL case. Like in CTRL, the CCS simulations reproduce the mechanism of in-cloud droplet activation induced by the reduction in N_d from collision-coalescence discussed above. This leads to a strong reduction in D_m and to an increase in N_d and ΔD_{99} near cloud top similar to CTRL. It is interesting that the largest values of σ are found at the boundary between the regions containing larger and smaller D_m values, respectively, suggesting the role of mixing in broadening the DSDs in the model (an aspect discussed in detail in section 4c). Similar to CTRL, decreasing the model grid spacing from 100 m to 30 m for the CCS case does not have any major influence on the characteristics of the simulated DSDs other than better resolving small-scale details in the

microphysical and macrophysical cloud structure and the narrowing of DSDs below 2.5 to 3 km with higher resolution.

Consistent with our conclusion from the CCS simulation regarding the dominant role of collision-coalescence on DSD evolution, turning off collision-coalescence leads to large changes in the cloud microphysical structure. In the TURB test, which neglects collision-coalescence, sedimentation and evaporation but has a turbulent-like flow, in-cloud activation is greatly reduced compared to CCS and CTRL. Thus, the broad region of small D_m , large σ , and $\Delta D_{99} > 30 \mu\text{m}$ in the upper part of the cloud from collision-coalescence and its inducement of droplet activation in CCS and CTRL does not occur in TURB (compare Fig. 10 with Fig. 8). Instead, the largest values of ΔD_{99} are located at the interface between relatively undilute cloudy air and the entrained air associated with the circulation of turbulent eddies near cloud top (Fig. 10d). Values of ΔD_{99} are generally greater than $10 \mu\text{m}$ in the upper part of the cloud, where smaller resolved eddies contribute to mixing. This contrasts with the laminar simulations described below that have much smaller values of ΔD_{99} in the upper cloud region. Correspondingly, the largest diagnosed collision-coalescence rates are in eddies near cloud top in TURB, but occur lower down in the cloud core in the other simulations with laminar flow.

The largest values of σ in TURB are located along the cloud boundary near cloud top (Fig. 10c), and are associated with relatively large N_d values. This suggests, indirectly, the role of droplet activation in broadening DSDs at cloud top. Although resolved turbulent motion enhances this feature, it is also seen in the laminar simulations described below. This is evidently caused by cloud dilution that reduces N_d and thereby promotes in-cloud droplet activation in ascending air along the cloud top (similar to the inducement of droplet activation from the reduction of N_d by collision-coalescence discussed above). We emphasize that cloud “dilution” here (and in the rest of the paper) specifically refers to the mixing of cloudy and cloud-free air without

necessarily involving evaporation. When evaporation is turned off, this is analogous to extremely inhomogeneous mixing, whereby N_d is reduced but the DSD mean size and shape are unaffected.

Also note that droplet activation along the cloud top may be augmented by the generation of high supersaturation due to the separated advection of temperature and water vapor (Stevens et al. 1996b; Grabowski and Morrison 2008), with contributions from evaporation as the cloud edge advects into non-cloudy grid cells depending on the ratio of the phase relaxation and advective timescales (Hoffmann 2016). However, sharp peaks in supersaturation right at cloud top are not evident in the supersaturation field (similar to results from the 3D simulation seen in Fig. 3a), outside of a few isolated grid points; thus, this mechanism appears to be relatively unimportant here.

Figure 11 illustrates results from the EVAP simulations, which have laminar flow and neglect collision-coalescence and sedimentation but include evaporation. The largest values of σ ($> 12 \mu\text{m}$) and ΔD_{99} ($> 30 \mu\text{m}$) are primarily restricted to the cloud top. The structure of σ is similar to that in TURB, and as we show later, also similar to tests of the same configuration except with evaporation turned off. As we discussed above, the peak in σ along the cloud top is evidently related to droplet activation that is induced by the reduction of N_d from dilution. Thus, the effects of evaporation on the DSD width appear to be small. Limited sensitivity to evaporation is broadly consistent with an analysis of evaporation rates from the 3D simulation in Fig. 3, which shows that the vast majority of cloudy points undergo condensation, with only a few points right along the cloud edge experiencing evaporation (mainly at points along the cloud top and lower down associated with the toroidal circulation). Like in the previous cases, decreasing the model grid spacing from 100 m to 30 m in the EVAP simulations better resolves the cloud structure, although the impact on the cloud microphysical structure is limited above 2.5 to 3 km.

To briefly summarize, these sensitivity tests suggest that the representation of collision-coalescence has a major impact on the evolution of the DSDs, which is not surprising given the strongly nonlinear drop growth by collision-coalescence once drops grow large enough for significant collision-coalescence to occur. We note, however, the effects of collision-coalescence on DSD width are augmented by in-cloud droplet activation that occurs in strong upward motion when collision-coalescence reduces N_d below the diagnosed number of activated CCN. Because the model does not explicitly track aerosol inside clouds and instead assumes that the concentration of unactivated aerosol is equal to that of the background aerosol minus the existing droplet concentration following Eq. (1), this effect may be overemphasized. Note that the diagnosed CCrate in the TURB and EVAP simulations, with collision-coalescence turned off, is about 10 times smaller than the CCrate from the simulations including collision-coalescence (CTRL and CCS). This reflects the smaller mean drop sizes and much narrower DSDs in the former two cases. However, because collision-coalescence is significant only after the DSDs reach a sufficient mean size and spread, it is important to understand mechanisms responsible for DSD broadening prior to the occurrence of significant collision-coalescence.

b. Effects of droplet activation

Here, we consider two additional cases designed to better understand the role of droplet activation on DSD broadening. In the first case, the original activation scheme is employed (ACT case) and, in the second case, droplet activation is allowed only inside a layer of 120-m depth at cloud base (CBACT case). In all simulations described below we turn off collision-coalescence, sedimentation, and evaporation, and the flow is laminar.

Figure 12 shows that in the ACT simulations, D_m increases nearly monotonically with height and the cloud-top peak of N_d is remarkably diminished compared to CTRL and CCS, which both

include collision-coalescence. This is true to an even greater extent for CBACT (Fig. 13) compared to CTRL and CCS. The fields of both D_m and N_d are also smoother and more uniform in ACT and especially CBACT. By turning off collision-coalescence, the source for large drops is limited. In-cloud activation is also suppressed when collision-coalescence is turned off, as discussed in the previous section, limiting the generation of small droplets in ACT (especially in the cloud core). In CBACT, droplet activation is simply turned off above cloud base. With the sources of both large and small drops limited (or turned off) above cloud base in ACT and CBACT, the DSDs in the upper part of the cloud core are narrower than in CTRL (i.e., smaller σ and ΔD_{99}); compare Figs. 12, 13c,d with 8c,d. The lack of a tail at large sizes and the weaker small mode of the DSDs are also seen when the DSDs in ACT and CBACT (Figs. 14,15) are compared to those from CTRL (Fig. 6). Also note that the upper part of the cloud core in ACT has ΔD_{99} values about one-half as large as those in TURB, with the peak collision-coalescence rate shifted downward relative to cloud top (compare Figs. 12d,e and 10c). This finding supports the earlier discussion on the role of turbulent motion in broadening DSDs near cloud top, since the only difference between TURB and ACT is that the resolved flow is turbulent in the former and laminar in the latter.

There are several major differences in DSD characteristics between ACT and CBACT, further highlighting the important role of droplet activation above cloud base. Much higher supersaturations occur in CBACT (for instance, supersaturation in the entrained region near the lateral cloud edge can be as high as 30% due to the lack of activation; similar values were reported by Lebo et al. 2012), leading to mean droplet sizes that are locally 2-3 times larger compared to ACT near the cloud edges. However, in the cloud core, σ is greater in ACT than CBACT by up to about a factor of 2. These results suggest that in-cloud droplet activation is important for broadening DSDs in the cloud core even when activation is not enhanced by the reduction of N_d from collision-coalescence. It is also seen that many areas of relatively large σ in the cloud core in ACT (Fig. 12) associated

with in-cloud droplet activation have relatively small ΔD_{99} ($< 10 \mu\text{m}$), consistent with the idea that droplet activation broadens DSDs to small sizes but has a weaker influence on the large drop tail.

While the DSDs are generally narrower (smaller σ) in CBACT than ACT, there is still substantial DSD broadening in CBACT, especially along the cloud edges (Fig. 13). This broadening can only occur from entrainment and mixing processes in conjunction with condensation because all other broadening mechanisms are turned off in CBACT. Interestingly, the DSDs are actually much broader along the *lateral* cloud edges in CBACT ($\sigma > 12 \mu\text{m}$ and $\Delta D_{99} > 35 \mu\text{m}$) than in ACT, indicating that droplet activation limits DSD broadening there in contrast to its influence on DSDs in the cloud core and along the cloud top. Evidently, this occurs because activation of droplets limits the effects of dilution in reducing N_d locally. In ACT, there is a region of relatively large N_d and small D_m along the lateral cloud edges at mid-levels (approximately 2 to 4 km height), whereas the opposite occurs in CBACT.

The results for the lateral cloud edges described above contrast with the situation along cloud top; in ACT, there are sharp maxima in σ and ΔD_{99} along the cloud top for all model resolutions tested (Fig. 12c,d). Whereas in CBACT, there are either weak local maxima in σ and ΔD_{99} at cloud top in the lower-resolution simulations (first, third, and fourth rows in Fig. 13) or no apparent maxima at all at cloud top in the 30×30 simulation (second row from the top in Fig. 13). This result again indicates the important role of droplet activation along the cloud top in broadening the modeled DSDs.

The diagnosed CCrate is generally smaller in CBACT than ACT (Fig. 12e)- 13e), consistent with smaller N_d , σ , and ΔD_{99} in the cloud core without any in-cloud activation. Nonetheless, there are some small, isolated regions of higher CCrate near the cloud edges in CBACT compared to ACT, reflecting the relatively broad DSDs there compared to those in ACT.

The results discussed above are further supported by comparing DSDs from selected locations inside the cloud at $t = 16$ min from ACT (Fig. 14) and CBACT (Fig. 15). It is seen that the DSDs are narrower in the cloud interior (left panels in the figures) for CBACT than ACT because of the absence of a tail to small sizes in the absence of in-cloud droplet activation. In contrast, DSDs are fairly wide in CBACT near the cloud edge (right panels in the figures) with a shoulder of the DSDs extending to large sizes; significant concentrations of droplets exceed a diameter of $100\ \mu\text{m}$. This DSD shoulder extending to large droplet sizes is absent in ACT. These DSD results are consistent with stronger condensational growth of droplets associated with much higher supersaturations along the lateral cloud edges in CBACT (not shown).

Varying the resolution in CBACT induces some differences in the dynamics of the cloud. Notable is the second thermal and associated toroidal circulation region that develops around 5 km height at 30 m grid spacing, leading to enhanced entrainment in this region. This second thermal-like feature is clearly seen in the microphysical fields in the 2nd and 3th rows of Fig. 13.

The role of varying the model grid spacing (vertical $\times$ horizontal), in terms of the horizontally averaged (in-cloud) profiles of D_m , σ , and ΔD_{99} for both the ACT and the CBACT simulations, is further highlighted in Figs. 16 and 17 for $t = 7$ and 16 min, respectively. Results show that the horizontally averaged D_m is mostly insensitive to the model resolution when the grid spacing is reduced from 100 m to 30 m. In general, both the ACT and CBACT simulations produce narrower DSDs (smaller σ and ΔD_{99}) with a decrease in *vertical* grid spacing at $t = 7$ min (Fig. 16), although the impact of changing grid spacing is somewhat smaller and less robust (i.e., more variable with height) in ACT than CBACT. There is much less sensitivity to changes in *horizontal* grid spacing. This picture is less clear at $t = 16$ min (Fig. 17). In regions of enhanced entrainment associated with inflow from the cloud's toroidal circulation around 3 km height (and also 5 km height in the 30 $\times$ 30 simulation with the secondary circulation), the higher-resolution simulations

have wider DSDs, whereas elsewhere they are generally narrower. At $t = 7$ min, the cloud is still very shallow, especially in the CBACT simulations, and entrainment and dilution are limited before the development of a coherent toroidal circulation. At $t = 16$ min, on the other hand, the cloud is three times deeper and well-developed toroidal circulations are evident (similar to those seen in Fig. 4). This enhances the inflow of environmental air into the cloud (i.e., entrainment), especially at mid-levels, in turn augmenting DSD broadening from dilution followed by in-cloud mixing. This mechanism is analyzed in more detail below.

c. The role of cloud dilution

Relationships between entrainment, dilution, and DSD characteristics are further examined through scatterplots and histograms of θ_e , N_d , σ , and ΔD_{99} . As shown by the histograms in Fig. 18b,h, in the ACT simulations, when the grid spacing is decreased from 100 m to 30 m, there is a shift toward slightly larger values of N_d . However, as illustrated by the scatterplots in the right columns of Fig. 18, the relationship between ΔD_{99} and N_d is rather weak at both resolutions. On the other hand, except for a cluster of points with nearly monodisperse DSDs at $\theta_e \sim 345$ K, σ generally increases with N_d at a given θ_e value. This is consistent with the effects of in-cloud droplet activation, which broadens the DSDs toward small sizes, increasing σ but having less influence on ΔD_{99} . What, then, drives the variability in ΔD_{99} ? The answer resides in the degree of cloud dilution.

At $t = 16$ min in Fig. 18, ΔD_{99} has a reasonably strong negative correlation with θ_e , which we consider as a proxy for the degree of dilution of the cloudy air. Thus, the DSDs with the broadest tails tend to occur at points that are more dilute (i.e., those with lower θ_e). This provides more evidence for the role of dilution in DSD broadening. A negative correlation between σ and θ_e is also evident when $N_d < \sim 150 \text{ mg}^{-1}$, consistent with DSD broadening from dilution. When the

model grid spacing is decreased from 100 m to 30 m, there is a small shift in the θ_e distribution toward higher values (Fig. 18a,g). This is likely related to better resolving fine-scale structures near the cloud edge, thereby slightly reducing cloud dilution. This leads to higher N_d values, as seen in Fig. 18b,h, although the reduction in N_d from dilution is counterbalanced by activation of droplets in entrained air in the ACT test. Again, the overall impact of changing the model grid spacing from 100 m to 30 m on N_d is small.

Without in-cloud droplet activation, in CBACT, there are clear decreases in σ and ΔD_{99} with increases in N_d (Fig. 19i-l). Larger σ and ΔD_{99} also tend to occur at points with lower θ_e , again highlighting the role of cloud dilution in DSD broadening. However, this relationship is less evident in the high-resolution simulation, plausibly because of greater spatial variability and delayed in-cloud mixing at high resolution. For example, small-scale entrainment in the high-resolution simulation could lead to points with low θ_e but relatively small ΔD_{99} prior to in-cloud mixing. There is again a shift toward higher θ_e values at higher model resolution in CBACT (Fig. 19a,g), similar to ACT, while there is a stronger shift toward higher values of N_d at higher resolution (Fig. 19b,h) associated with the reduced dilution. The difference in N_d between the low- and high-resolution simulations in CBACT is larger than in ACT because there is no droplet activation above cloud base in CBACT to compensate for the reduction in N_d from dilution.

Overall, these results show that the greatest DSD broadening and widest DSDs tend to occur in both the ACT and CBACT cases in cloud regions that experience more entrainment and mixing with the environment and hence greater dilution, especially at low model resolution. However, this relationship is strongly modulated by droplet activation. Thus, the variations of N_d in the ACT and CBACT simulations are determined by two competing processes: activation and dilution. Note that dilution (in the absence of evaporation) itself does not affect the DSD shape; it acts indirectly by creating the conditions for subsequent DSD broadening. Specifically, dilution reduces N_d ,

which results in locally increased supersaturations and droplet condensational growth rates when this diluted air rises within the updraft. DSD broadening ensues when this air is mixed with less dilute cloud core air containing droplets that experienced lower supersaturation and slower growth. This broadening mechanism from dilution and in-cloud mixing is similar to “eddy hopping” (e.g., Cooper 1989; Grabowski and Abade 2017; Abade et al. 2018), with broadening from the mixing of DSDs that have different growth histories. This process is greatly amplified by the toroidal circulation, which leads to inflowing air that enhances entrainment and dilution along the cloud edges, followed by lifting of this air as it moves toward the cloud core. This mechanism is able to generate embryo raindrop sizes (larger than $\sim 80 \mu\text{m}$) in CBACT even though condensation and mixing are the only processes acting on the DSDs above cloud base.

When in-cloud droplet activation above cloud base is allowed (as in ACT), the increase in N_d that occurs following dilution increases water vapor competition among droplets, which results in reduced supersaturation, limiting condensational growth. This reduces the potential for DSD broadening from subsequent in-cloud mixing with less dilute cloud air (i.e., it limits the dilution-driven “eddy hopping” broadening mechanism). However, by constituting a source of small droplets, such activation extends DSDs to small sizes, which itself broadens the DSDs particularly in terms of σ .

d. Simulations with fixed N_d

In the absence of collision-coalescence, sedimentation, evaporation, and droplet activation above cloud base (i.e., the CBACT case), cloud dilution evidently can still produce wide DSDs with $\sigma > 10$ μm and $\Delta D_{99} > 30 \mu\text{m}$, particularly in regions of enhanced entrainment associated with the cloud’s toroidal circulation(s). Above, we proposed a dilution-driven “eddy hopping”-like mechanism to explain this behavior. Here, we further investigate this mechanism and other DSD broadening

processes by removing the effects of dilution on N_d (ND100 simulations). To restrict the DSD broadening induced by dilution in the simulations, additional simulations are performed with N_d fixed to a constant value at every grid point inside the cloud, once the activation creates enough droplets to exceed our cloud mask threshold of $N_d = 1 \text{ mg}^{-1}$. At each time step, we set $N_d = 100 \text{ mg}^{-1}$ everywhere in the cloud by scaling the existing DSD uniformly in all size bins such that the DSD mean size, shape, and width are not affected. Although doing so violates water mass conservation, this simple test allows us to remove the direct effects of DSD dilution. For the ND100 simulations, we restrict droplet activation to occur only at the cloud base.

Figure 20 illustrates the effects of fixing N_d . It is seen that the DSD broadening is highly reduced in these simulations due to the lack of DSD dilution, compared to CBACT (compare Figs. 16-17 with 20). This comparison provides further evidence for the important role of the “eddy-hopping”-like mechanism whereby dilution reduces N_d locally, leading to spatial heterogeneity of droplet growth and subsequent broadening from in-cloud mixing of DSDs. Of course, even though N_d is constant in these simulations and hence there is no direct dilution of the DSDs, there is still dilution of the thermodynamic variables from entrainment and mixing with environmental air. This could also lead to spatial variability in supersaturation and droplet growth and hence DSD broadening after in-cloud DSD mixing—another “eddy-hopping”-like mechanism. However, given the large decrease in DSD width when N_d is fixed compared to the CBACT simulations (up to a factor of ~ 3 decrease in σ and ΔD_{99} above 3 km), the dilution of thermodynamic properties appears to be of secondary importance in broadening DSDs compared to direct dilution of the DSDs and reduction in N_d .

For the ND100 simulations, differences in the horizontally averaged profiles of σ and ΔD_{99} as a function of the model resolution remain similar to the other cases. At $t = 7$ min and below approximately 2.5 km height at $t = 16$ min, decreasing the *vertical* grid spacing from 100 m to 30

m reduces the horizontally averaged σ and ΔD_{99} values by approximately 20 to 50%. There is little sensitivity to changes in horizontal grid spacing. At later times above 2.5 km, the profiles of σ and ΔD_{99} are less sensitive to the model grid spacing. This likely reflects the more complicated cloud structure and effects of the toroidal circulations that develop at these later times, including the secondary toroidal circulation that develops in the 30×30 simulation.

Although we cannot isolate the effects of unphysical DSD broadening caused by vertical numerical diffusion, the narrowing of DSDs as vertical grid spacing is decreased, with little sensitivity to horizontal grid spacing, is consistent with such numerical broadening. For example, Morrison et al. (2018) showed a decrease in ΔD_{99} by about a factor of 1.5-2 when the vertical grid spacing was reduced from 40 to 20 m in their idealized one-dimensional tests that included only vertical advection and condensational growth (meaning that vertical numerical diffusion was the only broadening mechanism). The potential for physical DSD broadening from the “eddy hopping”-like mechanism increases when supersaturation gradients are larger, simply because this leads to more heterogeneity in droplet growth. Because details of cloud structure and the supersaturation field are better resolved as the grid spacing is decreased, with larger supersaturation maxima and greater spatial variability (not shown), one might anticipate that broader DSDs should occur with increasing resolution. However, the opposite behavior occurs here earlier in the simulations (e.g., at $t = 7$ min), and thus the broadening effects of better resolving spatial variability in the thermodynamic fields seem to be secondary to unphysical DSD broadening from vertical numerical diffusion. Later in the simulations (e.g., at $t = 16$ min) above ~ 2.5 km, where entrainment and mixing seem to play a more important role in the cloud evolution, there is less sensitivity of DSD width to changing the model grid spacing from 100 m to 30 m. In this instance, it is possible that the reduction in unphysical numerical broadening from increased model resolution is compensated by

increased physical DSD broadening from better resolving small-scale features and thermodynamic heterogeneities.

5. Summary and concluding remarks

The simulations analyzed in this study provide useful insights to understand the processes that lead to DSD broadening in an Eulerian bin microphysics model. Several “mechanism denial” experiments allowed us to identify the effects of different processes that influence DSDs in simulations of an idealized warm cumulus cloud. We showed that the modeled DSD evolution is strongly affected by collision-coalescence, reflecting the rapid drop growth that occurs once embryo raindrops are generated. Collision-coalescence rates were greatest in the upper part of the cloud core similar to Khain et al. (2013). The reduction in N_d from collision-coalescence also induced droplet activation in regions of strong ascent and high supersaturation within the cloud core, consistent with the results of Hall (1980). Thus, collision-coalescence broadened the DSDs to large sizes directly, and to small sizes from the subsequent in-cloud droplet activation. The result was a broad region in the upper part of the cloud with small mean droplet sizes but large DSD standard deviations ($\sigma > 10 \mu\text{m}$) and broad tails ($\Delta D_{99} > 30 \mu\text{m}$). The occurrence of in-cloud activation in regions of strong ascent has been discussed by previous studies (Heymsfield et al. 2009; Pinsky and Khain 2002; Segal et al. 2003; Yang et al. 2015; Fan et al. 2018). Slawinska et al. (2012) found that about 40% of all cloud droplets in a LES simulation of shallow cumulus are activated above the cloud base. However, because aerosols were not explicitly tracked in the model, this effect of in-cloud droplet activation may have been overemphasized in our simulations compared to reality.

Additional simulations were analyzed to examine further the effects of resolved turbulent motion, sedimentation, evaporation, and droplet activation on DSD evolution. Turbulent eddies enhanced DSD broadening near cloud top by influencing entrainment and DSD mixing processes, while

evaporation played only a minor role in DSD evolution and sedimentation had little effect. Droplet activation, especially along the cloud top, led to DSD broadening to small sizes and $\sigma > 10 \mu\text{m}$. This activation was associated with cloud dilution, which reduced N_d locally, inducing the activation of droplets when this air ascended. This mechanism was confirmed by simulations in which droplet activation above cloud base was turned off.

Even with collision-coalescence, sedimentation, evaporation, and in-cloud (above cloud base) droplet activation turned off in the model, and laminar flow, the dilution of parcels at the cloud interface from mixing with cloud-free environmental air still led to wide DSDs ($\sigma > 10 \mu\text{m}$) and large tails ($\Delta D_{99} > 40 \mu\text{m}$), particularly along the lateral cloud edges. From an analysis of mixing via θ_e , it was shown that the widest DSDs, in this case, tended to occur in parcels that were strongly diluted, especially for the low-resolution simulations. The reduction in N_d from dilution meant locally higher supersaturation and stronger condensational growth when this air ascended in the updraft (i.e., super-adiabatic growth, as discussed by Yang et al. 2016), leading to broad DSDs after mixing with DSDs from the less dilute cloud core. Thus, dilution created the conditions for DSD broadening by reducing N_d locally and introducing spatial heterogeneity in droplet growth rates. This result is consistent with the idea of “eddy hopping” as a DSD broadening mechanism, whereby DSDs undergoing different growth histories are broadened by their mixing (e.g., Cooper 1989; Grabowski and Abade 2017; Abade et al. 2018). This mechanism was confirmed by additional simulations also with collision-coalescence, sedimentation, evaporation, and in-cloud (above cloud base) droplet activation turned off in the model, and laminar flow, but N_d fixed at a constant value within the cloud by adjusting N_d without changing the DSD mean size, width, or shape. These simulations had much narrower DSDs and no sharp maxima in σ and ΔD_{99} along the cloud boundaries compared to the corresponding simulations that did not fix N_d .

We also examined the effects of horizontal and vertical model grid spacing on the simulated DSD characteristics. Overall, the main effect of decreasing the grid spacing from 100 m to 30 m, in all of the process sensitivity tests, was to better resolve fine-scale cloud macrophysical and microphysical structure. Impacts on the overall DSD features were generally small, but there was a noticeable decrease in σ and ΔD_{99} in the lower part of the cloud (below about 2.5 to 3 km) with decreased *vertical* grid spacing in all of the tests. This is a cloud region that was less affected by entrainment and dilution, collision-coalescence, and other physical DSD broadening processes. Narrower DSDs with decreased vertical grid spacing were also evident in the fixed- N_d simulations with collision-coalescence, sedimentation, evaporation and in-cloud activation turned off. This sensitivity of DSD width to the model vertical resolution is a signature of DSD broadening from vertical numerical diffusion, a non-physical feature detailed in Morrison et al. (2018). Quantifying this artificial numerical broadening directly is cumbersome given that it is flow dependent and difficult to separate from the other mixing processes (e.g., horizontal mixing). Nonetheless, decreasing the vertical grid spacing from 100 m to 30 m led to reductions in σ and ΔD_{99} by 20 to 50% in the fixed- N_d simulations at earlier times ($t = 7$ min), consistent with the behavior of DSD broadening from vertical numerical diffusion. However, above 2.5 km at later simulation times ($t = 16$ min) there was little sensitivity to vertical grid spacing, which was likely due to the dominant role of other processes in DSD broadening, especially horizontal mixing. Thus, while there was a signature of artificial DSD broadening from vertical numerical diffusion, the overall evolution of DSD width in this case was dominated by other physical mechanisms, particularly collision-coalescence, in-cloud droplet activation, and dilution, above ~ 2.5 km. Note these results apply specifically to simulations of an isolated, growing cumulus cloud. For other cloud types in which these physical processes may be less active, such as stratocumulus, artificial numerical

DSD broadening may play a more important role. We leave a detailed investigation of the DSD broadening mechanisms in bin model simulations of other cloud types to future work.

The analysis and interpretation of the simulations in this study allowed us to infer details regarding the evolution of DSDs in an Eulerian model with bin microphysics. An emerging approach in microphysical modeling is the Lagrangian particle-based approach, in which the hydrometeor population is represented by a sampling of point particles (often called “super-droplets”) that move following Lagrangian trajectories in the Eulerian modeled flow (e.g., Shima et al. 2009; Sölch and Kärcher 2010; Riechelmann et al. 2012; Unterstrasser et al. 2017; Grabowski et al. 2018; Jaruga and Pawlowska 2018; Seifert et al. 2019; Dziekan et al. 2019). This approach presents a much different methodology than bin microphysics (e.g., see the discussion in Grabowski et al. 2019). It would be useful to compare how DSDs evolve in Lagrangian particle-based versus bin schemes. This has been done for idealized simulations of a laboratory chamber (Grabowski 2020), and work is planned to extend this effort to simulations of a cumulus cloud. Although Lagrangian schemes have limitations related to the finite number of superdroplets that can be used, they do not have any numerical diffusion of DSDs. Therefore, such a comparison will be especially useful for evaluating the effects of numerical diffusion in bin schemes within Eulerian models.

Data availability statement. The atmospheric sounding employed to initialize the model was obtained from the Atmospheric Radiation Measurement (ARM) user facility (Holdridge et al. 2014). We employed the WRF model version 3.6.1 (<https://doi.org/10.5065/d6mk6b4k>). The TAU microphysics code is hosted by G. Feingold (<https://www.esrl.noaa.gov/csd/staff/graham.feingold/code/>). The WRF outputs and input namelists for the simulations employed in this study are available at <http://ftp.cptec.inpe.br/chuva/pesquisa/JAS2020/>.

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References

Abade, G. C., W. W. Grabowski, and H. Pawlowska, 2018: Broadening of cloud droplet spectra through eddy hopping: Turbulent entraining parcel simulations. *Journal of the Atmospheric Sciences*, **75** (10), 3365–3379, doi:10.1175/JAS-D-18-0078.1.

- 684 Ayala, O., B. Rosa, and L.-P. Wang, 2008a: Effects of turbulence on the geometric collision rate
685 of sedimenting droplets. Part 2. Theory and parameterization. *New Journal of Physics*, **10** (7),
686 075 016, doi:10.1088/1367-2630/10/7/075016.
- 687 Ayala, O., B. Rosa, L.-P. Wang, and W. W. Grabowski, 2008b: Effects of turbulence on the
688 geometric collision rate of sedimenting droplets. Part 1. Results from direct numerical simulation.
689 *New Journal of Physics*, **10** (7), 075 015, doi:10.1088/1367-2630/10/7/075015.
- 690 Baker, M. B., R. E. Breidenthal, T. W. Choullarton, and J. Latham, 1984: The effects of
691 turbulent mixing in clouds. *Journal of the Atmospheric Sciences*, **41** (2), 299–304, doi:
692 10.1175/1520-0469(1984)041<0299:TEOTMI>2.0.CO;2.
- 693 Baker, M. B., R. G. Corbin, and J. Latham, 1980: The influence of entrainment on the evolution
694 of cloud droplet spectra: I. A model of inhomogeneous mixing. *Quarterly Journal of the Royal
695 Meteorological Society*, **106** (449), 581–598, doi:10.1002/qj.49710644914.
- 696 Baker, M. B., and J. Latham, 1979: The evolution of droplet spectra and the rate of production
697 of embryonic raindrops in small cumulus clouds. *Journal of the Atmospheric Sciences*, **36** (8),
698 1612–1615, doi:10.1175/1520-0469(1979)036<1612:TEODSA>2.0.CO;2.
- 699 Blyth, A. M., 1993: Entrainment in cumulus clouds. *Journal of Applied Meteorology*, **32** (4),
700 626–641, doi:10.1175/1520-0450(1993)032<0626:EICC>2.0.CO;2.
- 701 Brenguier, J.-L., and W. W. Grabowski, 1993: Cumulus entrainment and cloud droplet spectra: A
702 numerical model within a two-dimensional dynamical framework. *Journal of the Atmospheric
703 Sciences*, **50** (1), 120–136, doi:10.1175/1520-0469(1993)050<0120:CEACDS>2.0.CO;2.

- Cecchini, M. A., and Coauthors, 2017: Illustration of microphysical processes in Amazonian deep convective clouds in the gamma phase space: Introduction and potential applications. *Atmospheric Chemistry and Physics*, **17** (23), 14 727–14 746, doi:10.5194/acp-17-14727-2017.
- Chen, S., M. K. Yau, and P. Bartello, 2018: Turbulence effects of collision efficiency and broadening of droplet size distribution in cumulus clouds. *Journal of the Atmospheric Sciences*, **75** (1), 203–217, doi:10.1175/JAS-D-17-0123.1.
- Cooper, W. A., 1989: Effects of variable droplet growth histories on droplet size distributions. Part I: Theory. *Journal of the Atmospheric Sciences*, **46** (10), 1301–1311, doi:10.1175/1520-0469(1989)046<1301:EOVDGH>2.0.CO;2.
- Cooper, W. A., S. G. Lasher-Trapp, and A. M. Blyth, 2013: The influence of entrainment and mixing on the initial formation of rain in a warm cumulus cloud. *Journal of the Atmospheric Sciences*, **70** (6), 1727–1743, doi:10.1175/JAS-D-12-0128.1.
- Dávila, J., and J. C. R. Hunt, 2001: Settling of small particles near vortices and in turbulence. *Journal of Fluid Mechanics*, **440**, 117–145, doi:10.1017/S0022112001004694.
- Devenish, B. J., and Coauthors, 2012: Droplet growth in warm turbulent clouds. *Quarterly Journal of the Royal Meteorological Society*, **138** (667), 1401–1429, doi:10.1002/qj.1897.
- Dodson, D. S., and J. D. Small Griswold, 2019: Droplet inhomogeneity in shallow cumuli: The effects of in-cloud location and aerosol number concentration. *Atmospheric Chemistry and Physics*, **19** (11), 7297–7317, doi:10.5194/acp-19-7297-2019.
- Dziekan, P., M. Waruszewski, and H. Pawlowska, 2019: University of Warsaw Lagrangian cloud model (UWLCM) 1.0: A modern large-eddy simulation tool for warm cloud modeling with

Lagrangian microphysics. *Geoscientific Model Development*, **12** (6), 2587–2606, doi:10.5194/
gmd-12-2587-2019.

Fan, J., and Coauthors, 2018: Substantial convection and precipitation enhancements by ultrafine
aerosol particles. *Science*, **359** (6374), 411–418, doi:10.1126/science.aan8461.

Feingold, G., S. Tzivion, and Z. Leviv, 1988: Evolution of raindrop spectra. Part I: Solution to the
stochastic collection/breakup equation using the method of moments. *Journal of the Atmospheric
Sciences*, **45** (22), 3387–3399, doi:10.1175/1520-0469(1988)045<3387:EORSPI>2.0.CO;2.

Franklin, C. N., P. A. Vaillancourt, M. K. Yau, and P. Bartello, 2005: Collision rates of cloud
droplets in turbulent flow. *Journal of the Atmospheric Sciences*, **62** (7), 2451–2466, doi:10.1175/
JAS3493.1.

Grabowski, W. W., 2014: Extracting microphysical impacts in large-eddy simulations of shallow
convection. *Journal of the Atmospheric Sciences*, **71** (12), 4493–4499, doi:10.1175/
JAS-D-14-0231.1.

Grabowski, W. W., 2020: Comparison of Eulerian bin and Lagrangian particle-based schemes in
simulations of Pi Chamber dynamics and microphysics. *Journal of the Atmospheric Sciences*,
77 (3), 1151–1165, doi:10.1175/JAS-D-19-0216.1.

Grabowski, W. W., and G. C. Abade, 2017: Broadening of cloud droplet spectra through eddy
hopping: Turbulent adiabatic parcel simulations. *Journal of the Atmospheric Sciences*, **74** (5),
1485–1493, doi:10.1175/JAS-D-17-0043.1.

Grabowski, W. W., P. Dziekan, and H. Pawlowska, 2018: Lagrangian condensation microphysics
with Twomey CCN activation. *Geoscientific Model Development*, **11** (1), 103–120, doi:10.5194/
gmd-11-103-2018.

- Grabowski, W. W., and H. Morrison, 2008: Toward the mitigation of spurious cloud-edge supersaturation in cloud models. *Monthly Weather Review*, **136** (3), 1224–1234, doi:10.1175/2007MWR2283.1.
- Grabowski, W. W., H. Morrison, S.-I. Shima, G. C. Abade, P. Dziekan, and H. Pawlowska, 2019: Modeling of cloud microphysics: Can we do better? *Bulletin of the American Meteorological Society*, **100** (4), 655–672, doi:10.1175/BAMS-D-18-0005.1.
- Grabowski, W. W., and L.-P. Wang, 2013: Growth of cloud droplets in a turbulent environment. *Annual Review of Fluid Mechanics*, **45** (1), 293–324, doi:10.1146/annurev-fluid-011212-140750.
- Gunthe, S. S., and Coauthors, 2009: Cloud condensation nuclei in pristine tropical rainforest air of Amazonia: Size-resolved measurements and modeling of atmospheric aerosol composition and CCN activity. *Atmospheric Chemistry and Physics*, **9** (19), 7551–7575.
- Hall, W. D., 1980: A detailed microphysical model within a two-dimensional dynamic framework: Model description and preliminary results. *Journal of the Atmospheric Sciences*, **37** (11), 2486–2507, doi:10.1175/1520-0469(1980)037<2486:ADMMWA>2.0.CO;2.
- Hartman, C. M., and J. Y. Harrington, 2005a: Radiative impacts on the growth of drops within simulated marine stratocumulus. Part I: Maximum solar heating. *Journal of the Atmospheric Sciences*, **62** (7), 2323–2338, doi:10.1175/JAS3477.1.
- Hartman, C. M., and J. Y. Harrington, 2005b: Radiative impacts on the growth of drops within simulated marine stratocumulus. Part II: Solar zenith angle variations. *Journal of the Atmospheric Sciences*, **62** (7), 2339–2351, doi:10.1175/JAS3478.1.

- Hernández Pardo, L., L. A. Toledo Machado, M. Amore Cecchini, and M. Sánchez Gácita, 2019: Quantifying the aerosol effect on droplet size distribution at cloud top. *Atmospheric Chemistry and Physics*, **19** (11), 7839–7857, doi:10.5194/acp-19-7839-2019.
- Heymsfield, A. J., A. Bansemer, G. Heymsfield, and A. O. Fierro, 2009: Microphysics of maritime tropical convective updrafts at temperatures from -20° to -60° . *Journal of the Atmospheric Sciences*, **66** (12), 3530–3562, doi:10.1175/2009JAS3107.1.
- Hoffmann, F., 2016: The Effect of Spurious Cloud Edge Supersaturations in Lagrangian Cloud Models: An Analytical and Numerical Study. *Monthly Weather Review*, **144** (1), 107–118, doi:10.1175/MWR-D-15-0234.1.
- Hoffmann, F., S. Raasch, and Y. Noh, 2015: Entrainment of aerosols and their activation in a shallow cumulus cloud studied with a coupled LCM–LES approach. *Atmospheric Research*, **156**, 43 – 57, doi:https://doi.org/10.1016/j.atmosres.2014.12.008.
- Holdridge, D., J. Kyrouac, and E. Keeler, 2014: Balloon-Borne Sounding System (SONDEWNPN). Atmospheric Radiation Measurement (ARM) user facility, accessed 05 April 2019, doi:10.5439/1021460.
- Hudson, J. G., and X. Da, 1996: Volatility and size of cloud condensation nuclei. *Journal of Geophysical Research: Atmospheres*, **101** (D2), 4435–4442, doi:10.1029/95JD00192.
- Jaruga, A., and H. Pawlowska, 2018: libcloudph++ 2.0: aqueous-phase chemistry extension of the particle-based cloud microphysics scheme. *Geoscientific Model Development*, **11** (9), 3623–3645, doi:10.5194/gmd-11-3623-2018.

Jensen, J. B., P. H. Austin, M. B. Baker, and A. M. Blyth, 1985: Turbulent mixing, spectral evolution and dynamics in a warm cumulus cloud. *Journal of the Atmospheric Sciences*, **42** (2), 173–192, doi:10.1175/1520-0469(1985)042<0173:TMSEAD>2.0.CO;2.

Jensen, J. B., and A. D. Nugent, 2017: Condensational growth of drops formed on giant sea-salt aerosol particles. *Journal of the Atmospheric Sciences*, **74** (3), 679–697, doi:10.1175/JAS-D-15-0370.1.

Khain, A., M. Ovtchinnikov, M. Pinsky, A. Pokrovsky, and H. Krugliak, 2000: Notes on the state-of-the-art numerical modeling of cloud microphysics. *Atmospheric Research*, **55** (3), 159 – 224, doi:https://doi.org/10.1016/S0169-8095(00)00064-8.

Khain, A., T. V. Prabha, N. Benmoshe, G. Pandithurai, and M. Ovchinnikov, 2013: The mechanism of first raindrops formation in deep convective clouds. *Journal of Geophysical Research: Atmospheres*, **118** (16), 9123–9140, doi:10.1002/jgrd.50641.

Korolev, A. V., 1995: The influence of supersaturation fluctuations on droplet size spectra formation. *Journal of the Atmospheric Sciences*, **52** (20), 3620–3634, doi:10.1175/1520-0469(1995)052<3620:TIOSFO>2.0.CO;2.

Lasher-trapp, S. G., W. A. Cooper, and A. M. Blyth, 2005: Broadening of droplet size distributions from entrainment and mixing in a cumulus cloud. *Quarterly Journal of the Royal Meteorological Society*, **131** (605), 195–220, doi:10.1256/qj.03.199.

Latham, J., and R. L. Reed, 1977: Laboratory studies of the effects of mixing on the evolution of cloud droplet spectra. *Quarterly Journal of the Royal Meteorological Society*, **103** (436), 297–306, doi:10.1002/qj.49710343607.

- 808 Lebo, Z. J., N. C. Johnson, and J. Y. Harrington, 2008: Radiative influences on ice crystal and
 809 droplet growth within mixed-phase stratus clouds. *Journal of Geophysical Research: Atmo-*
 810 *spheres*, **113** (D9), doi:10.1029/2007JD009262.
- 811 Lebo, Z. J., H. Morrison, and J. H. Seinfeld, 2012: Are simulated aerosol-induced effects on deep
 812 convective clouds strongly dependent on saturation adjustment? *Atmospheric Chemistry and*
 813 *Physics*, **12** (20), 9941–9964.
- 814 Levine, J., 1959: Spherical vortex theory of bubble-like motion in cumulus clouds. *Journal of*
 815 *Meteorology*, **16** (6), 653–662, doi:10.1175/1520-0469(1959)016<0653:SVTOBL>2.0.CO;2.
- 816 Lu, J., H. Nordsiek, and R. A. Shaw, 2010: Clustering of settling charged particles in turbulence:
 817 Theory and experiments. *New Journal of Physics*, **12** (12), 123 030, doi:10.1088/1367-2630/12/
 818 12/123030.
- 819 Machado, L. A. T., and Coauthors, 2014: The CHUVA project: How does convection vary across
 820 Brazil? *Bulletin of the American Meteorological Society*, **95** (9), 1365–1380.
- 821 Martin, S. T., and Coauthors, 2010: Sources and properties of Amazonian aerosol particles.
 822 *Reviews of Geophysics*, **48** (2).
- 823 Martin, S. T., and Coauthors, 2016: Introduction: Observations and modeling of the green
 824 ocean amazon (GoAmazon2014/5). *Atmospheric Chemistry and Physics*, **16** (8), 4785–4797,
 825 doi:10.5194/acp-16-4785-2016.
- 826 Morrison, H., 2016a: Impacts of updraft size and dimensionality on the perturbation pressure and
 827 vertical velocity in cumulus convection. Part I: Simple, generalized analytic solutions. *Journal*
 828 *of the Atmospheric Sciences*, **73** (4), 1441–1454, doi:10.1175/JAS-D-15-0040.1.

- Morrison, H., 2016b: Impacts of updraft size and dimensionality on the perturbation pressure and vertical velocity in cumulus convection. Part II: Comparison of theoretical and numerical solutions and fully dynamical simulations. *Journal of the Atmospheric Sciences*, **73** (4), 1455–1480, doi:10.1175/JAS-D-15-0041.1.
- Morrison, H., M. Witte, G. H. Bryan, J. Y. Harrington, and Z. J. Lebo, 2018: Broadening of modeled cloud droplet spectra using bin microphysics in an Eulerian spatial domain. *Journal of the Atmospheric Sciences*, **75** (11), 4005–4030, doi:10.1175/JAS-D-18-0055.1.
- NCAR, 2018: The NCAR Command Language (Version 6.5.0) [Software]. Boulder, Colorado: UCAR/NCAR/CISL/TDD., doi:10.5065/D6WD3XH5.
- Ovtchinnikov, M., and R. C. Easter, 2009: Nonlinear advection algorithms applied to interrelated tracers: Errors and implications for modeling aerosol–cloud interactions. *Monthly Weather Review*, **137** (2), 632–644, doi:10.1175/2008MWR2626.1.
- Peters, J. M., W. Hannah, and H. Morrison, 2019: The influence of vertical wind shear on moist thermals. *Journal of the Atmospheric Sciences*, **76** (6), 1645–1659, doi:10.1175/JAS-D-18-0296.1.
- Petters, M. D., and S. M. Kreidenweis, 2007: A single parameter representation of hygroscopic growth and cloud condensation nucleus activity. *Atmospheric Chemistry and Physics*, **7** (8), 1961–1971, doi:10.5194/acp-7-1961-2007.
- Phillips, V. T. J., and L. J. Donner, 2006: Cloud microphysics, radiation and vertical velocities in two- and three-dimensional simulations of deep convection. *Quarterly Journal of the Royal Meteorological Society*, **132** (621C), 3011–3033, doi:10.1256/qj.05.171.

- 850 Pinsky, M., A. Khain, and A. Korolev, 2016: Theoretical analysis of mixing in liquid clouds
851 – Part 3: Inhomogeneous mixing. *Atmospheric Chemistry and Physics*, **16** (14), 9273–9297,
852 doi:10.5194/acp-16-9273-2016, URL <https://www.atmos-chem-phys.net/16/9273/2016/>.
- 853 Pinsky, M. B., and A. P. Khain, 2002: Effects of in-cloud nucleation and turbulence on droplet
854 spectrum formation in cumulus clouds. *Quarterly Journal of the Royal Meteorological Society*,
855 **128** (580), 501–533, doi:10.1256/003590002321042072.
- 856 Pinsky, M. B., A. P. Khain, B. Grits, and M. Shapiro, 2006: Collisions of small drops in a turbulent
857 flow. Part III: Relative droplet fluxes and swept volumes. *Journal of the Atmospheric Sciences*,
858 **63** (8), 2123–2139, doi:10.1175/JAS3730.1.
- 859 Pinsky, M. B., A. P. Khain, and M. Shapiro, 2007: Collisions of cloud droplets in a turbulent
860 flow. Part IV: Droplet hydrodynamic interaction. *Journal of the Atmospheric Sciences*, **64** (7),
861 2462–2482, doi:10.1175/JAS3952.1.
- 862 Pöhlker, M. L., and Coauthors, 2016: Long-term observations of cloud condensation nuclei in
863 the Amazon rain forest – Part 1: Aerosol size distribution, hygroscopicity, and new model
864 parametrizations for CCN prediction. *Atmospheric Chemistry and Physics*, **16** (24), 15 709–
865 15 740.
- 866 Pruppacher, H. R., and J. D. Klett, 2012: *Microphysics of Clouds and Precipitation: Reprinted*
867 *1980*. Springer Science & Business Media.
- 868 Riechelmann, T., Y. Noh, and S. Raasch, 2012: A new method for large-eddy simulations of clouds
869 with Lagrangian droplets including the effects of turbulent collision. *New Journal of Physics*,
870 **14** (6), 065 008, doi:10.1088/1367-2630/14/6/065008.

- 871 Segal, Y., M. Pinsky, A. Khain, and C. Erlick, 2003: Thermodynamic factors influencing bimodal
872 spectrum formation in cumulus clouds. *Atmospheric Research*, **66** (1), 43 – 64, doi:[https://doi.org/10.1016/S0169-8095\(02\)00172-2](https://doi.org/10.1016/S0169-8095(02)00172-2).
873
- 874 Seifert, A., J. Leinonen, C. Siewert, and S. Kneifel, 2019: The geometry of rimed aggregate
875 snowflakes: A modeling study. *Journal of Advances in Modeling Earth Systems*, **11** (3), 712–
876 731, doi:10.1029/2018MS001519.
- 877 Shima, S., K. Kusano, A. Kawano, T. Sugiyama, and S. Kawahara, 2009: The super-droplet method
878 for the numerical simulation of clouds and precipitation: A particle-based and probabilistic
879 microphysics model coupled with a non-hydrostatic model. *Quarterly Journal of the Royal
880 Meteorological Society*, **135** (642), 1307–1320, doi:10.1002/qj.441.
- 881 Skamarock, W. C., and Coauthors, 2008: A description of the advanced research WRF version
882 3 (no. NCAR/TN-475+str). Tech. rep., University Corporation for Atmospheric Research. doi:
883 10.5065/D68S4MVH.
- 884 Slawinska, J., W. W. Grabowski, H. Pawlowska, and H. Morrison, 2012: Droplet Activation and
885 Mixing in Large-Eddy Simulation of a Shallow Cumulus Field. *Journal of the Atmospheric
886 Sciences*, **69** (2), 444–462, doi:10.1175/JAS-D-11-054.1.
- 887 Stevens, B., G. Feingold, W. R. Cotton, and R. L. Walko, 1996a: Elements of the microphysical
888 structure of numerically simulated nonprecipitating stratocumulus. *Journal of the atmospheric
889 sciences*, **53** (7), 980–1006.
- 890 Stevens, B., R. L. Walko, W. R. Cotton, and G. Feingold, 1996b: The spurious production of
891 cloud-edge supersaturations by Eulerian models. *Monthly Weather Review*, **124** (5), 1034–1041,
892 doi:10.1175/1520-0493(1996)124<1034:TSPOCE>2.0.CO;2.

- Sölch, I., and B. Kärcher, 2010: A large-eddy model for cirrus clouds with explicit aerosol and ice microphysics and Lagrangian ice particle tracking. *Quarterly Journal of the Royal Meteorological Society*, **136** (653), 2074–2093, doi:10.1002/qj.689.
- Tao, W.-k., J. Simpson, and S.-T. Soong, 1987: Statistical properties of a cloud ensemble: A numerical study. *Journal of the Atmospheric Sciences*, **44** (21), 3175–3187, doi:10.1175/1520-0469(1987)044<3175:SPOACE>2.0.CO;2.
- Turner, J. S., 1963: Model experiments relating to thermals with increasing buoyancy. *Quarterly Journal of the Royal Meteorological Society*, **89** (379), 62–74, doi:10.1002/qj.49708937904.
- Tzivion, S., G. Feingold, and Z. Levin, 1987: An efficient numerical solution to the stochastic collection equation. *Journal of the Atmospheric Sciences*, **44** (21), 3139–3149, doi:10.1175/1520-0469(1987)044<3139:AENSTT>2.0.CO;2.
- Tzivion, S., G. Feingold, and Z. Levin, 1989: The evolution of raindrop spectra. Part II: Collisional collection/breakup and evaporation in a rainshaft. *Journal of the Atmospheric Sciences*, **46** (21), 3312–3328, doi:10.1175/1520-0469(1989)046<3312:TEORSP>2.0.CO;2.
- Unterstrasser, S., F. Hoffmann, and M. Lerch, 2017: Collection/aggregation algorithms in Lagrangian cloud microphysical models: rigorous evaluation in box model simulations. *Geoscientific Model Development*, **10** (4), 1521–1548, doi:10.5194/gmd-10-1521-2017.
- Wang, L.-P., O. Ayala, B. Rosa, and W. W. Grabowski, 2008: Turbulent collision efficiency of heavy particles relevant to cloud droplets. *New Journal of Physics*, **10** (7), 075 013, doi:10.1088/1367-2630/10/7/075013.

- 913 Wang, S., and A. H. Sobel, 2011: Response of convection to relative sea surface temperature:
914 Cloud-resolving simulations in two and three dimensions. *Journal of Geophysical Research:*
915 *Atmospheres*, **116** (D11), doi:10.1029/2010JD015347.
- 916 Wendisch, M., and Coauthors, 2016: The ACRIDICON-CHUVA campaign: Studying tropical
917 deep convective clouds and precipitation over Amazonia using the new German research aircraft
918 HALO. *Bulletin of the American Meteorological Society*, (2016).
- 919 Wilhelmson, R., 1974: The life cycle of a thunderstorm in three dimensions. *Journal of the*
920 *Atmospheric Sciences*, **31** (6), 1629–1651, doi:10.1175/1520-0469(1974)031<1629:TLCOAT>
921 2.0.CO;2.
- 922 Yang, F., P. Kollias, R. A. Shaw, and A. M. Vogelmann, 2018: Cloud droplet size distribution
923 broadening during diffusional growth: Ripening amplified by deactivation and reactivation.
924 *Atmospheric Chemistry and Physics*, **18** (10), 7313–7328, doi:10.5194/acp-18-7313-2018.
- 925 Yang, F., R. Shaw, and H. Xue, 2016: Conditions for super-adiabatic droplet growth af-
926 ter entrainment mixing. *Atmospheric Chemistry and Physics*, **16** (14), 9421–9433, doi:
927 10.5194/acp-16-9421-2016, URL <https://www.atmos-chem-phys.net/16/9421/2016/>.
- 928 Yang, Q., and Coauthors, 2015: Aerosol transport and wet scavenging in deep convective clouds:
929 A case study and model evaluation using a multiple passive tracer analysis approach. *Journal of*
930 *Geophysical Research: Atmospheres*, **120** (16), 8448–8468, doi:10.1002/2015JD023647.
- 931 Yin, Y., Z. Levin, T. G. Reisin, and S. Tzivion, 2000: The effects of giant cloud condensation nuclei
932 on the development of precipitation in convective clouds — A numerical study. *Atmospheric*
933 *Research*, **53** (1), 91 – 116, doi:[https://doi.org/10.1016/S0169-8095\(99\)00046-0](https://doi.org/10.1016/S0169-8095(99)00046-0).

934 Zeng, X., W.-K. Tao, S. Lang, A. Y. Hou, M. Zhang, and J. Simpson, 2008: On the sensitivity of
935 atmospheric ensembles to cloud microphysics in long-term cloud-resolving model simulations.
936 *Journal of the Meteorological Society of Japan. Ser. II*, **86A**, 45–65, doi:10.2151/jmsj.86A.45.

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 946 a check mark ($\checkmark$) in the corresponding column.

Name	Coll-coal	Sed	Turb	Evap	Act	$N_d = C$
CTRL	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	-
CCS	$\checkmark$	$\checkmark$	-	-	$\checkmark$	-
TURB	-	-	$\checkmark$	-	$\checkmark$	-
EVAP	-	-	-	$\checkmark$	$\checkmark$	-
ACT	-	-	-	-	$\checkmark$	-
CBACT	-	-	-	-	-	-
ND100	-	-	-	-	-	$\checkmark$

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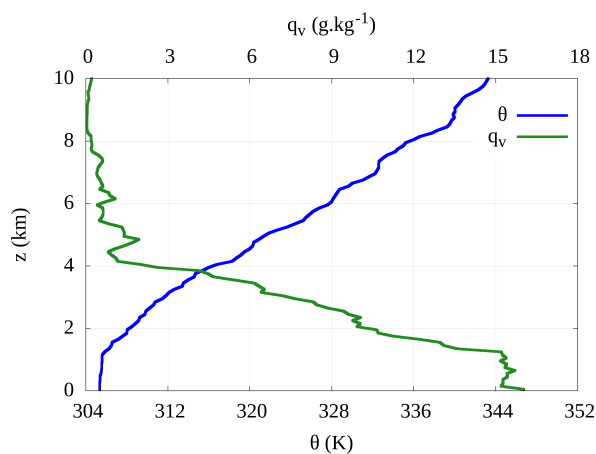


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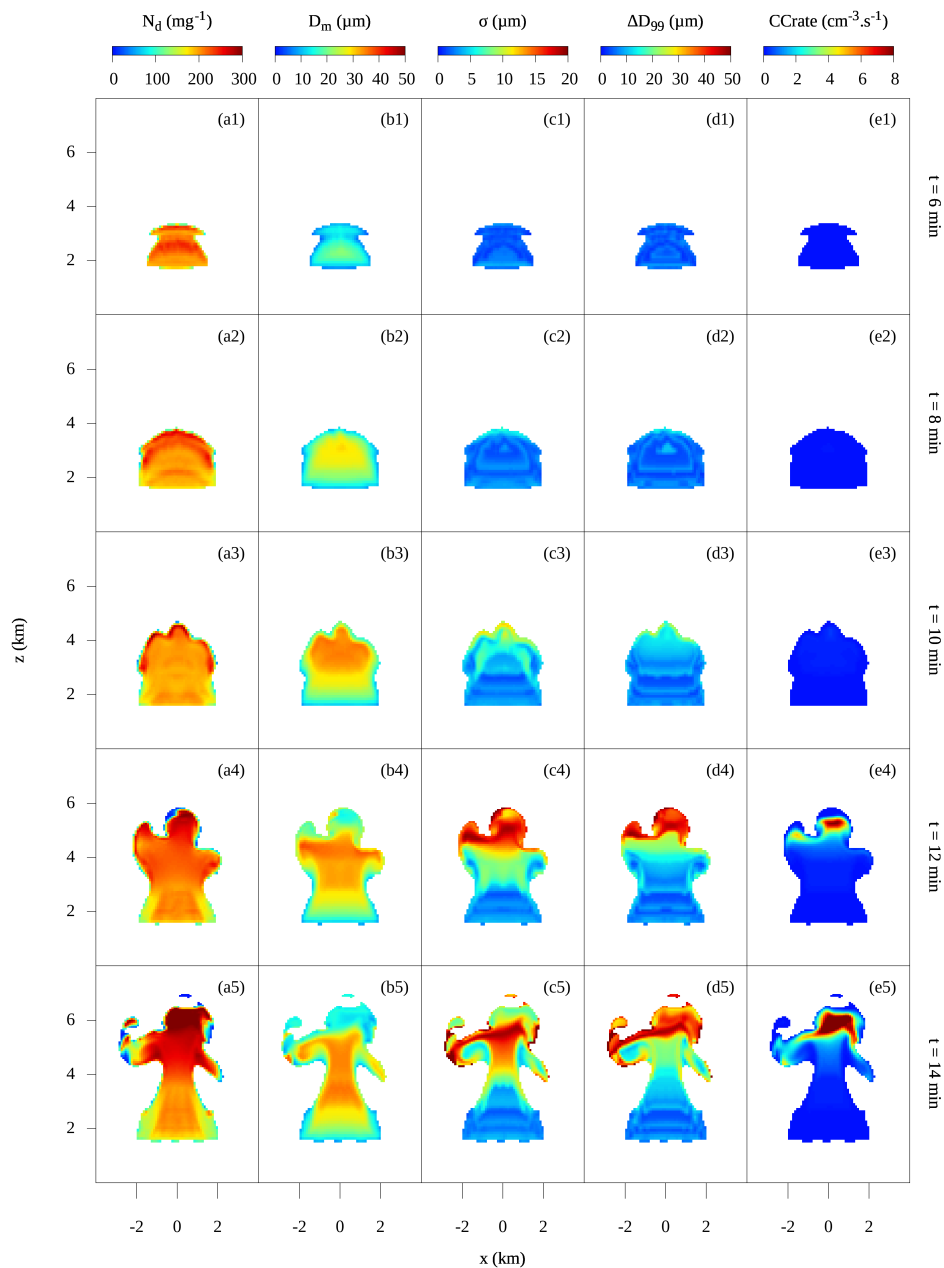


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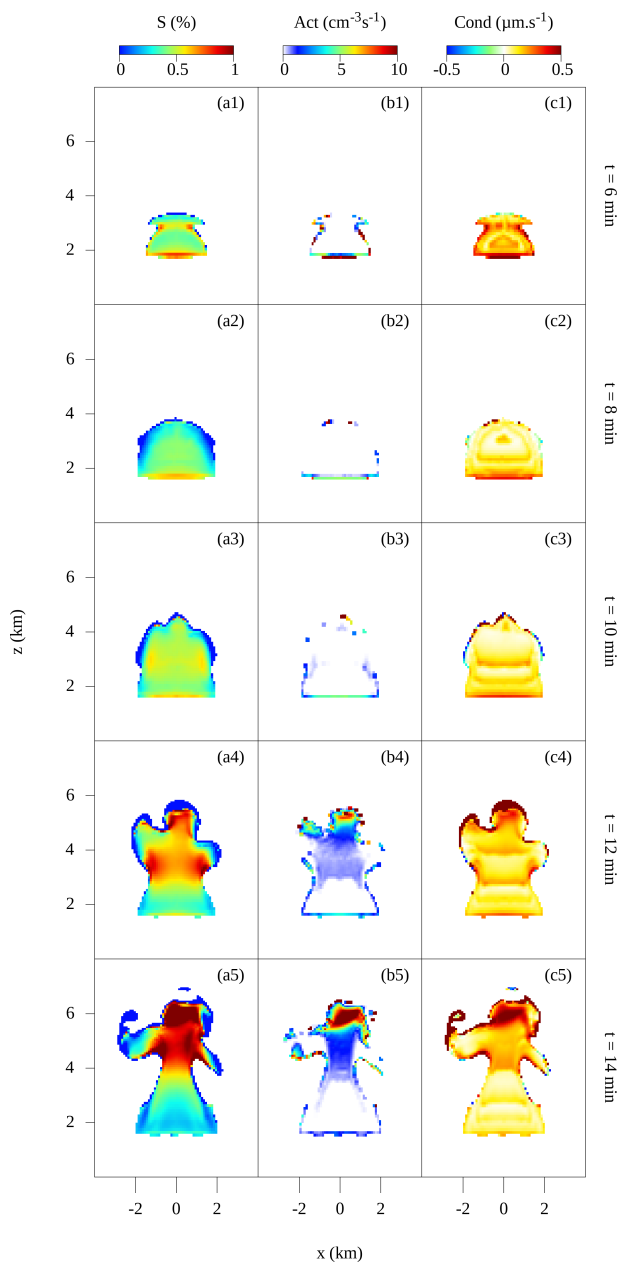


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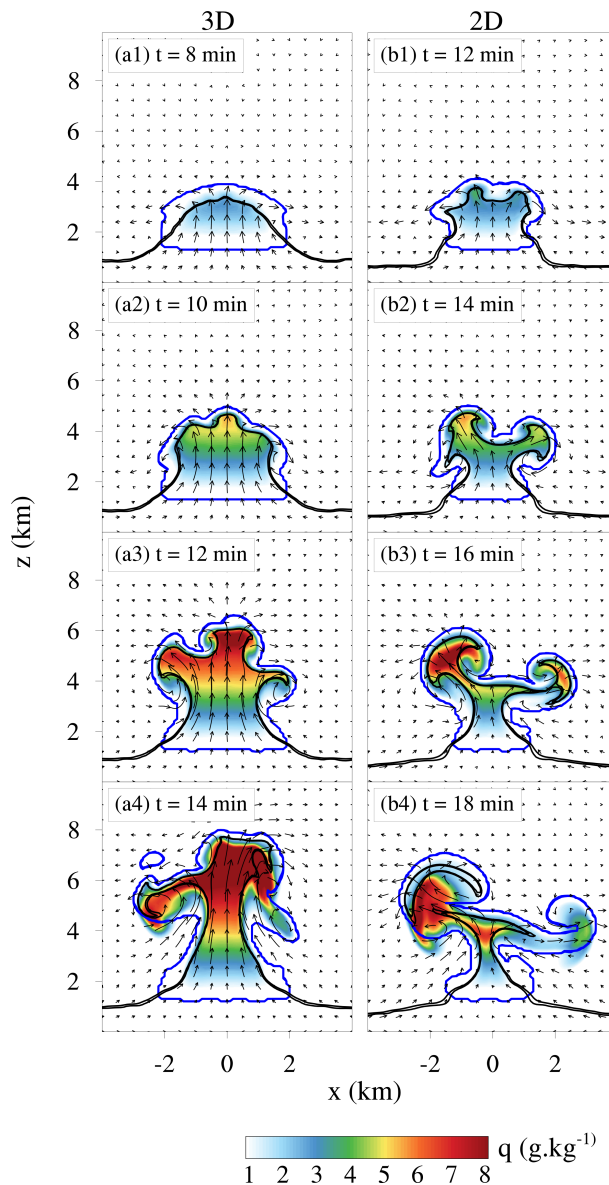


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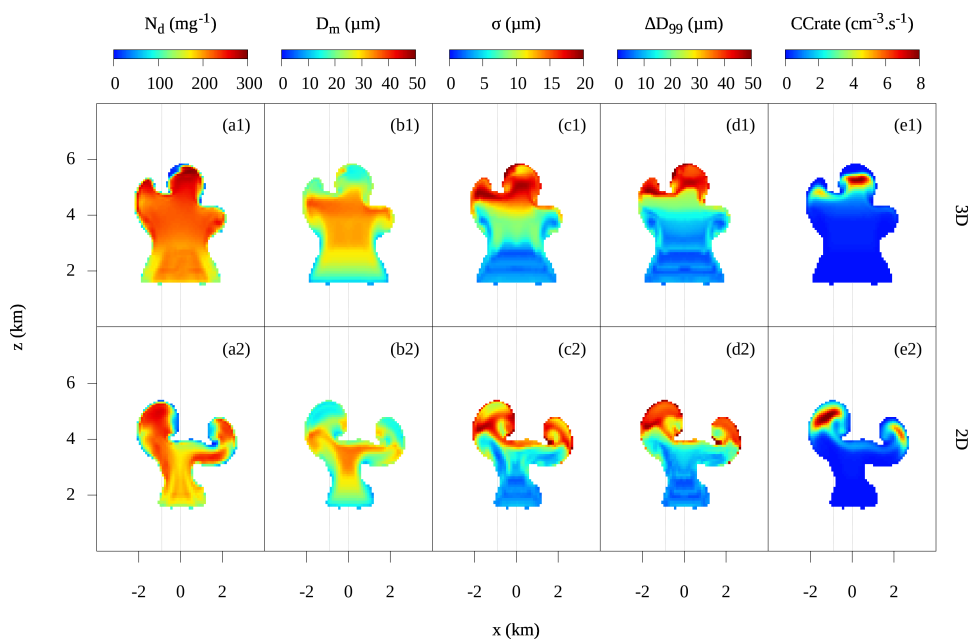


FIG. 5. Similar to Fig. 2, but for the base 3D simulation at $t = 12$ min (top row) and 2D simulation at $t = 16$ min (bottom row).

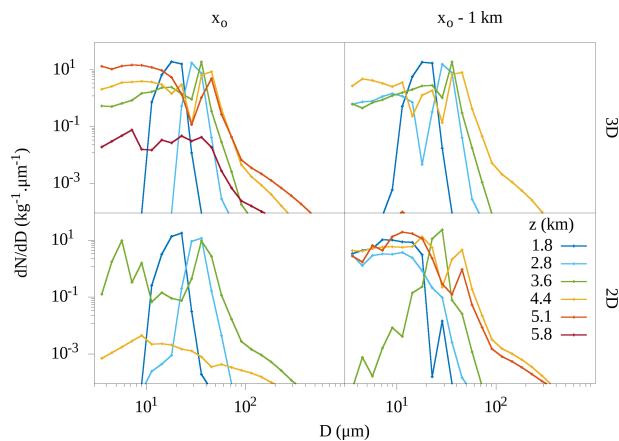


FIG. 6. Bin DSDs at different heights (colored lines) in the base 3D simulation at $t = 12$ min (top row) and 2D simulation at $t = 16$ min (bottom row). The left panels show DSDs at the cloud center (rightmost vertical lines in each panel of Fig. 5). The right panels show DSDs 1 km to the “left” of center (leftmost vertical lines in each panel of Fig. 5).

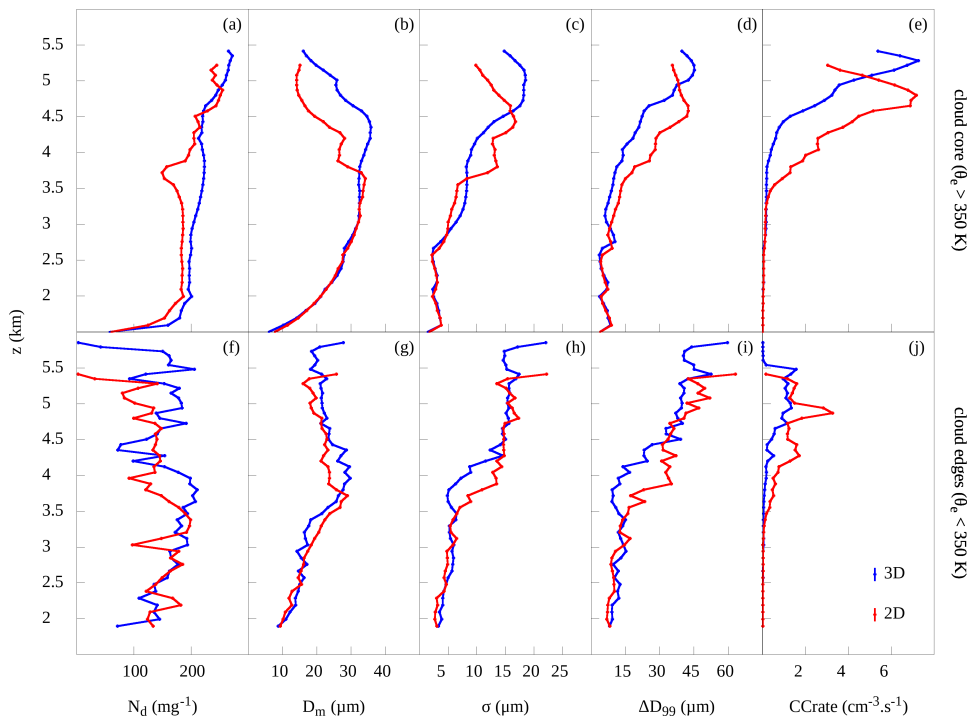


FIG. 7. Mean profiles of a) number mixing ratio (N_d), b) mean diameter (D_m), c) DSD standard deviation (σ), d) difference between the sizes of the 99th and 50th percentiles of the DSD (ΔD_{99}), and e) rate of collision-coalescence growth (CCrate), for $\theta_e > 350$ K (first row) and $\theta_e < 350$ K (second row) at $t = 12$ min in the base 3D simulation (blue lines) and $t = 16$ min in the 2D simulation (red lines).

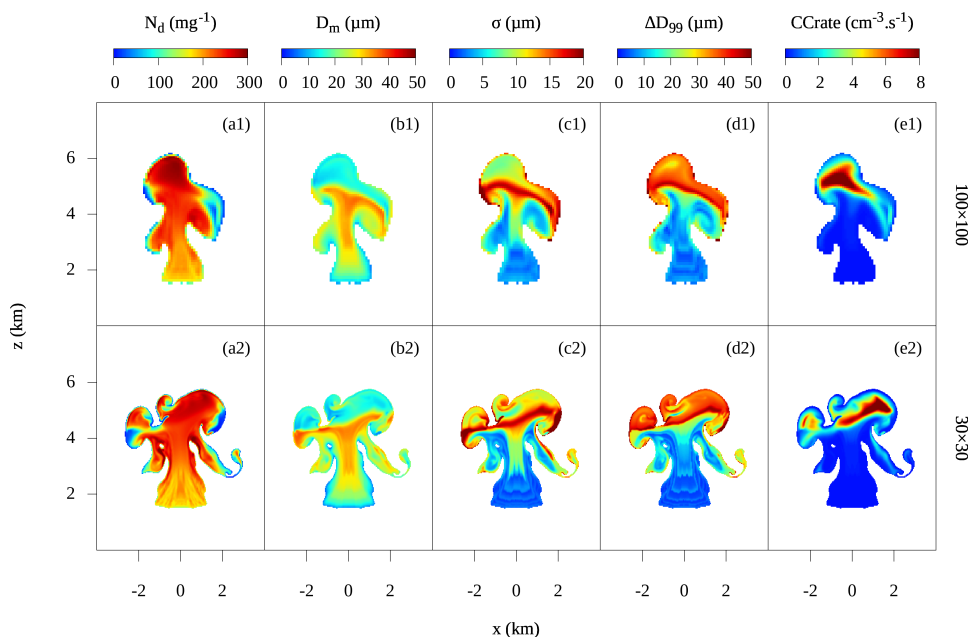


FIG. 8. Vertical cross-sections of a) number mixing ratio (N_d), b) mean diameter (D_m), c) DSD standard deviation (σ), d) difference between the sizes of the 99th and 50th percentiles of the DSD (ΔD_{99}), and e) collision-coalescence rate (CCrate) for the low-resolution 100×100 simulation (top row) and high-resolution 30×30 (bottom row) simulations for the CTRL case, at time $t = 16$ min.

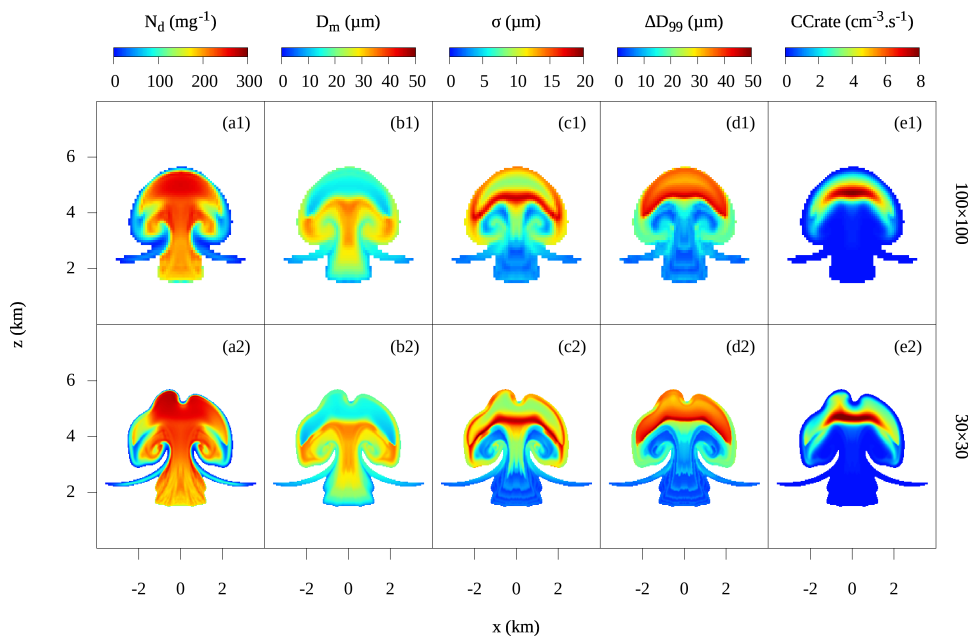


FIG. 9. Similar to Fig. 8, but for the CCS simulations.

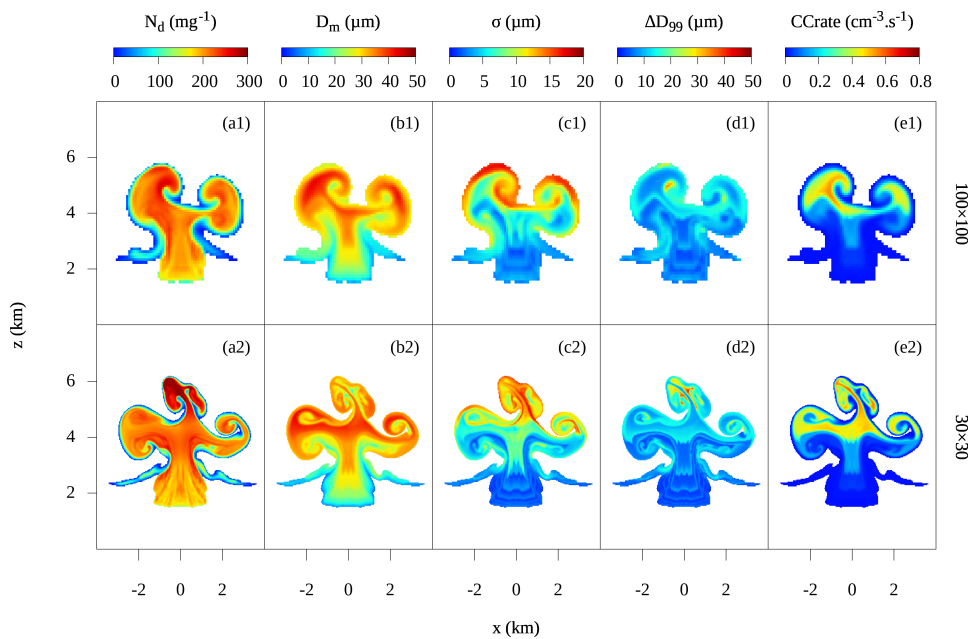


FIG. 10. Similar to Fig. 8, but for the TURB simulations. CCrate here constitutes a diagnosed rate because collision-coalescence is turned off in these simulations.

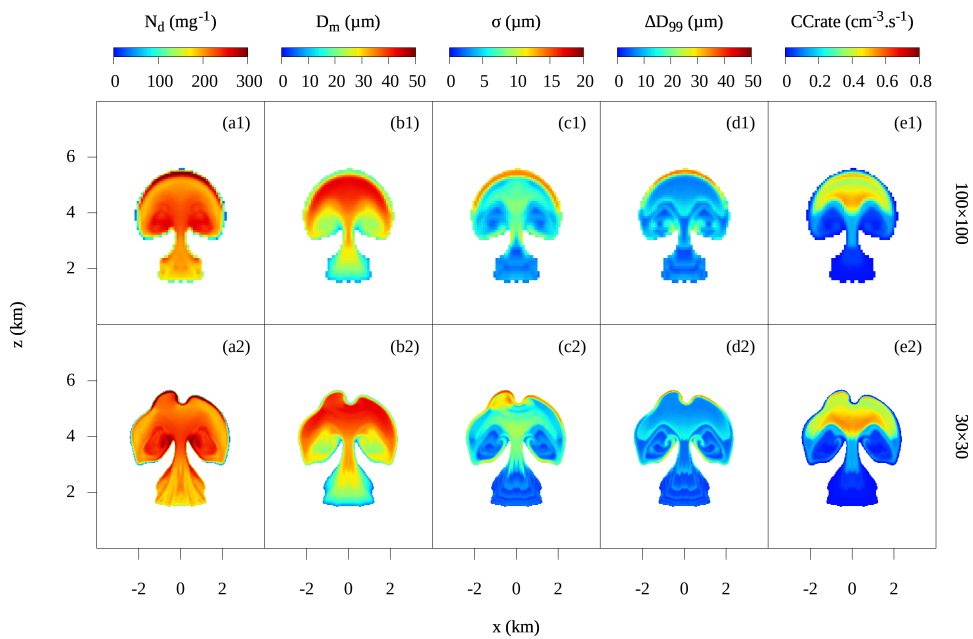


FIG. 11. Similar to Fig. 10, but for the EVAP simulations.

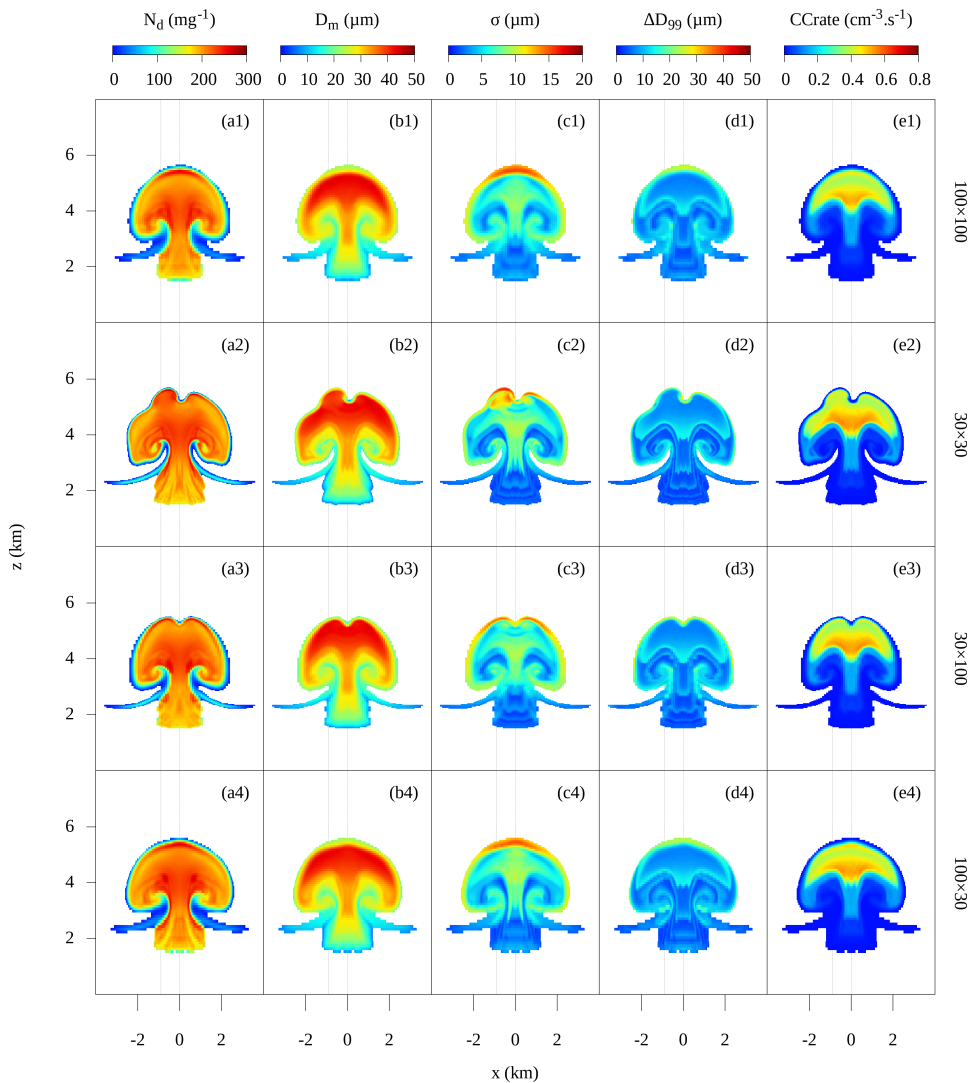


FIG. 12. Similar to Fig. 10, but for the ACT case, and with two extra rows for the simulations with high vertical resolution (30x100) and high horizontal resolution (100x30). CCrate here constitutes a diagnosed rate because collision-coalescence is turned off in these simulations.

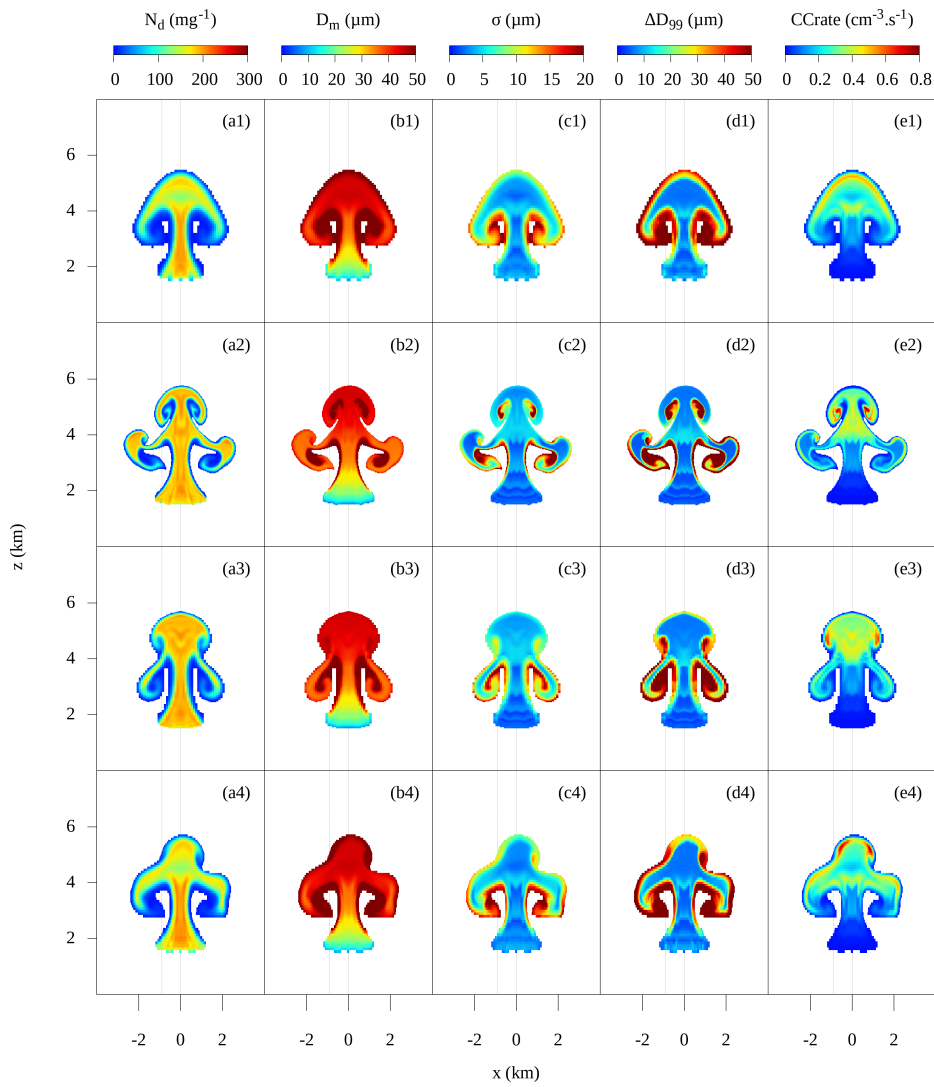


FIG. 13. As in Fig. 12, except for the CBACT simulations.

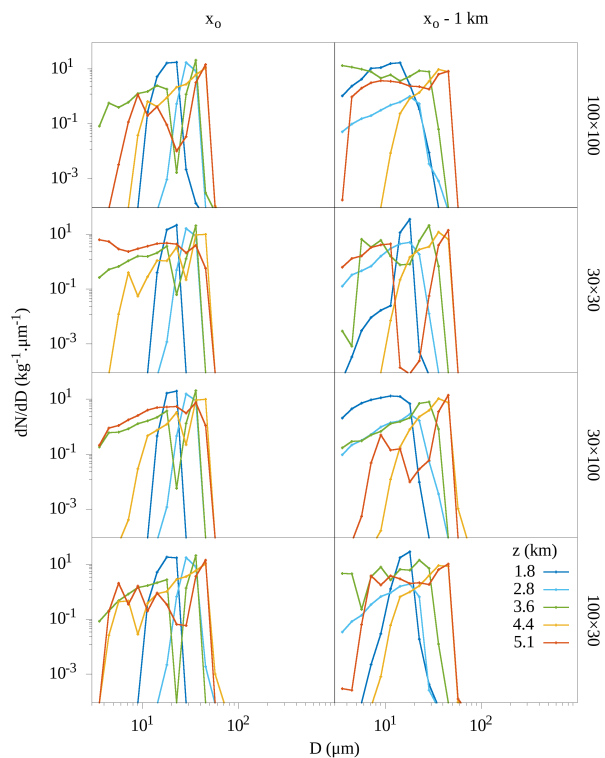


FIG. 14. As in Fig. 6, except for the bin DSDs at different heights at $t = 16 \text{ min}$ in the ACT simulations.

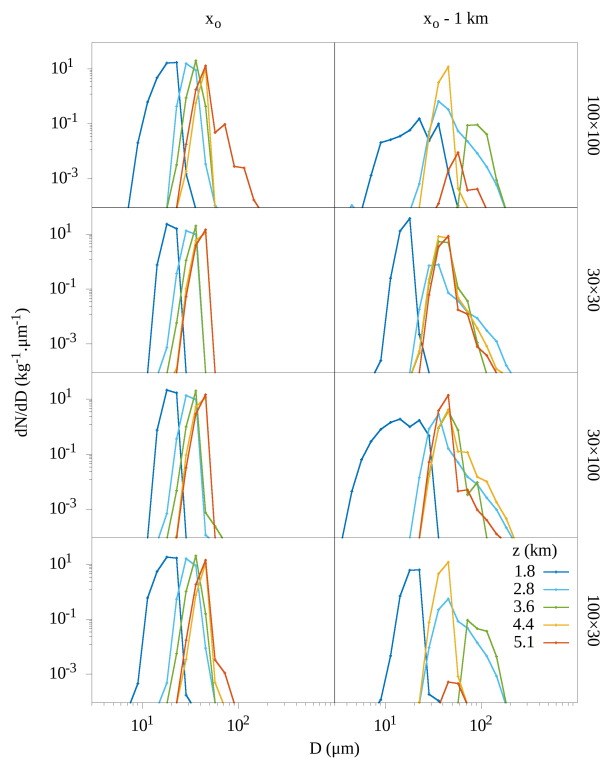


FIG. 15. As in Fig. 14, except for the CBACT simulations.

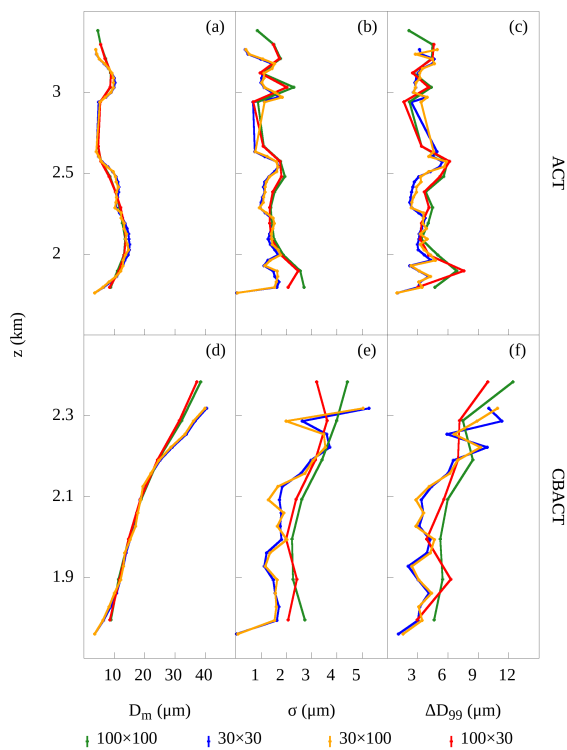


FIG. 16. Mean profiles of a,d) D_m , b,e) σ , and c,f) ΔD_{99} for different model resolutions (colored lines labeled at the bottom of the figure) in the ACT and CBACT simulations at $t = 7$ min.

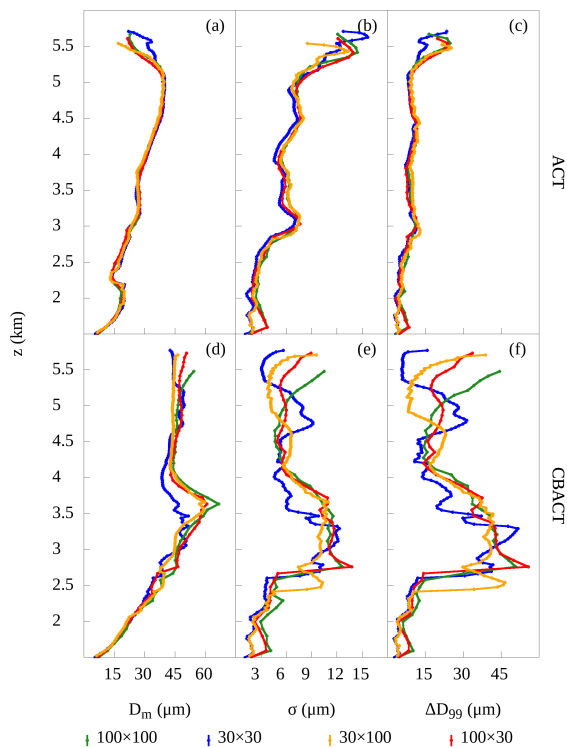


FIG. 17. Similar to Fig. 16, but for $t = 16$ min.

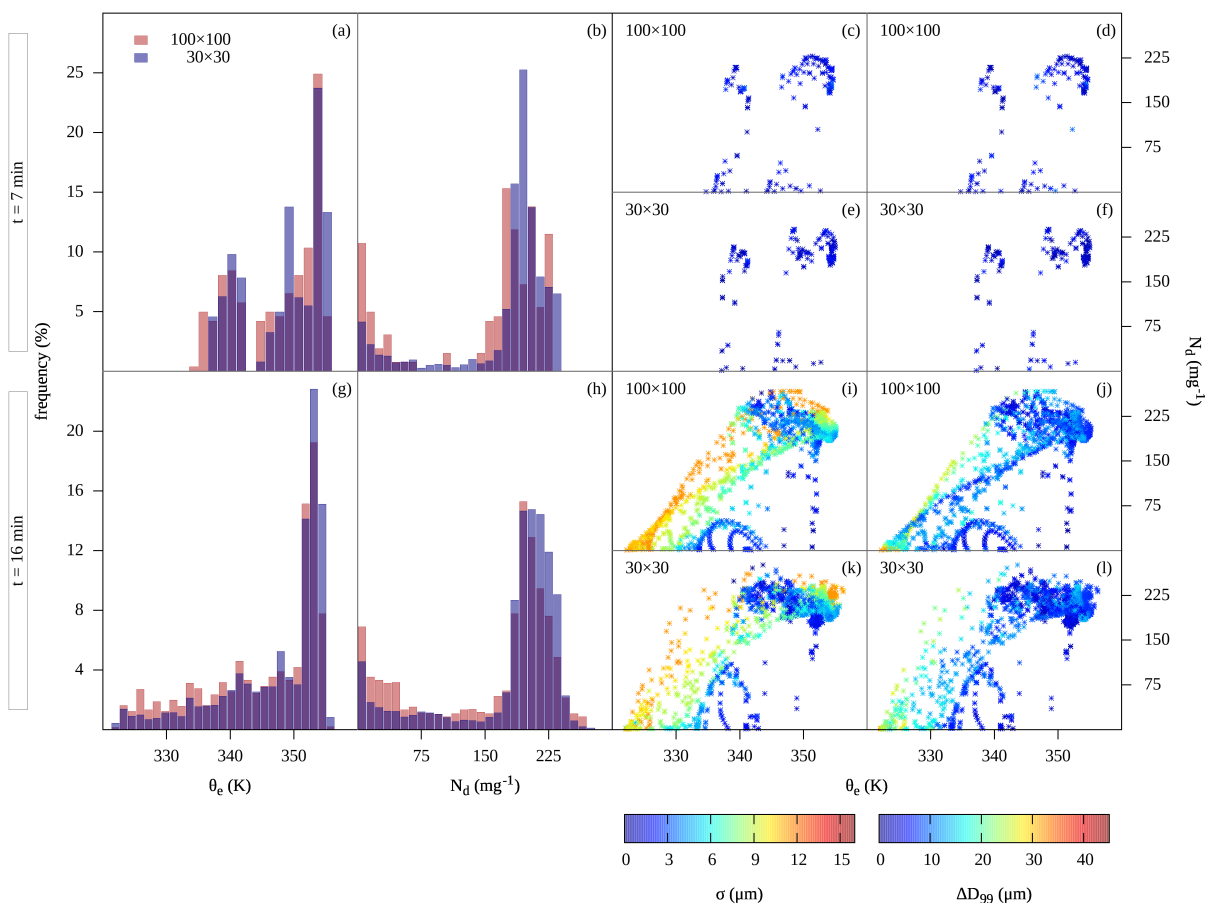


FIG. 18. Histograms of θ_e (a,g) and N_d (b,h) as well as scatterplots of σ (c,e,i,k) and ΔD_{99} (d,f,j,l) in the space of N_d versus θ_e , at $t = 7$ min (upper plots) and $t = 16$ min (lower plots) in the ACT simulations. The scatterplots in (c,d,i,j) and (e,f,k,l) correspond to the lower-resolution and higher-resolution simulations, respectively, as labeled in the plots. In the scatterplots of the high-resolution simulation, only one out of every three points is shown to facilitate the visualization.

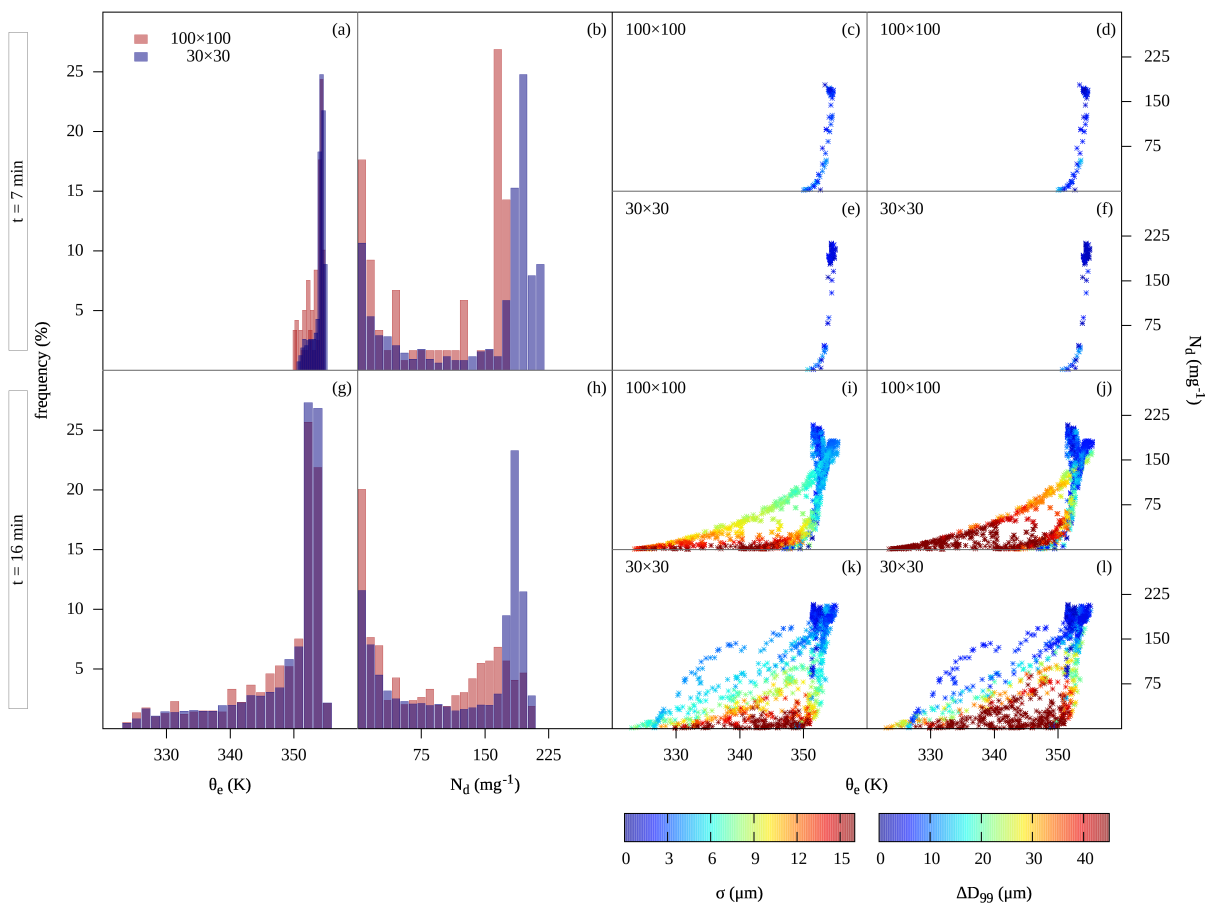


FIG. 19. Similar to Fig. 18, but for the CBACT simulations.

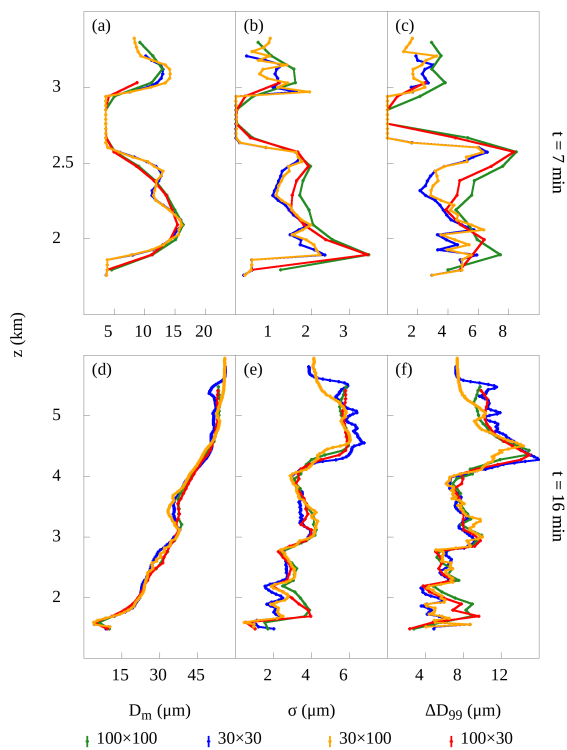


FIG. 20. Mean profiles of a,d) D_m , b,e) σ , and c,f) ΔD_{99} for different model resolutions (colored lines labeled at the figure bottom) at $t = 7$ min and $t = 16$ min for the ND100 simulations.