

A Comparison of Axiomatic Distance-Based Collective Intelligence Methods for Wireless Sensor Network State Estimation in the Presence of Information Injection

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Abstract—Wireless sensor networks are a cost-effective means of data collection, especially in areas which may not have significant infrastructure. There are significant challenges associated with the reliability of measurements, in particular due to their distributed nature. As such, it is important to develop methods that can extract reliable state estimation results in the presence of errors. This work proposes and compares methods based on collective intelligence ideas, namely consensus ranking and rating models, which are founded on axiomatic distances and intuitive social choice properties. The efficacy of these methods to assess a transmitted signal’s strength with varying quantity and quality of incompleteness in the network’s readings is tested.

I. INTRODUCTION

Wireless sensor networks have been used for a wide variety of estimation tasks including localization, power grid monitoring, and intrusion detection. Although significant technological improvements have been made in recent years, wireless sensor networks are still subject to degradation from interference, which may be due to non-intentional sources (e.g., static reflectors, hardware failure) or intentional sources (e.g., compromise by an adversary) [1]–[3]. This work considers that an adversary can inject a false sensor position into the network, while previous work considers the effect of erroneous sensor positions on source localization [4]–[7]. As such, when trying to perform state estimation tasks based on data collected by wireless sensor networks, it is critical that the chosen methods are robust and capable of achieving high estimation accuracy in spite of large quantities of incompleteness and errors in the reported data [8]. One approach that has begun to show promising results in recent research is the principled aggregation methods of Collective Intelligence.

Collective Intelligence has recently been adapted from a primarily socio-theoretical framework into an applicable tool

for decision-making in multiple domains. This work focuses on Collective Intelligence methods related to computational social choice, whose foundations overlap with the idea of the “wisdom of the crowd” [9]. It applies methods that are principled, that is, founded on axioms inspired by social choice properties. These principled aggregation methods have been successfully translated into distributed decision-making and data aggregation contexts, e.g., the monitoring of environments through distributed wireless sensor networks [1]. While several such methods have been proposed and tested, their effectiveness in handling highly incomplete and erroneous data has not been analyzed or evaluated.

In particular, the work discussed herein seeks to evaluate the effectiveness of several Collective Intelligence approaches at challenging state estimation tasks on wireless sensor networks in the presence of an adversary. This analysis will rely on recent works using axiomatic measures for incomplete ordinal and cardinal data aggregation and will evaluate the effectiveness of several methods on simulated networks.

II. PROBLEM

Missing sensor readings and high levels of noise are inherent to highly distributed wireless sensor networks. These problematic qualities are only amplified when the networks are subject to injection of false information by an adversary. Incompleteness in an individual sensor’s readings is often addressed via redundancy, considering only sensors with complete data, or by directly imputing the missing information [10]–[12]. Each of these approaches becomes less feasible as the level of incompleteness rises. Even existing robust Collective Intelligence tools, which have been applied mostly in group decision making as well as for environmental monitoring and detection of mobility patterns [13], [14], can struggle to handle such high levels of incompleteness.

This work experiments with a neutral treatment of incompleteness, where no inferences are made about the missing

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values of a sensor's readings. It adapts several recent Collective Intelligence tools to analyze the effectiveness of each at handling a state estimation task over a simulated wireless sensor network. The distance metrics used in these tools treat all sources of information as equally reliable, regardless of that source's level of incompleteness. Each tool is evaluated on its ability to recover a single source true value using a collection of sensors that faces both standard measurement reliability errors and increasing levels of incompleteness. The incompleteness in sensor readings is simulated to reflect both random unreliability and more systemic downtime in individual sensors, whether due to non-intentional or intentional sources of error.

To handle these aggregation tasks under significant noise and incompleteness, three primary models are considered: the separation-deviation model proposed by Hochbaum and Levin (SepDev) [15], [16], an adapted separation-deviation model proposed by Fishbain and Moreno (FM) [1] notably used for environmental time-series data aggregation via distributed low-cost sensors, and the ranking and rating model proposed by Escobedo et al. (RR) [9]. Each of these is an optimization model which is not necessarily convex. However, each model is solved using exact linearized models.

The SepDev model considers cardinal pairwise comparisons to generate an aggregate ranking that minimizes two factors: disagreement among actual readings and disagreement among the intensity of difference in pairs of readings. A general form of this optimization problem is:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \lambda_1 \sum_{k=1}^m \sum_{i \in V^k} |x_i - a_i^k| + \lambda_2 \sum_{k=1}^m \sum_{(i,j) \in \mathcal{A}^k} |(x_i - x_j) - p_{ij}^k| \quad (1) \\ \text{s.t.} \quad & L \leq x_i \leq U \quad i = 1, 2, \dots, n \quad (2) \\ & x_i \geq 0 \quad i = 1, 2, \dots, n \quad (3) \end{aligned}$$

Here, V^k represents the set of readings recorded by the k th sensor and \mathcal{A}^k is the set of pairs (i, j) s.t. sensor k reads at both times i and j . The measurements to be optimized are x_i , the preference intensity between two readings is $p_{ij}^k = a_i^k - a_j^k$, where a_i^k is the value provided by sensor k for reading i . The relative weights of the separation and deviation pieces of information in the model are given by λ_1 and λ_2 (both set to 1 for SepDev experiment runs). L and U represent the lower bound and upper bounds of allowed measurements, respectively, and μ is the minimum considered separation in values.

The FM model was designed to aggregate data from uncalibrated wireless sensor networks; the model is founded on a set of axioms for the robust aggregation of cardinal data (i.e., ratings) in group decision-making. Its general form is:

$$\min_{\mathbf{x}} \quad \sum_{k=1}^m d(\mathbf{a}^k, \mathbf{x}), \quad (4)$$

where d is a distance function between each input rating vector and subject to (2) and (3). For this study, the Normalized Projected Cook-Kress (NPCK) distance measure is used, as in [1]. This turns out to be a special case of the SepDev problem

with $\lambda_1 = 0$ and $\lambda_2^k = \left(4(U - L) \cdot \lceil \frac{|V^k|}{2} \rceil \cdot \lfloor \frac{|V^k|}{2} \rfloor\right)^{-1}$, i.e., a different separation penalty per sensor, normalized via a function of its number of readings. The linearized constraints are described within the RR model. A secondary optimization problem is solved to find the best constant c to add to the outputs from FM so as to approximate the magnitude of the readings. The specific statement of the calibration problem is provided here, but its application is also considered in some of the experimental results from the other two models. The model is given as:

$$\min_c \quad \sum_{k=1}^m \sum_{i \in V^k} |(x_i^* + c) - a_i^k|. \quad (5)$$

Finally, the RR model provides a similar aggregation method that can handle the joint aggregation of different qualities of information, i.e., cardinal and ordinal (or scale-free) information, allowing more general data to be processed. The general statement of this problem is:

$$\min_{\mathbf{x}} \quad \sum_{k=1}^m d_1(\mathbf{a}^k, \mathbf{x}) + \sum_{k=1}^m d_2(\mathbf{b}^k, \text{rank}(\mathbf{x})) \quad (6)$$

$$\text{s.t.} \quad 0 \leq x_i \leq (U - L)/\mu \quad i = 1, 2, \dots, n \quad (7)$$

$$x_i \in \mathbb{Z} \quad i = 1, 2, \dots, n \quad (8)$$

Here, each of the distances in the objective function is a pseudometric; specifically, each obeys most axioms of a metric with the exception that the triangle inequality is satisfied in the projected space of overlapping readings of the two vectors being compared. The distances used are d_{NPCK} and d_{NPKS} , respectively, where d_{NPCK} is the *normalized projected Cook-Kress* distance developed in [1] and d_{NPKS} is the *normalized projected Kemeny-Snell* distance developed in [17]. Additionally, the vectors \mathbf{b}^k are the input ranking vectors and $\text{rank}(\mathbf{x})$ is the ranking solution vector (obtained by sorting \mathbf{x}). The exact reformulation used for this model is developed and described in [18] and is given below:

$$\max_{\mathbf{s}, \mathbf{t}, \mathbf{x}, \mathbf{y}} \quad \sum_{k=1}^m -4\lambda_2^k \sum_{(i,j) \in \mathcal{A}^k} t_{ij}^k + \sum_{i=1}^n \sum_{j=1}^n 2\hat{B}_{ij} y_{ij} \quad (9)$$

$$\text{s.t.} \quad y_{ij} + y_{ji} \geq 1 \quad i, j = 1, \dots, n; i \neq j \quad (10)$$

$$y_{ij} - y_{kj} - y_{ik} \geq -1 \quad i, j, k = 1, \dots, n; i \neq j \neq k \quad (11)$$

$$t_{ij}^k \geq \mu(x_i - x_j) - p_{ij}^k \quad (i, j) \in \mathcal{A}^k, k = 1, \dots, m \quad (12)$$

$$t_{ij}^k \geq -\mu(x_i - x_j) + p_{ij}^k \quad (i, j) \in \mathcal{A}^k, k = 1, \dots, m \quad (13)$$

$$x_i - x_j \geq \mu - (\mu + M)y_{ij} \quad i, j = 1, \dots, n; i \neq j \quad (14)$$

$$x_i - x_j \leq M(1 - y_{ij}) \quad i, j = 1, \dots, n; i \neq j \quad (15)$$

$$0 \leq x_i \leq (U - L)/\mu \quad i = 1, \dots, n \quad (16)$$

$$x_i \in \mathbb{Z} \quad i = 1, \dots, n \quad (17)$$

$$y_{ij} \in \{0, 1\} \quad i, j = 1, \dots, n \quad (18)$$

$$t_{ij}^k \text{ unrestricted} \quad (i, j) \in \mathcal{A}^k, k = 1, \dots, m \quad (19)$$

where λ_2^k is defined as in the FM model, \hat{B}_{ij} is the sum of the scaled input ranking matrices [19] (summarizing the ordinal relationships expressed by all sensors over all pairs of readings) and $[y_{ij}]$ represents the solution ranking matrix.

Here, constraints (10) and (11) induce a valid complete ranking of the output readings. Constraints (12) and (13) and auxiliary variables t_{ij} are added to linearize the part of the objective function that corresponds to the FM model. Lastly, constraints (14) and (15) link the variables of the optimal ranking and rating solutions.

III. METHODOLOGY

The data generation method of the performed experiments is as follows. We consider the localization of a single stationary transmitter. A collection of sensors which are uniformly distributed in an X by Y area seeks to estimate the location of this emitter. Each sensor is assumed to be placed along a straight single line radiating from the transmitter to simplify the signal strength loss calculations [20]. Specifically, these sensors are assumed to measure the signal strength based on free space path loss. Each sensor is assumed to take between 5 and 20 readings at distinct times which will be used to generate a single estimate of the original signal strength. Results are only presented for 10 readings, as these were representative of the patterns seen across all readings recorded. That is, each sensor produces a 10-element vector, \mathbf{a}^k , of readings. After these sensor readings are generated, incompleteness is added to the system and aggregation is performed using each of the models described in section 2.

To analyze the effects of incomplete data, two distinct classes of experiment were performed with different assumptions about the nature of incompleteness. In the first class, referred to as *arbitrary incompleteness*, data points are removed at random, distributed uniformly across the full set of existing data points. The level of this incompleteness is measured via the parameter $0 \leq \gamma_1 \leq 1$, where $\gamma_1 = 0$ corresponds to a complete data set and $\gamma_1 = 1$ corresponds to a completely empty set. This process is repeated until a minimum of a fixed percentage of data points has been removed. The exception to this rule, is that each sensor must still report at least two readings, and for each iteration of readings across sensors, there must also be at least two reported readings. This incompleteness is tested at the values $\gamma_1 = 0.05, 0.1, \dots, 0.75$.

The second class, referred to as *systematic incompleteness*, generates incompleteness on a per-sensor basis, and any sensor that fails is considered to remain in a failure state for several consecutive readings. The level of this incompleteness is measured via the parameter $0 \leq \gamma_2 \leq m$, where m is the total number of sensors and $\gamma_2 \in \mathbb{Z}$. The number of such sensors that fail is uniformly random from 1 to $m - 1$. This incompleteness is tested at the values $\gamma_2 = 1, 2, \dots, m - 1$. The start time and duration of failure are similarly randomly generated, with the number of incomplete readings between one third and two thirds of the total readings per failed sensor.

Another set of experiments was performed to analyze the effectiveness of the models in the presence of false data. In this experiment set, two cases were considered: the presence of *distance measurement error* and the existence of *false signal strength*. In the first case, an incorrect distance measure was used by some of the sensors when trying to recover the

broadcasted signal strength in spite of a deliberate injection of false data. The erroneous distance values d_{error} were generated randomly between 10 and 30 kilometers, where d_{error} is different from the actual distance measure of the sensors. In the second case, an error was introduced by injecting false input signal values. In order not to arouse suspicion, which might trigger an alarm if the sensor readings exceed a predetermined threshold value, the strength of the false signal transmitted is restricted. In both cases, the percentage of sensors experiencing errors was measured via the parameter $0 \leq \gamma_3 \leq 1$, where $\gamma_3 = 0$ corresponds to none of the sensors experiencing errors and $\gamma_3 = 1$ corresponds to all of them being compromised. As it is unlikely that a large number of sensors will be experiencing errors at the same time without being detected, the only tested values of γ_3 were kept between 0.2 and 0.6 with a 0.1 increment.

For each combination of number of sensors and type and amount of incompleteness/error, problem data were generated and solved 100 times. Data generation and models were coded in Python 3.6; optimization models were reformulated as integer linear programs and solved in commercial mathematical programming solvers CPLEX 12.8 and Gurobi 8.0.

IV. RESULTS

The effectiveness of each model was first tested with the arbitrary incompleteness generation described in section 3. Figure 1 presents the estimates and the averages for each level of incompleteness produced from 100 repetitions of the data generation and state estimation procedure for each of the three primary models, in which only the FM model is calibrated. The incompleteness is described as the fraction of data which is missing. The large symbols represent the average of the 100 estimates performed. Although only $m = 20$ sensor results are presented, the results are similar for $m = 5, 10, 15, 25, 30$; that is, the standard deviation tends to decrease as the number of sensors increases, but the trends in estimate values and relative standard deviation remain similar.

In this first test, the FM model produced superior results to each of the other two models across any number of sensors, both in terms of accuracy and precision. In fact, even at only five sensors, the average estimate produced by the FM model was always less than 0.005 watts away from the assumed true broadcast signal strength, with a standard deviation of less than 0.02 watts. The SepDev model produced results that were almost as consistent, with approximately double the standard deviation, but averaged well below the true value of one watt. This average estimate did increase slightly with more incompleteness, which is to say that the estimated values improved with less data. However, because of the randomness in which data is removed, this may be incidental to the randomness of the data. Finally, the RR model produced very inconsistent results overall. For all sensors and incompleteness levels, the standard deviation of its estimates was significantly greater than either the SepDev or FM models. Its average estimates were always closer to the true value than the SepDev model, but also tended to underestimate the original signal strength.

20 Sensor Estimates, Uncalibrated

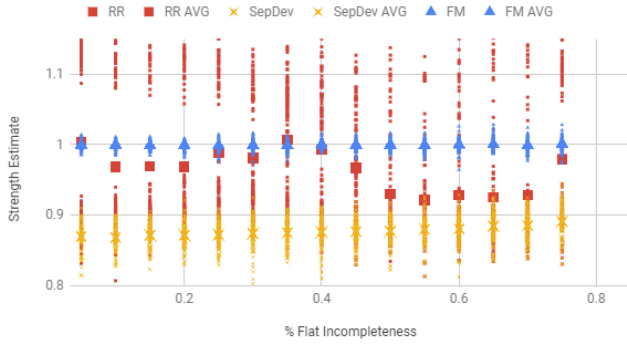


Fig. 1. Estimates for Arbitrary Incompleteness, No Calibration

20 Sensor Estimates, Calibrated

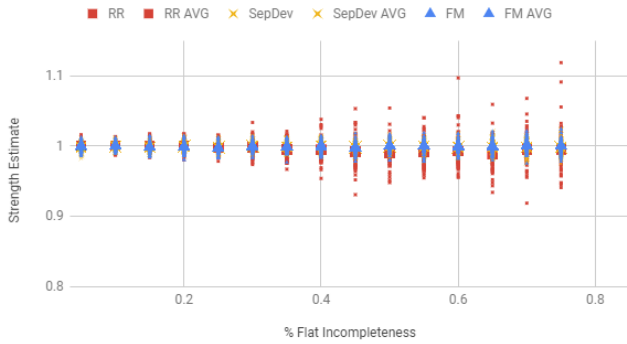


Fig. 2. Estimates for Arbitrary Incompleteness, Calibration

Unlike the other models, the RR model shows a tendency toward a bimodal distribution of its estimates, especially as the number of sensors increases.

Next, the effectiveness of each model was again tested with the arbitrary incompleteness generation; however, in this case, the calibration procedure described in section 2 was applied to all three models. The experiments were repeated for the FM model, even though it is always calibrated, in order to compare each model across the same generated datasets, including all sensors' initial readings and the generated incompleteness. Figure 2 is analogous to Figure 1 above. The main effects of this calibration are that they dramatically improve the accuracy of the estimations of both the SepDev and RR models, as well as reduce their standard deviations (not plotted here). This is most notable in the SepDev model, which produces results nearly identical to those produced by the FM model, with slightly more deviation across all numbers of sensors. The RR model still has the largest standard deviation and produces the least accurate estimates, but it performs much more closely to the quality of the other two models. Another notable difference in the calibrated versus the uncalibrated results is that the bimodal tendencies of the RR models estimates disappear.

In the next experiment set, all models were tested with the systematic incompleteness generation method described in section 3, and the experiments were run both with and without calibration in the same manner as above. For these results,

20 Sensor Estimates, Uncalibrated

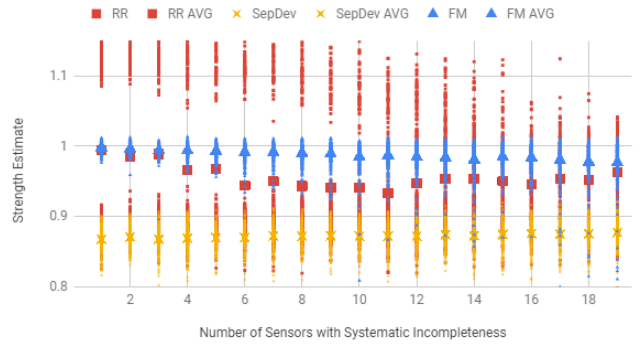


Fig. 3. Estimates for Systematic Incompleteness, No Calibration

20 Sensor Estimates, Calibrated

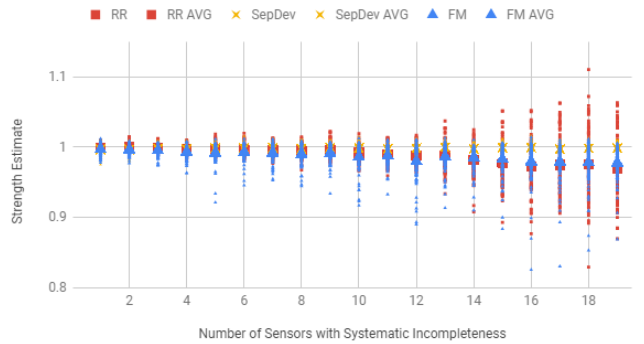


Fig. 4. Estimates for Systematic Incompleteness, Calibration

incompleteness is measured as the number of sensors with any missing readings. For the experiments run without calibration, the RR model performed similarly to the arbitrary incompleteness above, i.e., it had the highest standard deviation of estimates across all numbers of sensors and exhibited similar bimodal tendencies. However, the bimodal aspect disappeared as the levels of incompleteness increased, which corresponds to a decrease in the standard deviations in its estimates. The SepDev model had a similarly consistent underestimation of the ground truth, but the standard deviations of its estimates was nearly zero as the amount of incompleteness increased, so for high incompleteness it actually produced more precise estimates than did the FM model. The FM model also began to diverge from true value in its estimates as incompleteness increased, indicating that this model may not continue to produce accurate state estimates in cases of more systematic incompleteness. Without calibration, each of these models performed noticeably worse at estimating the true signal strength when the incompleteness was generated in this more systematic way. For the experiments run with calibration, the SepDev model became both the most accurate and the most precise. The average and standard deviation of its estimations barely changed with increased incompleteness, indicating it as the best performing model for estimating signal strength with this type of incompleteness. Here, the FM model's estimations have a higher standard deviation than the SepDev model's, and

20 Sensor Estimates, Uncalibrated, Random Distance Error

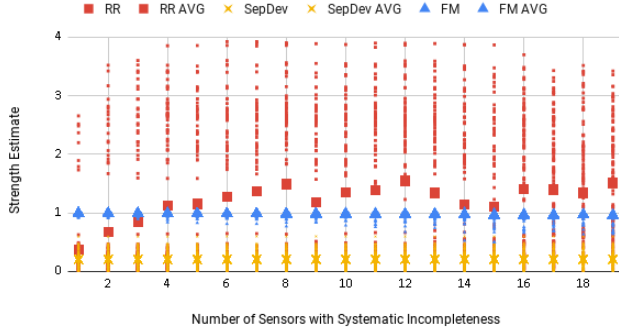


Fig. 5. Estimates for Systematic Incompleteness, No Calibration, 60% Sensor with Random Distance Error

20 Sensor Estimates, Uncalibrated, Random Signal Error

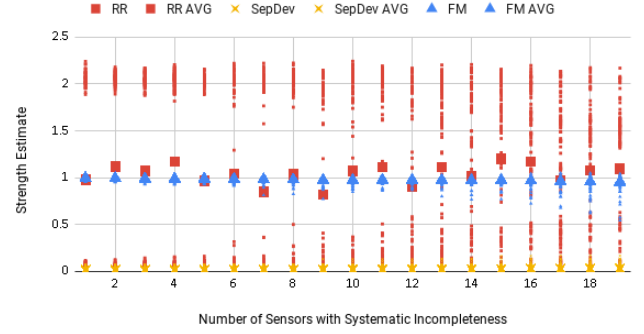


Fig. 7. Estimates for Systematic Incompleteness, No Calibration, 60% Sensor with Random Signal Error

20 Sensor Estimates, Calibrated, Random Distance Error

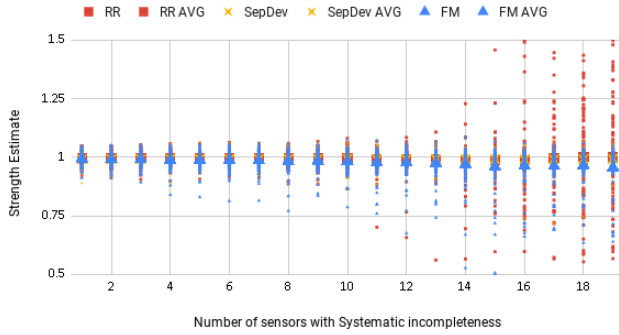


Fig. 6. Estimates for Systematic Incompleteness, Calibration, 60% Sensor with Random Distance Error

20 Sensor Estimates, Calibrated, Random Signal Error

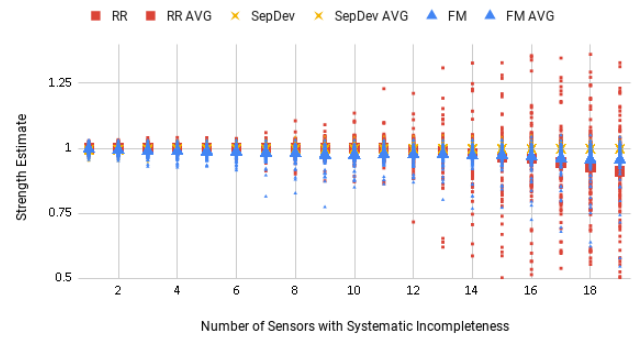


Fig. 8. Estimates for Systematic Incompleteness, Calibration, 60% Sensor with Random Signal Error

it exhibits the same behavior it showed without calibration of decreasing estimated values as incompleteness rises. The RR model shows very similar behavior to the FM model in both average and standard deviation of its estimates as incompleteness was increased.

In the final set of experiments, all models were tested with arbitrary and systematic incompleteness in the presence of both distance and signal errors. Due to the presence of errors, the standard deviations of the estimates obtained were much higher than when errors were not present. The induced false distance measure resulted in much larger deviation of signal estimates than the results obtained for false input signal. In each case, the models behaved similarly to cases when errors were not present. In the experiment sets where calibration was not performed for the RR and the SepDev models, the FM model produced a much more consistent strength estimate. Even with 60% of sensors compromised when introducing distance measurement errors, the standard deviation of the FM model for 20 sensors and 10 readings was less than 0.09 watts for random incompleteness and 0.02 for systematic incompleteness, whereas for the SepDev model it was around 0.12; the RR model resulted in standard deviation over 1 watts in both cases. When calibrated, the performance of

the three models became similar in the case of arbitrary incompleteness for both distance and signal errors. But in the case of systematic incompleteness, the performance of the SepDev model improved drastically, resulting in a much better signal estimate as can be seen in Figures 6 and 8. As the number of compromised sensors increased, the estimates obtained from the FM model started to fluctuate, but the performance of the SepDev model was much more consistent. In all cases, the RR model produced the largest standard deviation which increased significantly with incompleteness and the number of compromised sensors.

V. CONCLUSION

The effectiveness of three distinct Collective Intelligence models at estimating the location of an unknown emitter in the presence of incomplete data and random errors has been evaluated, where two of those models are augmented with an additional calibration step. Each model appears to have some merits in different circumstances, although the RR model appeared to provide middle of the road performance in all cases. For situations in which incompleteness is expected to occur in very short windows and very erratically, the FM model which has been used for environmental time series estimation

seems to perform with the highest levels of precision and accuracy, even with small numbers of sensors and readings. For situations in which incompleteness is expected to occur among individual sensors for a larger number of readings, e.g., scheduled or unscheduled outages, the SepDev model with calibration performs with comparable accuracy and precision. In both cases, the best choice model maintains similarly accurate estimates even as up to 75% of data is missing or nearly all sensors suffer from a period of non-responsiveness. Fictitious distances (used in sensor calculations) seem to have a much higher impact on the estimations than fictitious input signal strength values (i.e., the readings). But even with these corrupted readings, the performance of the models is similar to the first experiments, demonstrating the potential of Collective Intelligence at handling challenging state estimation tasks in the presence of false data injection.

Notably, these experiments have demonstrated two things. The first is that Collective Intelligence methods can be effectively applied to state estimation tasks with a high level of incompleteness and errors. These methods can be run on systems of varying sizes, maintaining much of their accuracy and precision even in smaller systems that produce few readings. The second is that further study is needed to determine the best matches between tools and applications. While each case had at least one method that produced highly accurate and precise state estimations, not every method performed equally well. Furthermore, the two most successful models (the FM and SepDev models) are solvable in polynomial time as long as their penalty functions are convex [21], as is the case here. That means these two models can be used to generate reliable estimations very quickly, even for large networks and observation spans.

VI. FUTURE WORK

Although some basic experimentation has been performed on these systems at this time, there are significant questions that remain, including some raised by the results of these experiments. These methods should be tested on 2- and 3-dimensional distributions of sensors. Furthermore, the potential effectiveness of these models should be tested for other forms of systematic false data injection. Additionally, one new question remaining to be tested is the robustness of the sensors' observation span overlap (e.g., counting the maximum number of hops required to compare values at any two reading times [22]). This could help analyze the cause of the improvement in the SepDev model with arbitrary incompleteness as incompleteness increased. The results from the systematic incompleteness tests also indicate that there could be a method for optimal sensor downtime scheduling (e.g. to preserve battery life) such that there is sufficient overlap in the sensors to guarantee accurate estimations.

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