

Updated universal relations for tidal deformabilities of neutron stars from phenomenological equations of state

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Equation of state (EOS) insensitive relations, so-called universal relations, between the neutron star (NS) compactness, its multipolar tidal deformability coefficients, and between the tidal parameters for binary systems are essential to break degeneracies in gravitational wave data analysis. Here, we validate and recalibrate these universal relations using a large set of almost 2 million phenomenological EOSs that are consistent with current observations. In doing so, we extend universal relations to a larger region of the EOS parameter space, most notably to softer EOSs and larger compactnesses. We show that waveform models that neglect higher-than-leading-order tidal deformations of the NSs accumulate as much as 3.5 radians of dephasing from 20 Hz to merger. We also perform a full Bayesian parameter estimation of the GW170817 data, and we compare the NS radius constraints produced using universal relations from the literature and the updated fits we propose here. We find that the new fits yield a NS radius that is smaller by about 500 meters. This difference is less than the statistical uncertainty on the radius at the signal-to-noise-ratio of GW170817, but it is significantly larger than the precision anticipated for next-generation detectors.

I. INTRODUCTION

There is still great uncertainty in the equation of state (EOS) that describes the incredibly dense nuclear matter of neutron stars (NSs) in the regime above nuclear saturation density ($\rho_{\text{nuc}} \simeq 2.7 \times 10^{14} \text{ g/cm}^3$) due to the highly non-perturbative nature of nuclear matter in this regime. Consequently, there is great uncertainty in the properties of NSs predicted by theory that are highly dependent on the EOS, such as the maximum stable mass for a non-rotating NS (M_{max}), as well as the relation between the mass of an NS and its radius (a.k.a. the mass-radius curve). The collection of current NS mass measurements shows that the lower bound on the value of M_{max} lies firmly within the range $1.9 - 2.0 M_{\odot}$ [1, 2], and the discovery of pulsar J0740+6620 ($M \simeq 2.14 M_{\odot}$) strongly indicates that the lower bound could be constrained even higher [3]. Constraints on M_{max} and measurements of NS radii have been combined to place constraints the EOS using both Bayesian/weighted [4–10] and unweighted [11–15] techniques. Upcoming precision NICER measurements of millisecond pulsar radii will likely constrain the EOS even further through these techniques [1, 11].

However, certain relations between bulk properties of NSs exhibit universality, meaning they are largely independent of the EOS. In the age of gravitational wave (GW) astronomy, some particularly important relations are those between the tidal deformability parameters (or,

simply, tidal deformabilities) of NSs, which are related to the tidal Love numbers. During a binary neutron star (BNS) inspiral, the gravitational field of each star causes a deformation on the other star through tidal forces. These deformations, which are described by the tidal deformability parameters, alter the trajectory of each star, which becomes imprinted in the resultant GW signal. Yagi [16] has demonstrated that a robust relation between several l -th order dimensionless electric tidal deformabilities, Λ_l , of non-rotating NSs exists across a variety of theoretical neutron star equations of state (EOS). Yagi and Yunes [17, 18] have also shown that a similar relation exists for BNSs between the symmetric and anti-symmetric combinations of each NS’s electric quadrupolar tidal deformability, Λ_2 .

A universal relation reduces a group of parameters to a single parameter family; that is, the measurement of one parameter yields all others through the relation, breaking the degeneracy between them. In this analysis, we are concerned with a set of universal relations that are important for GW astronomy and LIGO/VIRGO observations (and have notably been used in the LIGO/VIRGO analysis of GW170817 [19, 20]).

First, there are universal relations between various multipole tidal deformabilities Λ_l , the so-called “multipole Love relations.” Tidal deformabilities enter into the waveform of GW signals of BNS inspirals at different post-Newtonian (PN) orders. (The l th order electric tidal deformability enters into the GW signal at $2l + 1$ PN orders [16].) The dominant order is the quadrupole (Λ_2) term, followed by the much smaller octupole (Λ_3) and hexadecapole (Λ_4) terms. The Λ_3 and Λ_4 terms are difficult to measure accurately in the GW signal due to their small magnitudes and are often dropped to compute the leading order effect. However, the measurement

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of these quantities as well as the bias introduced by dropping them from calculations can be avoided entirely using universal relations. With the multipole relations, Λ_3 and Λ_4 can be computed directly using the more easily measurable Λ_2 , leading to a manifold increase in measurement accuracy of Λ_2 [16, 21]. These multipole relations, then, will be critical tools for the analysis of GW signals with upcoming third-generation GW detectors such as LIGO III and the Einstein Telescope [16, 22].

Next, there is a universal relation between Λ_2 and the compactness of a NS, $C \equiv M/R$, where M and R are the mass and radius of the NS respectively (we take $G = c = 1$). This relation essentially falls out of the definition for Λ_2 . The l -th order dimensionless electric tidal deformability Λ_l can be defined in terms of C and the l -th order electric tidal Love number k_l as [16, 23]

$$\Lambda_l \equiv \frac{2}{(2l-1)!!} \frac{k_l}{C^{2l+1}}. \quad (1)$$

We see also that the previous multipole relations follow from and have their physical origins in Eq. (1). It can be shown that k_2 goes roughly as C^{-1} , independent of the EOS, over the range of C values observed in Nature. Thus, overall, Λ_2 goes roughly as C^{-6} for all EOSs [14, 24]. There is a clear physical intuition for this relation. For a given NS mass M , the less compact the NS is (that is, the larger its radius R), the more easily it is deformed by a tidal potential, and thus the larger the value of Λ_2 . This relation, then, allows one to convert constraints on Λ_2 from GW observations to constraints on the radius of the NS (or even to compute the radius directly from Λ_2) as has been done in the LIGO/VIRGO analysis of GW170817 [14, 20].

Finally, there is a universal relation for BNSs between the symmetric and antisymmetric combinations of Λ_2 for each star, the so-called “binary Love relation.” Consider a NS binary with primary and secondary masses m_1 and m_2 ($m_1 \geq m_2$) and respective quadrupolar tidal deformabilities $\Lambda_{2,1}$ and $\Lambda_{2,2}$. The symmetric and antisymmetric combinations of $\Lambda_{2,1}$ and $\Lambda_{2,2}$ are

$$\Lambda_s \equiv \frac{\Lambda_{2,1} + \Lambda_{2,2}}{2}, \quad \Lambda_a \equiv \frac{\Lambda_{2,2} - \Lambda_{2,1}}{2}. \quad (2)$$

The individual tidal deformabilities, $\Lambda_{2,1}$ and $\Lambda_{2,2}$, are degenerate in the GW phase information. What is actually measured in the GW signal is really a combination of $\Lambda_{2,1}$ and $\Lambda_{2,2}$ [18, 21]. Just as with the multipole relations, the relation between Λ_s and Λ_a (which also involves a third parameter, the binary mass ratio $q \equiv m_2/m_1$) reduces the analysis to the estimation and measurement of a single parameter, Λ_s , from which Λ_a (and, thus, $\Lambda_{2,1}$ and $\Lambda_{2,2}$) can then be computed. Currently, this is the method by which Advanced LIGO is able to extract individual tidal deformability information from GW signals of BNSs [18, 20, 21]. The approximate universality of the relation between Λ_a and Λ_s follows from the approximate no-hair relations for compact objects, arising from

the approximate symmetry of isodensity self-similarity [21].

In their original analysis, Yagi [16] and Yagi and Yunes [17] validated these universal relations against a set of a few very diverse theoretical EOS models, but not over the entire space of EOSs allowed by astronomical observation and theoretical calculations. The motivation for the work in this paper, then, is to validate these relations over a much broader extent of the space of all possible EOSs.

In this paper, we update and recalibrate the fits to these universal relations using a large set of randomly generated phenomenological EOSs that satisfy astronomical observation and theoretical calculations. The structure of this paper is as follows. In Sec. II we will describe the parameterization of the four-piece polytrope EOS model and the algorithm by which the phenomenological EOSs are generated. In Sec. III, we analyze the universal Λ_3 - Λ_2 , Λ_4 - Λ_2 , and C - Λ_2 relations from the collection of phenomenological EOSs. We present the fitting parameters of these relations and compare them to previous fits. In Sec. IV, we analyze the Λ_a - Λ_s relation and compare the fitting parameters to previous fits, including the fit currently used by the LIGO/VIRGO collaboration. In Sec. V, we discuss the applications of the updated fits to GW modelling and parameter estimation. A concluding summary is given in Sec. VI.

II. METHODS

A. EOS Parameterization

In order to explore the space of all possible EOSs that satisfy known observational constraints and theoretical calculations (known as the EOS band), we employ a Markov chain Monte Carlo (MCMC) algorithm that generates random piecewise polytropic EOSs. We use a variation of the piecewise polytropic interpolation scheme developed by Read *et al.* [25]. The scheme models the EOS as a continuous piecewise function of four polytropes:

$$p(\rho) = \begin{cases} K_0 \rho^{\Gamma_0} & \rho \leq \rho_0 \\ K_1 \rho^{\Gamma_1} & \rho_0 < \rho \leq \rho_1 \\ K_2 \rho^{\Gamma_2} & \rho_1 < \rho \leq \rho_2 \\ K_3 \rho^{\Gamma_3} & \rho > \rho_2. \end{cases} \quad (3)$$

A four-piece model allows for a great diversity of EOSs (ex. hard/soft EOSs, EOSs with/without phase transitions, etc.) and ensures that the most extreme regions of the EOS band will be reached by the MCMC algorithm. The specific choice of a piecewise polytropic ansatz for the EOS, as opposed to known alternative schemes, does not significantly bias the resultant shape of the computed EOS band [13].

The first polytrope piece of the model corresponds to the presumed known EOS of the outer and inner crust up to around nuclear density, where $K_0 = 3.59389 \times 10^{13}$ [cgs] and $\Gamma_0 = 1.35692$ [26]. Here, nuclear density is

taken to be $\rho_{\text{nuc}} = 2.7 \times 10^{14} \text{ g/cm}^3$. This piece is fixed for all randomly generated EOSs. The specific choice of the low-density crust EOS does not significantly determine the bulk physical properties of the NS [7, 25, 27]

The K_i are determined by continuity; thus, the last three polytrope pieces of the EOS are specified by six parameters: three transition densities (ρ_0 , ρ_1 , and ρ_2) and three adiabatic indices (Γ_1 , Γ_2 , and Γ_3). The authors of [25] reduce this to four parameters by fixing the values of ρ_1 and ρ_2 , as an EOS with a smaller number of parameters can be reasonably constrained by a only a few astronomical observations. However, this imposes a prior assumption on the form of the EOS and narrows the parameter space to a much smaller region of the EOS band. Thus, to probe the entire EOS band (including extreme EOSs not ruled out by observation) without imposing assumptions about the true form of the EOS, we allow ρ_1 and ρ_2 to be free parameters as well. Therefore, each EOS is specified by the full set of six parameters: $\rho_0 \in [0.15\rho_{\text{nuc}}, 1.2\rho_{\text{nuc}}]$, $\rho_1 \in [1.5\rho_{\text{nuc}}, 8\rho_{\text{nuc}}]$, $\rho_2 \in (\rho_1, 8.5\rho_{\text{nuc}}]$, $\Gamma_1 \in [1.4, 5]$, $\Gamma_2 \in [0, 8]$, $\Gamma_3 \in [0.5, 8]$.

Though continuity is imposed on each EOS, the speed of sound within the NS (c_s) as a function of density for each EOS is not necessarily continuous at the transition densities, nor is it necessarily monotonic as a result of these jump discontinuities [11]. O’Boyle *et al.* [28] have recently developed a modified version of the piecewise polytropic scheme by Read *et al.* [25] used in this analysis which imposes continuity on c_s . We do not use this modified scheme, as it was published after our analysis. However, Kanakis-Pegios *et al.* [29] have shown that the effects of discontinuities in c_s on the bulk properties of a NS are negligible.

B. MCMC Algorithm

To probe the EOS band in a way that is both thorough and computationally efficient, we use a MCMC algorithm in the form of a random walk through the parameter space. The constraints of the EOS band define a path-connected region of the six-dimensional parameter space. A path between any two points in this region, then, corresponds to a smooth deformation of the EOS at one point to the EOS at the second point. Therefore, a series of small, random deformations of the parameterized EOS model would correspond to a random walk through the parameter space. This is the basic idea behind the algorithm.

Taking the logarithm of Eq. (3) converts the EOS to a piecewise linear function. A deformation can then be performed very easily by shifting the positions of just four points: the three transition points $\mathbf{r}_0 = \{\log(\rho_0), \log(p_0)\}$, $\mathbf{r}_1 = \{\log(\rho_1), \log(p_1)\}$, and $\mathbf{r}_2 = \{\log(\rho_2), \log(p_2)\}$; and an endpoint $\mathbf{r}_3 = \{15.5, \log(p_3)\}$. The density value of the endpoint is kept fixed at $\rho = 10^{15.5} \text{ g/cm}^3$, but this choice is arbitrary, as the only purpose of the endpoint is to define the slope of the last

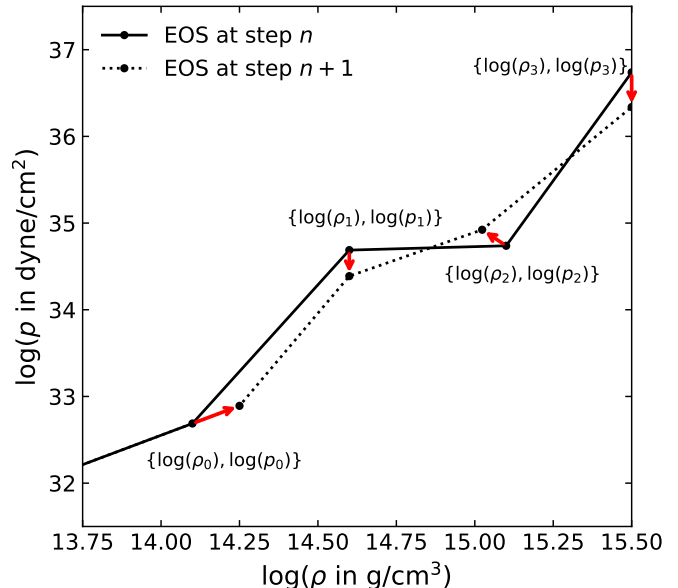


FIG. 1. A single step of the MCMC algorithm. Each of the four points used to define the current EOS (solid line) is independently shifted by a displacement vector of random magnitude (shown in red), creating a trial EOS (dashed line). Trial EOSs are repeatedly generated from the current EOS until one is found that satisfies observational constraints. When this occurs, the current EOS is updated to the trial EOS and the process repeats.

linear piece, Γ_3 . Both $\mathbf{r}_0$ and $\mathbf{r}_3$ are restricted to move one-dimensionally ($\mathbf{r}_0$ can only shift along the crust EOS line and $\mathbf{r}_3$ can only shift vertically up or down), while $\mathbf{r}_1$ and $\mathbf{r}_2$ have the full two degrees of freedom. Thus, the whole EOS still has six degrees of freedom.

A single step of the MCMC algorithm proceeds as follows and is illustrated in Fig. 1. The current EOS is defined by the four points $\mathbf{r}_i$ and corresponds to the current position of the algorithm in parameter space. For each $\mathbf{r}_i$, a displacement vector $\Delta\mathbf{r}_i$ is independently generated from a uniform distribution with a random direction (respecting the point’s degrees of freedom) and a random magnitude (up to a maximum size $\|\Delta\mathbf{r}_i\| \leq 0.05$). A trial EOS is then defined by the four new points, $\mathbf{r}'_i = \mathbf{r}_i + \Delta\mathbf{r}_i$. The transition densities and adiabatic indices of this trial EOS are then checked to see if they are within the bounds given in Sec. II A.

The physical properties of the trial EOS are then found by computing a sequence of solutions to the Tolman-Oppenheimer-Volkoff (TOV) equation. We utilize the publicly available TOVL code described in Bernuzzi and Nagar [30] and Damour and Nagar [23] to solve the TOV equation. The trial EOS is accepted if it satisfies three weak physical constraints that define the EOS band:

1. causality of the maximum mass NS (i.e. the sound speed is subluminal, $c_s < c$, below the maximum stable central density);

2. $M_{\max} > 1.97 M_{\odot}$;
3. $\Lambda_2 < 800$ for the $1.4 M_{\odot}$ NS.

The upper limit on Λ_2 is the 90%-credible upper bound derived in the LIGO/VIRGO analysis of GW170817 [19] and was chosen to place a minimally weak constraint on Λ_2 . If the trial EOS is accepted, its parameters and physical properties are then recorded, and the current EOS is updated to the new EOS, $\mathbf{r}_i \rightarrow \mathbf{r}'_i$.

The initial EOS is randomly selected from a set of EOSs that satisfy the constraints computed via a standard Monte Carlo analysis of the parameter space. The algorithm is then allowed to proceed until a specified number of steps have been completed.

III. MULTIPOLE AND COMPACTNESS RELATIONS

Using the MCMC algorithm, we generate a set of 1,966,225 phenomenological EOSs. The complete set of EOSs is plotted in Fig. 2 along with the associated mass-radius curve for each EOS. The plot reveals the approximate shape of the full EOS band defined by the three weak constraints in Sec. II B. The edges of the band are populated by the most extreme EOSs where $c_s = 1$. The upper limit for the value of $M_{\max}$ correlates strongly with Λ_2 for the $1.4 M_{\odot}$ NS [11]; consequently, the constraint that $\Lambda_2 < 800$ for the $1.4 M_{\odot}$ NS effectively functions as an upper constraint on the value of $M_{\max}$. The largest value of $M_{\max}$ in our data set is $2.9096 M_{\odot}$, below the theoretical upper limit from GR of $3.2 M_{\odot}$ [27], which can be seen in the mass-radius band in Fig. 2. Similarly, and quite understandably, the upper limit for the radius of the $1.4 M_{\odot}$ NS, $R_{1.4}$, also correlates strongly with Λ_2 for the $1.4 M_{\odot}$ NS [11]. The $\Lambda_2 < 800$ constraint then also functions as an upper limit for $R_{1.4}$. This explains why a kink can be seen on the right side of the mass-radius band at $M \sim 1.4 M_{\odot}$ in Fig. 2. The largest value of $R_{1.4}$ in our data set is 13.9 km.

The universal Λ_3 - Λ_2 , Λ_4 - Λ_2 , and C - Λ_2 relations are the nearly-EOS-independent relations that reduce Λ_3 , Λ_4 , and C to functions of the single parameter Λ_2 for any given NS. To analyze these relations across our set of phenomenological EOSs, we solve the TOV equation for each EOS at sixteen evenly spaced central density values in the range $3.09 \times 10^{14} \text{ g/cm}^3 \leq \rho_c \leq 3.09 \times 10^{15} \text{ g/cm}^3$ and then extract Λ_2 , Λ_3 , Λ_4 , and C from the NS solution at each density. (If the central density of the maximum mass NS for a given EOS falls below any of the density values, then the TOV equation is not solved for those density values.) A total of 30,133,746 NS solutions were computed.

In Fig. 3, we plot (a) Λ_3 vs. Λ_2 and (b) Λ_4 vs. Λ_2 for our set of NS solutions. The Λ_3 - Λ_2 and Λ_4 - Λ_2 relations

are fitted with the polynomial-like expression

$$\ln \Lambda_{3,4} = \sum_{k=0}^6 a_k (\ln \Lambda_2)^k. \quad (4)$$

This is an updated version of the fitting function used by Yagi [16]. The original fitting function is quartic in $\ln \Lambda_2$. However, with our larger data set, a quartic fit is insufficient to remove trends from the residual of each relation. Thus, we employ a fit that is 6th order in $\ln \Lambda_2$. The quality of the fits can be appreciated in Fig. 3, where the residuals of $\log_{10} \Lambda_3$ and $\log_{10} \Lambda_4$ are given with 95% prediction intervals. The outliers in the peak that can be seen in both residual plots between $\Lambda_2 = 1$ and $\Lambda_2 = 100$ are associated with small values of Γ_2 , indicating a softening in the EOS. The most extreme models, those with the largest deviations from the fit, have $\Gamma_2 < 1.5$. This can be seen clearly in Fig. 14 in Appendix A. In Figs. 4 and 5, we compare the 68%, 95%, and 99.7% relative errors of the fits in this work to those the original fits by Yagi [16]. At each percentage error level, our fits demonstrate a general improvement in accuracy over the original fits.

Over the range of tidal parameters most relevant to LIGO observations, $\Lambda_2, \Lambda_3, \Lambda_4 \in [1, 10^4]$, our Λ_3 - Λ_2 fit holds to a maximum error of $\sim 30\%$, with 95% of the errors below $\sim 13\%$. The original fit to this relation holds to a maximum error of $\sim 30\%$, with 95% of the error below $\sim 14\%$. Our Λ_4 - Λ_2 fit holds to a maximum error of $\sim 45\%$, with 95% of the errors below $\sim 25\%$. The original fit to this relation holds to a maximum error of $\sim 40\%$, with 95% of the error below $\sim 26\%$.

The fitting parameters $\vec{a} = \{a_k\}$ of the new fits and the original quartic fits by Yagi [16] are given in Table I. The leading-order terms of the fits are the constant ($k = 0$) and linear ($k = 1$) terms. (This is evident from the near-linear shape of the distribution of points in each log-log plot in Fig. 3.) Therefore, a direct comparison can be made between the leading-order terms of the original fits and our new fits. The coefficients a_0 and a_1 for our fits are in good agreement with those of the original fits. This demonstrates the validity of the original Λ_3 - Λ_2 and Λ_4 - Λ_2 fits across the EOS band.

Just like the multipole relations, the universal relation between C and Λ_2 reduces C to a function of Λ_2 . This relation essentially falls out of the definition of Λ_2 in Eq. (1). In Fig. 6, we plot C vs. Λ_2 for our set of NS solutions. The C - Λ_2 relation is fitted with a similar polynomial-like expression

$$C = \sum_{k=0}^6 a_k (\ln \Lambda_{2,3,4})^k \quad (5)$$

This fitting function is an updated version of the function used by Maselli *et al.* [31] and presented in [18]. The original fitting function is quadratic in $\ln \Lambda_2$. Once again, with our larger data set, we are required to go to 6th order in $\ln \Lambda_2$ to remove all trends from the residual of

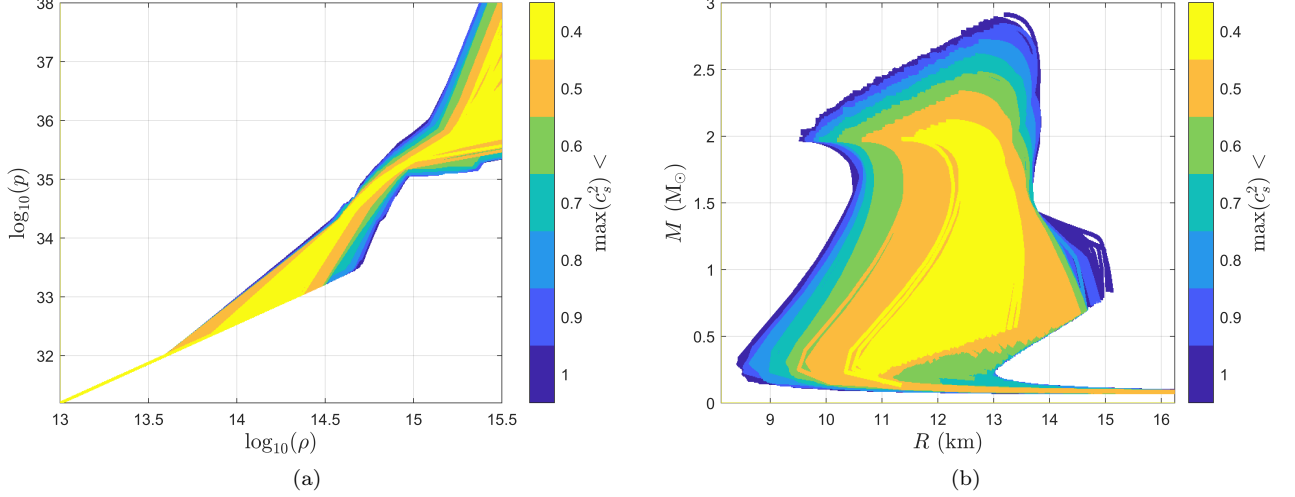


FIG. 2. The collection of 1,966,225 phenomenological EOSs (a) and associated mass-radius curves (b) computed using the MCMC algorithm. Color here is used to indicate the maximum sound speed c_s reached within the maximum mass NS of each EOS (EOSs with smaller $\max(c_s^2)$ are drawn on top of ones with larger $\max(c_s^2)$). The collection reveals the approximate shape of the EOS band. The most extreme EOSs that reach the edges of the band are those where $c_s = 1$.

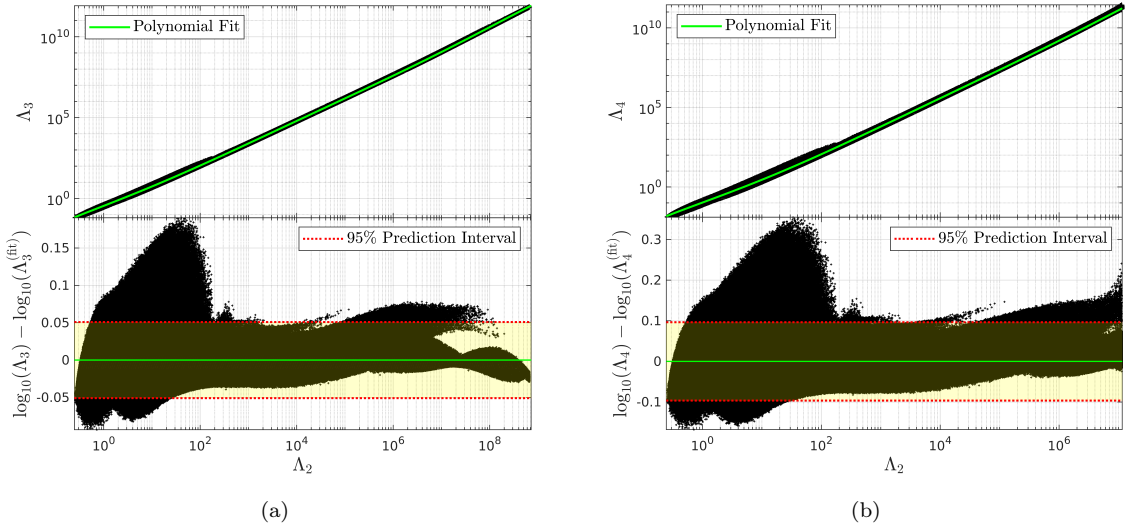


FIG. 3. Universal (a) Λ_3 - Λ_2 and (b) Λ_4 - Λ_2 relations for NSs from the collection of phenomenological EOSs. Sixteen NSs with central densities in the range $3.09 \times 10^{14} \text{ g/cm}^3 \leq \rho_c \leq 3.09 \times 10^{15} \text{ g/cm}^3$ were computed for each EOS. The relations are fitted with the polynomial expression in Eq. (4), with the fitting parameters for each relation given in Table I. The log residuals of both fits are shown with 95% prediction intervals.

our fit. The fitting parameters $\vec{a} = \{a_k\}$ are also given in Table I. Similar to the residuals in Fig. 3, there is a downwards-pointing peak between $\Lambda_2 = 1$ and $\Lambda_2 = 100$ in the residual in Fig. 6. These outliers are also associated with small values of Γ_2 , with the most extreme having $\Gamma_2 < 1.5$.

In Fig. 7, we compare the 68%, 95%, and 99.7% relative errors of the fit in this work to those of the original fit by Maselli *et al.* [31]. Our fit demonstrates a significant improvement in accuracy over the original fit at larger values of C .

As one can see from Fig. 5, the range of Λ_2 values most relevant to LIGO observations, $\Lambda_2 \in [1, 10^4]$, translates into a corresponding range $C \in [0.1, 0.35]$. Over this range, our fit holds to a maximum error of $\sim 10\%$, with 95% of the errors below $\sim 5\%$. The original fit to this relation holds to a maximum error of $\sim 15\%$, with 95% of the error below $\sim 14\%$. This is the largest improvement in accuracy out of all the fits in this work. We are able then to confirm the concerns raised by Kastaun and Ohme [32] that existing fits to the C - Λ_2 are unreliable at large C .

The fitting parameters of the quadratic fit by Maselli

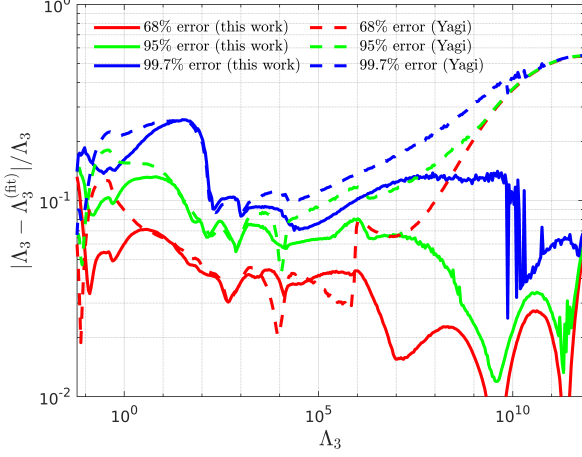


FIG. 4. 68%, 95%, and 99.7% relative error of the Λ_3 - Λ_2 fit as a function of Λ_3 . The solid lines represent the error of the new fit in this work, and the dashed lines represent the error of the original fit by Yagi [16].

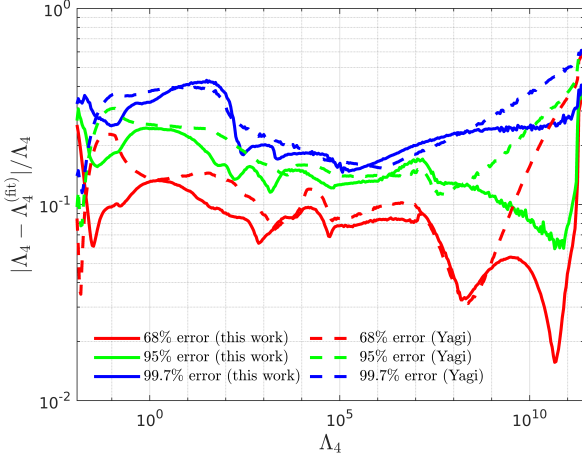


FIG. 5. 68%, 95%, and 99.7% relative error of the Λ_4 - Λ_2 fit as a function of Λ_4 . The solid lines represent the error of the fit in this work, and the dashed lines represent the error of the original fit by Yagi [16].

et al. [31] are given in Table II. The leading-order terms are once again the constant and linear terms, as can be seen in the near-linear shape of the distribution of points in Fig. 6. The coefficients a_0 and a_1 are in generally good agreement between the original and updated fits; however, there is a relatively large difference between the values of a_2 as compared to what is seen with the multiple fits. This is a consequence of the improvements to the fit made by using both a higher-order polynomial function and a larger data set over a greater extent of the EOS band.

In addition to the our fit to the C - Λ_2 relation, we also present entirely new fits to the C - Λ_3 and C - Λ_4 relations, the fitting parameters of which are also given in Table

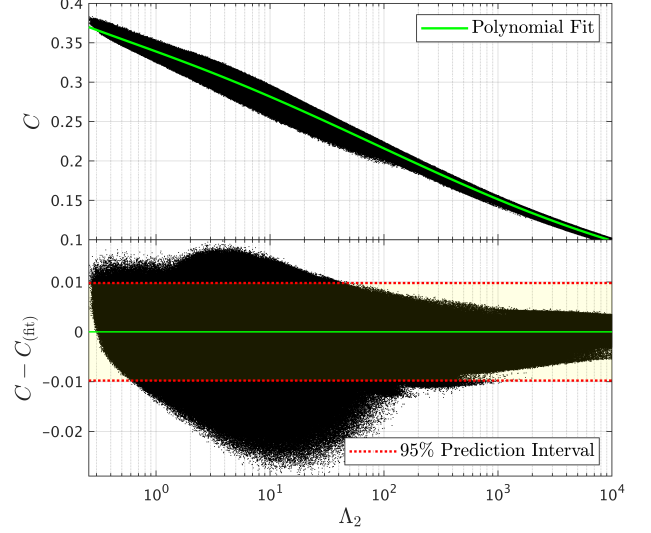


FIG. 6. Universal C - Λ_2 relation for NSs from the collection of phenomenological EOSs. The relation is fitted with the expression in Eq. (5), with the fitting parameters given in Table II. The residual of the fit is shown with the 95% prediction interval. The distribution of Λ_2 values roughly goes as C^{-6} .

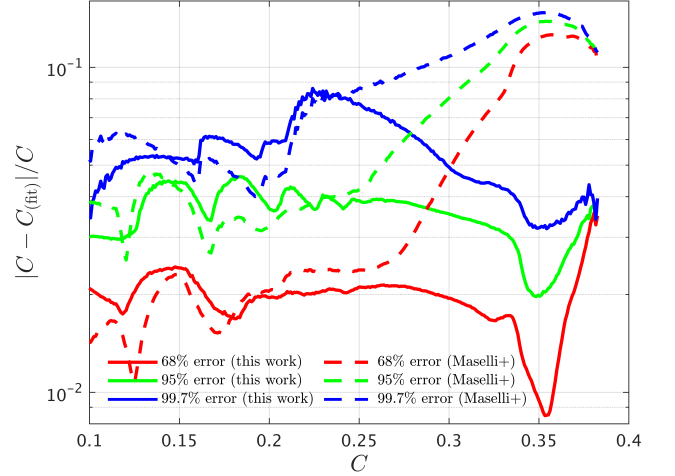


FIG. 7. 68%, 95%, and 99.7% relative error of the C - Λ_2 fit as a function of C . The solid lines represent the error of the fit in this work, and the dashed lines represent the error of the original fit by Maselli *et al.* [31]. Our fit has a drastically smaller error at larger values of C than the original fit has.

II. While these two relations can be derived from the multipole and C - Λ_2 fits, these explicit fits have a smaller error, comparable to that of the C - Λ_2 fit.

Fit	Relation	a_0	a_1	a_2	a_3	a_4	a_5	a_6
Yagi [16]	Λ_3 - Λ_2	-1.15	1.18	2.51×10^{-2}	-1.31×10^{-3}	2.52×10^{-5}	-	-
	Λ_4 - Λ_2	-2.45	1.43	3.95×10^{-2}	-1.81×10^{-3}	2.80×10^{-5}	-	-
This work	Λ_3 - Λ_2	-1.052	1.165	6.369×10^{-3}	5.058×10^{-3}	-7.268×10^{-4}	3.794×10^{-5}	-6.803×10^{-7}
	Λ_4 - Λ_2	-2.262	1.383	1.662×10^{-3}	1.225×10^{-2}	-1.752×10^{-3}	9.667×10^{-5}	-1.886×10^{-6}

TABLE I. Fitting parameters $\vec{a} = \{a_k\}$ for the Λ_3 - Λ_2 and Λ_4 - Λ_2 relations given in Eq. (5) from the original fits in [16] and from the fits in this work to the phenomenological EOSs.

Fit	Relation	a_0	a_1	a_2	a_3	a_4	a_5	a_6
Maselli <i>et al.</i> [31]	C - Λ_2	0.371	-3.91×10^{-2}	1.056×10^{-3}	-	-	-	-
This work	C - Λ_2	0.3389	-2.293×10^{-2}	-5.172×10^{-4}	-2.449×10^{-4}	5.161×10^{-5}	-3.03×10^{-6}	5.841×10^{-8}
	C - Λ_3	0.3180	-2.067×10^{-2}	-4.713×10^{-4}	3.779×10^{-5}	3.248×10^{-6}	-2.453×10^{-7}	4.253×10^{-9}
	C - Λ_4	0.3001	-1.764×10^{-2}	-1.518×10^{-4}	1.449×10^{-5}	2.228×10^{-6}	-1.487×10^{-7}	2.514×10^{-9}

TABLE II. Fitting parameters $\vec{a} = \{a_k\}$ for the C - Λ_2 , C - Λ_3 , and C - Λ_4 relations given in Eq. (5) from the original fit in [31] and from the fits in this work to the phenomenological EOSs.

IV. BINARY RELATION

The universal Λ_a - Λ_s relation is slightly more complicated than the multipole and compactness relations, as it reduces Λ_a to a function of two parameters: Λ_s and the binary mass ratio q for any given NS. We analyze the Λ_a - Λ_s relation across the set of phenomenological EOSs by computing sequences of random BNSs using a random sample of a quarter of the total number of EOSs. We use the convention that the primary and secondary masses are given by m_1 and m_2 respectively, so $m_1 \geq m_2$. The binary mass ratio is then defined as $q \equiv m_2/m_1$. For each EOS, twenty random BNSs were generated. For each BNS, two masses were selected uniformly from the range $[1 M_\odot, M_{\text{max}}]$, where M_{max} is the maximum stable mass for the given EOS. The larger mass and smaller mass were then set as m_1 and m_2 respectively. The TOV equation was then solved for both stars in the binary. The quadrupolar tidal deformability of each star was extracted from the solution. Here we define $\Lambda_{2,1}$ and $\Lambda_{2,2}$ as the tidal deformabilities of the primary and secondary respectively. The symmetric and antisymmetric combinations of $\Lambda_{2,1}$ and $\Lambda_{2,2}$ were then computed using the definitions in Eq. (2). A total of 5,454,778 BNS solutions were computed.

In Fig. 8, we plot Λ_a vs. Λ_s vs. q for our set of BNS solutions. We employed the same fitting function used in the original analysis by Yagi and Yunes [17, 18], which is a Padé approximant multiplied by a controlling factor:

$$\Lambda_a = F_n(q) \Lambda_s \frac{1 + \sum_{i=1}^3 \sum_{j=1}^2 b_{ij} q^j \Lambda_s^{i/5}}{1 + \sum_{i=1}^3 \sum_{j=1}^2 c_{ij} q^j \Lambda_s^{i/5}}. \quad (6)$$

The controlling factor $F_n(q)$ is derived from the Newtonian limit of the Λ_a - Λ_s relation where the EOS is treated as a single Newtonian polytrope with polytropic index n :

$$F_n(q) \equiv \frac{1 - q^{10/(3-n)}}{1 + q^{10/(3-n)}}. \quad (7)$$

If one computes this limit with an EOS that is *not* a single polytrope, n instead represents the *effective* polytropic index of the EOS. When considering *multiple* EOSs in the context of the universal relation, n is replaced with $\bar{n}$, which is the *average* effective polytropic index across the set of EOSs. Yagi and Yunes [17] originally analyzed the Λ_a - Λ_s relation using a set of theoretical EOSs with an average effective polytropic index of $\bar{n} = 0.743$. Chatziioannou *et al.* [33] reproduced the fit using three theoretical EOSs with diverse physical properties while still using $\bar{n} = 0.743$. The reproduced fit is the version that was used in the LIGO/VIRGO analysis of GW170817 [20]. In this analysis, we also take $\bar{n} = 0.743$ to allow for a direct comparison between our fit and the two previous fits. The fitting parameters $\vec{b} = \{b_{ij}, c_{ij}\}$ for all three fits are given in Table III. We use the parameters of the fit by Chatziioannou *et al.* [33] as the initial parameter values for our fit.

The residual of the fit is shown from two perspectives in Fig. 9. In Fig. 10, we compare the 68%, 95%, and 99.7% relative errors of the this fit to those the original fit by Yagi and Yunes [17, 18]. (We verify that the relative error of the fit by Chatziioannou *et al.* [33] is identical to that of the original fit.) Our fit holds to a maximum error of $\sim 40\%$, with 95% of the error below $\sim 29\%$. The original fit holds to a maximum error of $\sim 43\%$, with 95% of the error below $\sim 31\%$.

We demonstrate the improvement our updated fits provide over the original fits to GW analysis by comparing the accuracy with which the radii of NS can be recovered from GW signals. The reduced tidal parameter

$$\tilde{\Lambda} = \frac{16}{13} \left(\frac{(m_1 + 12m_2)m_1^4 \Lambda_{2,1}}{(m_1 + m_2)^5} + \frac{(m_2 + 12m_1)m_2^4 \Lambda_{2,2}}{(m_1 + m_2)^5} \right) \quad (8)$$

can be recovered the GW waveform [34–36] and is used by LIGO to compute the radii of both NSs in a binary merger. First, we compute the value of $\tilde{\Lambda}$ for each BNS solution using Eq. (8). Using the definitions in Eq. (2) and the Λ_a - Λ_s relation, we can re-express Eq. (8) in

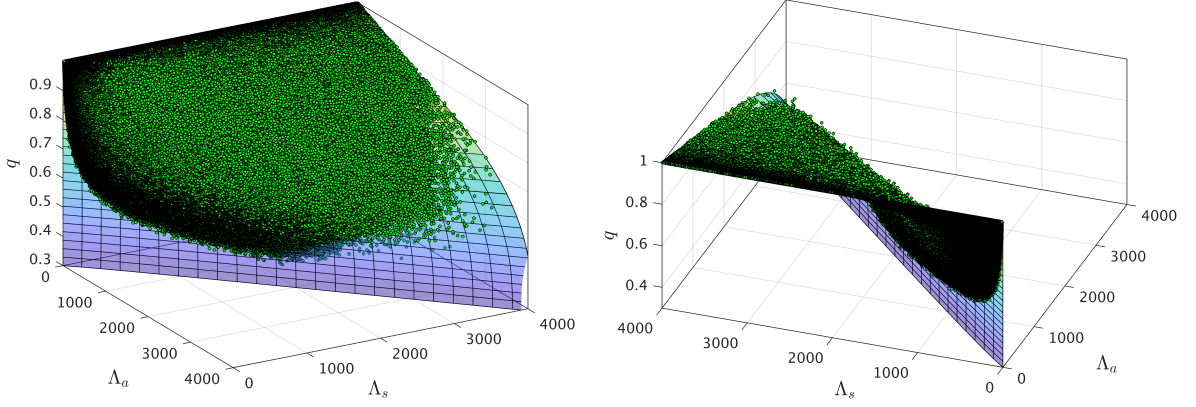


FIG. 8. Approximate universal Λ_a - Λ_s relation for NSs from the collection of phenomenological EOSs, shown from two different angles. Twenty random binaries with $1 M_\odot \leq m_2 \leq m_1 \leq M_{\text{max}}$ were computed for each EOS. The relation was fitted with the expression in Eq. (6), with the fitting parameters given in Table III.

Fit	b_{11}	b_{12}	b_{21}	b_{22}	b_{31}	b_{32}	c_{11}	c_{12}	c_{21}	c_{22}	c_{31}	c_{32}
Yagi and Yunes [17, 18] ^a	-29.60	11.22	138.41	-43.06	-207.95	180.26	-27.13	7.915	105.18	7.494	-97.48	-17.48
Chatziioannou <i>et al.</i> [33]	-27.74	8.42	122.69	-19.76	-175.50	133.71	-25.56	5.585	92.03	26.86	-70.25	-56.31
This work	-18.32	3.875	28.06	11.08	43.56	17.3	-18.37	1.338	15.99	55.07	98.56	-135.1

^a The parameter a from these papers has been factored out of these parameter values.

TABLE III. Fitting parameters $\vec{b} = \{b_{ij}, c_{ij}\}$ for the Λ_a - Λ_s relation given in Eq. (6) from the original fit in [17, 18], from the updated fit in [33] used by LIGO/VIRGO, and from the fit in this work to the phenomenological EOSs. For all three fits, the average effective polytropic index is taken to be $\bar{n} = 0.743$.

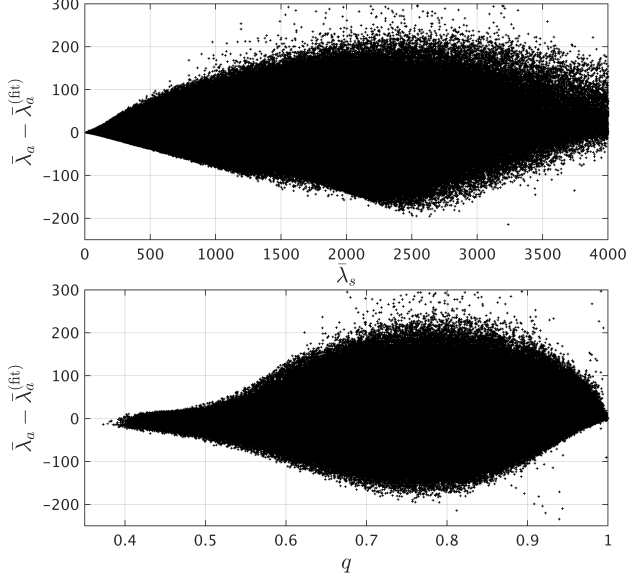


FIG. 9. Residual of the Λ_a - Λ_s relation fit for $\Lambda_s < 4000$ seen from two perspectives. The fit becomes increasingly better as $\Lambda_s \rightarrow 0$ and as $q \rightarrow 1$.

terms of Λ_s and q . Taking m_1 and m_2 to be known, we then solve Eq. (8) numerically for Λ_s and recover a measurement of Λ_s for each BNS solution. The Λ_a - Λ_s

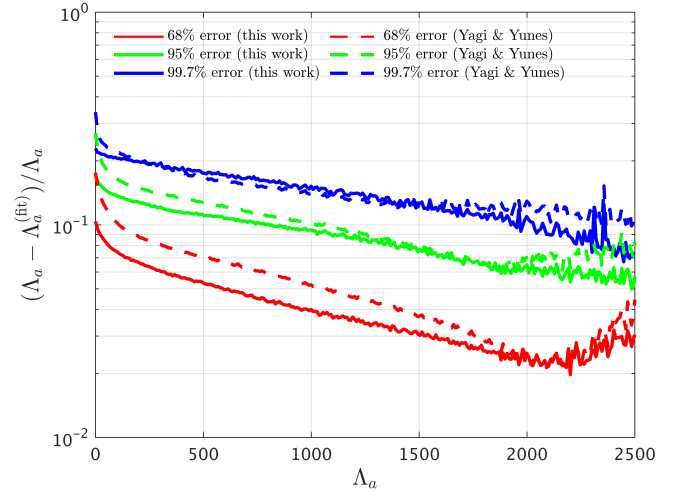


FIG. 10. 68%, 95%, and 99.7% relative error of the Λ_a - Λ_s fit as a function of Λ_a . The solid lines represent the error of the fit in this work, and the dashed lines represent the error of the original fit by Yagi and Yunes [17, 18].

relation then yields a measurement for Λ_a , allowing us to recover $\Lambda_{2,1}$ and $\Lambda_{2,2}$. Finally, we use the C - Λ_a to compute C for each NS, which when combined with m_1 and m_2 gives us the radius of each NS. In Fig. 11, we compare the 68%, 95%, and 99.7% relative errors of the radii recovered using the fits in this work to those recovered us-

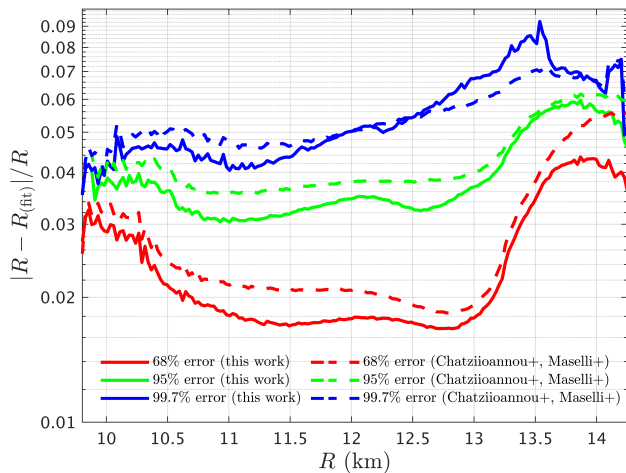


FIG. 11. 68%, 95%, and 99.7% relative error of NS radii recovered using the fits to the Λ_a - Λ_s and C - Λ_2 relations as a function of the radius. The solid lines represent the error of the fits in this work, and the dashed lines represent the error of the fits by Chatzioannou *et al.* [33] and Maselli *et al.* [31].

ing the fits by Chatzioannou *et al.* [33] and Maselli *et al.* [31]. At the 68% and 95% error levels especially, our recovery show a definite improvement in accuracy over the original recovery. This is primarily a result of the drastic improvement in accuracy afforded by the updated C - Λ_s fit. Our recovery has a maximum error of $\sim 10\%$, with 95% of the error below $\sim 6\%$. The original recovery has a maximum error of $\sim 8\%$, with 95% of the error below $\sim 6.5\%$.

V. APPLICATION TO WAVEFORM MODELING AND PE

We now discuss the impact of the new quasiniversal relations in GW modeling and PE using the state-of-art effective-one-body model **TEOBResumS**, that provides us with multipolar tidal waveforms for the full inspiral-merger regime [39–41]. Specifically, the tidal sector of **TEOBResumS** includes the $\ell = 2, 3, 4$ gravitoelectric and the $\ell = 2$ gravitomagnetic tidal terms in the binary interaction potential at the highest known post-Newtonian order, and additionally implements a resummation for the $\ell = 2, 3$ ($\ell = 2$) gravitoelectric (magnetic) terms based on gravitational-self-force results [41–43]. The quasinuniversal relations among the tidal polarizability parameters with different ℓ are employed to obtain the octupolar and hexadecapolar from the quadrupolar parameters.

The importance of including the $\ell = 3, 4$ corrections in the waveform model is highlighted in the left panel of Fig. 12. The figure shows the contribution of the octupolar and hexadecapolar gravitoelectric terms for 10^3 binaries that are identified by different values of the symmetric mass ratio and the reduced tidal parameter $\tilde{\Lambda}$. These tidal terms accelerate the inspiral and give an overall con-

tribution of 0.5 radians to one cycle for a starting GW frequency of 20 Hz, Cf. [44]. The smallest phase differences are found for smaller reduced tidal parameters and equal masses binaries. The phase differences are accumulated from GW frequencies $\gtrsim 500$ Hz (Cf. [45]), corresponding to the last orbits before merger. We remark that current differences between **TEOBResumS** and numerical-relativity waveforms are precisely of order 0.5-1 rad and are comparable to the numerical-relativity error [39–41]. Thus, this analysis indicates that an accurate modeling of higher multipoles in the binary interaction potential can be relevant to capture the merger waveform.

The accuracy of the fitting formulas for the Λ_3 - Λ_2 and Λ_4 - Λ_2 relations can impact significantly the GW phase. The GW phase differences induced by the use of the new fits of Eq. (4) and those of [37] is shown in the right panel of Fig. 12. Differences of order 10% in the fitting formulas result in differences up to one radian in the GW (again we use an initial frequency of 20 Hz). These differences are relevant for GW observations at signal-to-noise ratio $\gtrsim 80$, at which the universal relations can, among other modeling choices, make a difference towards obtaining faithful waveform models [46].

Finally, we perform Bayesian parameter estimation of GW170817 using **TEOBResumS** and the **pbilby** [38, 47] infrastructure with the **dynesty** [48] sampler. Strain data is downloaded from the Gravitational Waves Open Science Center (GWOSC) [49], and the noise curves used are those provided in [50]. Our analysis is identical to the one performed in [46], to which we refer for all the technical details, except for the use of the quasinuniversal relations developed here. We just recall that we use a sampling rate of 2048 Hz, that implies a cutoff maximum frequency of 1024 Hz for the analysis. This conservative choice ensures that no high-frequency systematic effect will affect our estimates [46], and distinguishes our analysis from that of e.g [38, 50]. In Fig. 13 we report the marginalized one-dimensional posteriors for the reduced tidal parameter and the NS radius for the original run of [46] that used the Yagi relations (red inline) and for the one performed here with the new quasinuniversal relations (black inline). The two $\tilde{\Lambda}$ distributions are largely compatible, with the newly computed posteriors displaying slightly more support for low values ($\tilde{\Lambda} \lesssim 300$). This difference is negligible with respect to statistical fluctuations, but can nonetheless be understood by observing that the new fits predict stronger tidal effects than the ones of Yagi and Yunes. The reduced tidal parameter distributions can then, together with the mass ratio and component mass posteriors [51], be mapped into estimates of the radii of the stars. By combining Eq. (6) with the definition of the reduced tidal parameter and Eq. (5) we estimate the distribution of the radius of the lighter star R_2 . Using the coefficients collected in Tab. II and III we obtain $R_2^{\text{Godzieba+}} = 11.9^{+1.2}_{-2.1}$ and $R_2^{\text{Yagi+}} = 12.4^{+1.2}_{-1.7}$. While the two results lie well within the statistical uncertainty of the other, their difference ($R_2^{\text{Godzieba,median}} - R_2^{\text{Yagi,median}} \sim 0.5$ km) can be fully

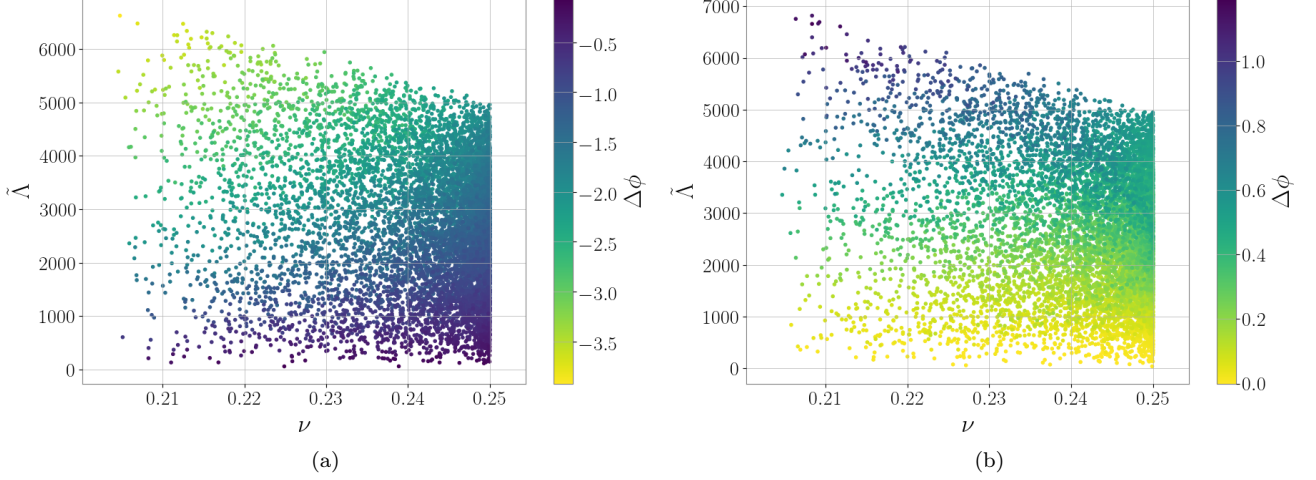


FIG. 12. Left: impact on the GW phase of the $\ell = 3, 4$ corrections in the binary interaction potential. The plot shows the phase contribution of these corrections for different binaries, identified by different values of the symmetric mass ratio and the reduced tidal parameter ($\nu, \tilde{\Lambda}$), evolving from a GW frequency of 20 Hz to merger. Right: GW phase differences given by the fitting relations $\tilde{\Lambda}_2(\tilde{\Lambda}_{3,4})$ and those given by [37]

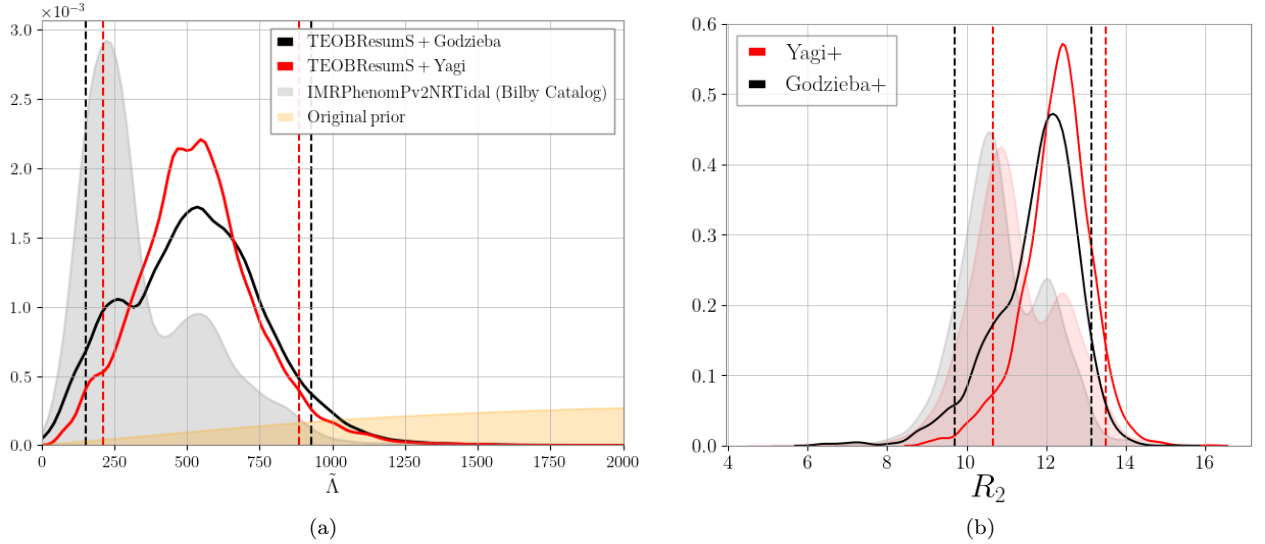


FIG. 13. Left panel: the marginalized one dimensional posterior distributions of $\tilde{\Lambda}$, recovered with the **TEOBResumS** approximant and the fits of Yagi (red) or those presented in this paper (black). We additionally compare our results with those obtained in [38] (shaded gray area). Although statistical fluctuations are larger than any systematic effect due to the choice of quasi-universal relation, the use of the new phenomenological relations slightly increases the support at values of $\tilde{\Lambda} \lesssim 300$. Right panel: the radius of the lighter component of the binary, estimated through quasiuniversal relations from the reduced tidal parameter and mass distributions. The shaded distributions correspond to the values obtained when mapping the **bilby** catalog $\tilde{\Lambda}$ posteriors into radii values with either the new fits (gray) or the Yagi formulae. We find that the newly fitted coefficients for Eq. (5) lead to lower radii values than those predicted by Yagi and Yunes.

attributed to quasiuniversal relations, in particular the $C\text{-}\Lambda_2$ fit. The discrepancy found, negligible for current events, will become very relevant when analyzing GW data from loud BNS events detected by advanced and third generation detectors. The source parameters recovered for such systems will be affected by small statistical fluctuations. A simple error propagation, applied to the

fit of $R_{1.4M_\odot}$ of [14], gives

$$\sigma_{R_{1.4M_\odot}} = (11.2 \pm 0.2) \frac{\mathcal{M}}{4800} \left(\frac{\tilde{\Lambda}}{800} \right)^{-5/6} \sigma_{\tilde{\Lambda}} \quad (9)$$

where we assumed that the error on the chirp mass $\mathcal{M}$ is negligible. Therefore, $\sigma_{\tilde{\Lambda}} \sim 20$ (value expected for SNRs ~ 300) translates into $\sigma_{R_{1.4M_\odot}} \sim 50$ m for a

($\mathcal{M}, \tilde{\Lambda} = 1.18, 800$) BNS system. This value amounts to approximately one tenth of the difference found above due to universal relations.

VI. CONCLUSION

We present updated fits to several universal relations between bulk properties of NSs relevant to current and future GW astronomy. The original fits can be found in [16–18, 31]. The updated fits are produced by sampling from a larger volume of the space of all possible NS EOSs not yet excluded by astronomical observation than had been done in the original fits. We sample the space of possible EOSs using an MCMC algorithm, which has a transition rate determined by three general physical constraints. Our results confirm and extends previously identified universal relations to a much larger set of EOSs.

First, we update fits to relations among three of the l -th order electric tidal deformabilities, Λ_2 , Λ_3 , and Λ_4 . On the whole, the update decreases the relative errors of the fits by $\sim 1\%$ compared to those of the original fits.

Next, we update the fit to the relation between the compactness C and Λ_2 . For small C ($C < 0.25$), our fit has a relative error comparable to that of the original fit. However, for large C ($C > 0.25$), the error of our fit is as much as $\sim 10\%$ smaller than that of the original fit. We can attribute this increase in accuracy at least in part to better sampling of EOSs that admit NSs with $C > 0.3$. This is the largest improvement in accuracy among our updated fits.

Finally, we update the fit to the relation between the symmetric (Λ_s) and antisymmetric (Λ_2) combinations of Λ_2 for each star in a BNS. On the whole, the update decreases the relative error of the fit by $\sim 1\%$ compared to that of the original fit. To demonstrate the improvements updating these fits make to GW analysis, we use the C - Λ_2 and Λ_a - Λ_s relations to recover the radii of BNSs. We get $\sim 0.5\%$ decreased relative error in the recovered radii using our updated fits versus using the original fits. This is due almost entirely to the drastic improvement in the accuracy of the C - Λ_2 fit.

We also discuss the implications of universal relations and the updated fits for GW waveform modelling and parameter estimation. Higher-order ($l > 2$) multipole corrections in the waveform model are important for determining the GW phase of a merger. The $l = 3, 4$ corrections in particular contribute as much as 3.5 radians

of accumulated dephasing at merger for a starting GW frequency of 20 Hz. Thus, accurate modelling of $l > 2$ multipoles is relevant for capturing the merger waveform faithfully. When using the original and updated multipole fits to recover Λ_3 and Λ_4 in waveform modelling, we find that the phase difference between the merger waveforms computed from each fit is of about 0.5 radians from 20 Hz to merger.

We perform a Bayesian parameter estimation of GW170817 using both the original and updated C - Λ_2 and Λ_a - Λ_s fits, and we compare the distributions for the radius of the secondary in the merger yielded by each set of fits. The results from the updated fits are consistent with those of the original fits; however, the updated fits slightly favor a smaller radius, with the difference in the medians of the two distributions being ~ 0.5 km. This is due almost entirely to the increased accuracy of the updated C - Λ_2 fit. Advanced and third generation GW detectors will be sensitive enough to measure NS radii to comparable accuracy. Thus, with increased sensitivity, the accuracy of fitting formulas for universal relations will become very relevant.

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Appendix A

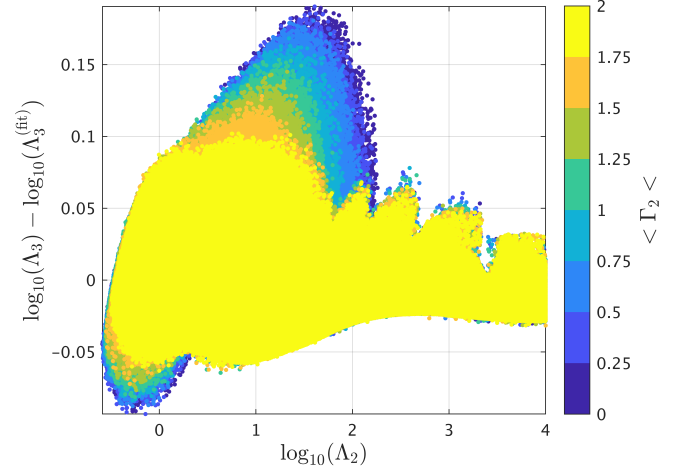


FIG. 14. The log residual of the fit to the universal Λ_3 - Λ_2 relation, with each point colored according to the value of the third adiabatic index Γ_2 of the associated EOS model. The NSs that deviate the farthest from the relation are those whose associated EOS model are soft, having $\Gamma_2 < 1.5$.