

Limiting masses and radii of neutron stars and their implications

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We combine the equation of state of dense matter up to twice nuclear saturation density n_{sat} obtained using chiral effective field theory (χ EFT), and recent observations of neutron stars to gain insights about the high-density matter encountered in their cores. A key element in our study is the recent Bayesian analysis of correlated EFT truncation errors based on order-by-order calculations up to next-to-next-to-next-to-leading order in the χ EFT expansion. We refine the bounds on the maximum mass imposed by causality at high densities, and provide stringent limits on the maximum and minimum radii of $\sim 1.4 M_{\odot}$ and $\sim 2.0 M_{\odot}$ stars. Including χ EFT predictions from n_{sat} to $2 n_{\text{sat}}$ reduces the permitted ranges of the radius of a $1.4 M_{\odot}$ star, $R_{1.4}$, by ~ 3.5 km. If observations indicate $R_{1.4} < 11.2$ km, our study implies that either the squared speed of sound $c_s^2 > 1/2$ for densities above $2 n_{\text{sat}}$, or that χ EFT breaks down below $2 n_{\text{sat}}$. We also comment on the nature of the secondary compact object in GW190814 with mass $\simeq 2.6 M_{\odot}$, and discuss the implications of massive neutron stars $> 2.1 M_{\odot}$ ($2.6 M_{\odot}$) in future radio and gravitational-wave searches. Some form of strongly interacting matter with $c_s^2 > 0.35$ (0.55) must be realized in the cores of such massive neutron stars. In the absence of phase transitions below $2 n_{\text{sat}}$, the small tidal deformability inferred from GW170817 lends support for the relatively small pressure predicted by χ EFT for the baryon density n_B in the range $1 - 2 n_{\text{sat}}$. Together they imply that the rapid stiffening required to support a high maximum mass should occur only when $n_B \gtrsim 1.5 - 1.8 n_{\text{sat}}$.

I. INTRODUCTION

The maximum mass, M_{max} , and radii of neutron stars (NSs) are related to each other by the equation of state (EOS) of dense matter and both can be accessed by observations. Primary constraints on M_{max} come from observations and have a number of astronomical and physical implications. M_{max} is predominately determined by the EOS at densities higher than three times nuclear saturation density, $n_{\text{sat}} \simeq 0.16 \text{ fm}^{-3}$ [1], and is therefore a probe of the nature of high-density matter. Pinning down M_{max} enables the exploration of the phases of cold and dense matter in the strongly coupled region of quantum chromodynamics (QCD) as well as the determination of the pressure vs energy density relation (or the EOS) of such phases. The radii of canonical NSs with masses $\simeq 1.4 M_{\odot}$, on the other hand, are largely determined by the EOS at densities less than $3 n_{\text{sat}}$ [2].

M_{max} also fixes the minimum mass of a stellar mass $\mathcal{O}(M_{\odot})$ black hole (BH). It is therefore a crucial factor in determining the final fate of core-collapse supernovae and binary neutron star (BNS) mergers. In core-collapse supernovae, the formation of a BH will depend on the

amount of fall-back matter and will be sensitive to the nature of the progenitor and neutrino emission after the initial formation of a proto-neutron star. In BNS mergers, the formation of a BH depends on the total inspiralling mass, mass ejection, and the extent of rotational and magnetohydrodynamic support [3, 4]. Now that at least a few mergers involving NS have been detected through gravitational-wave (GW) radiation, and many more are anticipated in the near future, improved constraints on M_{max} will become available. As the high-frequency capabilities of GW detectors are improved, the detection of post-merger radiation will profoundly influence our knowledge of M_{max} . Already, knowledge of M_{max} would determine the nature of the components of the recently observed mergers GW190425 and GW190814, both of which show indications of having a component with a mass larger than $2 M_{\odot}$ that either could be a heavy NS or a light BH. If concomitant electromagnetic (EM) signals are also detected from future GW events, as they were in the BNS merger GW170817 [5–7], additional information about M_{max} becomes available [3, 4].

On the theoretical front, M_{max} plays a crucial role in determining both the minimum and maximum radius as a function of the NS mass M . Therefore, besides the important contributions from radio and X-ray binary pulsar observations that have accurately measured several NS masses and provided a lower bound $M_{\text{max}} \gtrsim 2 M_{\odot}$ [8–13], GW and X-ray data that can simultaneously determine NS masses and radii offer important constraints. So far, the radii inferred from X-ray observa-

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tions (see Ref. [14] for a review) of quiescent low-mass X-ray binaries (QLMXBs) [15], photospheric radius expansion bursts (PREs) [16], and pulse-profiles from rotation-powered millisecond pulsars [17], together with the first GW detection of the BNS merger GW170817 [5, 6], have mostly been of NSs with canonical masses around $1.4 M_\odot$. Consequently, the *Neutron Star Interior Composition ExploreR* (NICER) proposal [18] to measure the radii of relatively massive NSs such as PSR J1614-2230 ($M \simeq 1.91 M_\odot$ [8, 10, 11]) and PSR J0740+6620 ($M \simeq 2.14 M_\odot$ [12]) is of considerable interest. The same is true of future radio observations using the *Square Kilometre Array* (SKA) telescope [19], etc. from binary pulsars that could reveal even more massive NSs.

The purpose of this paper is to explore the interplay between M_{max} and NS radii and to confront theoretical expectations with currently available observational constraints. An earlier study [2] showed that the radii of $\simeq 1.4 M_\odot$ NSs are strongly correlated with the pressure of matter in the density range $1 - 3 n_{\text{sat}}$. In the important regime $\lesssim 2 n_{\text{sat}}$, chiral effective field theory (χ EFT) with pion and nucleon degrees of freedom [20–23] has become the dominant microscopic approach to describing nuclear interactions. χ EFT has enabled significant progress in predicting the EOS of infinite nuclear matter and the structure of neutron stars with quantifiable theoretical uncertainties (see Refs. [24–27] for recent reviews). An important step toward the full uncertainty quantification of the EOS has been achieved recently. The *Bayesian Uncertainty Quantification: Errors in Your EFT* (BUQEYE) collaboration [28] has introduced a Bayesian framework [29, 30] for quantifying and propagating correlated EFT truncation errors in infinite-matter calculations using Gaussian Processes (GPs). They also conducted a statistical analysis of the zero-temperature EOS based on χ EFT nucleon-nucleon (NN) and three-nucleon (3N) interactions and inferred posterior distributions for nuclear saturation properties as well as key quantities for neutron stars, including the nuclear symmetry energy and its density dependence. This study was motivated by recent advances in many-body perturbation theory (MBPT) [31] that have enabled improved χ EFT predictions of the pure neutron matter (PNM) EOS and first order-by-order calculations in symmetric nuclear matter (SNM) up to next-to-next-to-next-to-leading order (N^3 LO) in the chiral expansion [29, 31, 32].

In this paper, we use BUQEYE’s analysis of the EOS in the limits of PNM and SNM at baryon densities $n_B \leq 2 n_{\text{sat}}$ to construct the EOS of charge neutral and beta-stable neutron-star matter (NSM). This is coupled to a standard NS crust for $n_B \lesssim 0.5 n_{\text{sat}}$ and extrapolations for $n_B \gtrsim 2.0 n_{\text{sat}}$ to assess the overall impact on NS structure. One goal of this study is to address quantitatively the extent to which EOS knowledge at $\sim 2.0 n_{\text{sat}}$ can inform us about the NS maximum mass, and how it can be combined with observations of massive NSs to constrain the properties of matter encountered at the highest densities in their cores. Another goal is to derive model-

independent bounds on the radii of NSs with masses in the range $1 - 2 M_\odot$.

As the squared speed of sound c_s^2 reflects the stiffness of the EOS, we probe both maximum and minimum radius bounds by matching the N^3 LO results, including possible extrapolations up to $3 n_{\text{sat}}$, with a constant sound speed beyond a matching density n_m . The existence of nuclei, observations of accreting NSs that implicate the presence of neutron-rich nuclei in the NS crust, and heavy ion collisions (HICs) at intermediate energies together provide compelling circumstantial evidence to indicate that $n_m > n_{\text{sat}}$, and in this work we consider $n_m = 1 - 3 n_{\text{sat}}$. The use of the maximally stiff EOS with $c_s^2 = 1$ (the causal limit) for $n_B > n_m$ establishes firm upper bounds both on M_{max} and the radius as a function of mass. In addition, we also consider energy density discontinuities at n_m to refine minimum bounds on radii as functions of mass for specified values of M_{max} . We also explore models with smaller c_s^2 at high density to ascertain maximum possible sound speeds from values of M_{max} and mass-radius (M - R) observations.

The discovery of a massive secondary compact object with mass $\sim 2.6 M_\odot$ through GW observations of the binary merger GW190814 generated a flurry of articles addressing if this object can be a NS, and, if so, its possible implications [33–40]. Our results complement earlier studies, but go beyond in several aspects. Most significantly,

- (i) we consistently include statistically meaningful EFT truncation errors in the EOS of NSM up to N^3 LO, and determine its range of applicability, to provide a framework for constraining M_{max} and NS radii,
- (ii) we identify correlations of NS radii and tidal deformabilities with M_{max} , together with their possible implications for the EOS at $n_B \gtrsim 2 n_{\text{sat}}$, and
- (iii) we show how these correlations and future observations can tighten current bounds on NS masses and radii.

This paper is organized as follows. Section II contains details of the various EOSs used along with the rationale for their choice. Our results and their discussion in light of the current observational constraints and possible future findings are presented in Sec. III. An overall discussion and comparison with pertinent recent works are contained in Sec. IV. Our concluding remarks are given in Sec. V. Appendix A examines the most conservative bounds and the scaling relations for the masses and radii of NSs imposed by causality. The current shortcomings and prognosis for future improvements to χ EFT are discussed in Appendix B. Appendix C quantifies the density ranges for which $R_{1.4}$ and $R_{2.0}$, the radii of $1.4 M_\odot$ and $2.0 M_\odot$ stars, respectively, and the neutron star maximum mass M_{max} are most sensitive. We use natural units in which $\hbar = c = 1$ unless explicitly specified.

II. CONSTRUCTION OF THE EOS

A. General considerations

Since the pressure-energy density relation, which we call the EOS, completely determines the neutron star M – R relation through the general relativistic TOV equations, bounds of the allowed M – R space are determined by assumptions concerning the EOS. From the perspective of this paper, the three most important regions for the EOS of a NS are the crust, outer core, and inner core. The EOS up to the outer core-crust boundary at $n_{cc} \approx 0.5 n_{\text{sat}}$ is generally considered to be well-understood [41, 42]. Because nucleons contribute $\lesssim 10\%$ to the crust pressure, uncertainties in the NN potential only weakly propagate into the crust EOS. The proton fraction x in the uniform nucleonic matter at densities higher than n_{cc} in the outer core is relatively small, so that the EOS in the vicinity of n_{sat} is dominated by that of PNM. The admixture of protons and leptons produces small corrections, which are effectively minimized because of the requirement that NSM be in beta equilibrium; that is, the total energy is minimized with respect to x .

A causal maximum radius bound $R_{\text{max},c}(M)$, as detailed in Appendix A, can be obtained by assuming a causal EOS at densities greater than that of a fiducial density n_0 , generally greater than that of the core-crust boundary, while below n_0 , the pressure and energy density are taken to be zero. $R_{\text{max},c}$ will depend on the values of the associated fiducial energy density, ε_0 . This calculation explicitly ignores the existence of a crust. Appendix A also highlights the important role M_{max} plays in determining bounds on the radii of neutron stars. In the most extreme case, in which only causality is assumed with the EOS $\varepsilon = \varepsilon_0 + P$, absolute upper bounds on $M_{\text{max}} \simeq 4.09 M_\odot$ and $R_{M_{\text{max}}} \simeq 17.1 \text{ km}$ exist as long as $\varepsilon_0 > \varepsilon_{\text{sat}}$ (see Eq. (A4) and Eq. (A5)). Firm lower bounds on $R_{\text{min}}(M)$ and $R_{M_{\text{max}}}$ that scale with M_{max} can also be established. For the case that $M_{\text{max}} = 2.0 M_\odot$, $R_{\text{min}}(1.4 M_\odot) = 8.2 \text{ km}$ and $R_{M_{\text{max}}} = 8.4 \text{ km}$.

However, there are no observations that indicate the absence of a crust. More realistic bounds to the allowed M – R space (as well as for other relations such as M – Λ for the tidal deformability or M – $\bar{I}$ for the moment of inertia) are obtained by including the presence of the neutron star crust and also imposing theoretical limits to the properties of neutron-rich matter in the outer core up to a matching density n_m , with associated energy density ε_m and pressure P_m (which replace ε_0 and $P_0 = 0$, respectively). Above the matching density, in now what is effectively the inner core, a constant sound speed EOS is assumed, and the maximum radius bounds occur when this sound speed is the speed of light.

Initially, we will explore radius bounds assuming the validity of theoretical studies up to the transition density $n_m = 2.0 n_{\text{sat}}$ and imposing the causal EOS at higher densities. If the causal EOS is imposed exactly at n_m ,

one obtains maximum radius contours $R_{\text{max}}(M)$ and the greatest value for M_{max} . The artificial introduction of a first-order phase transition between the two densities n_m and $n_u > n_m$ with the imposition of the causal EOS for $n_B > n_u$, on the other hand, results in a smaller value of M_{max} but a minimum radius contour $R_{\text{min}}(M)$ unique to that M_{max} (or n_u). We will also explore how R_{min} and R_{max} change if the value of n_m is changed, or if the EOS above n_m or n_u is assumed to have a subluminal sound speed.

B. The EOS of the outer core

To construct the EOS of charge-neutral, beta-equilibrated NSM in the outer core between n_{cc} and $\sim 2.0 n_{\text{sat}}$, we use the standard approximation of keeping only the quadratic term in the nuclear energy expanded in the isospin asymmetry parameter $\beta = 1 - 2x$, where $x = n_p/n_B$ is the proton fraction, and n_p the proton density. The total energy per baryon of NSM is then

$$E_{\text{NSM}} = E_{\text{PNM}}(1-2x)^2 + E_{\text{SNM}} 4x(1-x) + E_e + E_\mu, \quad (1)$$

where E_{PNM} and E_{SNM} (E_e and E_μ) are the energies per baryon of PNM and SNM (electrons and muons), respectively. Microscopic calculations of asymmetric matter based on chiral NN and 3N interactions at $n_B \lesssim n_{\text{sat}}$ have confirmed that the quadratic expansion Eq. (1) is a reasonable approximation of the full isospin dependence of the EOS [43–47]. Beta equilibrium follows then from the condition that the total charge-neutral energy be minimized with respect to x , i.e.,

$$\frac{\partial E_{\text{NSM}}}{\partial x} = 0, \quad (2)$$

or in terms of the associated chemical potentials

$$\mu_n - \mu_p = 4E_{\text{sym}}(1-2x) = \mu_e = \mu_\mu. \quad (3)$$

Using the Jupyter notebooks [28] provided by the BUQEYE collaboration we extract the mean values, standard deviations (encoding the EFT truncation errors), and correlation information of the energy per particle, pressure and speed of sound in PNM and SNM, and also the symmetry energy. These data sets form the microscopic input of our interpolation to NSM.

Specifically, BUQEYE's EFT truncation error analysis [29, 30] is based on recent order-by-order MBPT calculations in PNM and SNM with chiral NN and 3N interactions up to $N^3\text{LO}$ [29, 31, 32]. More details on the underlying nuclear interactions can be found in Appendix B. The range in density covers $n_B = 0.05 - 0.34 \text{ fm}^{-3}$. These calculations significantly improved previous MBPT studies in PNM at $N^3\text{LO}$ [48–50], and assessed, for the first time, the SNM EOS with NN and 3N interactions order-by-order up to $N^3\text{LO}$. The high-order MBPT calculations were performed by the novel Monte Carlo framework introduced in Ref. [31], which enables MBPT calculations of the EOS with controlled many-body uncertainties for these χEFT interactions. The statistical analysis

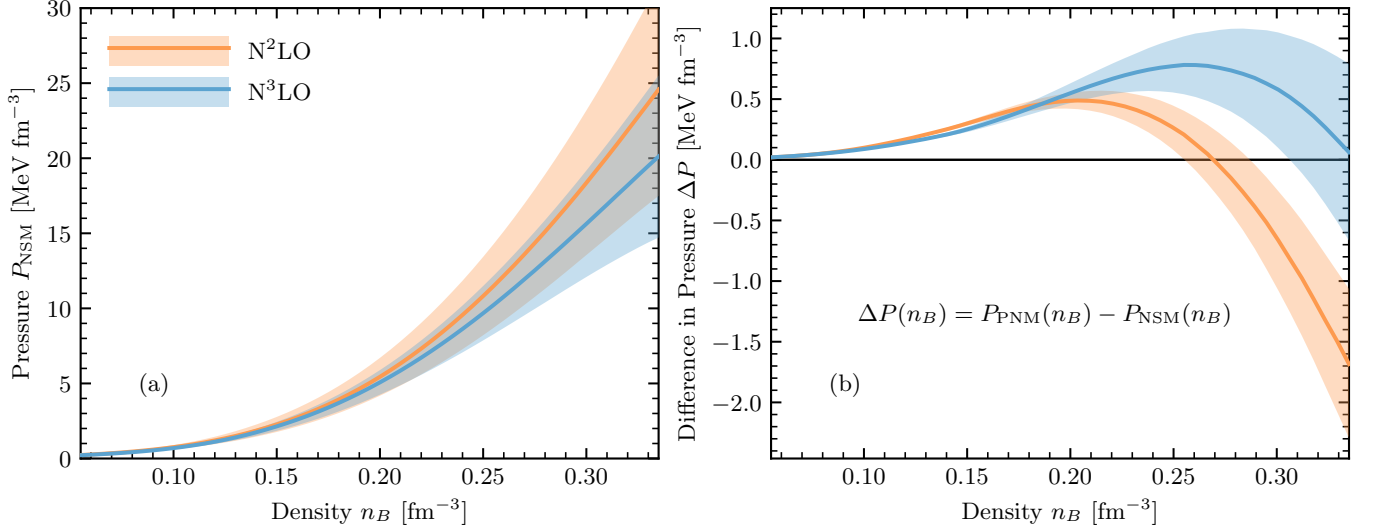


FIG. 1. Panel (a): pressure of neutron-star matter (NSM) in the outer core as a function of the baryon number density at N^2 LO (orange-shaded band) and N^3 LO (blue-shaded band) in the chiral expansion; panel (b): differences of the PNM and NSM in the outer core with the same notation. Uncertainty bands depict 1σ confidence regions.

indicates that the EFT truncation error is strongly correlated. In other words, perturbing the EOS at one point in the density (or the proton fraction) perturbs neighboring points as well. In general, the range of these correlations, called the correlation length, depends on the density and the underlying nuclear interactions. Chiral 3N forces make for important contributions to the EOS in PNM and SNM at $n_B \gtrsim n_{\text{sat}}$, and typically have a markedly different density-dependence than NN contributions. The correlation lengths inferred are comparable to the k_F associated with n_{sat} in PNM and SNM, respectively. Without including these correlations, uncertainties in derived quantities of the EOS, such as the nuclear symmetry energy, can be overestimated.

Our approach considers correlations between the EOSs in PNM and SNM explicitly, neglecting correlations in density.¹ The extracted observables are then given by independent normal distributions sampled on a fine grid in density using the GPs; e.g.,

$$E_{\text{PNM}} \sim \mathcal{N}(\mu_{\text{PNM}}, \sigma_{\text{PNM}}^2), \quad (4)$$

$$E_{\text{SNM}} \sim \mathcal{N}(\mu_{\text{SNM}}, \sigma_{\text{SNM}}^2). \quad (5)$$

The nuclear symmetry energy is defined as

$$E_{\text{sym}} = E_{\text{PNM}} - E_{\text{SNM}} \sim \mathcal{N}(\mu_{\text{NSM}}, \sigma_{\text{NSM}}^2), \quad (6)$$

and, hence, has mean and variance (see, e.g., Ref. [51]):

$$\mu_{\text{sym}} = \mu_{\text{PNM}} - \mu_{\text{SNM}}, \quad (7)$$

$$\sigma_{\text{sym}}^2 = \sigma_{\text{PNM}}^2 + \sigma_{\text{SNM}}^2 - 2\rho\sigma_{\text{PNM}}\sigma_{\text{SNM}}, \quad (8)$$

¹ Such correlations could be implemented in future work by directly sampling from the GPs, which the BUQEYE collaboration uses to model the correlated EFT truncation errors.

where ρ is the correlation coefficient between the energies per particle in PNM and SNM. For subsequent discussion, we introduce here also the usual parameters S_v and L in the density expansion of the nuclear symmetry energy Eq. (6),

$$E_{\text{sym}} = S_v + \frac{L}{3} \left(\frac{n_B - n_{\text{sat}}}{n_{\text{sat}}} \right) + \dots \quad (9)$$

The correlation between the coefficients in the χ EFT expansions for the PNM and SNM energy per particle was quantified to be $\rho^* = 0.934$, corresponding to *very strong* correlations [52, 53]. A detailed discussion can be found in Sec. IV A of Ref. [30]. We have checked that $\rho \simeq \rho^*$ by comparing E_{sym} against the values obtained in Ref. [30]: the maximum deviation between the mean values of two approaches is 37 keV (340 keV for its $\pm 1\sigma$ bounds) at the highest density, $n_B = 0.34 \text{ fm}^{-3}$, which is negligible compared to the overall EFT truncation error at that density.

We also found that numerical integration of the pressure of PNM and SNM agreed well with the energy found in the GP approach, the maximum deviation of the mean values being 3 keV and 1 keV for PNM and SNM, respectively (290 keV and 500 keV for their respective $\pm 1\sigma$ bounds) at the highest density. There are mainly two related reasons why finite differencing for the pressure, discrete integration for the energy, and subtraction for the symmetry energy, works so well. First, the correlation length of the EOS is much longer than the length scale used for finite differencing. That means numerical differentiation follows closely the curves $\mu \pm \sigma$, which are two realizations of the underlying GP. Secondly, the raw EOS data has already been preprocessed by BUQEYE's

truncation error model. Numerical noise from the many-body method has been smoothed out, and the EOS has been sampled on a fine grid in density using the GP interpolant. This underlines that GP interpolants are efficient tools for analyzing χ EFT calculations of the EOS.

Propagating the EFT uncertainties to E_{NSM} associated with Eq. (1) is straightforward because of the condition (2). We obtain

$$\sigma_{E_{\text{NSM}}}^2 = \left(\frac{\partial E_{\text{NSM}}}{\partial E_{\text{PNM}}} \right)^2 \sigma_{E_{\text{PNM}}}^2 + \left(\frac{\partial E_{\text{NSM}}}{\partial E_{\text{SNM}}} \right)^2 \sigma_{E_{\text{SNM}}}^2 + 2\rho \frac{\partial E_{\text{NSM}}}{\partial E_{\text{PNM}}} \frac{\partial E_{\text{NSM}}}{\partial E_{\text{SNM}}} \sigma_{E_{\text{PNM}}} \sigma_{E_{\text{SNM}}}, \quad (10)$$

with the derivatives $\partial E_{\text{NSM}}/\partial E_{\text{PNM}} = (1 - 2x)^2$ and $\partial E_{\text{NSM}}/\partial E_{\text{SNM}} = 4x(1 - x)$.

Figure 1 (a) shows the pressure of NSM (including contributions from the leptons) $P_{\text{NSM}} = n_{\text{B}}^2 (dE_{\text{NSM}}/dn_{\text{B}})$ in the outer core. The blue (orange) uncertainty band corresponds to the N³LO (N²LO) results at the 1σ level. Panel (b) displays the difference in pressures between PNM and NSM. The zero crossings indicate where the pressure of NSM equals that of PNM. Depending on the chiral order, these crossings occur at $n \approx 1.6 - 2.1 n_{\text{sat}}$. They are due to a softening of E_{sym} at the higher densities; nevertheless, E_{NSM} is always less than that of E_{PNM} . In no case does x exceed about 0.055 for $n_{\text{B}} \leq 0.34 \text{ fm}^{-3}$.

III. RESULTS

A. Minimum and maximum radius bounds with χ EFT and causality

Earlier work has shown that canonical-mass ($1.1 - 1.7 M_{\odot}$) neutron star radii are most sensitive to the EOS in the density interval $1.5 - 3.0 n_{\text{sat}}$ [2], and this is further quantified in Appendix C.² As a result, calculations up to $\lesssim 2.0 n_{\text{sat}}$ are adequate to place stringent bounds on the NS radius [1, 54, 55]. We assume a typical crust EOS [41, 42] below $0.5 n_{\text{sat}}$, the EOS for NSM based on MBPT- χ EFT calculations [30] in the outer core, and a matching linear EOS $P(\varepsilon) = P_{\text{m}} + c_s^2(\varepsilon - \varepsilon_{\text{m}})$ characterized by c_s^2 in the inner core.

Figure 2 (a) shows the M - R relation for N³LO-NSM, N²LO-NSM outer core EOSs in Fig. 1 (a) matched at $n_{\text{m}} = 2.0 n_{\text{sat}}$ to the stiffest linear EOS ($c_s^2 = 1$); the solid colored curves refer to the central values and the color-shaded bands refer to $\pm 1\sigma$ uncertainties in the MBPT- χ EFT calculations. Results for matching at a lower density $n_{\text{m}} = 1.5 n_{\text{sat}}$ are shown in Fig. 2 (b). As expected,

for a given value of n_{m} , the largest radii result from the largest matching pressure P_{m} , and thus N²LO $+1\sigma$; note that N²LO- 1σ shows little difference compared to N³LO- 1σ . In general, the lower is n_{m} , the larger are the maximum radii (Figure 2 (b)). Any discontinuities in the energy density for $n_{\text{B}} \geq n_{\text{m}}$, such as from a phase transition, would serve to decrease $R(M)$, emphasizing the results in these figures as being upper bounds. The extreme case, described in Appendix A, self-bound (crustless) stars with $P_0 = 0$ and $c_s^2 = 1$ for a given M_{max} represent the “maximally compact” configurations that exhibit the smallest possible radii at all masses, and for comparison their mass-radius relations are also displayed (black solid lines). The causal limit $c_s^2 = 1$ in all cases shown leads to maximum masses as high as $\approx 2.93 M_{\odot}$, as predicted by

$$M_{\text{max}} \simeq 4.09 \sqrt{\frac{\varepsilon_{\text{sat}}}{\varepsilon_0}} M_{\odot}, \quad (11)$$

using $\varepsilon_0 = \varepsilon_{\text{m}} \simeq 2.0 \varepsilon_{\text{sat}}$ (see derivation in Eq. (A4)). Differences at low densities, e.g., between N³LO and N²LO, have negligible effects on M_{max} , as already noted in the crustless case of Appendix A. For a given value of $c_{s,\text{match}}^2$, M_{max} is essentially determined by n_{m} and is relatively insensitive to P_{m} . With smaller values of $c_{s,\text{match}}^2$ for a given n_{m} , the maximum mass decreases. It can be seen that the upper bounds on $R_{1.4}$ (where the bands intersect with the $M = 1.4 M_{\odot}$ horizontal line) are about 12.9 km (13.6 km) if $n_{\text{m}} = 2.0 n_{\text{sat}}$ ($n_{\text{m}} = 1.5 n_{\text{sat}}$). Although M_{max} is not sensitive to the low-density EOS (see also Appendix C), $R_{\text{max}}(M)$ for canonical-mass neutron stars ($1.1 - 1.7 M_{\odot}$) is. The relatively soft N³LO EOS up to $2.0 n_{\text{sat}}$ guarantees that the typical NS radius $\lesssim 13$ km, even with very stiff matter at higher densities that can lead to $M_{\text{max}} > 2.6 M_{\odot}$.

In contrast to R_{max} , it is possible to deduce a minimum radius R_{min} for a given low-density ($n_{\text{B}} < n_{\text{m}}$) EOS by introducing a finite discontinuity in the energy density $\Delta\varepsilon_{\text{m}}$ at n_{m} . Above the density $\varepsilon_{\text{m}} + \Delta\varepsilon_{\text{m}}$, the EOS is assumed to be the causal EOS with $c_s^2 = 1$. The larger is $\Delta\varepsilon_{\text{m}}$, the smaller is the resulting value of M_{max} , which has a one-to-one relation with it. If the pressure at n_{m} is vanishingly small, this effectively gives the $R_{\text{min},c}(M)$ relation for the maximally compact EOS of self-bound stars as described in Appendix A but with $\varepsilon_0 = \varepsilon_{\text{m}} + \Delta\varepsilon_{\text{m}}$. With finite pressure at n_{m} based on χ EFT calculations, $R_{\text{min}}(M)$ is larger and is the minimum radius for normal NSs. In the self-bound case, the magnitude of $\Delta\varepsilon_{\text{m}}$ is related to the maximum mass according to Eq. (11) by imposing $\varepsilon_0 = \varepsilon_{\text{m}} + \Delta\varepsilon_{\text{m}}$. Even in the case with a crust, since the maximum mass is reached at very high densities, this relation remains relatively accurate. For $M_{\text{max}} = 2.0 M_{\odot}$, we find that $\Delta\varepsilon_{\text{m}} \approx \varepsilon_{\text{nuc}}(n_{\text{m}}) \simeq 2.0 \varepsilon_{\text{sat}}$. To accommodate a maximum mass of $2.6 M_{\odot}$, for example, requires a much smaller discontinuity, $\Delta\varepsilon_{\text{m}} \approx 0.25 \varepsilon_{\text{nuc}}(n_{\text{m}})$. Furthermore, all the trajectories within any $\pm 2\sigma$ band for each value of M_{max} have nearly identical values of $\Delta\varepsilon_{\text{m}}$ resulting from the fact that $P_{\text{m}} \ll \varepsilon_{\text{m}}$. The relation be-

² Appendix C also quantifies the sensitivity of the key observables $R_{1.4}$, $R_{2.0}$, and M_{max} to the pressure as a function of density $P(n_{\text{B}})$. The highest correlations, i.e., the most sensitive regions, involve the density ranges $1.0 - 3.0 n_{\text{sat}}$, $1.5 - 4.0 n_{\text{sat}}$, and $2.0 - 6.0 n_{\text{sat}}$, respectively.

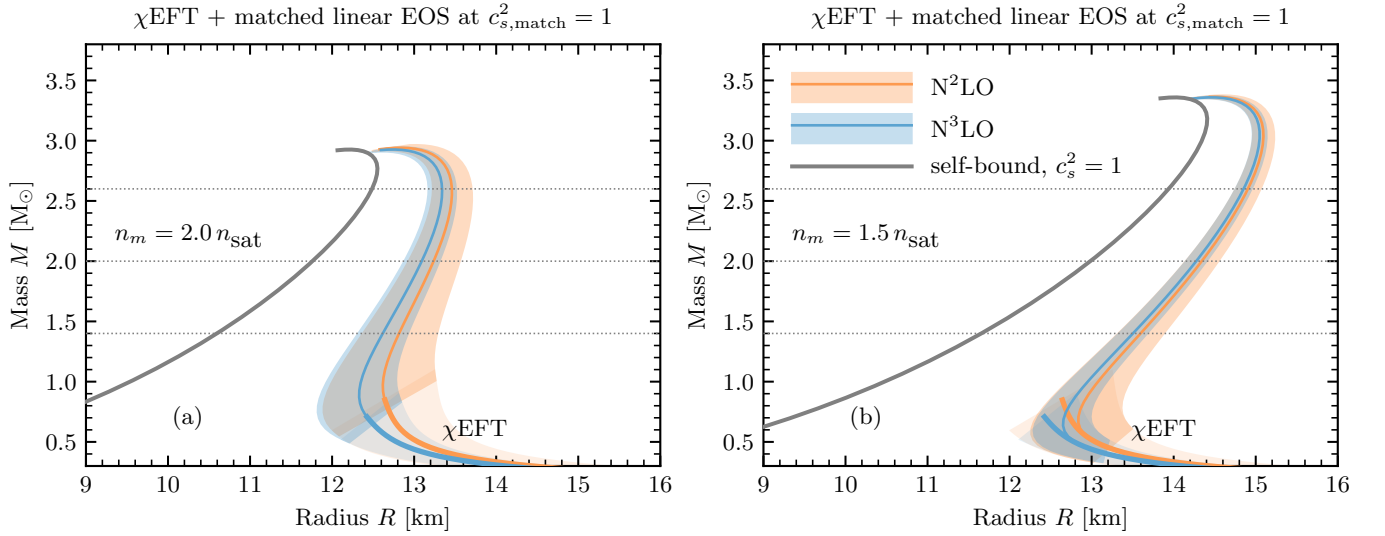


FIG. 2. Panel (a): M - R diagram for NSM based on MBPT calculations shown in Fig. 1 (a), including N^2 LO-NSM (orange-shaded band) and N^3 LO-NSM (blue-shaded band) in the outer core matched to a linear causal ($c_{s,\text{match}}^2 = 1.0$) EOS at $n_m = 2.0 n_{\text{sat}}$, and the maximally compact EOS for self-bound stars with the same value of M_{max} (black solid). Horizontal lines indicate $M = 1.4, 2.0, 2.6 M_{\odot}$. The colored bands above $n_m = 2.0 n_{\text{sat}}$ represent upper bounds on the NS radius for a given mass, as the high-density matter is assumed maximally stiff without discontinuities in the overall EOS (see detailed discussions in Sec. III A). Panel (b): similar to (a) but with a lower matching density, $n_m = 1.5 n_{\text{sat}}$.

tween $\Delta\varepsilon_m$ at $2.0 n_{\text{sat}}$ and M_{max} is indeed relatively insensitive to the low-density EOS.

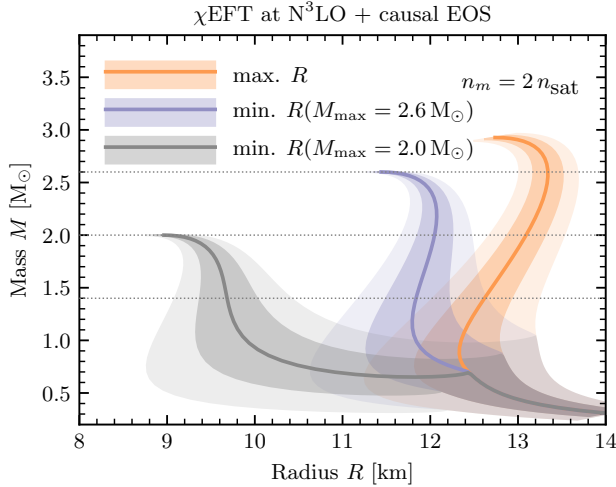


FIG. 3. Radius bounds obtained by combining N^3 LO- χ EFT predictions up to $n_m = 2.0 n_{\text{sat}}$ and maximum-mass information is shown. The orange bands show the upper bound on the NS radius, while the black and purple bands depict the lower bounds corresponding to $M_{\text{max}} = 2.0 M_{\odot}$ and $M_{\text{max}} = 2.6 M_{\odot}$, respectively.

Figure 3 shows the combined minimum and maximum radius bounds. The central values of the minimum radii $R_{\text{min}}(M)$ for $M_{\text{max}} = 2.0 M_{\odot}$ and $M_{\text{max}} = 2.6 M_{\odot}$ are shown as black and purple solid curves, respectively, while the darker and lighter bands reflect 1σ and 2σ un-

certainties, respectively. To 2σ confidence, the minimum radius of a $1.6 M_{\odot}$ star ranges from 9.2–12.2 km as M_{max} is varied from $2.00 M_{\odot}$ to $2.93 M_{\odot}$; roughly, the minimum value of $R_{1.6} \propto M_{\text{max}}^{3/4}$. Similarly, the minimum values of $R_{M_{\text{max}}}$ vary from 9.0–12.6 km. It is interesting to compare these results with claims that $R_{1.6} > 10.68$ km and $R_{M_{\text{max}}} > 8.6$ km from observations of GW170817 [56] using empirical relations established in hydrodynamical simulations that relate R , M_{max} , and the threshold binary mass M_{thres} for prompt collapse of a merger remnant. We can therefore provide a more restrictive bound for $R_{M_{\text{max}}}$ since M_{max} is believed to be $\geq 2.0 M_{\odot}$.

Figure 3 demonstrates how future discoveries of NSs with large masses could constrain the radii of all NSs. Several interesting insights can be gleaned from this figure. A striking, albeit expected, feature is the convergence of the upper and lower radius bounds with increasing M_{max} . This is in accordance with the facts that the discontinuity $\Delta\varepsilon_m$ leading to the minimum radii has to decrease to achieve a higher M_{max} [57] and that the limit $\Delta\varepsilon_m \rightarrow 0$ defines the maximum radii. For example, the uncertainty in theoretical predictions for the radius of a $1.4 M_{\odot}$ NS would be reduced from about 3 km when $M_{\text{max}} = 2.0 M_{\odot}$ to about 0.5 km when $M_{\text{max}} = 2.6 M_{\odot}$. Another feature worth noting is the evolution of the 2σ lower bound on the NS radius. It increases by about 2 km, from 9.2 km for $M_{\text{max}} = 2.0 M_{\odot}$ to 11.2 km when $M_{\text{max}} = 2.6 M_{\odot}$. Comparing the black and purple bands shows that the radii of heavier neutron stars are even more tightly constrained with increasing M_{max} . Future observational constraints on NS radii in the mass range 1.4–2.0 $M_{\odot}$ could be valuable in this regard since X-

ray and GW observations are best suited to provide radius information at the level of 5% uncertainty in this mass range [58]. Results in Fig. 3 also demonstrate that an upper bound of about 13 km for $R_{1.4}$ obtained from GW170817 is consistent with NSs with $M_{\max} \simeq 2.6 M_{\odot}$.

The trends seen in Fig. 3 also have important implications for the EOS of matter at the highest densities encountered in the NS inner core. Our results imply that $M_{\max} > 2.5 M_{\odot}$ and/or radii > 12.5 km for neutron-star masses $\simeq 1.4 M_{\odot}$ can only be achieved if $c_s^2 \simeq 1$ over a wide density range encountered in the NS core. We emphasize here that this insight relies on the relatively soft EOS predicted by N³LO- χ EFT calculations. Improving the EOS, especially the EFT truncation errors in the vicinity of $n_B \simeq 2.0 n_{\text{sat}}$, will be critical in extracting better constraints on the EOS at higher densities in the core if future observations favor these large radii or masses. Supporting $c_s^2 \simeq 1$ from $2-5 n_{\text{sat}}$ requires a form of strongly interacting relativistic matter that poses significant challenges for dense-matter theory and QCD [59].

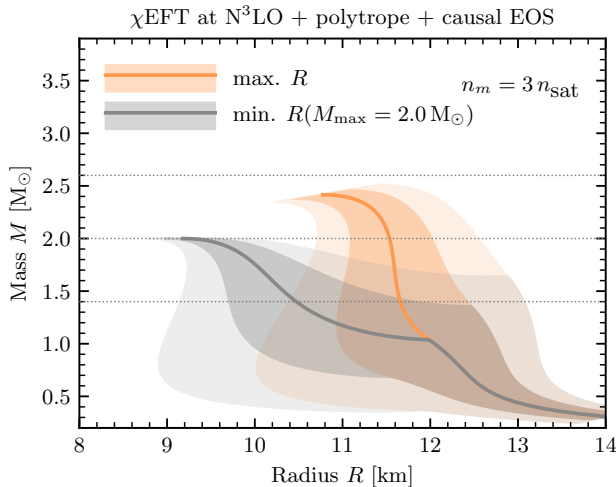


FIG. 4. Similar to Fig. 3 but obtained using the polytropic extrapolation of the χ EFT EOS up to $n_m = 3.0 n_{\text{sat}}$.

B. Consequences of increasing n_m or decreasing c_s^2

Encouraged by the apparent convergence of χ EFT calculations over the density interval $1-2 n_{\text{sat}}$, it is natural to ask if a nuclear physics based description of dense matter can be extended to higher density. Extrapolating the EOS from $2.0 n_{\text{sat}}$ to $3.0 n_{\text{sat}}$ will be model-dependent, even in the absence of phase transitions to non-nucleonic matter, since we presently do not have reliable calculations at higher densities. We climb this rung of the density ladder with some reservation to motivate and explore the impact of future calculations of the EOS in this density interval.

We first consider a polytropic model where $P = \kappa \varepsilon^\gamma$ in which the parameters κ and γ are determined by fitting to

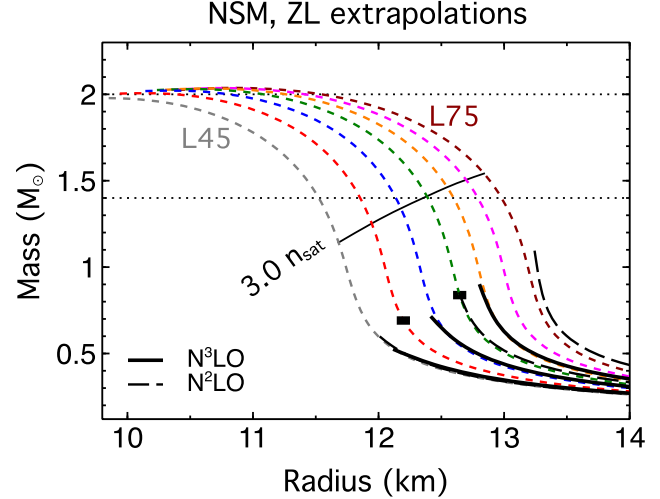


FIG. 5. M - R relations for NSM EOSs extrapolated to $n_B \geq 2.0 n_{\text{sat}}$ beyond the χ EFT calculations using the ZL parametrization [60]; the thin black line indicates where the NS central densities are $3.0 n_{\text{sat}}$. From left to right, the colored dotted curves represent $L = 45$ MeV to $L = 75$ MeV in increments of 5 MeV, and the black-solid (black-dashed) curves refer to χ EFT-N³LO (N²LO) with $\pm 1\sigma$ uncertainties. The $L = 50$ MeV (red) and $L = 60$ MeV (green) ZL EOSs are used in Fig. 11 because they best represent $\pm 1\sigma$ bounds.

the behavior predicted by χ EFT calculations in the density interval $1.9-2.1 n_{\text{sat}}$ to extrapolate the EOS from $2.0 n_{\text{sat}}$ to $3.0 n_{\text{sat}}$. This choice is somewhat arbitrary and is chosen to approximately capture the key features of the density dependence of the EOS predicted by χ EFT. The resulting radius bounds are shown in Fig. 4. We have also found that an alternative parametrization [60] of NSM matter, which has a single parameter corresponding to the symmetry energy coefficient L , to be a convenient extrapolation tool, referred to hereafter as the ZL parameterization. Figure 5 shows M - R curves for the ZL EOSs together with a standard crust. For example, $L = 45$ MeV (65 MeV) successfully tracks N³LO, while $L = 45$ MeV (75 MeV) tracks N²LO, for $-\sigma$ ($+\sigma$). We have checked that alternate extrapolations using the polytropic model, with parameters chosen to suitably match the χ EFT results at $2.0 n_{\text{sat}}$, do not significantly alter our conclusions.

A comparison between the results shown in Fig. 3 with those in Fig. 4 reveals the following insights. First, the increase in n_m does not alter the bounds on $R_{\min}(M)$ (including $R_{M_{\max}}$), as a function of $M_{\max}$, except that in the extrapolated case M and $M_{\max}$ cannot exceed about $2.5 M_{\odot}$. These bounds are therefore particularly robust for $M < 2.5 M_{\odot}$.

The increase in n_m results in more stringent upper bounds on the NS radius for masses in the range

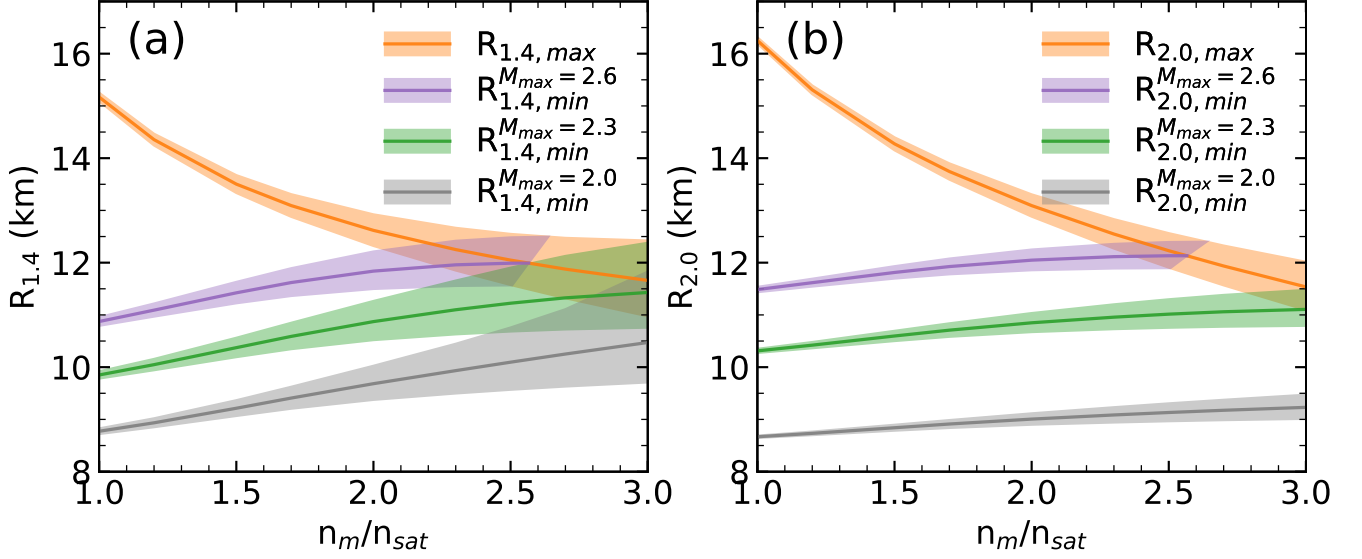


FIG. 6. Similar to Figs. 3 and 4, but displaying the minimum and maximum radii of $1.4 M_\odot$ (panel (a)) and $2.0 M_\odot$ (panel (b)) stars as a function of the matching density $n_m = 1.0 - 3.0 n_{\text{sat}}$. Additionally, R_{min} contours and uncertainty bands for the case $M_{\text{max}} = 2.3 M_\odot$ are shown.

$1.4 - 2.5 M_\odot$. For example, the polytropic extrapolation to $3.0 n_{\text{sat}}$ predicts $R_{\text{max}}(1.4 M_\odot) = 11.6^{+0.8}_{-0.6}$ km, which is to be contrasted with $R_{\text{max}}(1.4 M_\odot) = 12.5^{+0.3}_{-0.2}$ km obtained using $n_m = 2.0 n_{\text{sat}}$. This reduction has implications for the interpretation of future radius measurements which aim for an accuracy of better than 5% [58]. If these observations favor NSs in this mass range to have radii > 12 km, it would require new mechanisms to rapidly stiffen the EOS below $3.0 n_{\text{sat}}$.

It is also apparent, if the secondary component in GW190814 were to be confirmed to be a massive NS, new mechanisms would also be implicated at a low density, since the extrapolated EOS up to $3.0 n_{\text{sat}}$ predicts M_{max} in the range $2.32 - 2.53 M_\odot$ at $\pm 2\sigma$.

The results shown in Figs. 3 and 4 are summarized in Fig. 6 for the specific cases of $R_{1.4}$ and $R_{2.0}$, with a broader range of n_m explored between $1.0 - 3.0 n_{\text{sat}}$.

Figure 6 also conveniently illustrates the dramatic effect of increasing the lower bound on M_{max} for the allowed ranges between $R_{\text{min}}(M)$ and $R_{\text{max}}(M)$, which improves (shrinks) the $R_{1.4}$ ($R_{2.0}$) bounds by an average $3 \text{ km}/M_\odot$ ($5 \text{ km}/M_\odot$); these limits could be further restricted by forthcoming observations. We note that $R_{1.4}$ or $R_{2.0} < 10.7 \text{ km}$ would be incompatible with $M_{\text{max}} > 2.3 M_\odot$ (assuming $n_m = 2.0 n_{\text{sat}}$). In addition, if future measurements from different sources and messengers, e.g., X-ray data from QLMXBs or PREs (or GW detections of mergers by LIGO) vs NICER targets, were to exhibit discrepancies in the radius inference close to or larger than the gaps between the minimum and maximum bands shown on this figure, then these are hints of a large energy-density discontinuity $\Delta\epsilon_m$ in the EOS

(accompanied with high-density stiff matter) occurring at $n_B \lesssim n_m$ [61].

It is important to recall that M_{max} depends monotonically on n_m (or $\epsilon_m + \Delta\epsilon_m$) for fixed $c_{s,\text{match}}^2$. So far, we have only considered causal EOSs ($c_{s,\text{match}}^2 = 1$). However, it is almost certain that the EOS in this high-density region will be subluminal. To keep the discussion straightforward, we now consider the consequences of fixing the sound speed in this region to a constant value $c_{s,\text{match}}^2 \leq 1$. Therefore, assuming a crust EOS, the validity of χEFT up to n_m , and a constant sound speed for the highest density region, implies that M - R trajectories, and M_{max} , will depend on three quantities: n_m , $\Delta\epsilon_m$ and $c_{s,\text{match}}^2$. Instead of using the polytropic parameterization, we extend the nucleonic EOS to $3.0 n_{\text{sat}}$ with the ZL parameterization. We find that the ZL EOSs corresponding to $L = 60 \text{ MeV}$ and 50 MeV , respectively, smoothly join the $M_{\text{max}}(n_m)$ relations for the $\text{N}^3\text{LO} + 1\sigma$ and $\text{N}^3\text{LO-cen}$ EOSs, even though those corresponding to $L = 65 \text{ MeV}$ and $L = 45 \text{ MeV}$ seem to match the $\pm 1\sigma$ M - R results below $2.0 n_{\text{sat}}$ ³; the reason is that the

³ Note that $M_{\text{max}}(n_m)$ for the extrapolated EOSs will eventually bend upwards at sufficiently large n_m , which is a generic feature whenever a “standard” nucleonic-like EOS (i.e. gradually increasing c_s^2 without kinks or discontinuities that naturally extends from low-density e.g. χEFT calculations) is switched to a linear EOS at some critical density, with or without discontinuities in ϵ (see, e.g., Fig. 5 in Ref. [57]). However, we limit our studies to $n_m \lesssim 3.0 n_{\text{sat}}$, as there is little guidance for the validity of nucleonic degrees of freedom at higher densities from theory.

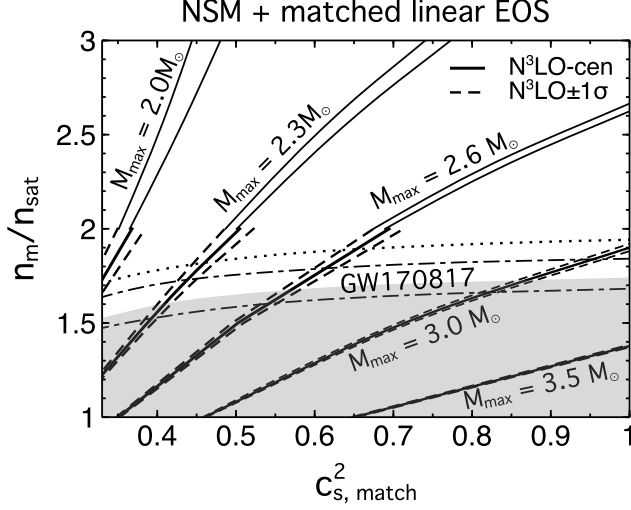


FIG. 7. $M_{\max}$ contours on the $(c_{s,\text{match}}^2, n_m)$ plane, obtained when $\Delta\varepsilon_m = 0$. For each value of $M_{\max}$, the central solid curve shows results with the central value of $\chi\text{EFT-N}^3\text{LO}$; dashed lines indicate $\pm 1\sigma$ bounds. Extensions to $n_m > 2.0 n_{\text{sat}}$ for the central and $+\sigma$ bound are also shown as solid curves. The grey-shaded region is excluded by the binary tidal deformability constraint $\tilde{\Lambda}_{1.186} \leq 720$ from GW170817 at the 90% credibility level [6] if $\text{N}^3\text{LO-cen}$ is assumed; the dot-dashed lines refer to constraints with the $\text{N}^3\text{LO} \pm 1\sigma$ boundaries. The GW170817 bounds will be shifted downwards if there is a first-order transition at such low densities.

masses of stars with central density $n_{\text{cent}} = 2.0 n_{\text{sat}}$ are similar in both cases. If $\chi\text{EFT-N}^3\text{LO}$ is assumed valid up to $2.0 n_{\text{sat}}$, the upper and lower bounds on NS radii are substantially tightened in comparison with using $\chi\text{EFT-N}^3\text{LO}$ only up to n_{sat} : for example if $M_{\max} \geq 2.0 M_{\odot}$ then $R_{1.4}$ must lie between $12.5^{+0.3}_{-0.2}$ km and $9.7^{+0.4}_{-0.3}$ km at the 1σ level, which is consistent with earlier studies in Ref. [55]. Radius constraints are further tightened if $\chi\text{EFT-N}^3\text{LO}$ is assumed valid to higher densities, but there is a diminishing return.

Figure 7 shows how $M_{\max}$ depends on n_m and $c_{s,\text{match}}^2$. We find, for example, that $M_{\max} \geq 2.6 M_{\odot}$ requires $c_{s,\text{match}}^2 > 0.35$ (i.e., the conformal limit $c_s^2 \leq 1/3$ is violated) if $n_m = n_{\text{sat}}$, and $c_{s,\text{match}}^2 > 0.7$ if $n_m = 2.0 n_{\text{sat}}$. The conformal limit is also violated for $n_m > 1.7 n_{\text{sat}}$, even if $M_{\max}$ is as low as $2.0 M_{\odot}$. If $M_{\max} > 2.45 M_{\odot}$, n_m must not exceed $3.0 n_{\text{sat}}$ no matter what the value of $c_{s,\text{match}}^2$ is. The calibrated uncertainties in $\chi\text{EFT-N}^3\text{LO}$ lead to relatively small uncertainties, less than $0.1 M_{\odot}$, in $M_{\max}(n_m, c_{s,\text{match}}^2)$.

There has been speculation that the speed of sound in QCD at finite baryon density may be bounded by the conformal limit which requires $c_s^2 < 1/3$ [62]. This speculation is in part based on strong-coupling calculations of $\text{SU}(N_c)$ gauge theories for which a holographic or gravity dual exist. In these theories the speed of sound can be

calculated at finite baryon density in the large- N_c limit using classical supergravity methods in a curved spacetime [63], and for a large class of such theories (for exceptions, see Refs. [64, 65]) $c_s^2 < 1/3$ [62, 66]. In addition, at finite temperature and zero baryon density, where lattice QCD calculations provide reliable predictions, $c_s^2 < 1/3$ at all temperatures. The sound speed increases rapidly in the hadronic phase (dominated by pions) reaching a maximum value $c_s^2 \simeq 0.2$, then decreases across hadron-quark cross-over region, corresponding to temperatures in the range 100–200 MeV, and eventually increases again to reach its asymptotic value of $c_s^2 \simeq 1/3$ at $T \simeq 500$ MeV [67].

Motivated by the discussion above, we briefly comment on the astrophysical implications of the conjecture that $c_s^2 < 1/3$ in QCD [62] in light of our results. It was already noted in Refs. [68, 69] that it is difficult to accommodate $c_s^2 < 1/3$ at high density and $M_{\max} > 2.0 M_{\odot}$ while still allowing for a soft EOS at intermediate density needed to ensure that $R_{1.4} < 13$ km. This is also evident from Fig. 7 which shows that when $c_s^2 < 1/3$, it is impossible, at the 1σ level, to simultaneously satisfy the tidal deformability constraint from GW170817 and $M_{\max} > 2.0 M_{\odot}$ if $\chi\text{EFT-N}^3\text{LO}$ is valid beyond $1.8 n_{\text{sat}}$.

Figure 8 shows how the bounds on the radius are influenced when $c_s^2 < 1/3$ at high density. The rapid decrease in the maximum value of $R_{1.4}$ with n_m is striking and implies that if $c_s^2 < 1/3$ and $\chi\text{EFT-N}^3\text{LO}$ is valid up to $1.5 n_{\text{sat}}$, then $R_{1.4}$ must lie between $12.4^{+0.2}_{-0.3}$ km and $13.1^{+0.3}_{-0.3}$ km at the 1σ level. Further, requiring that $M_{\max} > 2.0 M_{\odot}$ excludes a significant fraction of the $\chi\text{EFT-N}^3\text{LO}$ predicted range for the pressure for densities between $1.5 - 2.0 n_{\text{sat}}$. A tiny sliver of high pressure close to the edge of the 2σ boundary remains, and implies that $R_{1.4} = 13.1 \pm 0.1$ km! Predictions for $R_{2.0}$ are shown in the right panel. In Fig. 9 we show the maximum and minimum bounds on $R_{1.4}$ and $R_{2.0}$ obtained by imposing an intermediate limit of $c_s^2 \leq 1/2$. In this case for $n_m = 2.0 n_{\text{sat}}$, we find that $11.5^{+0.3}_{-0.3}$ km $< R_{1.4} < 12.5^{+0.3}_{-0.2}$ km and $M_{\max} < 2.29 \pm 0.04 M_{\odot}$ (Fig. 11 (b)), to 1σ confidence. The corollary to this implies that measurements of $R_{\sim 1.4}$ that are smaller than 11.2 km would favor a stiff EOS with $c_s^2 \geq 1/2$ above $2.0 n_{\text{sat}}$, or that $n_m < 2.0 n_{\text{sat}}$. This is particularly interesting because a recent analysis of the tidal deformability constraints from GW170817 in Ref. [70] suggests $11.0^{+0.9}_{-0.6}$ km (90% credible interval).

C. Tidal deformability constraints

Gravitational waveform fitting using the standard PhenomPNRT model [6, 71] directly sets constraints on the binary chirp mass $\mathcal{M} = 1.186 \pm 0.001 M_{\odot}$ and the binary tidal deformability $\tilde{\Lambda} \leq 720$ (90% credibility). In what follows, we will denote this constraint as $\tilde{\Lambda}_{1.186} \leq 720$,

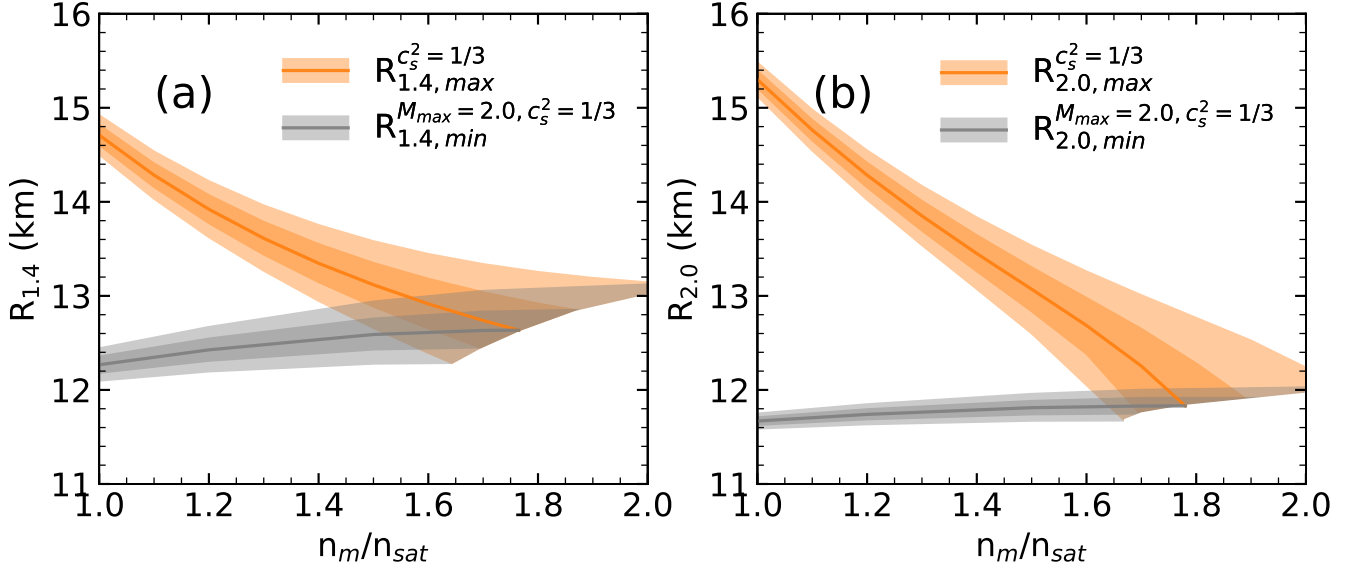


FIG. 8. The maximum (orange) and minimum (black) bounds on $R_{1.4}$ and $R_{2.0}$ assuming $c_s^2 \leq 1/3$ above $n_B = n_m$; $\chi\text{EFT-N}^3\text{LO}$ uncertainties are indicated (darker bands: $\pm 1\sigma$; lighter bands: $\pm 2\sigma$). The bands merge and terminate at critical matching densities above which $M_{\text{max}} < 2.0 M_{\odot}$.

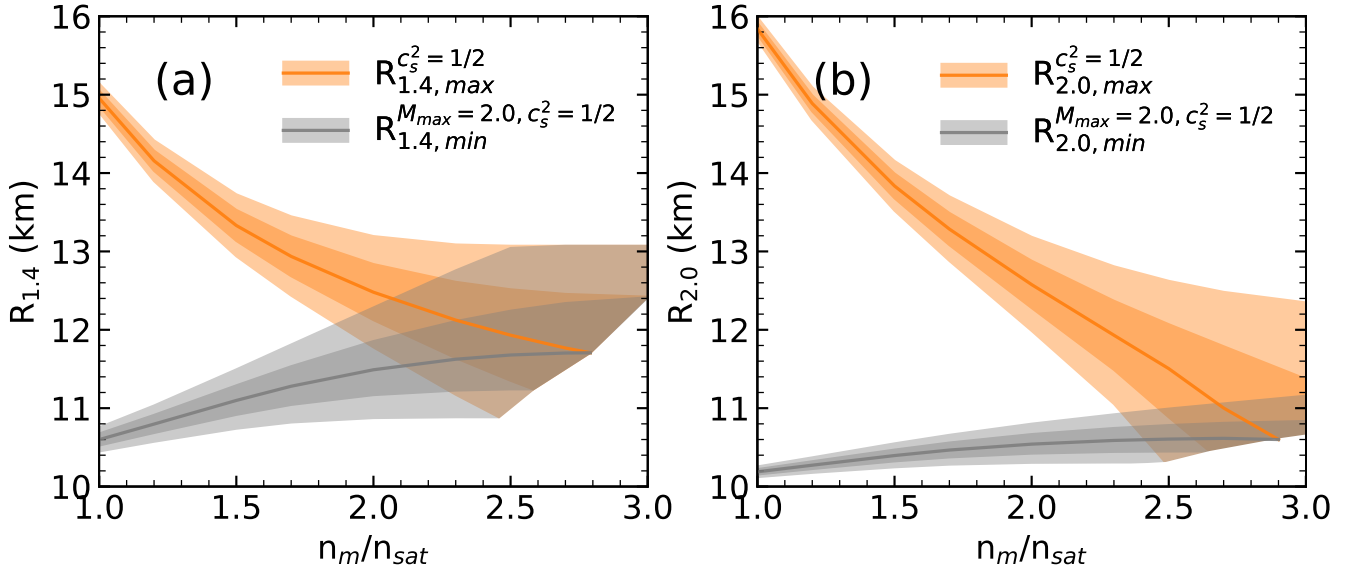


FIG. 9. Similar to Fig. 8, but assuming $c_s^2 \leq 1/2$ above $n_B = n_m$.

where the chirp mass is

$$\mathcal{M} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}}, \quad (12)$$

and the binary tidal deformability is defined as

$$\tilde{\Lambda} = \frac{16 \Lambda_1 m_1^4 (m_1 + 12m_2) + \Lambda_2 m_2^4 (12m_1 + m_2)}{(m_1 + m_2)^5}. \quad (13)$$

Here Λ_1 and Λ_2 refer to the individual deformabilities of the binary components with masses m_1 and m_2 , respectively. It can be shown [72] that Λ is approximately proportional to $(R/M)^6$ and $\tilde{\Lambda}$ is approximately proportional to $(\bar{R}/\mathcal{M})^6$, where $\bar{R}$ is the average radius of stars with masses constrained by $\tilde{\Lambda} \simeq 1.2 M_{\odot}$ and $q > 0.7$, where $q = m_2/m_1$, i.e., the component masses are confined to the interval between $1.1 M_{\odot}$ and $1.6 M_{\odot}$. There-

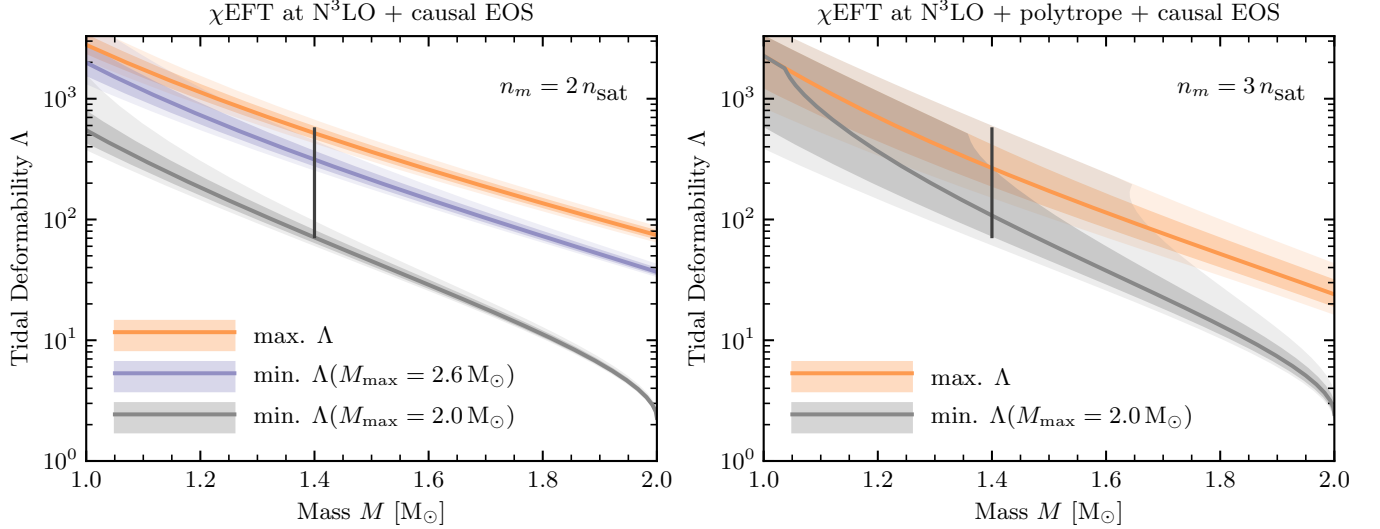


FIG. 10. The left panel shows bounds on the tidal deformability Λ obtained using χ EFT N^3 LO EOS up to $n_m = 2.0 n_{\text{sat}}$, and the right panel extends the low-density EOS to $n_m = 3.0 n_{\text{sat}}$ using the polytropic extrapolation. As in Fig. 3, the orange bands show the upper bound, while the lower bounds corresponding to $M_{\text{max}} = 2.0 M_\odot$ and $M_{\text{max}} = 2.6 M_\odot$ are shown by the black and purple bands, respectively. The vertical solid line depicts the constraint inferred from GW170817, $70 \leq \Lambda_{1.4} \leq 580$. When $n_m = 3.0 n_{\text{sat}}$, $M_{\text{max}} < 2.6 M_\odot$.

fore, the maximum radius $R_{\text{max}}(M)$ bound is tantamount to a maximum $\tilde{\Lambda}$ bound, and vice-versa. The $\tilde{\Lambda}$ – $\mathcal{M}$ constraint can be translated to a constraint on Λ at the mass M , Λ_M , but it is subject to small additional uncertainties from the poorly determined mass ratio q of GW170817 and EOS systematics. Using the resulting quasi-universal EOS relation $\Lambda_1 = q^6 \Lambda_2$, which is valid to 10%–20% for $\mathcal{M} = 1.186 M_\odot$ and $q > 0.7$ [72], one finds

$$\Lambda_M \simeq 2^{6/5} (\mathcal{M}/M)^6 \tilde{\Lambda}_M, \quad (14)$$

valid to a few percent. Absolute bounds from causality on the tidal deformability Λ can be derived in the same way as radius bounds: upper bounds are determined by smoothly matching a low-density EOS to a causal EOS at n_m [73], whereas lower bounds are determined by introducing a discontinuity $\Delta\epsilon_m$ (which lowers M_{max}) [72, 74]. The bounds for the N^3 LO- χ EFT EOS with $n_m = 2.0 n_{\text{sat}}$ are shown in Fig. 10. The role of M_{max} is clear from comparison of the $M_{\text{max}} = 2.0 M_\odot$ and the $M_{\text{max}} = 2.6 M_\odot$ cases. This figure also shows the effects of increasing n_m using the polytropic extrapolation from χ EFT from $2.0 n_{\text{sat}}$ to $3.0 n_{\text{sat}}$. In this case, $M_{\text{max}} < 2.6 M_\odot$. The fact that uncertainties in the GW170817 constraint of Λ extend almost precisely between the lower ($M_{\text{max}} = 2.0 M_\odot$ with a large discontinuity $\Delta\epsilon_m$ at n_m) and upper bounds ($c_{s,\text{match}}^2 = 1$ without discontinuity) to within 2σ for both $n_m = 2.0 n_{\text{sat}}$ and $n_m = 3.0 n_{\text{sat}}$ cases is not a coincidence. It is a consequence of the fact that for those values of n_m , $\tilde{\Lambda}_{1.186} < 720$ is always satisfied for all values of $c_{s,\text{match}}^2 \leq 1$ (see Fig. 11).

A comparison between the results shown in Fig. 10 provides quantitative insights into how access to the EOS at higher density will impact predictions for the tidal deformability Λ , especially for more massive NSs. It illus-

trates how constraints on Λ from future GW detections from binaries with massive NSs can provide insights on the evolution of c_s^2 in the density interval $2 - 3 n_{\text{sat}}$. For example, if $\Lambda_{2.0} \gtrsim 100$, it would pose a serious challenge for χ EFT predictions even in the density interval $1 - 2 n_{\text{sat}}$, and $\Lambda_{2.0} \gtrsim 50$ would be difficult to accommodate without new mechanisms to significantly stiffen the EOS in the density interval $2 - 3 n_{\text{sat}}$. On the other hand, if $\Lambda_{1.4} \lesssim 100$, it would imply a soft EOS between $1 - 3 n_{\text{sat}}$, a near-causal EOS at higher densities, and M_{max} not significantly larger than $2 M_\odot$.

Results for M_{max} using subluminal sound speeds for the high-density EOS are shown in Fig. 11 (a) for the cases $n_m = 1, 1.5, 2.0 n_{\text{sat}}$. This figure, in $M_{\text{max}} - c_{s,\text{match}}^2$ space, is a permutation of Fig. 7 that instead shows M_{max} contours in $n_m - c_{s,\text{match}}^2$ space. The dotted curve at $2.6 M_\odot$ intersects the contours for those cases for $c_{s,\text{match}}^2 = 0.35, 0.502$ and 0.695 , respectively.

The derived bounds on n_m and $c_{s,\text{match}}^2$ illuminate the importance of including nuclear-matter calculations in the density range $1 - 3 n_{\text{sat}}$. Standard extrapolations based on nucleonic models, similar to the ZL parametrization, are usually associated with a more gradual profile of $c_s^2(n_B)$ at low-to-intermediate densities, which cannot reconcile the small radii and/or small tidal deformabilities inferred for canonical-mass NSs with large maximum masses. The necessary rapid change in the sound speed guided by the simple matching scheme serves to indicate the breakdown of such extrapolations at high densities. A very high NS mass, e.g., $\gtrsim 2.45 M_\odot$ ($2.6 M_\odot$), would be in conflict with causality and standard extrapolation up to $3.0 n_{\text{sat}}$ ($2.66 n_{\text{sat}}$); therefore indicating something unusual in the EOS should be taking

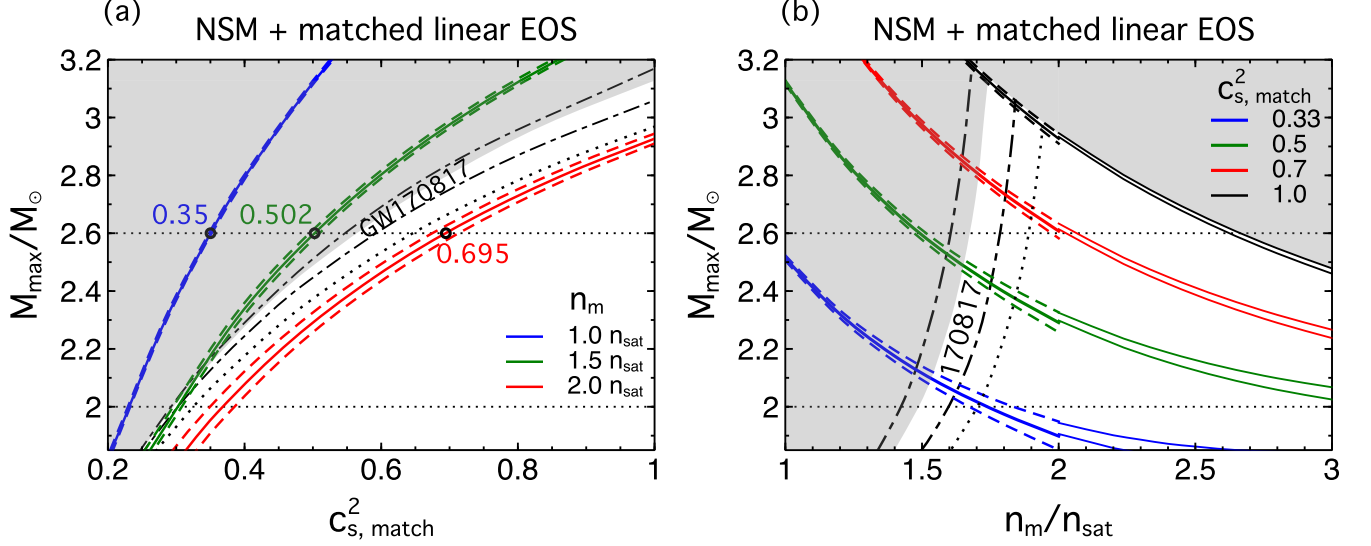


FIG. 11. Panel (a): the solid lines show contours of n_m in the $M_{\max}$ - $c_{s,\text{match}}^2$ plane, and the dashed lines bracket $N^3\text{LO} \pm 1\sigma$ uncertainties. The upper horizontal line indicates $M_{\max} = 2.6 M_\odot$; see also examples later in Fig. 14. The grey-shaded region is excluded by the binary tidal deformability constraint $\tilde{\Lambda}_{1.186} \leq 720$ from GW170817 at the 90% credibility level [6] if $N^3\text{LO}$ -cen is assumed; the dot-dashed lines refer to constraints with the $N^3\text{LO} \pm 1\sigma$ boundaries. The thin dotted line indicates a lower upper bound with $N^3\text{LO}$ -cen and $\tilde{\Lambda}_{1.186} \leq 600$. Panel (b): same as panel (a), except that contours of $c_{s,\text{match}}^2$ are displayed in the $M_{\max}$ - n_m plane; the upper-right grey-shaded region is excluded by causality. For $n_m \in [2.0, 3.0] n_{\text{sat}}$, extrapolations from χEFT using ZL models with $L = 50 \text{ MeV}$ and $L = 60 \text{ MeV}$ are applied (see Fig. 5).

place near this density. This is consistent with the findings of Refs. [33, 34].

A more conservative estimate for the maximum mass, such as $2.2 - 2.3 M_\odot$, increases the allowed range for n_m and $c_{s,\text{match}}^2$ to be consistent with data; the generic trend is shown in Fig. 11. Specifically, Fig. 11 (a) demonstrates how $M_{\max}$ scales with $c_{s,\text{match}}^2$ using the $N^3\text{LO}$ -NSM EOS for $n_m = 1.0, 1.5, 2.0 n_{\text{sat}}$. The solid curves correspond to results for $N^3\text{LO}$ -cen and the dashed ones with $\pm 1\sigma$ uncertainties. The dots indicate the intersections of the central curves with $M_{\max} = 2.6 M_\odot$ for the same EOSs as shown later in Fig. 14 (b). The χEFT uncertainties at the respective densities only slightly broaden these correlations. Together with GW170817, the constraint $M_{\max} \geq 2.1 M_\odot$ rules out very weakly-interacting matter ($c_s^2 \approx 0.33$) at high densities, whereas $M_{\max} \geq 2.5 M_\odot$ rules out matter with $c_s^2 \lesssim 0.5$.

The third permutation of Fig. 7 is displayed in Fig. 11 (b). It is noteworthy that the GW170817 boundary (edge of the grey-shaded region) for $N^3\text{LO}$ -cen is nearly parallel to the n_m contours. For matching densities $\lesssim 1.5 - 1.8 n_{\text{sat}}$, all constructed EOSs result in $\tilde{\Lambda}_{1.186} > 720$ and can be therefore considered ruled out by GW170817 (see also examples later in Fig. 15 (a)). If an even lower upper bound on $\tilde{\Lambda}_{1.186}$ were to be established,

the excluded region would become larger, increasing the threshold of minimally allowed n_m .

Compatibility with GW170817 is readily satisfied if the χEFT calculations (with uncertainties) are assumed valid up to $2.0 n_{\text{sat}}$ consistent with previous studies [55]. The evolution of $M_{\max}$ with $c_{s,\text{match}}^2$ has been known [75–78], but it was unclear how the uncertainty in the low-density EOS translates to an uncertainty in the derived upper bound. As shown in Fig. 11 (b), we find that for $n_m = 2.0 n_{\text{sat}}$, the uncertainty in $M_{\max}$ ranges from $\approx 0.1 M_\odot$ for $c_{s,\text{match}}^2 = 0.33$ (blue-dashed line) to $\approx 0.05 M_\odot$ for $c_{s,\text{match}}^2 = 1$ (black-dashed line) with $N^3\text{LO} \pm 1\sigma$ inputs at low densities.

In summary, satisfying the GW170817 tidal deformability constraint $\tilde{\Lambda}_{1.186} < 720$ and imposing $M_{\max} > 2.1 M_\odot$ requires $n_m > 1.5 n_{\text{sat}}$ and $c_s^2 > 0.35$. This limit is not very sensitive to $M_{\max}$. Even if $M_{\max} > 2.6 M_\odot$, it is required that $1.7 < n_m/n_{\text{sat}} < 2.6$ and $c_{s,\text{match}}^2 > 0.55$. The existence of a $2.6 M_\odot$ star evidently requires a significant change from normal hadronic EOSs to a much stiffer EOS between $1.7 n_{\text{sat}}$ and $2.6 n_{\text{sat}}$. In the presence of a discontinuity in ε , the lower bound $n_m \gtrsim 1.7 n_{\text{sat}}$ can decrease, whereas the upper bound $n_m \lesssim 2.6 n_{\text{sat}}$ remains unaffected as it is imposed by causality.

For stars with a normal crust, refined upper limits to $R_{M_{\max}}$ can be found using the GW170817 constraint and an assumed value for $M_{\max}$, while lower limits follow from the causal EOS: $9 \text{ km} < R_{M_{\max}} < 12.2 \text{ km}$ for $M_{\max} \leq$

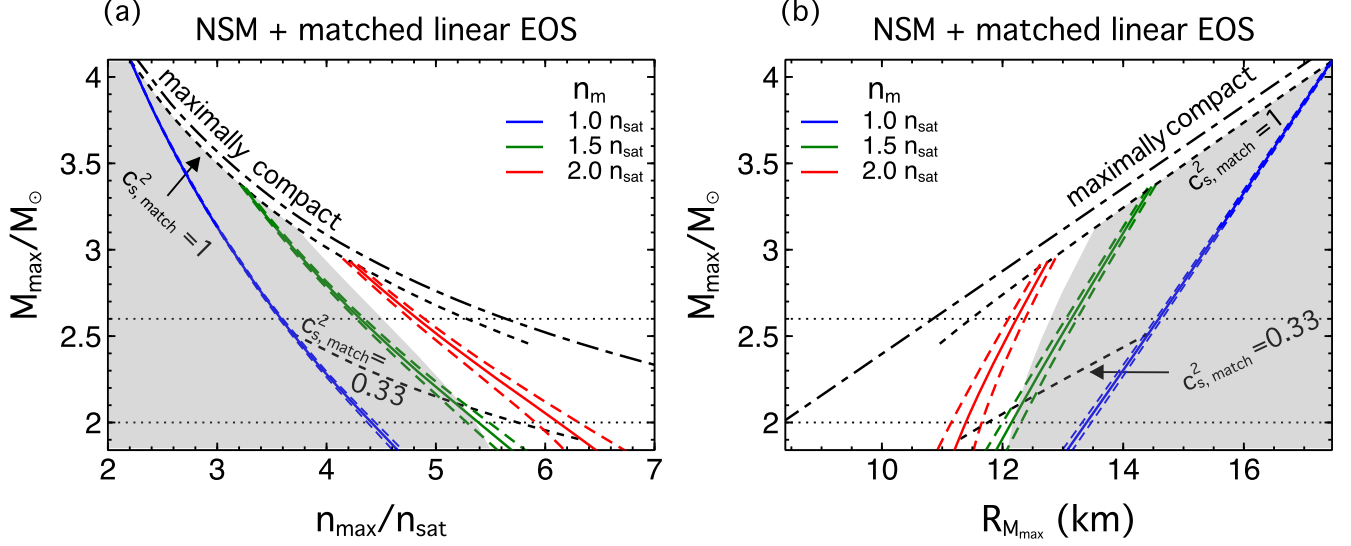


FIG. 12. Panel (a): scaling relations between $M_{\max}$ and $n_{\max}$; panel (b): scaling relations between $M_{\max}$ and $R_{M_{\max}}$. Both relations, shown as dot-dashed lines, follow from the maximally compact EOS (see Appendix A). The black dashed curves correspond to the presence of a low-density nuclear mantle (crust + N³LO EOS) for $n_B \leq n_m$, with fixed sound speeds $c_{s,\text{match}}^2 = 0.33$ and $c_{s,\text{match}}^2 = 1.0$ for $n_B > n_m$. The grey-shaded region is excluded by GW170817 ($\Lambda_{1.186} \leq 720$ and N³LO-cen). The solid colored curves show contours of $n_m = 1.0, 1.5, 2.0 n_{\text{sat}}$ for N³LO-cen; dashed colored curves show $\pm 1\sigma$ uncertainties. For EOSs that accommodate $M_{\max} \geq 2.6 M_{\odot}$, the permitted ranges of $n_{\max}$ and $R_{M_{\max}}$ are severely restricted.

$2.1 M_{\odot}$ and $11.3 \text{ km} < R_{M_{\max}} < 12.8 \text{ km}$ for $M_{\max} \leq 2.6 M_{\odot}$.

D. $M_{\max}$ scalings compared to the maximally compact case

In Fig. 12 (a) we show the absolute upper limit on $M_{\max}$ (see e.g. Eq. (A7)) as a function of $n_{\max}$, the highest possible baryon density from the maximally compact EOSs, as represented by the dot-dashed boundary. The slightly lower black dashed boundary matches the maximally compact EOS to a low-density nuclear EOS at some density n_m varying from n_{sat} to about $3.0 n_{\text{sat}}$ (from left to right). The relatively small difference between these two boundaries suggests that effects on the absolute upper bound on $n_{\max}$ and $M_{\max}$ from the low density EOS is small, and for $M_{\max} \geq 2.6 M_{\odot}$, $n_{\max}$ should be smaller than $5.3 - 5.6 n_{\text{sat}}$. This is in good agreement with $\approx 5 n_{\text{sat}}$ obtained in Ref. [33]. For $n_m \leq 2.0 n_{\text{sat}}$, we employ χ EFT calculations with uncertainties, and the ZL parametrizations (see Fig. 5) are applied for n_m between $2.0 - 3.0 n_{\text{sat}}$. If the high-density matter is assumed to be much softer with $c_{s,\text{match}}^2 = 0.33$, matching it to the nuclear EOS at different matching densities n_m gives rise to the predicted $M_{\max}$ - $n_{\max}$ relation shown by the lower dashed curve. The grey-shaded region is ruled out by tidal deformability constraints inferred from GW170817, prohibiting small values of n_m below $1.5 - 1.8 n_{\text{sat}}$. As a result, $c_{s,\text{match}}^2 \lesssim 0.33$ is incompatible

with $M_{\max} \gtrsim 2.1 M_{\odot}$; see also Fig. 11. Furthermore, imposing $M_{\max} \geq 2.0 M_{\odot}$ leads to $5.23 < n_{\max}/n_{\text{sat}} < 5.79$.

The colored curves in Fig. 12 indicate where the matching densities are fixed at $n_m/n_{\text{sat}} = 1.0$ (blue), 1.5 (green), and 2.0 (red), and they track decreasing values of $c_{s,\text{match}}^2$ from 1 to below 0.33. In each case, the highest $M_{\max}$ as well as the smallest $n_{\max}$ correspond to where they end at the $c_{s,\text{match}}^2 = 1$ upper boundary (black dashed line). The N³LO $\pm 1\sigma$ uncertainty at $2.0 n_{\text{sat}}$ translates to $\approx 0.4 n_{\text{sat}}$ uncertainty in $n_{\max}$ ($5.9 - 6.3 n_{\text{sat}}$) if $M_{\max} = 2.0 M_{\odot}$, and $\approx 0.1 n_{\text{sat}}$ uncertainty for $M_{\max} = 2.6 M_{\odot}$. Beyond $n_m \gtrsim 2.0 n_{\text{sat}}$, extrapolation of the χ EFT calculations is needed for which the curves would move to the lower-right while remaining under the $c_{s,\text{match}}^2 = 1$ bound. Using the ZL parametrization to extrapolate up to $3.0 n_{\text{sat}}$ (not shown), we obtain $n_{\max} \leq 5.71 - 5.92 n_{\text{sat}}$ and $M_{\max} \leq 2.45 - 2.48 M_{\odot}$.

As discussed in Appendix A, the maximally compact EOS with $c_s^2 = 1$ determines the smallest possible radius at a given mass. Figure 12 (b) displays the absolute bound on the radius of the maximum mass star, $R_{M_{\max}}$, as well as a more realistic bound taking into account the low-density EOS below n_m . Assuming χ EFT up to $n_m = 2.0 n_{\text{sat}}$ and $M_{\max} = 2.0 M_{\odot}$, the N³LO $\pm 1\sigma$ uncertainties induce an uncertainty $\approx 0.5 \text{ km}$ in $R_{M_{\max}} = 11.14 - 11.66 \text{ km}$. For $M_{\max} = 2.6 M_{\odot}$, an uncertainty $\approx 0.3 \text{ km}$ is found with $R_{M_{\max}} = 12.09 - 12.38 \text{ km}$. Extrapolating to higher densities $n_m \gtrsim 2.0 n_{\text{sat}}$, $M_{\max} \geq 2.6 M_{\odot}$ leads to $R_{M_{\max}} \geq 11.49 \text{ km}$. The

tidal deformability constraint inferred from GW170817 instead corresponds to limits on the radii of canonical-mass stars. With the simple matching condition used here, that constraint simultaneously rules out too large $R_{M_{\max}}$, e.g., $R_{M_{\max}} \leq 12.18 \text{ km}$ if $M_{\max} = 2.0 M_{\odot}$ and $R_{M_{\max}} \leq 12.79 \text{ km}$ if $M_{\max} = 2.6 M_{\odot}$.

On the other hand, introducing a finite discontinuity in ε would decrease $R_{M_{\max}}$ and increase $n_{\max}$, but to reach the same $M_{\max}$ necessitates the transition density to be smaller than the matching density n_m when there is no discontinuity [61]. The overall effect is that larger $n_{\max}$ and smaller $R_{M_{\max}}$ are possible but must still lie within the bounds set by the maximally compact EOSs.

IV. DISCUSSION

It is worth mentioning that so far we have largely avoided finite discontinuities in the energy density ε , except when located at n_m , which would otherwise introduce an additional parameter that characterizes the strength of a sharp first-order phase transition. In that scenario, the $M_{\max}$ bounds will be shifted downwards due to the softening induced by the phase transition, while GW170817 boundaries may become more complicated depending on the possible formation of disconnected branches at intermediate densities on the M - R diagram [74, 79]. However, given the systematic uncertainties involved in obtaining $\tilde{\Lambda}$ from gravitational waveform data, the previously inferred bounds should still apply [72]. In any case, as discussed in Appendix A, useful information on the minimal radii $R_{\min}(M)$ can be obtained from matching to the causal EOS with a discontinuity $\Delta\varepsilon_m$ specified by $M_{\max}$, and we have elaborated on these lower bounds on R with χ EFT inputs up to n_m in Sec. III A.

A. Current and future constraints

To shed light on the properties of dense matter, the observational constraints used in this work are taken from (i) a handful of well measured NS masses from radio observations [8–12], (ii) the chirp and combined masses as well as bounds on tidal deformabilities of NSs deduced from GW detections in the binary NS-NS merger event GW170817 [5–7], and (iii) radius estimates from NICER for a NS of mass $\simeq 1.4 M_{\odot}$ [80, 81]. An upper bound of $M_{\max} \lesssim 2.3 M_{\odot}$ on the maximum gravitational mass of a cold, spherical NS was inferred from several studies using EM and GW data from GW170817 [3, 4, 82–84], but an upper bound on $M_{\max}$ itself does not provide further limits on the sound speed or bounds to NS radii since the EOS could suddenly soften above n_m .

The NICER M - R constraints on J0030+0451, namely, $R = 13.02^{+1.24}_{-1.19} \text{ km}$, $M = 1.44^{+0.15}_{-0.14} M_{\odot}$ [81] and $R = 12.71^{+1.14}_{-1.19} \text{ km}$, $M = 1.34^{+0.15}_{-0.16} M_{\odot}$ [80], and some EM observations of GW170817 [85–87] favor larger radii than

indicated by GW observations from GW170817, 10 – 13 km [6, 7], but the degree of tension is slight. Joint analyses of these data yield tighter but still consistent constraints on the typical NS radius $\sim 12.3 \text{ km}$ [88–91]; Ref. [92] found $11.8^{+1.0}_{-0.7} \text{ km}$ to 68.3% confidence.

It is fortunate that NICER targets also include several pulsars for which the masses are independently measured to high precision, e.g., PSR J1614-2230 $\simeq 1.91 M_{\odot}$ and PSR J0740+6620 $\simeq 2.14 M_{\odot}$, and PSR J0437-4715 [93] with mass $\simeq 1.44 M_{\odot}$. The possibility to measure radii of both intermediate as well as very massive NSs opens up the possibility to contrast the radii of $\sim 2.0 M_{\odot}$ stars, $R_{2.0}$, and more typical $\sim 1.4 M_{\odot}$ stars, $R_{1.4}$, to further constrain the EOSs [61, 94].

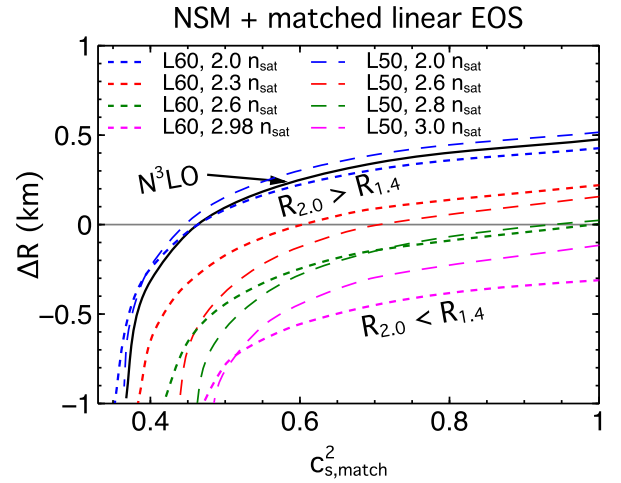


FIG. 13. Radius differences $\Delta R = R_{2.0} - R_{1.4}$ using the ZL extrapolations with $L = 50 \text{ MeV}$ and $L = 60 \text{ MeV}$ joined continuously to linear EOSs at n_m between $2.0 n_{\text{sat}}$ and $3.0 n_{\text{sat}}$.

We show in Fig. 13 the difference $\Delta R = R_{2.0} - R_{1.4}$ for stars with the $N^3\text{LO}$ EOS up to $2.0 n_{\text{sat}}$, ZL EOS extrapolations up to a range of matching densities $n_m = 2.0 - 3.0 n_{\text{sat}}$, and various linearly matched EOSs with different $c_{s,\text{match}}^2$ at higher densities. The ZL extrapolation with $L = 50 \text{ MeV}$ indicates that roughly above $n_m \gtrsim 2.8 n_{\text{sat}}$, all values of $c_{s,\text{match}}^2$ lead to $R_{2.0} \leq R_{1.4}$. The boundary between positive and negative ΔR shifts a bit when using the slightly stiffer ZL extrapolation with $L = 60 \text{ MeV}$: in this case $n_m \gtrsim 2.6 n_{\text{sat}}$ will guarantee $R_{2.0} \leq R_{1.4}$; note that $2.98 n_{\text{sat}}$ is already the central density of a $1.4 M_{\odot}$ star. We also checked radii differences between $2.1 M_{\odot}$ and $1.4 M_{\odot}$ stars, $\Delta R' = R_{2.1} - R_{1.4}$, and found that $\Delta R'$ is generally less than ΔR , with the largest decreases of a few tenths of a km occurring for the smaller values of $c_{s,\text{match}}^2$. For $c_{s,\text{match}}^2 \gtrsim 0.7$, there are negligible differences. ΔR or $\Delta R'$ being negative is typical when extrapolations to even higher densities are applied, or if there is additional softening in the EOS before reaching the central density of the maximum-mass star.

Should observations suggest $R_{2.0} > R_{1.4}$ or $R_{2.1} > R_{1.4}$, standard extrapolations such as ZL-models predict some unusual stiffening should occur below $\lesssim 2.6 - 2.8 n_{\text{sat}}$. Furthermore, if ΔR turns out to be greater than 0.5 km, then we should expect that this stiffening occurs for $n_{\text{m}} \lesssim 2.0 n_{\text{sat}}$, which suggests a very high M_{max} and less compatibility with radius constraints from GW170817; see Fig. 11 (b). However, NICER observations may not achieve the needed $\mathcal{O}(0.5 \text{ km})$ resolutions in the near future. Since central densities of $\sim 2.0 n_{\text{sat}}$ correspond to $0.5 - 1.0 M_{\odot}$ within 1σ uncertainties of χEFT calculations (Fig. 3), it will be greatly helpful if radii of very low-mass NSs $\sim 1.1 M_{\odot}$ can be obtained through X-ray observations, or tidal deformability measurements of binary systems with very low chirp masses.

From a different perspective, more accurate experimental determinations of S_v and L at n_{sat} from e.g., PREX, CREX, and FRIB/MSU, will be important to test χEFT predictions of properties of neutron-rich matter. At the present time, S_v and L are believed to be understood to the 10% and 40% levels, respectively [95]. For $n_{\text{B}} > n_{\text{sat}}$, constraints from the analyses of the collective flow of matter in HICs could be informative.

The best available information for the present comes from the analysis of HICs of Au nuclei using Boltzmann-type kinetic equations. The elliptic and sideways flow observables from these collisions are sensitive to the mean-field potential and to in-medium NN collisions at central densities of $2 - 5 n_{\text{sat}}$, and suggest SNM pressures of 7.5 MeV fm^{-3} to 14 MeV fm^{-3} at $2.0 n_{\text{sat}}$ [96]. In comparison, N³LO calculations for SNM predict somewhat larger pressures of 10.5 MeV fm^{-3} to 18.5 MeV fm^{-3} at $2.0 n_{\text{sat}}$ [29], which are, nevertheless, consistent within their stated 1σ uncertainties. However, the predictions from HICs involve model-dependent assumptions concerning the density- and momentum-dependencies of the assumed nuclear interactions, which have not been systematically explored; see Ref. [97] and references therein for the relevance of single-particle potentials in HICs. In addition to these uncertainties, HICs probe nearly symmetric matter, and to apply their observables to NSM requires an additional extrapolation involving the symmetry energy at supra-nuclear densities.

To improve the current status, heavy-ion facilities across the world, such as RHIC, FAIR, NICA, J-PARC, and HIAF, have launched programs to map out the QCD phase diagram of strongly interacting matter. The study of more neutron-rich matter in HICs, together with improved, systematic, modeling would be very valuable for dense-matter physics, not only for cold neutron stars, but also for understanding mergers involving NSs. As the analyses of HIC data have largely been done with nucleonic degrees of freedom, it would be also interesting and desirable to extend such analyses to include quark degrees of freedom and their subsequent hadronization as in RHIC and CERN experiments at higher energies.

B. $2.6 M_{\odot}$ neutron stars and the nature of the components of GW190425 and GW190814

It is also of interest to examine what matching conditions relating n_{m} and $c_{s,\text{match}}^2$ ensue from a restriction such as $M_{\text{max}} = 2.6 M_{\odot}$. Fig. 14 (a) depicts the M - R relations for n_{m} and $c_{s,\text{match}}^2$ that lead to $M_{\text{max}} = 2.6 M_{\odot}$, and the corresponding c_s^2 profiles are explicitly shown in panel (b). The required values of $c_{s,\text{match}}^2$ are indicated in the plot (solid horizontal lines for the N³LO-central (denoted as N³LO-cen) and dashed for $\pm 1\sigma$ uncertainties), which increase with the matching density n_{m} . At fixed matching density indicated by the vertical dotted lines, the variation in $c_{s,\text{match}}^2$ above n_{m} is consistent with the uncertainties in c_s^2 from χEFT calculations at n_{m} , and a softer EOS (smaller c_s^2) at low densities is compensated by a stiffer EOS (larger $c_{s,\text{match}}^2$) at higher densities.

The simple linear parametrization of high-density EOS used here can be viewed as a guide to assess the stiffness required at higher densities to achieve $M_{\text{max}} \geq 2.6 M_{\odot}$. Assuming χEFT -N³LO is valid up to $n_{\text{m}} = 2.0 n_{\text{sat}}$ ($1.5 n_{\text{sat}}$), to reach $2.6 M_{\odot}$ the “averaged” c_s^2 above $2.0 n_{\text{sat}}$ ($1.5 n_{\text{sat}}$) has to be greater than ~ 0.7 (~ 0.5). This is probably not achievable by using standard extrapolations of nonrelativistic nucleonic models (for which c_s^2 is gradually increasing) without violating causality below the central density of the maximum mass star.

Fig. 15 (a) shows an application of the deformability constraints from GW170817 (see discussions in Sec. III C) in the case that M_{max} is fixed to $2.6 M_{\odot}$. As mentioned before, a small matching density n_{m} results in a large radius for a given $c_{s,\text{match}}^2$. An EOS stiffening drastically from N³LO below $1.5 n_{\text{sat}}$ ends up violating $\tilde{\Lambda}_{1.186} \leq 720$ if $M_{\text{max}} \gtrsim 2.6 M_{\odot}$ (green band in Fig. 15 (a)).

Even considering the q and EOS uncertainties, one sees that $n_{\text{m}} \lesssim 1.5 n_{\text{sat}}$ violates the GW170817 constraint (Fig. 15 (b)). There exists a minimum $n_{\text{m}} \approx 1.7 n_{\text{sat}}$ for N³LO-NSM to survive the $\tilde{\Lambda} \leq 720$, or $\Lambda \leq 580$ constraint (when $M_{\text{max}} \geq 2.6 M_{\odot}$ is assumed), and an even smaller upper bound e.g., $\tilde{\Lambda} \simeq 600$ [70, 98, 99] which would increase the minimum required n_{m} . It is noteworthy that the posteriors of $\tilde{\Lambda}$ for GW170817 suggest a peak value around ≈ 225 , noticeably smaller than the upper bound of 720 (90% credible level).

The ranges of $1.6 - 2.5 M_{\odot}$ in GW190425 [100] and $2.59_{-0.09}^{+0.08} M_{\odot}$ in GW190814 [101] for one of the components in these merger events have raised the possibility that those compact objects could be NSs as opposed to being low-mass BHs. The data from GW190425 was inconclusive concerning the nature of the inspiralling binary [100], but some works favored the scenario in which the more massive component is a BH instead of a very heavy NS [102]. If it is *a priori* assumed that $M_{\text{max}} \lesssim 2.3 M_{\odot}$, a possibility motivated by EM and GW data from GW170817, the interpretation that it was a BNS merger instead statistically favors masses of approximately $1.5 \pm 0.2 M_{\odot}$ and $1.9 \pm 0.2 M_{\odot}$, while a neutron-

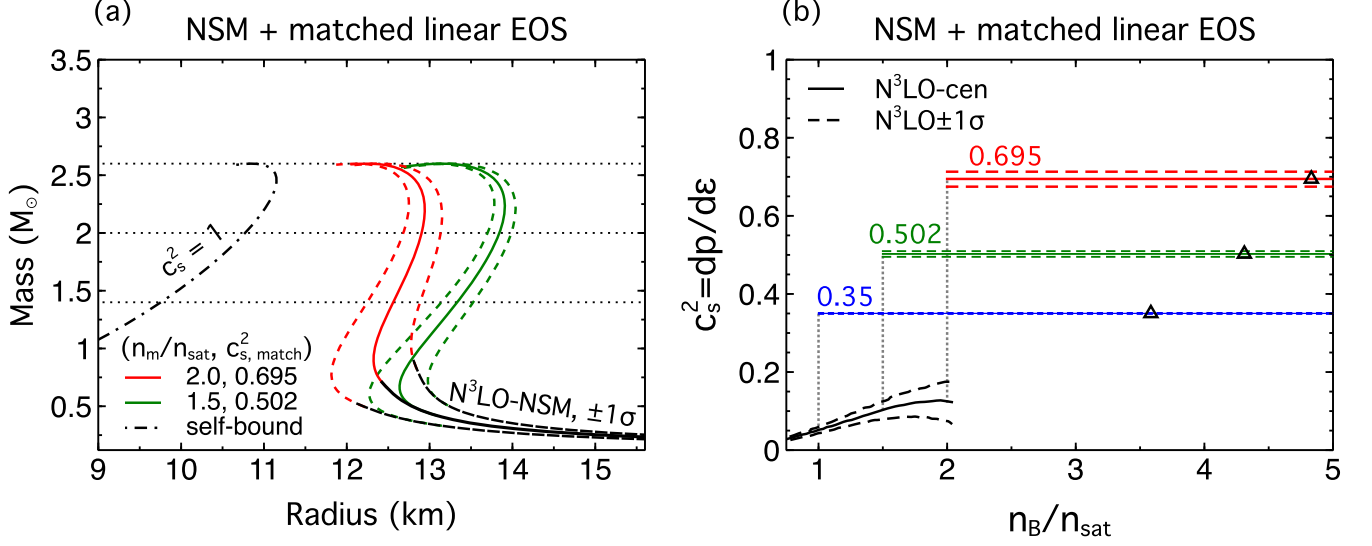


FIG. 14. Panel (a): M - R diagram for matched linear EOSs that give rise to $M_{\text{max}} = 2.6 M_\odot$ with $N^3\text{LO-NSM} (\pm 1\sigma)$ applied for low densities $\leq 2.0 n_{\text{sat}}$. Corresponding values of n_m and $c_{s,\text{match}}^2$ are indicated (see also Fig. 11 (a)). Panel (b): sound speed profiles $c_s^2(n_B)$ for $N^3\text{LO-NSM}$ only (black-solid for the central value and black-dashed for $\pm 1\sigma$ uncertainties), and matched linear EOSs with different values of $c_{s,\text{match}}^2$ associated with $M_{\text{max}} = 2.6 M_\odot$ in panel (a) (colored horizontal lines). The open triangles mark the central densities of the maximum-mass stars $M_{\text{max}} = 2.6 M_\odot$.

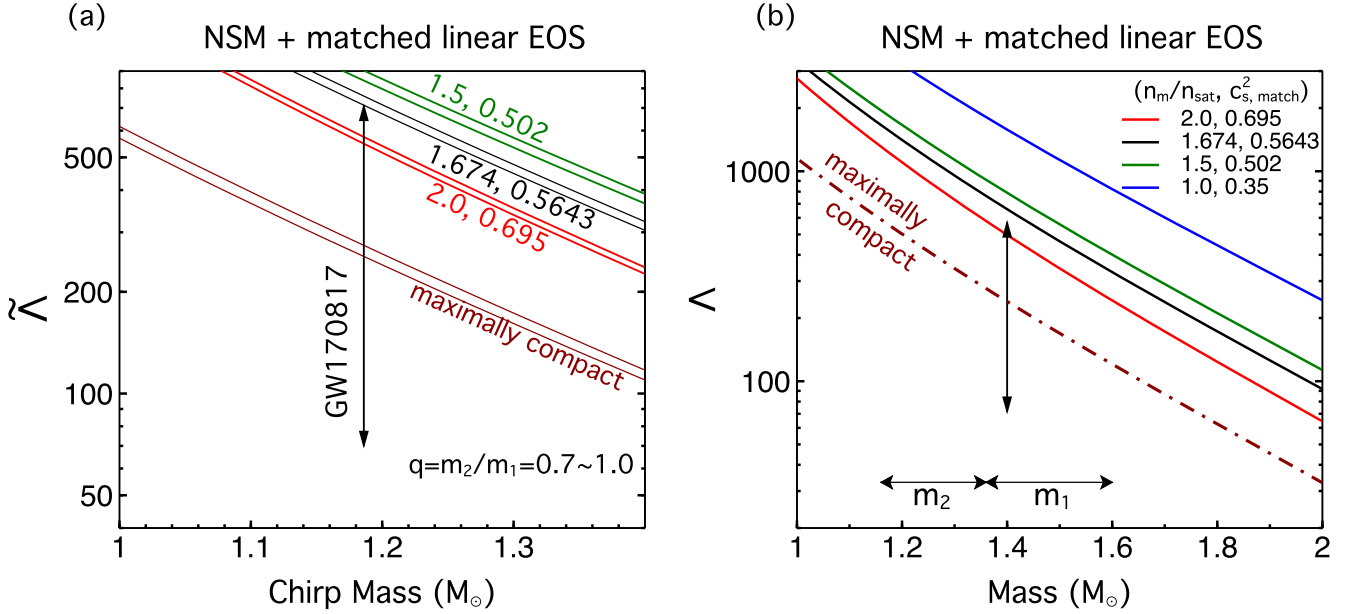


FIG. 15. $\tilde{\Lambda}$ - $\mathcal{M}$ and Λ - M relations confronted with constraints from GW170817 [6, 7] (vertical lines with arrows), with fixed $M_{\text{max}} = 2.6 M_\odot$ as an example. Parameters for matched EOSs are the same as in Fig. 14, except for the special case with $n_m = 1.674 n_{\text{sat}}$ (with $c_{s,\text{match}}^2 = 0.5643$), which refers to the minimum matching density that survives $\tilde{\Lambda}(\mathcal{M} = 1.186 M_\odot) \leq 720$.

star-black-hole (NSBH) merger interpretation favors a $1.3 \pm 0.1 M_\odot$ NS and a $2.2 \pm 0.2 M_\odot$ BH [102]. While both scenarios are statistically equally likely, the fact that the BNS masses are incompatible with those of observed galactic BNS systems, while the NS mass in the NSBH

scenario is compatible, seems to favor the NSBH interpretation. However, in either scenario according to this analysis, GW190425 would likely not contain a NS $> 2.1 M_\odot$. In the case of GW190814, there is no additional information, aside from one's assumption about M_{max} , to decide

if the primary is a high-mass NS or a low-mass BH. However, statistical analyses suggest that the probability of its secondary being a NS is very low [35, 36, 101]. If either GW190425 or GW190814 contains a $\sim 2.5 - 2.6 M_\odot$ NS, questions to address are: *What is the physical state of dense matter that could support such a heavy NS, and what radius constraints would follow?*

The scenario that GW190814's secondary component was an approximately $2.6 M_\odot$ NS does not itself violate theoretical limits from causality and the GW170817 constraint that $\tilde{\Lambda} < 720$ for $\mathcal{M} = 1.186 M_\odot$, but challenges remain finding physical mechanisms that can connect very stiff high-density matter with the relatively soft nuclear matter at $\lesssim 2.0 n_{\text{sat}}$ predicted from modern χ EFT calculations. As Fig. 11 shows, the conformal limit $c_s^2 \leq 1/3$ must be violated [88] below the central density of the maximum-mass star even by the requirements from pulsar timing that $M_{\text{max}} \gtrsim 2.1 M_\odot$ and from GW170817's tidal deformability constraint. Standard extrapolations that assume gradually increasing c_s^2 profiles are unlikely to be compatible with $M_{\text{max}} \geq 2.6 M_\odot$ [101].

In particular, the requirement that c_s^2 remains above ~ 0.6 for a wide range of densities $\gtrsim 2.0 n_{\text{sat}}$ is hard to explain. Extrapolations of non-relativistic potential models generally result in steadily increasing sound speeds with density, and it becomes problematic to prevent them from becoming acausal within NSs. At densities relevant to the center of very massive NSs, it is reasonable to expect the emergence of exotic degrees of freedom. A sharp first-order transition to stiff quark matter at some intermediate density is capable of reconciling small radii and high masses $\gtrsim 2.4 M_\odot$ (see examples of $R_{\text{min}}(M)$ in Sec. III A). With an increasing lower bound on M_{max} and/or smaller assumed values of c_s^2 at high densities, the transition threshold has to be pushed downward approaching $1.5 - 2.0 n_{\text{sat}}$ (similar to the results shown in Fig. 11 but involving a discontinuity $\Delta\epsilon_m$ that further decreases M_{max} and favors lower values of n_m [61]).

For most microscopic quark-matter models, for example the original MIT bag model [103], the original Nambu-Jona-Lasinio (NJL) model [104], and their variations, perturbative QCD matter [105], and quartic polynomial parametrizations [106], the speed of sound turns out to be weakly density-dependent. To be consistent with massive pulsars $\sim 2 M_\odot$, strong repulsive interactions that stiffen the quark EOS, possibly reaching $c_s^2 \geq 0.4$, have been implemented [107–109]. The maximally achievable c_s^2 is model-dependent, and requiring $c_s^2 \gtrsim 0.6$ on average in quark matter is expected to push model parameters to extreme values.

In contrast to sharp phase transitions, hadron-to-quark crossovers as in quarkyonic models [59, 60, 110] or with interpolation schemes [111] provide a natural stiffening to support high masses, but can also induce large radii. Quarkyonic models generate large values of c_s^2 by restricting the nucleonic momentum phase space when quarks appear, and in some cases are capable of simultaneously reaching $> 2.5 M_\odot$ and satisfying the GW170817 con-

straint $\tilde{\Lambda}_{1.186} < 720$. Some versions [60], in which quarks come to rapidly dominate the composition, leading to a high, but narrow, c_s^2 peak behavior, cannot jointly satisfy these conditions, reaching at most $M_{\text{max}} \simeq 2.4 M_\odot$. However, we find that other versions [59, 110], in which the quark abundances grow more slowly and that can retain large abundances of nucleons at high density, can simultaneously achieve these conditions.

Using extrapolation functions in terms of c_s^2 and μ , Annala *et al.* [112] found that the risk of hadronic EOSs violating causality at high-enough densities ($\gtrsim 4.0 n_{\text{sat}}$) to achieve high masses is remedied if a transition to perturbative QCD-like (soft $c_s^2 \approx 1/3$) quark matter occurs at high densities. However, considering that current calculations in perturbative QCD itself are only valid at densities $n_B \gtrsim 40 n_{\text{sat}}$, interpolations down to NS densities are problematic. The main feature of such a transition can be reproduced by simply requiring $c_s^2 \rightarrow 1/3$ for $n_B \gtrsim 6 n_{\text{sat}}$, but at intermediate densities the conformal limit $c_s^2 \leq 1/3$ being violated is strongly favored [88]. Moreover, despite the fact that hadronic matter breaking the causal limit is never a necessity, it is nearly impossible to distinguish such high-density transitions using observations of the M – R relation or tidal deformabilities due to the masquerade problem [106].

C. Comparison with other works

As noted earlier, the uncertain nature of the less compact object in GW190814 with mass $\simeq 2.6 M_\odot$ has piqued the interest of the dense-matter and nuclear-physics communities. Below we briefly discuss how our study differs from or complements the findings of several other recent articles [33–40] that have addressed the implications of the possible existence of NSs with such high masses.

Several of these articles, including Refs. [33, 35, 37], have relied on nuclear physics based EOSs to describe matter in the crust and outer core to show that the existence of a $2.6 M_\odot$ NS would require $c_s^2 \geq 0.6$ in the inner core. The authors of Ref. [40] use the upper bound on the tidal deformability of NSs set by GW170817 to further strengthen the need for a large c_s^2 in the inner core. Most notably, Ref. [39] derives strict upper bounds on the maximum mass of NSs that depend only on bulk properties of NSs, such as the radii and the tidal deformabilities to find that a NS in GW190814 would not be inconsistent with present astronomical constraints if c_s^2 is large in the inner core. Our finding suggests that a $2.6 M_\odot$ NS would require $c_s^2 \geq 0.55 - 0.6$ (see Fig. 11 (a)) in the inner core, which is in general agreement with these earlier studies. A unique feature of our study is the use of the N³LO- χ EFT EOS that allows us to properly incorporate EFT truncation errors at $n_B \leq 2.0 n_{\text{sat}}$.

Lim *et al.* [34] combine nuclear models valid in the vicinity of normal nuclear densities and a maximally stiff EOS at higher density to show that $2.5 - 2.6 M_\odot$ NS can exist without strongly affecting the properties such

as radius, tidal deformability, and moment of inertia of canonical NSs with mass $\sim 1.4 M_\odot$. They argue that properties of NSs with masses $\sim 2 M_\odot$ such as $R_{\sim 2.14}$ would be significantly different depending on whether the secondary component of GW190814 was a black hole or a NS. Our results support these findings, but go beyond by delineating how the lower and upper bounds on the radii of NSs in the mass range $1.4 - 2 M_\odot$ would be constrained if future observations were to confirm the existence of NSs with masses $\simeq 2.5 - 2.6 M_\odot$.

Using FSU-type relativistic mean field-theoretical (RMFT) models, Fattoyev *et al.* [38] found that the rapid increase in pressure with density required to support a $2.6 M_\odot$ NS, while barely accommodating the deformability constraint from the first analysis of GW170817 data that indicates $\Lambda_{1.4} \leq 800$ [5] but not the updated bounds $70 \leq \Lambda_{1.4} \leq 580$ [7] (see Ref. [113] for a similar study), is inconsistent with energy density functionals tuned to reproduce properties of nuclei and flow data from HICs. Note that Fattoyev *et al.* [38] only applied $\Lambda_{1.4}$ constraint without a comparison of the binary tidal deformability $\hat{\Lambda}$. We have confirmed that FSU-like RMFT interactions cannot accommodate both $\hat{\Lambda}_{1.186} \leq 720$ and $M_{\max} \geq 2.54 M_\odot$ [92].

Other recent works studied hyperonic matter in the EOS and/or rapid rotations that stabilize more massive stars than non-rotating configurations, which may or may not be consistent with GW190814 [114–117]; we do not consider these effects in the present paper.

V. CONCLUSION AND OUTLOOK

We determined the NSM EOS in beta-equilibrium from MBPT calculations of PNM and SNM up to N³LO in χ EFT. For a given n_B , the NSM EOS always has a lower ε than the PNM EOS. The pressure of NSM is less than PNM at the same n_B , typically by $< 1 \text{ MeV fm}^{-3}$, except for $n_B \gtrsim 2.0 n_{\text{sat}}$ when it becomes greater (Fig. 1 (b)). The proton fraction below $2.0 n_{\text{sat}}$ never exceeds the critical minimum value required for the direct URCA process of enhanced neutrino emission [118, 119].

The existence of the NS crust together with a nucleonic EOS below a matching density n_m establishes $R_{\max}(M)$. Extremes are again found by assuming $c_{s,\text{match}}^2 = 1$ for densities above n_m , for which the EOS is now $\varepsilon = \varepsilon_m + P - P_m$. Assuming $\varepsilon_m = \varepsilon_{\text{sat}}$, and that P_m is given by χ EFT-N³LO, the upper bounds are $R_{1.4,\text{max}} \approx 15.1 \text{ km}$ and $R_{2.0,\text{max}} \approx 16.2 \text{ km}$ (see Fig. 6 where $n_m = n_{\text{sat}}$), which are nearly identical to the case shown in Fig. 16 with a slightly different value of P_m at $\varepsilon_m = \varepsilon_{\text{sat}}$. These values are not in tension with observations, and with increasing n_m , the corresponding upper bounds on $R_{1.4}$ and $R_{2.0}$ decrease. For the same ε_m or n_m , $M_{\max}$ is not sensitive to the value of P_m or the nucleonic EOS between the crust and n_m , and is close to that of the case $P_0 = 0$ (self-bound stars) for the causal EOS; see also Fig. 2.

The merger events GW190425 and GW190814 are each consistent with at least one component $\gtrsim 2.5 M_\odot$ which could be either a massive NS or a low-mass BH, although GW190425 could instead involve two $\sim 1.7 M_\odot$ NSs. Should either system contain a NS with $M \gtrsim 2.5 M_\odot$, the implications would be that the conformal limit $c_s^2 \leq 1/3$ is almost certainly violated (since n_m is likely larger than n_{sat}); if $n_m > 1.5 n_{\text{sat}}$ ($2.0 n_{\text{sat}}$), the average c_s^2 above n_m should be > 0.5 (0.67). More importantly, in order to also satisfy the small binary tidal deformability inferred from GW170817, $n_m \gtrsim 1.65 n_{\text{sat}}$ (could be lowered if there is sudden softening in the EOS induced by a strong first-order transition) and $c_{s,\text{match}}^2 \gtrsim 0.6$ are necessary. These conditions are typically not satisfied by most microscopic quark models unless parametrizations with explicit large sound speeds, or some crossover-like transitions that can be realized in, e.g., quarkyonic matter, are assumed. Even in the crossover scenario, severe constraints would follow and require fine-tuning of model parameters.

Assuming $M_{\max} \geq 2.6 M_\odot$, we find $R_{\min}(1.4 M_\odot) > 9.75 \text{ km}$ and $R_{\min}(2.0 M_\odot) > 10.8 \text{ km}$ (Table I). If instead an upper limit $c_s^2 < 1$ is assumed so that $\varepsilon = \varepsilon_0 + P/c_s^2$, then $R_{\min}(M)$ and $M_{\max}$ depend sensitively on c_s^2 and decrease with it. For the case $c_s^2 = 1/3$ and $\varepsilon_0 = \varepsilon_{\text{sat}}$, for example, $M_{\max} = 2.48 M_\odot$, $R_{\min}(1.4 M_\odot) = 12.8 \text{ km}$, and $R_{M_{\max}} = 13.3 \text{ km}$ (Fig. 16).

We showed that positive values of $\Delta R = R_{2.0} - R_{1.4}$, potentially possible with NICER, would indicate low matching densities $\lesssim 2.0 - 2.5 n_{\text{sat}}$ and relatively large values of $c_{s,\text{match}}^2 \gtrsim 0.45 - 0.6$, which would also imply large values of $M_{\max}$. In the absence of a dramatic stiffening of the EOS near $2.0 n_{\text{sat}}$, the expectation is that $\Delta R < 0$. This is usually the case if extrapolations based on nucleonic-like models are used up to even higher densities and/or there is extra softening below $M_{\max}$.

Our studies have highlighted the interplay of $M_{\max}$, the radii of NSs, and the role of the nucleonic EOS for densities beyond n_{sat} . We also have illustrated that systematic order-by-order calculations up to N³LO in the χ EFT expansion provide an EOS for NSM up to $\sim 2.0 n_{\text{sat}}$ whose EFT truncation errors [29, 30] are small enough to have relatively minor influence on our major conclusions. Nevertheless, our results also reveal that theoretical studies at $n_B \gtrsim 2 n_{\text{sat}}$ can have a significant impact on NS properties, especially on the correlation between $M_{\max}$ and the NS radii. Detailed studies of EFT truncation errors at these higher densities and for a wide range of chiral interactions would be valuable. This requires the development of improved order-by-order χ EFT NN and 3N potentials within different regularization schemes [120–122]. Further, models that include additional degrees of freedom such as pions, hyperons, and quarks (while still being able to accommodate massive NSs) can provide new insights but need to be improved. Work along these lines is in progress. The advances in nuclear-matter calculations from χ EFT at low densities (see, e.g., Refs. [31, 123]) combined with Bayesian uncertainty

quantification (see, e.g., Refs. [29, 124, 125]) will enable astrophysical applications over a wide range in density and proton fraction, which would soon be confronted with X-ray, radio, and GW observations.

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Appendix A: Bounds Imposed by Causality

The assumption of causality, i.e., that the maximum sound speed $c_s = \sqrt{dP/d\varepsilon}$ is unity in units of c , can establish relations limiting both minimum and maximum radii, as functions of mass, for NS. These limits will explicitly depend on assumptions concerning the NS maximum mass $M_{\max}$. These causal bounds can be improved with the consideration of nuclear physics inputs as will be discussed in Sec. III. The causality limit is imposed by using the EOS

$$P(\varepsilon) = P_0 + (\varepsilon - \varepsilon_0) \quad (\text{A1})$$

for the pressure $P > P_0$ and the energy density $\varepsilon > \varepsilon_0$.

The minimum radius as a function of mass $R_{\min}(M)$ for any EOS is conjectured [126] to result from using Eq. (A1) with $P_0 = 0$, $P = 0$ for $\varepsilon \leq \varepsilon_0$ (i.e., a self-bound star). In this case, the EOS has a single parameter (ε_0) and solutions of the Tolman-Oppenheimer-Volkoff (TOV) equation [127, 128] scale with it. Letting m be the mass enclosed within the radius r , one can define

$$r = x \frac{c^2}{\sqrt{G\varepsilon_0}}, \quad m = y \frac{c^4}{\sqrt{G^3\varepsilon_0}}, \quad \text{and} \quad P = z\varepsilon_0, \quad (\text{A2})$$

where $y(x)$ and $z(x)$ are dimensionless functions, with the boundary conditions $y_c = y(x=0) = 0$ and $z_c = z(x=0) > 0$ at the stellar center, and $y_s = y(x=x_s)$ and $z(x=x_s) = 0$ at the stellar surface x_s . The quantities y_s and x_s depend on z_c . For small x_s , $y_s \propto x_s^3$, as expected.

It should also be noted that the EOS Eq. (A1) implies that the baryon number density is

$$n_B = n_0 \sqrt{\frac{\varepsilon + P}{\varepsilon_0 + P_0}}, \quad (\text{A3})$$

with $n_0 = (\varepsilon_0 + P_0)/\mu_0$ and μ_0 being the baryon chemical potential at ε_0 .

In the case that $P_0 = 0$, the central baryon density is $n_{\text{cent}} = n_0 \sqrt{1 + 2z_c}$. Also, the maximum mass configuration occurs for $dy_s/dx_s = 0$, or when $x_{\max,s} = 0.2405$, $y_{\max,s} = 0.08513$, and $z_{\max,c} = 2.023$ (and therefore $n_{\max,c}/n_0 = 2.246$). The maximum mass can then be expressed as

$$M_{\max} = \frac{y_{\max,s} c^4}{\sqrt{G^3\varepsilon_0}} \simeq 4.09 \sqrt{\frac{\varepsilon_{\text{sat}}}{\varepsilon_0}} M_{\odot}, \quad (\text{A4})$$

and the radius of the maximum mass configuration is

$$R_{M_{\max}} = \frac{x_{\max,s} c^2}{\sqrt{G\varepsilon_0}} \simeq 17.1 \sqrt{\frac{\varepsilon_{\text{sat}}}{\varepsilon_0}} \text{ km}. \quad (\text{A5})$$

The central energy density for the maximum mass configuration is $\varepsilon_{\max,c} = (z_{\max,c} + 1) \varepsilon_0$, or using Eq. (A4) to eliminate ε_0 ,

$$\varepsilon_{\max,c} \simeq 50.8 \left(\frac{M_{\odot}}{M_{\max}} \right)^2 \varepsilon_{\text{sat}}, \quad (\text{A6})$$

where $\varepsilon_{\text{sat}} \simeq 150 \text{ MeV fm}^{-3}$ is the energy density at n_{sat} . This must be the largest energy density found in any NS and it scales with $M_{\max}^{-2}$. The maximum baryon density is

$$n_{\max,c} \simeq 37.6 \frac{m_B}{\mu_0} \left(\frac{M_{\max}}{M_{\odot}} \right)^2 n_{\text{sat}}, \quad (\text{A7})$$

where $\mu_0 \sim m_B$, the baryon mass. As an example, if one assumes that $M_{\max} = 2.6 M_{\odot}$ and $\mu_0 = m_B$, it is found that $\varepsilon_0 = 2.475 \varepsilon_{\text{sat}}$, $\varepsilon_{\max,c} = 7.48 \varepsilon_{\text{sat}}$ and $n_{\max,c} = 5.56 n_{\text{sat}}$.

The dimensionless M - R curve for the causal self-bound configuration is thus defined by $y_s(x_s)$. Its *dimensionful* radius, as a function of mass, is conjectured to be the minimum radius for any configuration, $R_{\min}(M)$. It scales with ε_0 and therefore with the assumed value of the maximum mass:

$$R_{\min} = \frac{GM_{\max}x_s}{y_{\max,s}c^2} = \frac{GM_{\max}}{y_{\max,s}c^2} y_s^{-1} \left(y_{\max,s} \frac{M}{M_{\max}} \right), \quad (\text{A8})$$

where $y_s^{-1} = x_s$ is the inverse function. $R_{\min}$ increases as $M_{\max}$ increases. The cases with $M_{\max} = 2.6 M_{\odot}$ and $M_{\max} = 4.09 M_{\odot}$ for which ε_0 is $2.48 \varepsilon_{\text{sat}}$ and ε_{sat} , respectively, are shown in Fig. 16. For the case that $M_{\max} = 2.0 M_{\odot}$ for which $\varepsilon_0 = 4.2 \varepsilon_{\text{sat}}$, we obtain $R_{\min}(1.4 M_{\odot}) = 8.2 \text{ km}$ and $R_{M_{\max}} = 8.4 \text{ km}$.

If the assumed maximum sound speed is less than c , $R_{\min}(M)$ will increase. Assuming the sound speed never exceeds a given value of c_s , $R_{\min}(M)$ can be found using

$$P = P_0 + c_s^2(\varepsilon - \varepsilon_0), \quad (\text{A9})$$

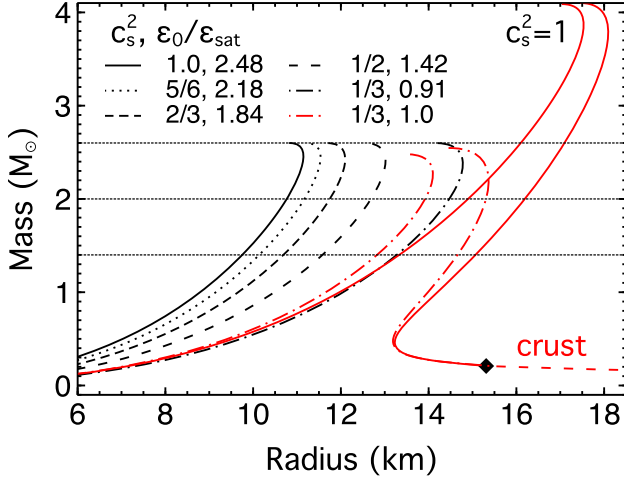


FIG. 16. The mass as a function of the radius for the EOS Eq. (A9) with $P_0 = 0$, $P = 0$ for $\varepsilon < \varepsilon_0$, and various values for c_s^2 with fixed $M_{\max} = 2.6 M_\odot$, are shown as five black curves (see legend). These curves correspond to the minimum possible radius $R_{\min}(M)$, for different maximum values of the sound speed. The four red curves correspond to $\varepsilon_0 = \varepsilon_{\text{sat}}$, either $c_s^2 = 1$ (and $M_{\max} \simeq 4.09 M_\odot$) or $c_s^2 = 1/3$ (and $M_{\max} \simeq 2.48 M_\odot$) for $P > P_0$, and either $P_0 = 0$ (self-bound) or $P_0 = 0.02 \varepsilon_0 \simeq 3 \text{ MeV fm}^{-3}$ and a normal crust EOS for $P < P_0$ (maximum possible radii $R_{\max}(M)$); the configuration where $\varepsilon_c = \varepsilon_{\text{sat}}$ is indicated by a diamond.

with $P_0 = 0$ and $P = 0$ for $\varepsilon < \varepsilon_0$. Once again, the TOV equation can be rendered into dimensionless form using Eq. (A2). Now, however, the baryon number density becomes

$$n_B = n_0 \left(\frac{P + \varepsilon}{P_0 + \varepsilon_0} \right)^{1/(1+c_s^2)} \quad (\text{A10})$$

and

$$n_{\text{cent}} = n_0 [1 + z_c (1 + c_s^{-2})]^{1/(1+c_s^2)}. \quad (\text{A11})$$

The dimensionless M - R curve $y_s(x_s)$ changes, as do the properties of the maximum mass configuration $x_{\max,s}$, $y_{\max,s}$ and $z_{\max,c}$. Figure 16 shows M - R solutions for $c_s^2 = 1, 5/6, 2/3, 1/2$, and $1/3$, all scaled so that $M_{\max} = 2.6 M_\odot$. In the case $c_s^2 = 1$, one finds

$$R_{\min,1.4} = 9.75 \text{ km and } R_{\min,2.0} = 10.8 \text{ km.} \quad (\text{A12})$$

Approximately, the minimum radii for smaller values of c_s scale as $c_s^{-1/2}$ [75], and for $c_s^2 = 1/3$, one finds that

$$R_{\min,1.4} \simeq 13.3 \text{ km and } R_{\min,2.0} \simeq 14.5 \text{ km.} \quad (\text{A13})$$

$\varepsilon_{\max,c}/\varepsilon_{\text{sat}}$ is proportional to $z_{\max,c} + 1$, which for $c_s^2 < 1$, is seen to scale roughly as $c_s^{3/2}$. Relevant properties of these solutions are given in Table I.

TABLE I. Maximum mass solutions for the EOS Eq. (A9) with $P_0 = 0$. The last two columns give the minimum radii in km for $1.4 M_\odot$ and $2.0 M_\odot$ stars, respectively, assuming $M_{\max} = 2.6 M_\odot$.

c_s^2	$x_{\max,s}$	$y_{\max,s}$	$z_{\max,c}$	$R_{\min,1.4}$	$R_{\min,2.0}$
1	0.2405	0.08513	2.023	9.75	10.8
5/6	0.2329	0.07992	1.884	10.1	11.2
2/3	0.2234	0.07328	1.705	10.7	11.7
1/2	0.2105	0.06439	1.499	11.6	12.7
1/3	0.1908	0.05169	1.277	13.3	14.5

Stars with $P_0 = 0$ are often referred to as *self-bound* stars. In contrast, normal NSs have a low-density crust with $P_0 > 0$. For normal stars, $R_{\min}(M)$ will be larger than those shown in Fig. 16. Generally, the radius will increase with the assumed values of ε_0 and P_0 for a given value of c_s , and, to a lesser degree, will also depend on the crust EOS for $P < P_0$. Most importantly, since $M_{\max}$ and ε_0 remain closely related, $R_{\min}(M)$ will be very sensitive to the lower limit to $M_{\max}$. Details and implications are discussed in Sec. III A.

Ironically, the maximum radius as a function of mass $R_{\max}(M)$ can also be found by appending the same EOS Eq. (A9) at a matching density n_m or ε_m onto an assumed lower-density (crust) EOS. This is because Eq. (A9) is the stiffest possible EOS for an assumed maximum value of the sound speed c_s . Although the same EOS is used, the $R_{\min}(M)$ bound involves a finite surface energy density $\varepsilon_0 = \varepsilon_m$, while the $R_{\max}(M)$ bound is assumed to lack a discontinuity in ε when appending the crust⁴. The resulting $R_{\max}(M)$ trajectory, and $M_{\max}$, will depend on the matching density ε_m and pressure P_m , the crust EOS, and assumed maximum sound speed c_s , and both roughly scale as $\varepsilon_m^{-1/2}$. Since there is no evidence that a transition to a non-hadronic EOS occurs for densities smaller than ε_{sat} , a limiting set of $R_{\max}(M)$ curves is found assuming $\varepsilon_m = \varepsilon_{\text{sat}}$. As the matching pressure P_m is not negligibly small, $P_m \simeq 0.02 \varepsilon_{\text{sat}}$ for $\varepsilon_m \simeq \varepsilon_{\text{sat}}$, the M - R curve is considerably altered, and forms a maximum radius trajectory $R_{\max}(M)$ which lies at a larger radius for each mass than $R_{\min}(M)$, as can be seen by comparing the two solid red curves for $c_s^2 = 1$ in Fig. 16. $R_{\max}$ for $c_s^2 = 1$ can be safely assumed to give, approximately, the largest possible radii for normal NS (it varies with the assumed EOS below ε_m). It is interesting that the maximum masses with $\varepsilon_m = \varepsilon_{\text{sat}}$ for a self-bound star (left red solid curve) and for a normal star with a crust (right red solid curve) are nearly identical and are substantially larger than $2.6 M_\odot$, for

⁴ Note that if a discontinuity in ε is assumed at ε_m , a smaller $R_{\max}$ trajectory is obtained, but one with a correspondingly smaller $M_{\max}$ as well. This situation is briefly discussed in Sec. III D.

example. Lower maximum masses are obtained if the matching density is increased, which decreases $R_{\max}(M)$ as well. An observed upper limit on $M_{\max}$ below $4.09 M_{\odot}$ will automatically alter the $R_{\max}$ boundary, however, because in this case either ε_m would have to increase or c_s would have to decrease to correspondingly reduce $M_{\max}$.

The situation is similar if a lower fixed sound speed is assumed. Figure 16 also displays $R_{\min}(M)$ and $R_{\max}(M)$ trajectories for $c_s^2 = 1/3$ for the self-bound and realistic crust cases (the left and right red dot-dashed curves, respectively), which have smaller radii and $M_{\max}$ values than for $c_s^2 = 1$. Note that $R_{\max}(M)$ for $c_s^2 = 1/3$ (right red dot-dashed curve) can become smaller than $R_{\min}(M)$ for $c_s^2 = 1$ and $P_0 = 0$ (left red solid curve) for $M \gtrsim 2.3 M_{\odot}$, suggesting that $c_s^2 = 1/3$ is incompatible with the assumption that $M_{\max} = 2.6 M_{\odot}$; the maximum value of c_s^2 must be larger than $1/3$ in the interior of a $2.6 M_{\odot}$ star, or $P_0 > 0$ (i.e., there is a crust), or $M_{\max} < 2.6 M_{\odot}$.

A more realistic maximum radius boundary will depend on both the matching density and the EOS below that density. In the next section we discuss realistic constraints on this portion of the EOS stemming from theoretical studies of NSM.

Appendix B: Chiral interactions used and their nuclear saturation properties

The chiral nuclear interactions this work is based on were constrained in Ref. [31] as follows: NN potentials by Entem, Machleidt, and Nosyk [129] up to $N^3\text{LO}$ were combined with 3N forces at the same order and momentum cutoff so as to construct a set of order-by-order NN and 3N interactions. The two 3N low-energy couplings c_D and c_E , which govern the intermediate- and short-range 3N contributions, respectively, at $N^2\text{LO}$ were constrained by the triton binding energy and the empirical saturation point of SNM. Several combinations of c_D and c_E with reasonable saturation properties could be obtained at $N^2\text{LO}$ and $N^3\text{LO}$ for the momentum cutoffs $\Lambda = 450$ and 500 MeV. A momentum cutoff is a typical scale in the regulator function that is applied to χEFT interactions to suppress contributions from high-momentum modes. Note that the EFT breakdown scale Λ_b is a physical scale inherent to the EFT, whereas the results should not be sensitive to the artificial scale Λ ; in practice, however, this has not yet been achieved in χEFT for infinite matter. The BUQEYE collaboration found that their results do not significantly dependent on which c_D and c_E combination is chosen for a given momentum cutoff. Furthermore, the 3N contributions proportional to c_D and c_E vanish in PNM for nonlocal regulator functions [130]. Consequently, they considered only one combination for each cutoff, and focused their analysis on the Hamiltonian with $\Lambda = 500$ MeV, while the results for the $\Lambda = 450$ MeV interaction were provided in the Supplemental Material there.

We follow this strategy here, and note that the residual cutoff dependence is well within the EFT truncation-error estimates at the 1σ level; i.e., for $\Lambda = 450$ MeV, $P_{\text{PNM}}(2.0 n_{\text{sat}}) = 17.29 \pm 4.56 \text{ MeV fm}^{-3}$ and $E_{\text{PNM}}(2.0 n_{\text{sat}}) = 42.86 \pm 5.01 \text{ MeV}$, whereas for $\Lambda = 500$ MeV, $P_{\text{PNM}}(2.0 n_{\text{sat}}) = 18.53 \pm 5.14 \text{ MeV}$ and $E_{\text{PNM}}(2.0 n_{\text{sat}}) = 41.55 \pm 5.77 \text{ MeV fm}^{-3}$.

Experimental validation of χEFT predictions for the EOS of bulk matter relies on comparisons to the empirical saturation point, and constraints on the nuclear symmetry energy and its derivative with respect to density at n_{sat} . While the region in the S_v - L plane predicted by the nuclear interactions used in this work are well within the joint experimental constraint [29], the $\Lambda = 500$ MeV Hamiltonians—as discussed in Ref. [31]—actually do not saturate inside the empirical range for the saturation point, $n_{\text{sat}} = 0.164 \pm 0.007 \text{ fm}^{-3}$ with $(E/A)_{\text{sat}} = -15.86 \pm 0.57 \text{ MeV}$. Note, however, that this empirical range was obtained in Refs. [31, 44] from a set of energy density functionals, and thus only has limited statistical meaning. The predicted 2σ confidence ellipses for the nuclear saturation point at $N^2\text{LO}$ and $N^3\text{LO}$ are shown in Fig. 9 of Ref. [30].

In contrast to the properties of neutron-rich NSM EOS, nuclear saturation in SNM is sensitive to the short- and intermediate-range 3N interactions at $N^2\text{LO}$ that do not contribute to the PNM EOS; e.g., the 3N contact interaction ($\propto c_E$) is Pauli-blocked in PNM [130]. Together with the fact that the proton fraction is small, this means that the nuclear saturation properties are of relatively minor importance for constructing the NSM EOS. Nonetheless, a better understanding of nuclear saturation properties may help identify and quantify systematic uncertainties in the nuclear interactions. This might also lead to a better understanding of the link between (saturation) properties of infinite matter and medium-mass to heavy nuclei [120, 121] to explain why χEFT potentials generally tend to underestimate charge radii [122, 131, 132]. In this context, it is worth noting that systematic EFT calculations of the EOS of NSM, which is characterized by a small proton fraction, would obviate the need to rely on the quadratic expansion Eq. (1) (see, e.g., Ref. [133] in which the energy of adding a proton to PNM was calculated). When such calculations become available one can gauge the extent to which the EOS of NSM is correlated with the empirical properties of SNM.

Appendix C: Sensitivity to EOS density ranges

It is apparent that the limits to NS radii and tidal deformabilities are sensitive to the EOS in the density range $1 - 3 n_{\text{sat}}$, precisely where the restrictions from χEFT are important. This is not surprising given the tight correlation between $R_{1.4}$ and the NSM pressure for $1 - 2 n_{\text{sat}}$ discovered by Ref. [2]. However, up to this point, we have assumed fixed sound speeds above n_m . In this section, we demonstrate that this correlation is insensitive to the

details of the assumed EOS at all relevant densities; furthermore, we quantify this correlation and extend it to include the quantities $R_{2,0}$ and $M_{\max}$.

We evaluate these correlations by considering several parametrization schemes to construct families of high-density NSM EOSs at densities larger than about $0.5 n_{\text{sat}}$, the assumed core-crust boundary. All configurations are assumed to have a crust modeled with the SLy4 EOS [134]. Each EOS is given as a function of n_B only and is implicitly considered to represent beta-equilibrium matter. The parameters for each parametrization scheme are constrained to ensure causality, $c_s^2 \geq 0$, a minimum value $M_{\max} = 2.0 M_{\odot}$, a lower limit to the neutron-matter energy and pressure suggested by the unitary-gas conjecture [135] at all supra-nuclear densities, and upper limits to the NSM energy and pressure at n_{sat} implied by experimental limits of $S_v = 36 \text{ MeV}$ and $L = 80 \text{ MeV}$ [95]. Note that the latter two constraints are broader than the NSM- $\chi\text{EFT} \pm 1\sigma$ constraints, so that the correlations we find are conservatively expressed. Also, for each parametrization, we have ensured a minimum of 15,000 realizations that satisfy our constraints. We quantify a correlation in terms of the covariance between two quantities A and B ,

$$\text{cov}(A, B) = \sum_{i,j} \frac{(A_i - \bar{A})(B_j - \bar{B})}{\sigma_A \sigma_B}. \quad (\text{C1})$$

The σ 's represent standard deviations. We take $A = P(n_B)$ and $B = R_{1,4}$, $R_{2,0}$, or $M_{\max}$. Here, j ranges over all realizations of a given parameterized EOS and i over all values of n_B smaller than the central density of the relevant configuration for B .

Figure 17 shows the correlations between the pressure $P(n_B)$ and $R_{1,4}$, $R_{2,0}$ and $M_{\max}$ as functions of the baryon number density n_B for a variety of NSM parametrizations in common use. The parametrizations “n-EXP” and “k-EXP” are three-parameter Taylor expansions of the NSM energy in terms of n_B and $n_B^{1/3}$ [135], respectively. “n-EXP” is commonly used to model the nuclear energy around saturation; we take a Taylor expansion up to the fourth-order term $[(n_B - n_{\text{sat}})/n_{\text{sat}}]^4$. Two of the coefficients are set to match the crust EOS, leaving three free parameters. “k-EXP” contains a kinetic term $\propto (n_B/n_{\text{sat}})^{2/3}$ and a higher-order term up to $(n_B/n_{\text{sat}})^{7/3}$. It also has three free parameters after using two coefficients to match the crust EOS. “Spectral4” is the four-parameter spectral decomposition method [136–138]. “Quarkyonic” has two parameters, Λ and κ , specifying the quarkyonic momentum shell thickness and the transition density, and one parameter (effectively controlling L) for the nucleon potential [59]. “PP3+1” is a four-parameter piecewise-polytrope with three segments appended to the crust [139]. The density n_1 separating the first two segments is a parameter, while n_2 and n_3 are chosen to scale as $n_2 = 2n_1$ and $n_3 = 2n_2$. The corresponding bounding pressures P_1 , P_2 , and P_3 are the other

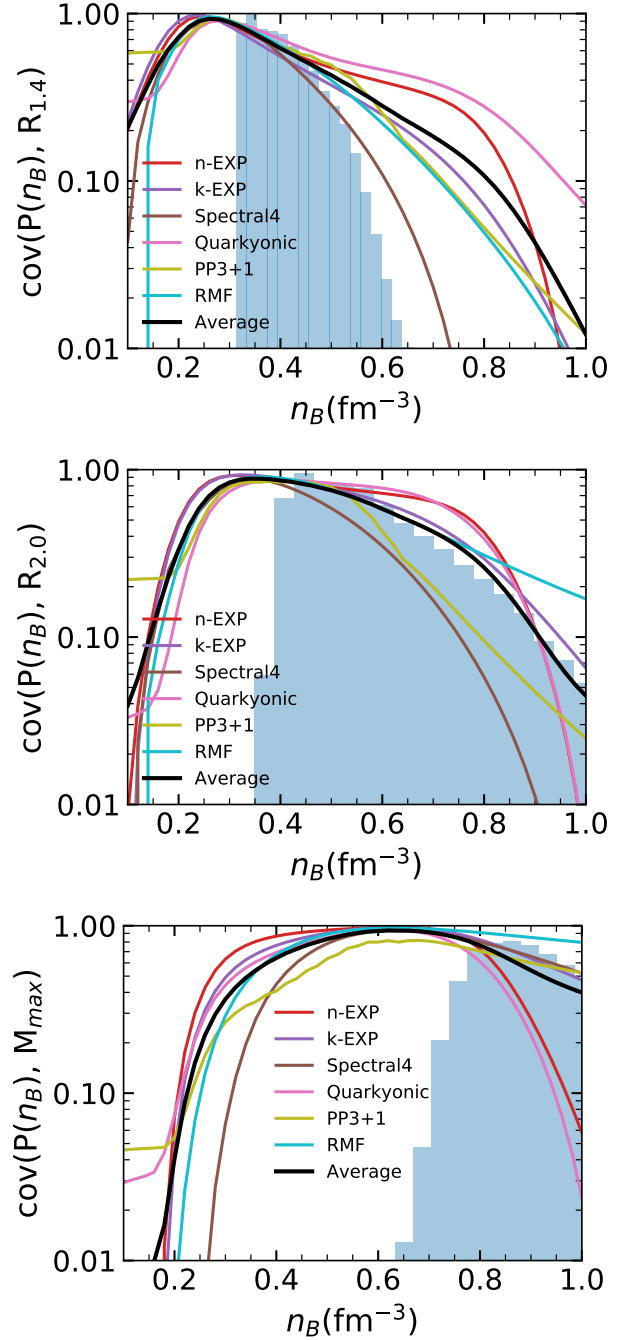


FIG. 17. Correlations among $P(n_B)$, $R_{1,4}$, $R_{2,0}$ and $M_{\max}$ for 6 EOS parametrizations (see text for details). “Average” refers to the mean of all models. Blue histograms show the summed distributions of the central densities of the relevant stars.

three free parameters⁵. “RMF” is a relativistic mean field model based on the FSU2 EOS [140] and contains σ , ω ,

⁵ The additional parameter n_1 greatly increases the flexibility of PP3+1 compared to the three-parameter (P_1, P_2, P_3) set PP3 often employed [139].

and ρ meson exchanges. It has seven coupling constants, of which three are fixed by saturation properties of SNM; the remaining four free parameters can be mapped to S_v , L , the effective nucleon mass at the saturation density, M^* , and the ω self-interaction coupling ζ .

The covariance parameter $\text{cov}(P(n_B), R_{1.4})$ peaks around $n_B = 1.65^{+1.32}_{-0.68} n_{\text{sat}}$, whereas $\text{cov}(P(n_B), R_{2.0})$ and $\text{cov}(P(n_B), M_{\text{max}})$ peak around $n_B = 2.17^{+2.14}_{-0.81} n_{\text{sat}}$ and $n_B = 3.90^{+2.00}_{-1.81} n_{\text{sat}}$, respectively. The uncertainties correspond to 50% of the peak covariance. Figure 17 also quantifies the extent to which the central baryon densities, and the width of their distributions, increase with the NS mass. Notably, the central baryon number densities peak at about 30% higher density than do the peak covariance in all three cases, but the widths of the central density distributions rapidly increase with NS mass.

The correlation between the pressure $P(n_B)$ and $R_{1.4}$ is strongest between n_{sat} and $3.0 n_{\text{sat}}$, as expected, and that between the pressure and $R_{2.0}$ is strongest at about 40%

higher densities. Significantly, these results appear to be relatively insensitive to the details of the parametrizations. The standard deviations of both $\text{cov}(P(n_B), R_{1.4})$ and $\text{cov}(P(n_B), R_{2.0})$ for the six parametrizations are small, being $\sigma_{\text{cov}, R} < 0.2$ for all densities and $\sigma_{\text{cov}, R} < 0.05$ near the covariance peaks. The bottom line is these results demonstrate, at present, that χEFT greatly constrains $R_{1.4}$ and, to a slightly lesser degree, $R_{2.0}$. The situation is somewhat different for M_{max} , where pressures at densities between $2.0 n_{\text{sat}}$ and $6.0 n_{\text{sat}}$ dominate. In addition, the standard deviation of $\text{cov}(P(n_B), M_{\text{max}})$ among the six parametrizations are somewhat larger, being $\sigma_{\text{cov}, M_{\text{max}}} < 0.25$ at all densities and $\sigma_{\text{cov}, M_{\text{max}}} < 0.1$ near the covariance peak. Thus, the M_{max} results are more model-dependent, and the significant densities likely lie above the validity range for χEFT . However, further refinement of EFT techniques at high densities combined with Bayesian uncertainty quantification might change that situation by providing improved constraints on all three quantities, although the EFT truncation error increases rapidly beyond n_{sat} .

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