On Energy Efficient Uplink Multi-User MIMO with Shared LNA Control

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***Abstract*—Implementation cost and power consumption are two important considerations in large-scale multi-antenna systems where the number of individual radio-frequency (RF) chains may be significantly larger than before. In this work, we propose to deploy a single low-noise amplifier (LNA) on the uplink multiple-input-multiple-output (MIMO) receiver to cover all antennas. This architecture, although favorable from the perspective of cost and power consumption, introduces challenges in the LNA gain control and user transmit power control. We formulate an energy efficiency maximization problem under practical system constraints, and prove that it is a constrained quasi-concave optimization problem. An efficient algorithm, *Bisection – Gradient Assisted Interior Point (B-GAIP)*, is proposed to solve this optimization problem. The optimality and complexity of B-GAIP are analyzed, and further corroborated via numerical simulations. In particular, the performance loss due to using a shared LNA as opposed to separate LNAs in each RF chain, when using B-GAIP to determine the LNA gain and user transmit power, is very small in both centralized and distributed MIMO systems.**

I. Introduction

Energy efficiency of communication systems is of significantly practical importance. From the operators' perspective, reducing both the operation cost and carbon dioxide emissions is becoming essential to their business bottom line. This is especially important with the commercialization of massive multiple-input-multiple-output (MIMO) [1] in 5G standards. When the number of antennas is large, hardware cost and power consumption increase substantially, which has motivated extensive studies on massive MIMO with inexpensive hardware components such as low-resolution Analog-to-Digital Converter (ADC) [2], Digital-to-Analog Converter (DAC) [3], mixers and oscillators [4].

In this paper, we follow the same design philosophy and study an attractive low-complexity MIMO receiver structure, where a *single* low-noise amplifier (LNA) [5] is used for *all* receive RF chains at the base station (BS). This is followed by low-resolution ADCs for each RF chain, as is typically done in massive MIMO [6]. This architecture has the benefits of reduced implementation cost and lower power consumption, compared to the separate LNA approach where each RF chain uses an independent LNA for gain control. Previously, this shared-LNA structure has been used in multi-channel communications [5] where signals from different channels are non-overlapping in the frequency domain. This feature mostly relies on LNA's wider bandwidth and more relaxed saturation point compared to other RF components like Analog-to-Digital Converter (ADC). However, we will show in this work that the shared-LNA structure can also be adopted in MIMO even when signals are mixed in the same spectrum.

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The challenge, however, comes from the single LNA operating point for multiple RF chains. For the separate LNA structure, each antenna will have an independent LNA to adjust the power of the received signal. This gain can be optimized based on the *individual* receive power of the RF chain, resulting in maximum flexibility. For the shared LNA structure, however, the single LNA gain control must accommodate *all* receive antennas. Hence, performance may degrade if this single gain control cannot meet the requirements for all RF chains. Intuitively, such loss can be significant when the dynamic power range across all antennas is large.

In this paper, we address this challenge by focusing on the joint design of shared LNA gain control and user transmit power control. We formulate this problem as maximizing the overall system energy efficiency subject to several engineering constraints that arise from practical multi-user MIMO systems, and then prove that this is a constrained quasi-concave optimization problem. An efficient algorithm, *Bisection – Gradient Assisted Interior Point (B-GAIP)*, is proposed and its optimality is studied. Furthermore, we analyze its convergence and complexity with the help of an equivalent interpretation of B-GAIP. Numerical simulation results are provided to evaluate the benefits of shared LNA.

The rest of this paper is organized as follows. Section II presents the system model, and the implementation is given in Section III. We formulate the energy efficiency optimization problem in Section IV. In Section V we give details of the proposed B-GAIP algorithm. Section VI presents numerical simulations, and Section VII concludes the paper.

II. System Model

Consider an uplink single-cell MU-MIMO system with a circular coverage area centered around the BS, where the radius is R_0. In the system, K user equipments (UEs) are randomly and uniformly distributed in the coverage area, and each UE is equipped with a single antenna. The BS deploys M antennas which are located entirely at the cell center. We

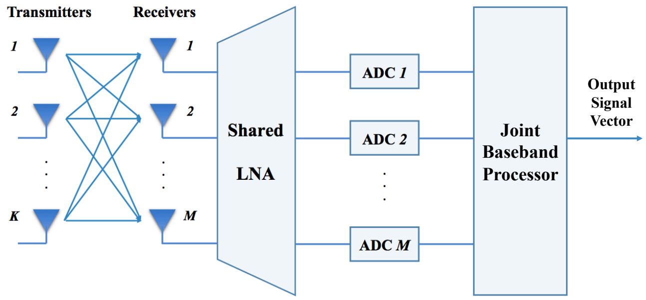


Fig. 1. The receiver uplink MU-MIMO structure with shared LNA control.

denote the set of all BS antennas as $\mathcal{M}$ and the set of UEs as $\mathcal{K}$, with cardinality $|\mathcal{M}| = M$ and $|\mathcal{K}| = K$, respectively.

Assume that all UEs simultaneously transmit data to the base station, the received singal at the BS can be written as

$$\mathbf{y} = \mathbf{G}\mathbf{P}\mathbf{x} + \mathbf{z}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{M\times 1}$ is the signal vector at the BS receive antennas and $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma_N^2 \mathbf{I}_M) \in \mathbb{C}^{M\times 1}$ is an additive white Gaussian noise (AWGN) vector with mean $\mathbf{0}$ and covariance $\sigma_N^2\mathbf{I}_M$, with $\mathbf{I}_M$ denoting the identity matrix with dimension M. $\mathbf{P} = \text{diag}(\sqrt{p_1}\ , \ldots, \sqrt{p_K})$ is the real-valued diagonal transmit amplitude matrix, and $\mathbf{x} \in \mathbb{C}^{K\times 1}$ is the power-normalized transmitted vector of K UEs. $\mathbf{G} \in \mathbb{C}^{M\times K}$ is the channel matrix between K UEs and M BS antennas, whose elements is $g_{mk} \triangleq [\mathbf{G}]_{mk}$. The channel matrix $\mathbf{G}$ models the independent fast fading, geometric attenuation, and log-normal shadow fading. As a result, element g_{mk} is given by

$$g_{mk} = h_{mk}\sqrt{\beta_{mk}}, \tag{2}$$

where h_{mk} is the fast fading coefficient which follows a circularly symmetric complex Gaussian distribution with zero mean and unit variance; $\sqrt{\beta_{mk}}$ represents the geometric attenuation and shadow fading which are assumed to be independent and constant over the coherent intervals. We adopt the WINNER II path loss model [7], where the path loss in dB domain is

$$\beta_{mk}^{\text{dB}} = 46 + 20\log_{10}(d_{mk}) + V_{mk}. \tag{3}$$

In model (3), d_{mk} is the distance from UE k to BS antenna m and V_{mk} denotes the shadow fading which follows the log-normal distribution. Note that $\beta_{mk} = 10^{(-\beta_{mk}^{\text{dB}}/10)}$.

The proposed receiver structure of a single LNA is illustrated in Fig. 1, where a *common* LNA is applied to amplify the signals of *all* receive antennas. The gain of this common LNA is denoted as Ω^{dB} in the dB domain and $\Omega = 10^{(\Omega^{\text{dB}}/10)}$. The amplified received signal vector can be written as

$$\tilde{\mathbf{y}} = \sqrt{\Omega}\mathbf{y}. \tag{4}$$

We consider a finite range with discrete values for parameter Ω^{dB}, i.e., $\Omega^{\text{dB}}_{\min} \leqslant \Omega^{\text{dB}} \leqslant \Omega^{\text{dB}}_{\max}$ and Ω^{dB} is an integer.

Following the shared LNA, each component of the signal vector will pass through an individual low-resolution ADC. We adopt the fixed ADC noise model[1] as in [8]:

$$\hat{\mathbf{y}} = \tilde{\mathbf{y}} + \mathbf{n}_q, \tag{5}$$

where the additive noise vector $\mathbf{n}_q \in \mathbb{C}^{M\times 1}$ is uncorrelated with the ADC input $\tilde{\mathbf{y}}$, and its elements are modeled as independent complex Gaussian random variables with zero mean and variance σ^2_{ADC}.

We assume BS has perfect knowledge of CSI. By using a zero-forcing (ZF) detector $\mathbf{F} \triangleq (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H$ where $\mathbf{G}^H$ denotes the Hermitian of $\mathbf{G}$, $\hat{\mathbf{y}}$ is processed as follows:

$$\mathbf{r} = \mathbf{F}\hat{\mathbf{y}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\hat{\mathbf{y}}. \tag{6}$$

Since we have $\mathbf{F}\mathbf{G} = \mathbf{I}_K$, $\mathbf{r}$ is given by

$$\mathbf{r} = \sqrt{\Omega}\mathbf{P}\mathbf{x} + \sqrt{\Omega}\mathbf{F}\mathbf{z} + \mathbf{F}\mathbf{n}_q. \tag{7}$$

Take the kth component as an example, we have

$$r_k = \sqrt{\Omega p_k}x_k + \sqrt{\Omega}\mathbf{f}_k\mathbf{z} + \mathbf{f}_k\mathbf{n}_q, \tag{8}$$

where $\mathbf{f}_k$ denotes the kth row of matrix $\mathbf{F}$. As a result, the signal-to-noise ratio (SNR) of the kth UE at the output of the BS receiver can be calculated as

$$\Gamma_k = \frac{\Omega p_k}{(\Omega\sigma_N^2 + \sigma^2_{\text{ADC}})\|\mathbf{f}_k\|^2}, \tag{9}$$

where $||\cdot||$ denotes the l_2 norm. By using SNR Γ_k, we define the spectral efficiency (SE) via modified Shannon capacity:

$$R_k = \begin{cases} \log_2(1 + A_d * \Gamma_k), & \Gamma_k < \Gamma_{\max} \\ \log_2(1 + A_d * \Gamma_{\max}), & \Gamma_k \geqslant \Gamma_{\max} \end{cases} \tag{10}$$

where A_d denotes the coding gain and possibly multi-antenna diversity gain, which in practice is obtained via off-line fitting via link adaptation simulations. $\Gamma_{\max}$ is the maximum achievable SNR at the receiver, which is often dominated by phase noise and IQ mismatch.

Finally, the **energy efficiency** is defined as the ratio between spectral efficiency and consumed power of the system [10]:

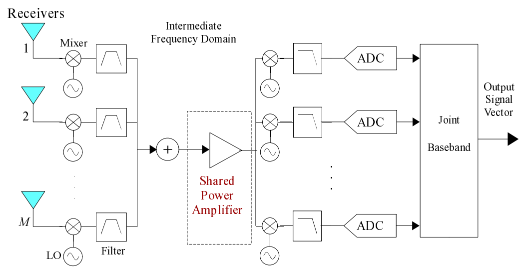
$$U(\mathbf{p}, \Omega) = \frac{\sum_{k=1}^K R_k}{P_c + \sum_{k=1}^K p_k/\eta}, \tag{11}$$

where P_c denotes the circuit power of both the transmitters and the receivers, η is the power amplifier efficiency, and $\mathbf{p} = [p_1, p_2, ..., p_K]$ is the power allocation vector. Moreover, we define SE vector under configuration $\mathbf{p}$ and Ω as $\mathbf{R} = [R_1, R_2, ..., R_K]$. We further use $U(\Omega)$ to denote the maximum energy efficiency under all feasible power vectors.

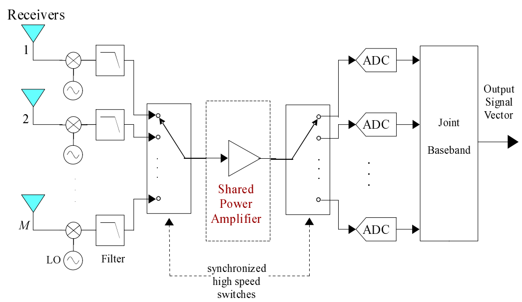
III. Implementation Considerations

It is not straightforward to determine how the shared LNA structure in Fig. 1 can be used in multi-antenna receivers,

[1]Note that under most of the ADC models, such as the additive quantization noise model (AQNM), the power of quantization noise changes with the power of ADC input signals. However, since LNA is used to control the power gain, it is convenient and appropriate to assume a fixed ADC noise [5], [8], [9].



(a) Super-heterodyne implementation



(b) High-speed switch implementation

Fig. 2. Two possible shared amplifier implementation structures.

where signals from different antennas are on the same frequency. We hereby discuss two possible implementations that can effectively and efficiently realize a shared LNA for a multi-antenna receiver.

The first implementation is to leverage the RF framework proposed in [11], which is built on a super-heterodyne receiver. The detail of this implementation is depicted in Fig. 2(a). In this structure, by programming the frond-end mixers and filters, these receive (Rx) paths can be tuned as non-overlapping in the frequency domain at the input of the shared amplifier. In addition, mixers and filters after the amplifier can further isolate the shared-amplifier output from each baseband path.

An alternative implementation is shown in Fig. 2(b), where a direct-conversion receiver is used and synchronized switches at the input and output of shared amplifier are applied. Assuming sample-and-hold type ADCs and power amplifiers, if the switches at the input and output of shared amplifier have well synchronized timing to ensure the same Rx path, and switching periodicity is aligned with the ADC sampling rate, then the baseband processor can collect the right digital samples from different Rx paths using one amplifier.

A final comment is that although we focus on LNA in this paper, our work can be directly extended to any power amplifiers in the Rx path of the receiver, such as Intermediate Frequency (IF) amplifier or baseband amplifier. Besides these two possible implementations, other novel RF structures for shared power amplifiers can be further developed, which is an open topic in the (beyond) 5G RF research.

IV. Problem Formulation and Analysis

The following engineering limitations in practical systems are captured in the problem formulation.

1) Each UE's transmit power p_k is subject to a maximum power value, and is non-negative:

$$0 \leqslant p_k \leqslant P_{\max}, \;\; k \in \mathcal{K}. \tag{12}$$

2) To avoid ADC saturation, each ADC's input power is capped by a maximum value $P_{\max}^{\text{ADC}}$:

$$\Omega(\mathbf{g}_m \mathbf{P}^2 \mathbf{g}_m^H + \sigma_N^2) \leqslant P_{\max}^{\text{ADC}}, \;\; m \in \mathcal{M}, \tag{13}$$

where $\mathbf{g}_m$ denotes the mth row of matrix $\mathbf{G}$.

3) Since the effective SNR and SE at the receiver are capped, the limitation on transmit power can be presented as

$$\frac{\Omega p_k}{(\Omega\sigma_N^2 + \sigma_{\text{ADC}}^2)\|\mathbf{f}_k\|^2} \leqslant \Gamma_{\max}, \;\; k \in \mathcal{K}, \tag{14}$$

which is an equivalent interpretation of (10).

With these practical limitations, the joint power and shared LNA optimization problem can be formally presented as

$$\begin{aligned} &\underset{\mathbf{p},\,\Omega}{\text{maximize}} \;\; U(\mathbf{p}, \Omega) \\ &\text{subject to} \;\; (12), (13), (14). \end{aligned} \tag{15}$$

Note that for Ω, since only a finite set of values can be used, we always have

$$\underset{\mathbf{p},\,\Omega}{\text{maximize}} \;\; U(\mathbf{p}, \Omega) = \underset{\Omega}{\text{maximize}}\{\underset{\mathbf{p}}{\text{maximize}} \;\; U(\mathbf{p}, \Omega)\}. \tag{16}$$

We first analyze the properties of the three constraints. Due to space limitation, the proofs are omitted and are reported in the journal version.

Lemma 1. *Under a fixed LNA gain Ω,* (12)*,* (13) *and* (14) *are all linear constraints on the power vector* $\mathbf{p}$*, and therefore form a convex set with respect to* $\mathbf{p}$*.*

Then, Theorem 1 below states that $U(\mathbf{p}, \Omega)$ is a strictly quasi-concave function under a fixed LNA gain Ω, and we immediately have that the local maximum of $U(\mathbf{p}, \Omega)$ is also the global maximum. Finally, Proposition 1 shows that the objective function is concave in Ω. These results lay the theoretical foundation of the proposed algorithm in Section V.

Theorem 1. *The objective function $U(\mathbf{p}, \Omega)$ is strictly quasi-concave with respect to* $\mathbf{p}$*. Thus, for a given LNA gain Ω, problem* (15) *is equivalent to the following constrained quasi-concave optimization problem:*

$$\begin{aligned} &\underset{\mathbf{p}}{\text{maximize}} \;\; U(\mathbf{p}, \Omega) \\ &\text{subject to} \;\; (12), (13), (14). \end{aligned} \tag{17}$$

Proposition 1. *$U(\Omega)$ is concave in the LNA gain Ω.*

V. The B-GAIP Algorithm

We propose the Bisection – Gradient Assisted Interior Point (B-GAIP) algorithm that solves (15). It is essentially a two-step implementation of (16) as follows. First, we fix the LNA gain Ω and design a gradient assisted interior-point (GAIP) algorithm to optimize the power vector, leveraging the strict quasi-concavity property established in Theorem 1. On top of GAIP, we use a bisection search method to find the optimal LNA gain for the maximum energy efficiency, based on its concavity in Ω as shown in Proposition 1.

A. GAIP: Optimizing Power Allocation under Fixed LNA Gain

The heuristic gradient-based optimization methods are commonly used in energy efficient power allocation problems [5]. This method is well-known and widely-used due to its effectiveness and succinctness. However, the optimization objective is a strictly quasi-concave function with convex constraints. As the system dimension becomes large, so does the number of constraints. Therefore, a straightforward adoption of the gradient descent algorithm [12] will have very slow convergence, or not converge at all within reasonable time.

To cope with this challenge, we resort to the interior-point method [12], which transfers constrained optimization problems into unconstrained ones. The main idea is to construct a penalty function which "punishes" the objective function when it approaches or falls out of the boundary of the feasible set. In particular, we chose a logarithmic penalty function, which is concave. Since we consider a fixed LNA gain Ω in this subsection, we simply write $U(\mathbf{p}, \Omega)$ as $U(\mathbf{p})$ for convenience. The penalty function can then be written as

$$\begin{aligned}\varphi(\mathbf{p}, \xi) = U(\mathbf{p}) + \xi \sum_{k=1}^{K} \Big[\ln p_k + \ln(P_{\max} - p_k) \\ + \ln\left(\Gamma_{\max} - \frac{\Omega p_k}{(\Omega\sigma_N^2 + \sigma_{\text{ADC}}^2)\|\mathbf{f}_k\|^2}\right) \Big] \\ + \xi \sum_{m=1}^{M} \ln\left[P_{\max}^{\text{ADC}} - \Omega\left(\sum_{k=1}^{K} |g_{mk}|^2 p_k + \sigma_N^2\right)\right].\end{aligned} \tag{18}$$

Note that $B(\mathbf{p})$ represents the penalty for approaching the boundaries, while ξ is the factor that decides the intensity of penalty. Intuitively, as the penalty factor ξ approaches zero, the penalty function $\varphi(\mathbf{p}, \xi)$ approaches $U(\mathbf{p})$.

We then resort to the gradient descent method to find the optimal value of the unconstrained optimization function in (18). In particular, the partial derivative of the penalty function with respect to p_k can be derived as

$$\begin{aligned}\frac{\partial \varphi(\mathbf{p}, \xi)}{\partial p_k} = \frac{\partial U(\mathbf{p})}{\partial p_k} + \xi\left(\frac{1}{p_k} - \frac{1}{P_{\max} - p_k} - \frac{T_k}{\Gamma_{\max} - T_k p_k}\right) \\ + \xi \sum_{m=1}^{M} \frac{-\Omega|g_{mk}|^2}{P_{\max}^{\text{ADC}} - \Omega\left(\sum_{k=1}^{K} |g_{mk}|^2 p_k + \sigma_N^2\right)},\end{aligned} \tag{19}$$

where we define $T_k = \frac{\Omega}{(\Omega\sigma_N^2 + \sigma_{\text{ADC}}^2)\|\mathbf{f}_k\|^2}$, and the first term $\frac{\partial U(\mathbf{p})}{\partial p_k}$ in (19) is given by

$$\frac{\partial U(\mathbf{p})}{\partial p_k} = \frac{A_d T_k}{\ln 2(1 + A_d\Gamma_k)(P_c + P_{\text{sum}})} - \frac{R_{\text{sum}}}{\eta(P_c + P_{\text{sum}})^2}. \tag{20}$$

We further define the gradient metric over power vector $\mathbf{p}$ as $\nabla\varphi(\mathbf{p}, \xi) = [\frac{\partial\varphi}{\partial p_1}, \ldots, \frac{\partial\varphi}{\partial p_K}]$. Finally, the proposed GAIP algorithm is compactly presented in Algorithm 1.

B. B-GAIP: Optimizing Both LNA Gain and Power Allocation

The GAIP algorithm only optimizes the power values under a fixed LNA gain. A naive approach is to apply Algorithm 1 to *all* possible values of Ω and obtain the optimal energy efficiency. However, this approach may have high complexity if the set of feasible Ω is large, and it does not utilize the concavity of the objective function. Accordingly, we propose to solve this problem using a bisection search method, which has lower complexity than linear sweeping, achieves the same optimal value, and leverages the concavity to guarantee optimality (see Propsition 1). The overall algorithm that solves (15) is presented in Algorithm 2.

C. Complexity Analysis

From the procedure above, we can conclude that the two-step algorithm has three main layers as follows.

- The outer layer: Optimize the LNA gain via the bisection search method;
- The middle layer: Under a given LNA gain, transfer the constrained problem into an unconstrained optimization problem via the interior-point method;
- The inner layer: Find the optimal value of the unconstrained problem via the gradient descent method.

To analyze the complexity of Algorithm 1 and 2, we individually analyze the complexity of each layer. For the inner layer, we perform $L_{\max}$ iterations and within each iteration, the partial derivative of each UE is calculated separately, resulting in a complexity scaling $\mathcal{O}(KL_{\max})$. For the middle layer, the number of iterations will change according to the required accuracy, and therefore, it is a function of the error limit ϵ. We denote the number of iteration times as T_ϵ and the complexity scaling of this layer should be $\mathcal{O}(T_\epsilon)$. Finally, for the outer layer, the complexity scaling of the bisection search is $\mathcal{O}\left(\log_2\left(\Omega_{\max}^{\text{dB}} - \Omega_{\min}^{\text{dB}}\right)\right)$. Putting all three layers together, the overall complexity of B-GAIP is of the order:

$$\mathcal{O}\left(KL_{\max}T_\epsilon \log_2\left(\Omega_{\max}^{\text{dB}} - \Omega_{\min}^{\text{dB}}\right)\right). \tag{21}$$

Qualitatively, as discussed before, in the scenario with large number of BS antennas and UEs, it becomes time-consuming to determine whether the boundary limitations are violated. Fortunately, this difficulty is circumvented in Algorithm 1 as it converts the engineering constraints into penalty items and therefore transfers a constrained optimization problem to an unconstrained one, greatly reducing the complexity, especially when the system dimension is large. In the meanwhile, Algorithm 2 utilizes a bisection approach which reduces the search

Algorithm 1: Gradient Assisted Interior Point Method

Parameters: initial penalty factor $\xi^{(0)}$; coefficient c; error limit ϵ; maximum loop count $L_{\max}$; step size t_l
Input: Ω, channel coefficients
Output: $\mathbf{p}_{\text{opt}}$ and $U_{\text{opt}} = U(\mathbf{p}_{\text{opt}})$

Randomly choose the initial $\mathbf{p}^{(0)}$ from the feasible set;
Set initial penalty function value $\varphi(\mathbf{p}^{(0)}, \xi^{(0)})$ using (18);
Set iteration index $i = 0$;
do
  $\mathbf{p}_{\text{curr}} = \mathbf{p}^{(i)}$; $\varphi_{\text{opt}} = \varphi(\mathbf{p}^{(i)}, \xi^{(i)})$;
  for $l = 1$ **to** $L_{\max}$ **do**
    Calculate $\mathbf{g}_l = \nabla\varphi(\mathbf{p}_{\text{curr}}, \xi^{(i)})/||\nabla\varphi(\mathbf{p}_{\text{curr}}, \xi^{(i)})||$;
    Update power vector as $\mathbf{p}_{\text{next}} = \mathbf{p}_{\text{curr}} + t_l \mathbf{g}_l$;
    if $\varphi(\mathbf{p}_{\text{next}}, \xi^{(i)}) > \varphi_{\text{opt}}$ **then**
      Set $\mathbf{p}_{\text{curr}} = \mathbf{p}_{\text{next}}$;
      Set $\varphi_{\text{opt}} = \varphi(\mathbf{p}_{\text{next}}, \xi^{(i)})$;
    end
  end
  i++; $\xi^{(i)} = \xi^{(i-1)} * c$;
  $\mathbf{p}^{(i)} = \mathbf{p}_{\text{opt}}$; $\varphi(\mathbf{p}^{(i)}, \xi^{(i)}) = \varphi_{\text{opt}}$;
while $\left|\frac{\varphi(\mathbf{p}^{(i)}, \xi^{(i)}) - \varphi(\mathbf{p}^{(i-1)}, \xi^{(i-1)})}{\varphi(\mathbf{p}^{(i-1)}, \xi^{(i-1)})}\right| > \epsilon$;

time exponentially compared with the intuitive linear search method. Accordingly, Algorithm 1 and 2 are more efficient than heuristic solutions and applicable for large scale systems.

VI. Simulation Results

We resort to system-level simulations of an uplink MIMO system to numerically evaluate the B-GAIP algorithm. Common simulation parameters can be found in Table I.

A. Comparison between B-GAIP and Brute Force Search

It is crucial to verify whether the proposed B-GAIP algorithm indeed converges to the globally optimal solution of the original problem, which can be obtained by a brute force search. We choose a small system dimension with 2 UEs and 4 BS antennas due to the high complexity of brute force search. We try all possible transmit power values and the LNA gain in dB domain with 0.1dB and 1dB step-size, respectively. We perform 2000 realizations of the channel parameters including UE positions, fast fading and shadow fading. Fig. 3 illustrates the average maximum energy efficiency obtained by the B-GAIP algorithm and the brute force search under different cell radii. We can see that the B-GAIP algorithm has the same energy efficiency as the brute force solution and therefore converges to the optimal value.

B. Comparison to Heuristic Algorithms

In addition, we compare B-GAIP with two commonly adopted heuristic algorithms [5]. Intuitively, higher transmit power and LNA gain shall result in higher SNR, and therefore higher energy efficiency performance. Correspondingly, we consider the following two heuristic algorithms: (1) use the

Algorithm 2: The B-GAIP Algorithm

Input: $\Omega^{\text{dB}}_{\text{min}}$, $\Omega^{\text{dB}}_{\text{max}}$ and channel coefficients
Output: optimal power allocation vector $\mathbf{p}_{\text{opt}}$; optimal LNA gain $\Omega^{\text{dB}}_{\text{opt}}$; global maximum energy efficiency U_{max}

Set $\Omega^{\text{dB}}_{\text{left}} = \Omega^{\text{dB}}_{\text{min}}$ and $\Omega^{\text{dB}}_{\text{right}} = \Omega^{\text{dB}}_{\text{max}}$;
while $\Omega^{\text{dB}}_{\text{left}} \neq \Omega^{\text{dB}}_{\text{right}}$ **do**
  $\text{LB} = \lfloor(\Omega^{\text{dB}}_{\text{left}} + \Omega^{\text{dB}}_{\text{right}})/2\rfloor$;
  $\text{UB} = \lceil(\Omega^{\text{dB}}_{\text{left}} + \Omega^{\text{dB}}_{\text{right}})/2\rceil$;
  if *LB* == *UB* **then**
    Set $\text{UB} = \text{UB} + 1$;
  end
  $(\mathbf{p}_{\text{opt1}}, U_{\text{opt1}}) \leftarrow$ **Algorithm 1** with input $\Omega^{\text{dB}} = \text{LB}$ and channel coefficients;
  $(\mathbf{p}_{\text{opt2}}, U_{\text{opt2}}) \leftarrow$ **Algorithm 1** with input $\Omega^{\text{dB}} = \text{UB}$ and channel coefficients;
  if $U_{\text{opt1}} > U_{\text{opt2}}$ **then**
    Set $\Omega^{\text{dB}}_{\text{right}} = \text{LB}$;
  else
    Set $\Omega^{\text{dB}}_{\text{left}} = \text{UB}$;
  end
  if $\Omega^{\text{dB}}_{\text{left}} == \Omega^{\text{dB}}_{\text{right}}$ **then**
    Set $\Omega^{\text{dB}}_{\text{opt}} = \Omega^{\text{dB}}_{\text{left}}$; $U_{\text{max}} = \max\{U_{\text{opt1}}, U_{\text{opt2}}\}$;
    Choose $\mathbf{p}_{\text{opt}}$ according to U_{max};
  end
end

TABLE I
SIMULATION PARAMETERS

Background noise σ_N^2	-104 dBm
Shadowing V	Log-normal with 8dB standard deviation
ADC noise σ_{ADC}^2	-60 dBm
LNA gain range $[\Omega^{\text{dB}}_{\text{min}}, \Omega^{\text{dB}}_{\text{max}}]$	[1, 70] dB
Diversity gain A_d	1
Circuit power P_c	0.1 W
Power amplifier efficiency η	50%
Maximal transmit power P_{max}	20 dBm
Maximal ADC input power $P^{\text{ADC}}_{\text{max}}$	-20 dBm
Maximal SNR Γ_{max}	35 dB
Step size t_l	$0.01/l$

maximal LNA gain combined with the corresponding optimal transmit power vector; and (2) use the maximal transmit power vector combined with the corresponding optimal LNA gain. Note that the engineering constraints in (12), (13), and (14) are still enforced when using these heuristic algorithms.

Fig. 4 illustrates the energy efficiency performance of three algorithms under small system dimensions with $K = 2$, $M = 4$. The performance gap is significant, suggesting that invoking B-GAIP to solve the original optimization problem is necessary. Statistically, at the maximum point, the average maximum energy efficiency value achieved by B-GAIP is 158% higher than that of the heuristic algorithm; while at the minimum point, the advantage is still 40%.

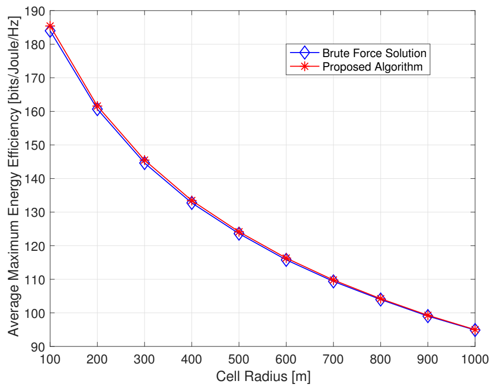


Fig. 3. Energy efficiency comparison between the B-GAIP algorithm and the brute force solution.

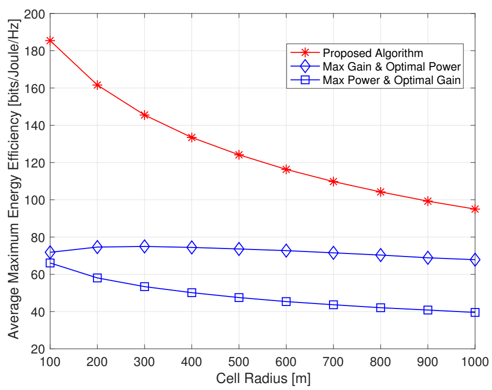


Fig. 4. Comparison between B-GAIP and the heuristic algorithms.

C. Comparison between Shared LNA and Separate LNA

Our system model uses one shared LNA to amplify the BS received signals in order to save both implementation cost and power consumption. One natural question is how much performance sacrifice we are incurring compared with using a separate LNA for each RF chain. In this subsection, we aim to address this question via system simulations. In particular, we adopt a small system dimension where the numbers of UEs and the BS antennas are set to 2, and the optimal gain values for all separate LNAs are chosen by brute force algorithm.

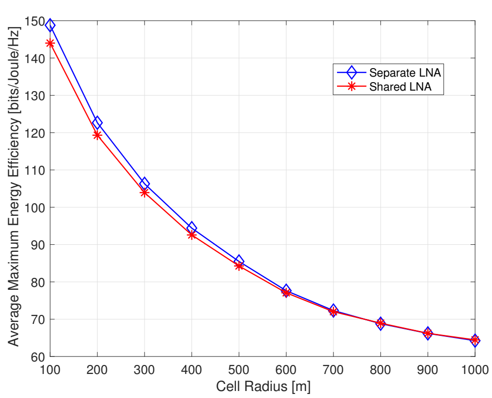


Fig. 5. Comparison between shared LNA and separate LNA structures.

Fig. 5 reports the comparison of energy efficiency with separate and shared LNA structures. We conclude from the figure that while the separate LNA structure achieves a better performance, using a shared LNA structure can very closely approach the performance of the separate LNA. Taking a deeper look at the statistics, we have that the maximum performance loss is only 3.21. It is worth noting that we use the same circuit power, i.e., $P_c = 0.1$ W, in both LNA structures, while in reality the separate LNA structure should have more power consumption than the shared LNA structure, which may further degrade its energy efficiency. As a result, using a shared LNA can significantly reduce the hardware cost and power consumption, while sacrificing very little energy efficiency. This result sheds important light on the design of RF front-end power amplifiers in practical MIMO systems.

VII. Conclusions

In this paper, we have proposed a *shared LNA* structure and showed that combined with low-resolution ADCs, this architecture saves hardware costs and reduces power consumption, while achieving near-optimal performance. In particular, we formulated the energy efficiency maximization problem under practical constraints, and proposed the Bisection – Gradient Assisted Interior Point (B-GAIP) algorithm that solves the optimization problem precisely and efficiently. Simulation results validated the convergence and effectiveness of B-GAIP.

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