Growing TTULPs Through Your Lessons

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ABSTRACT: *All students have strengths, which can be leveraged through universally designed instruction. We share a lesson planning tool, called the Thinking Through Universal Lesson Protocol (TTULP), that teachers used to create and enact mathematics lessons that support students with and without learning disabilities.*

The National Council of Teachers of Mathematics (2014) takes a clear stance that each and every student should have rich experiences in mathematics classrooms. Every student, including those with and without learning disabilities, has learning strengths; hence, instruction should leverage those strengths to foster deep learning (Kobett & Karp, 2020). "It is more productive to understand these students [with disabilities] as having developmental or learning differences…By focusing on access, we [educators] shift our gaze from identifying the 'problem' within the student to identifying the issues of access in the instruction or mathematics itself" (Lewis, 2018, pp. 37-38).

In 2015, the Every Student Succeeds Act noted that Universal Design for Learning (UDL) is an evidence-based, scientifically valid framework that fosters access to instruction for all children (Every Student Succeeds Act, 2015). A key step in fostering rich mathematics for each and every student is implementing mathematics tasks with high cognitive demand and multiple entry points (i.e., low-floor and high-ceiling tasks). Such tasks give students the opportunity to problem solve and engage in reasoning and critical thinking (Smith et al., 2008). Smith and her co-authors provide a comprehensive tool called the Thinking Through a Lesson Protocol (TTLP) that aims to help teachers make intentional choices about tasks and their implementation, and to encourage productive reflection that leads to action (Smith et al., 2008).

Our team used the TTLP as part of yearlong school-based professional development, and teachers found it helpful and rewarding. A goal within that professional development was to foster teachers' lesson planning so that it incorporates engagement in the Standards for Mathematical Practice (Common Core State Standards Initiative [CCSSI], 2010) for each and every student. Mr. Charlie, one of the intervention specialists who taught mathematics in self-contained and co-taught classes, expressed a concern about the TTLP during the professional development: "I like the TTLP because it helps me think about problem solving. How can I facilitate better instruction for students with learning disabilities? Where is UDL in the TTLP?" Other teachers agreed with Charle. They felt the current TTLP did not explicitly connect to UDL and in turn provide access for all learners, especially those with learning differences. Ms. Laura, a fifth-grade teacher, asked "What could we do to integrate

UDL into the TTLP? Can we change it?" To that end, we set out to revise the TTLP to encourage lessons that foster greater access for each and every learner, drawing on their strengths and learning differences. We describe a way that teachers leveraged a revised lesson planning guide to integrate UDL into their mathematics lessons. This revised lesson planning protocol helps teachers attend to the needs of diverse learners, aligns with UDL principles, and encourages use of rich mathematical tasks.

UDL: A Primer

The fundamental premise behind UDL is that lessons can be planned in ways that readily adapt to the variability among students, rather than expecting students to adapt to – or teachers to retro-fit – lessons to meet students' needs. The UDL Guidelines (CAST, 2018) offer a framework that teachers can apply across disciplines to ensure that all learners participate in meaningful, challenging learning opportunities. These guidelines are organized across three UDL principles: (a) multiple means of engagement, (b) multiple means of representation, and (c) multiple means of action and expression (see Figure 1).

<<Insert Figure 1 about here>>

Figure 1. The UDL principles and associated guidelines distributed through an Accessibility–Expertise Continuum. Questions under each guideline represent checkpoints that can guide teachers in lesson planning. Figure adapted from CAST (2018), Universal Design for Learning Guidelines version 2.2. available at http://udlguidelines.cast.org

Within each principle, the guidelines progress from suggestions that foster access to tasks (second column), to options that enhance students' learning within tasks (third column), and finally to practices and skills that help students become expert learners within and beyond tasks (fourth column; CAST, 2018). You can learn more about UDL through the videos, starting with an overview in[SD1] video 1 [SD2]and the UDL principle in video 2[SD3].

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Video 2. The UDL principle explained.

For example, the engagement principle draws teachers' attention to the affective dimensions of learning that impact students' motivation. First, to ensure access, teachers must recruit students' interest. Next, to build students' ability to learn, teachers need to weave options into tasks that encourage students' effort and persistence. Finally, to help students develop internalized engagement, teachers need to incorporate components in lessons that enhance self-regulation. This pattern is mirrored in the UDL principle: video 3, where UDL Guidelines start with access options that ensure all learners can perceive new information, move to options that

build students' capacity to use academic language and symbols within tasks, and finally develop students' comprehension through helping them internalize key patterns represented in learning tasks. In the UDL principle: video 4, *access* emphasizes options for physical movement within tasks, *building learning* emphasizes options for ways students communicate during tasks, and *internalization* emphasizes options to encourage students' executive functions. These guidelines are helpful; yet, they are not necessarily clear for mathematics teachers seeking to incorporate UDL into daily instruction.

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Video 3[SD4]**.** Multiple means of representation.

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Video 4.[SD5] Multiple means of action and expression.

Thinking Through Universally-designed Lessons Protocol (TTULP)

Revisions to the TTLP occurred through a process of collaboration with educators across multiple districts. As a result of this work, we share the Thinking Through Universally-designed Lessons Protocol (TTULP; pronounced like the word "tulip") that is shown in the online Appendix (see supplementary material[SD6]s). The revision process started as a mathematics educator, Jonathan, and intervention specialist, Brooks, reviewed the UDL guidelines and the TTLP. Teachers provided insight on how they saw the two merging, which led to an initial TTULP. Then, teachers used the TTULP for one month and offered suggestions for future improvements. Those improvements were made and ultimately, teachers across multiple districts voluntarily chose to use it to better foster access and equity while addressing learning differences in their classrooms. This was a collaborative experience and one upper elementary/middle grades teacher, Tim, shares details about how to integrate the TTULP into your instruction. The ideas apply to all grade levels; Tim and a high school math teacher, Sarah, provide further discussion in videos they made for readers.

There are three parts to this article. First, we describe revisions from the TTLP to the TTULP and the three parts of the TTULP. Next, we highlight a few parts within each facet of the TTULP. Then, we share teachers' reflections from it. Finally, we offer suggestions for those using TTULP for the first time.

Revising the TTLP to become the TTULP

There are three parts to the TTULP. Part one focuses on task selection and preparing for its instructional use. Part two is designed for thinking about students' task engagement. Part three involves how students communicate their mathematical work, particularly with peers. While these three parts align with the three UDL

principles noted in Figure 1, we decided that the actual prompts in the original TTLP could be modified to make connections with the UDL framework even more clear. We describe the changes made and rationale for them in the following paragraphs.

Part 1 included a few revisions around task selection and preparation for instruction. One additional question was added to address student errors in thinking. Teachers commented that prior experiences informed common error patterns so thinking ahead has potential to increase students' comprehension (see Figure 1). Another three questions were added about student expectations and competition (e.g., How will students show their understanding of the topic?). Teachers felt a strong need to better address the UDL components of language and expression as well as expression and communication. Collectively, these additional questions were useful for promoting three different UDL aspects.

Part 2 revisions about students' problem solving were intended to promote mathematical work during the task. Minor revisions were made to encourage multimedia use (UDL: expression and communication) such as "by providing auditory or visual support". Promoting transfer beyond the current lesson also guided the addition of "What questions will you ask to help students make connections beyond the current lesson?" Promoting active and consistent engagement in the task through multiple questions were intended to help students internalize expertise (e.g., What activities will you do to build academic stamina? What will you do to guide students self-monitoring and reflection?). General and special education teachers both agreed that it was important to add aspects that encourage engagement and self-actualization while problem solving.

The third part of the TTULP had just one revision, "Understand the key ideas that you want them to learn". Teachers and us agreed that it was important to consider questions that drive at the key ideas within a specific lesson. While one lesson may include many important topics, special education teachers commented that some students might struggle to understand what they should take away from the lesson and transfer to other topics. Teachers agreed that helping students recognize the mathematical residue (Davis, 1992) was important. This mathematical residue may consist of mathematical procedures, patterns of reasoning, mathematical structures, or other essential topics. Helping students perceive the key ideas and make connections to prior knowledge offers more students greater accessibility to the classroom learning.

Highlighting facets of the TTULP

The focus in part 1 of the TTULP is selecting a rich mathematics task that has a low-entry point and a high-ceiling. Teachers said that carefully considering learning goals pushed lesson planning in ways that naturally addressed UDL. They considered multiple forms of assessment that could demonstrate students'

understanding of the task. “If I [Mr. Duard] ask myself to think about my students and my objectives first, then naturally I wonder how I can structure my teaching and assessment so that students have choices.” In part 2 of the protocol, the focus turns from task selection to students’ exploration. Teachers felt it was important to offer differentiated supports so that each student felt increased independence during the task. They also agreed that the UDL guideline about sustaining effort and persistence connected with Mathematics Teaching Practice “Support Productive Struggle in Learning Mathematics” (NCTM, 2014, p. 48) and Standard for Mathematical Practice #1, “Make sense of problems and persevere in solving them” (CCSSI, 2010, p. 6). In part 3 of the protocol, the focus is communication. The original TTLincluded options for communication, so few revisions were made. One revision that teachers wanted was to focus class’ discussion around the conceptual understanding embedded in tasks, not just solution paths. The subsequent question seen in the TTULP is to consider “What is the order of ideas that will be shared or presented and why that order?” Ms. Caridad told us after using the TTULP for a few months that “Planning ahead for what ideas students might have helped me better structure class discussions around each and every students’ strengths and learning differences…I can better help them use prior knowledge for my current lesson and even check their thinking.” Thinking about part 3 in this manner connected to ways to internalize learning (see figure 1). In sum, adjustments in lesson planning using the TTULP guided teachers to better engage in UDL-focused instruction that provided better access for all students while still focusing on high cognitive demand tasks.

Example of Incorporating UDL into A Lesson with the TTULP

Tim, a classroom teacher, implemented the Locker Task with students across grades four through six and used the TTULP in his planning process to best address each and every student’s strengths. This section describes his use of the TTULP with one task through his perspective.

The Locker Task is often presented in the context of 100 lockers, but it can be adapted to include as many lockers as desired. One version of this task states:

> There are 100 students in the fourth grade and they decide to play with their lockers. At the beginning of the game, every locker is closed and unlocked. The first student opens every locker. The second student closes every other locker, starting with the second locker. The third student changes the state of every third locker, starting with the third locker. Changing the state implies that open lockers are closed while closed lockers are opened. This process continues until all 100 students have had a chance to play the game. What lockers are open? What lockers are closed?

A simpler but parallel-in-nature version of the Locker Task uses 20 lockers, which makes it feasible to solve the task by physically completing all iterations of opening and closing locker doors (e.g., https://illustrativemathematics.org/math-curriculum/k-5-math/). Factors and multiples are new concepts for

students in grade four (see CCSSI, 2010) and can be taught effectively with concrete representations such as actual lockers or two-sided counters. However, students in grade six are expected to apply these concepts more abstractly when analyzing equivalence among algebraic expressions. Changing the number of lockers in the task allows teachers to present a version of the task that is developmentally appropriate for their students, yet still aligns to standards 4.OA.4 and/or 6.NS.B4 of the Common Core State Standards in mathematics (CCSSI, 2010). The fourth-grade standard indicates that students should be able to

> Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite (CCSSI, 2010, p.29).

The sixth-grade standard extends learning about factors and multiples from fourth grade

> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor (CCSSI, 2010, p.42).

The Locker Task has potential to deepen student understanding of factors and multiples. Factors and multiples are often taught procedurally and in isolation from other topics, as seen in the textbooks used by teachers involved in our professional development. This superficial instruction can, in turn, lead to misconceptions such as when middle school students misconstrue the terms *greatest common factor* and *least common multiple* as greatest common multiple and least common factor.

Just as Smith et al. (2008) described, teachers should not necessarily respond to every question for every lesson. They may intentionally select questions from the TTULP to build a repertoire of universally designed lessons and focus their planning for a given lesson. To illustrate this use of TTULP, we highlight three TTULP questions that Tim felt were particularly helpful while planning to teach these standards using the Locker Task.

TTULP Part 1, Question 3

What resources or tools will students have to use in their work that will meet the needs of different learners?

This question draws attention to the UDL principle of action and expression. Tim chose this question because it prompted him to anticipate how students may be able to use different tools or resources to engage in, and navigate, the problem. He made video 5 to explain why he chose this TTULP question. For example, he provided access to manipulatives such as two-color counters in the Locker Task. The use of manipulatives gave all students a tool that may help with the construction of their responses, which in turn represents the UDL Guideline for Expression and Communication. This in turn has potential for students to use appropriate tools

strategically, which promotes the fifth Standard for Mathematical Practice (CCSSI, 2010). Even with access to tools for constructing the response, teachers may have to guide students with questions that foster thinking about strategic tool use. Tim asked his students, "What does the red side of the counter represent in the problem?" and, "How did that manipulative help you think about the problem?" Teacher guidance is important in helping students decontextualize the task. Decontexualization is a process of stripping away the problem's situational context and generating a mathematical representation, which students can act on using a known mathematical procedure. The teacher may also need to support students while they contextualize their results. Contextualization implies the process of pushing a mathematical result back into the situational context of a mathematics problem. In this case, it means determining whether the numbers represent open or closed lockers. Grade four students may simply need to recall what the color-counter chip represents in the context of the task. Grade six students may require guiding questions such as, "What do the factors of each number represent in regards to the problem?" This decontextualization and contextualization in turn, promotes the second Standard for Mathematical Practice, "Reason abstractly and qualitatively" (CCSSI, 2010, p. 6).

<KGL: INSERT 'Locker problem video screenshot.tiff' HERE>

Video 5. Tim explains why he chose this TTULP question.

Tim also chose to provide students with access to a multiplication chart. Because the goal of the lesson centers on the relationship between factors and multiples, not calculation, tools that foster strategic thinking may help students better engage in the task. A multiplication chart – as opposed to a four-function calculator – may better help students see 20 as a multiple of *both* 4 and 5 and therefore recognize 4 and 5 are factors of 20. The multiplication chart was helpful in both the fourth- and sixth-grade classrooms. In this case, a multiplication chart did not reduce the cognitive demand of the task, but rather serves as a scaffold for the concept development by providing access to tools students might leverage while problem solving and further promote the fifth Standard for Mathematical Practice.

TTULP Part 2, Question 3

What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance in solving the task?

While the questions noted in this section were designed to guide students in concept development, the assistance prompted in this TTULP question aligns with the UDL principle of Engagement. Specifically, the UDL Guideline for Sustaining Effort and Persistence encourages teachers to guide frustrated students and

optimize their progress during a task. Tim asked students questions meant to foster engagement while implementing the Locker Task.

- What goal(s) are we trying to accomplish in the problem?
- Let's just focus on the first question, "What lockers remain open?" How might we determine that? How could we act out the problem?
- Could you and your classmates find tools in order to act out the problem?

These questions acknowledge that the task itself may appear too daunting in its entirety at first, requiring more strategic competence and adaptive reasoning than students may be ready for. But, because this sort of frustration can be anticipated and fostering productive struggle, the UDL Guidelines incorporated into TTULP recommend questions that scaffold students through planning in a way that maintains high cognitive demand in the task while still allowing for a low-floor entry into the task. For example, helping students focus on a single question posed by the Locker Task may assist in optimizing an appropriate level of challenge for them. Some students in Tim's classroom also asked if they might physically open and close a set of twenty lockers to solve the problem. Students may need additional guidance in organizing tools to represent the lockers and or working with peers to facilitate problem solving.

TTULP Part 3, Question 1

What ideas do you want to have shared during the class discussion? What order will ideas be shared/presented? Why that order?

This question draws attention to the UDL principle of representation. The "sharing of ideas" referenced in TTULP refers to how the lesson may highlight patterns represented in the task. Tim chose to respond to this question because it encouraged him to develop a plan for the sharing of student solutions. More specifically, he considered how the sequencing of solution strategies would help make meaningful connections between concrete representations of the problem and the concepts of factors and multiples.

<< INSERT FIGURE 2 ABOUT HERE >>

Figure 2. Students' representation of the locker problem with two-color counters.

Tim expected the initial discussion to focus on which lockers remain open. This became an opportunity to discuss the use of tools, validating the use of concrete solution strategies to solve mathematical problems. He purposefully selected those who used concrete representations for discussion initially because he wanted to draw out concrete representations, followed by pictures and tables, and ending the discussion with abstract representations and their connections to the task. A goal was to promote students' reflection on the mathematical representations and procedures. Figure 2 depicts the work of a group of students that developed a concrete

representation of the locker problem using two-color counters. These students chose to represent open lockers with the red-side of the counters, and closed lockers are represented with the white-side of the counter. This representation was unique in the way the students drew a grid to look for patterns while completing all iterations of the task. The grid's X-axis displays the locker number, and the grid's Y-axis keeps track of how many individuals touched each locker. This method helped the students conclude that open lockers (i.e., the red counters) were touched by an odd number of people. Thus, when Tim considered the sequence of solution strategies to be shared, this group was purposely selected to share their ideas immediately prior to more abstract representations. This in turn helped elicit the connections between concrete representations and abstract representations of the locker problem.

Next, the teacher should build the discussion towards an intended mathematical structure or pattern. Tim encouraged students to consider the commonalities of open lockers, connecting solution strategies with the underlying mathematics structures (e.g., factors and multiples, prime and composite numbers, as well as even and odd numbers). With this task, it is likely that students will consider possible commonalities while they are still working on the task. This is especially true for students working with a large number of lockers. Early on in the task with 20 lockers, some students claimed that, "odd lockers will stay open, and the even lockers will be closed." As students problem solve, they are likely to determine that the open lockers were touched by an odd number of people. It is possible for students to solve the problem without any consideration of factors or multiples, specifically if the teacher opted for only 20 lockers in the riddle. Tim was concerned about this. If students can only describe representing open lockers as one side of the counter and closed lockers as the reverse side, then they may have a superficial understanding of concepts represented in this task. To overcome this, he posed a few questions:

- Suppose there are 30 lockers and 30 people, how could we determine the status of the lockers without using the color-counters?
- List all the people who will open or close locker 25, what do you notice?
- What lockers are touched exactly twice?

This extension to the problem helped students recognize that they are thinking about multiples when considering which lockers a person opened or closed, but are thinking about factors when considering who opened or closed a specific locker. It is through a planned, purposeful, orchestrated discussion of these patterns that students may develop a deeper conceptual understanding of the underlying mathematical structures and how they are related.

Final Thoughts: Strengths-based Instruction and TTULP

All students have the right to rich mathematics experiences that engage them in the Standards for Mathematical Practice. Each student brings strengths to the classroom that can be leveraged to promote every students' mathematical proficiency (Kobett & Karp, 2020). Put simply, mathematics instruction should foster meaningful learning for each and every student. While we conveyed details about Tim's work with thee TTULP, the ideas in this article transcend across grade levels. Sarah has been using the TTULP in her high school algebra (audio 1), geometry, algebra 2, and pre-calculus courses. She shares information about using the TTULP in her video. Our goal in this article is to help teachers attend to the needs of diverse learners throuh the UDL principles, and to engage students in rich mathematics tasks.

<KGL: INSERT 'High school TTULP video screenshot.tiff' HERE[SD7]>

Audio 1[SD8]**.** Sarah explains why she's using the TTULP in her high school algebra, geometry, algebra 2, and precalculus courses.

The TTULP is grounded in UDL and in turn, supports a strengths-based approach to teaching. UDLing lessons, like Tim and Sarah did, fostered students' engagement in rich tasks that promote problem solving and reasoning. In Tim's case, it was about important mathematical structures like factors and multiples. Students who wanted a resource provided through the UDL'd lesson - such as simulating lockers opening and closing – used the resources strategically. Those who did not want the resource were able to demonstrate their understanding and there was no change in the cognitive demand or rigor. Tim and Sarah witnessed that each and every student in their classes demonstrated understanding of the desired grade-level standards after UDLing lessons. Encouraging multiple forms of engagement, representation, and action and expression in lessons through the TTULP has power to foster more inclusive learning for each and every child.

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Watch Video 1: UDL overview.

[Here is an example from April 2021's Front and Center:]

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