Chapter 18
UV Origin of Discrete Symmetries

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Abstract We discuss the possible UV origin of discrete symmetries. We review the (i) interpretation of discrete R symmetries as discrete remnants of the Lorentz group; (ii) additional discrete transformations arising in orbifold compactifications, some of which have only been found recently; (iii) the stringy/gauge origin of family symmetries; (iv) $\mathcal{CP}$ violation from strings. These notes are based on an invited talk by the author at FHEP 2019 in Hyderabad.

18.1 Discrete Symmetries in Particle Physics

Discrete symmetries play a key role in our understanding of particle physics. The perhaps most prominent examples are the discrete transformations $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$. There are many more examples such as the matter parity in supersymmetric models (a.k.a. R parity), and the left–right parity of left–right symmetric and Pati–Salam models. In addition, attempts to solve the flavor puzzle often utilize discrete symmetries.

This raises several questions. How reliable are these symmetries? Are they also symmetries of the quantum theory? Our current understanding strongly suggests that all discrete symmetries need ultimately to be gauged [1]. Therefore it is imperative to seek a better understanding of the UV origin of discrete symmetries.

Mathematically all continuous gauge symmetries entail extra dimensions. That is, they correspond to "movements" along the fiber of a fiber bundle. Moreover, in strings e.g. the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ may be thought of being the result of 16 extra dimensions compactified on a Narain lattice [2]. These observations suggest that one may obtain a similar understanding (or interpretation) of discrete symmetries.

The original version of this chapter was revised: The corrections have been incorporated throughout the chapter. The correction to this chapter is available at https://doi.org/10.1007/978-981-15-6292-1_62

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A. Giri and R. Mohanta (eds.), *Workshop on Frontiers in High Energy Physics 2019*, Springer Proceedings in Physics 248, https://doi.org/10.1007/978-981-15-6292-1_18

An obvious option is to obtain discrete symmetries by breaking a continuous gauge symmetry (spontaneously) to a discrete subgroup. The emerging discrete symmetry is then clearly gauged. This breaking can occur by a field acquiring a vacuum expectation value (VEV), or by compactification, which is also a spontaneous breaking (if done consistently), just not in four dimensions.

18.2 Discrete Symmetries to Complete the MSSM

Let us start with a discussion of the role of discrete symmetries in the context of the minimal supersymmetric extension of the standard model (MSSM). Discrete R symmetries are instrumental for supersymmetric phenomenology. To see this, consider the most general superpotential that is consistent with the standard model gauge symmetries up to dimension 5,

$$\begin{aligned}\mathscr{W}_{\text{gauge invariant}} &= \mu\,\boldsymbol{h}_d\boldsymbol{h}_u + \kappa_i\,\boldsymbol{\ell}_i\boldsymbol{h}_u \\ &\quad + Y_e^{gf}\,\boldsymbol{\ell}_g\boldsymbol{h}_d\boldsymbol{e}_f^{\mathcal{C}} + Y_d^{gf}\,\boldsymbol{q}_g\boldsymbol{h}_d\boldsymbol{d}_f^{\mathcal{C}} + Y_u^{gf}\,\boldsymbol{q}_g\boldsymbol{h}_u\boldsymbol{u}_f^{\mathcal{C}} \\ &\quad + \lambda_{gfk}\,\boldsymbol{\ell}_g\boldsymbol{\ell}_f\boldsymbol{e}_k^{\mathcal{C}} + \lambda'_{gfk}\,\boldsymbol{\ell}_g\boldsymbol{q}_f\boldsymbol{d}_k^{\mathcal{C}} + \lambda''_{gfk}\,\boldsymbol{u}_g^{\mathcal{C}}\boldsymbol{d}_f^{\mathcal{C}}\boldsymbol{d}_k^{\mathcal{C}} \\ &\quad + \kappa_{gf}\,\boldsymbol{h}_u\boldsymbol{\ell}_g\,\boldsymbol{h}_u\boldsymbol{\ell}_f + \kappa_{gfk\ell}^{(1)}\,\boldsymbol{q}_g\boldsymbol{q}_f\boldsymbol{q}_k\boldsymbol{\ell}_\ell + \kappa_{gfk\ell}^{(2)}\,\boldsymbol{u}_g^{\mathcal{C}}\boldsymbol{u}_f^{\mathcal{C}}\boldsymbol{d}_k^{\mathcal{C}}\boldsymbol{e}_\ell^{\mathcal{C}}\;. \end{aligned} \tag{18.1}$$

Here, the boldface math letters represent the MSSM superfields in a suggestive convention. The κ_i terms in the first line have to vanish, or at least to be very small. The so–called μ parameter needs to be roughly of the order of the electroweak scale, and will be discussed below in more detail. The couplings in the second line need to be all present since they are (up to threshold corrections and multiplication by the ratio of Higgs VEVs $\langle\boldsymbol{h}_u\rangle/\langle\boldsymbol{h}_d\rangle$) given by the Yukawa couplings of the standard model. On the other hand, the terms in the third line need to vanish or to be very small.[1] All the κ_i, λ_{gfk}, λ'_{gfk} and λ''_{gfk} terms may be forbidden by imposing R parity [3].[2] This raises the question of where this $\mathbb{Z}_2$ symmetry, which we will denote $\mathbb{Z}_2^{\mathcal{M}}$ in the following, comes from. What is more, and what is sometimes not appreciated very much, $\mathbb{Z}_2^{\mathcal{M}}$ is *not* the full story. Rather, some of the $\kappa_{gfk\ell}^{(i)}$ terms in the last line need to be suppressed as much as $\lesssim 10^{-8}/M_\mathrm{P}$ [5]. On the other hand, the $\kappa_{gf}\,\boldsymbol{h}_u\boldsymbol{\ell}_g\,\boldsymbol{h}_u\boldsymbol{\ell}_f$ term is the so–called Weinberg operator, and the leading candidate for an operator that gives rise to realistic, suppressed neutrino masses.

This raises the question of how one can control the dangerous operators while keeping the desired ones. It turns out that, under arguably rather moderate assumptions, the choices are highly restricted. In detail, let us make the following assumptions and requirements:

1. SO(10) unification of matter is not an accident;
2. the μ term is forbidden by a symmetry but appears after SUSY breaking;
3. want to preserve gauge coupling unification;

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[1] One may allow one of the λ_{gfk}, λ'_{gfk} or λ''_{gfk} to be relatively unsuppressed.

[2] Despite its name, this symmetry is not a true R symmetry, but equivalent to matter parity [4].

Table 18.1 $\mathbb{Z}_4^R$ charges

	$\boldsymbol{q}$	$\boldsymbol{u}^{\mathcal{C}}$	$\boldsymbol{d}^{\mathcal{C}}$	$\boldsymbol{\ell}$	$\boldsymbol{e}^{\mathcal{C}}$	$\boldsymbol{h}_u$	$\boldsymbol{h}_d$	$\boldsymbol{\nu}^{\mathcal{C}}$
$\mathbb{Z}_4^R$	1	1	1	1	1	0	0	1

4. standard model Yukawa couplings and Weinberg operator are allowed.

It turns out that, under these assumptions, the symmetry is unique [6, 7]: it is an order four R symmetry, $\mathbb{Z}_4^R$, which has been first proposed in [8].[3] The charge assignment is very simple, see Table 18.1.

They are obviously consistent with SO(10) grand unification, where matter fields sit in one irreducible representation, the **16**–plet, and the Higgs fields come from the **10**–plet.

It is instructive to see how anomaly matching [10, 11] works for $\mathbb{Z}_4^R$, or, more generally, $\mathbb{Z}_M^R$ symmetries. Assume you start from a unified gauge group, SU(5) or higher. Then, at this level there is only one anomaly coefficient,

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta \ , \tag{18.2}$$

where $A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}$ is the contribution from the matter fields, $A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}$ a possible extra contribution, and $5q_\theta$ the contribution from the gauginos.[4] Now assume some mechanism breaks SU(5) (or larger) down to the standard model gauge symmetry $G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU2}_\mathrm{L} \times \mathrm{U}(1)_Y$. Then the anomaly coefficients become

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + g2\frac{1}{2}\cdot 2\cdot 2\cdot q_\theta \ , \tag{18.3a}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + g4\frac{1}{2}\cdot 2\cdot 3\cdot q_\theta \ . \tag{18.3b}$$

Here we have kept but crossed out the contributions from the extra gauginos which are in SU(5) but not G_{SM} (and which are sometimes called X and Y bosons). We see that if something breaks SU(5) down to G_{SM}, and removes the contributions from the extra gauginos, due to anomaly matching we need to have massless fields that do not come in complete GUT representations. That is, 't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

Where can such R symmetries come from? It can be shown with elementary group–theoretical methods that they can *not* arise from 4–dimensional models of grand unification [12]. In more detail, assuming (i) a GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$, (ii) GUT symmetry breaking is spontaneous, and (iii) there is

[3]The fact that only R symmetries can forbid the μ term has been motivated in [9].

[4]Our conventions are such that the superpotential has R charge 2.

only finite number of fields, one can show that one either has to break the R symmetry at the high scale, or has light exotic charged states.

What does light mean? Light means of the order of R symmetry breaking. Why and how is the R symmetry broken? R symmetries are necessarily broken because, in order to warrant an almost vanishing vacuum energy, the superpotential needs to acquire a VEV. This VEV determines the gravitino mass,

$$\langle \mathscr{W} \rangle \sim m_{3/2}\, M_{\mathrm{P}}^2 \quad \curvearrowright \quad m_{3/2} \sim \frac{\langle \mathscr{W} \rangle}{M_{\mathrm{P}}^2}\,. \tag{18.4}$$

However, as the superpotential carries R charge 2, there is a residual $\mathbb{Z}_2$ symmetry, which, in the case of $\mathbb{Z}_4^R$, coincides with matter parity. Let us briefly discuss the implications for the $\mathscr{W}_{\text{gauge invariant}}$ of (18.1). We see that the R parity violating couplings κ_i, λ_{gfk}, λ'_{gfk} and λ''_{gfk} are zero because of the exact residual symmetry. By construction, the standard model Yukawa couplings and Weinberg operator are allowed, and one can easily check that each of these terms carries R charge 2. What about the μ term and the $\kappa_{gfk\ell}^{(i)}$ couplings? They appear after R symmetry breaking. However, since the order parameter of R symmetry breaking is the gravitino mass, one finds that, in the framework of gravity mediation,

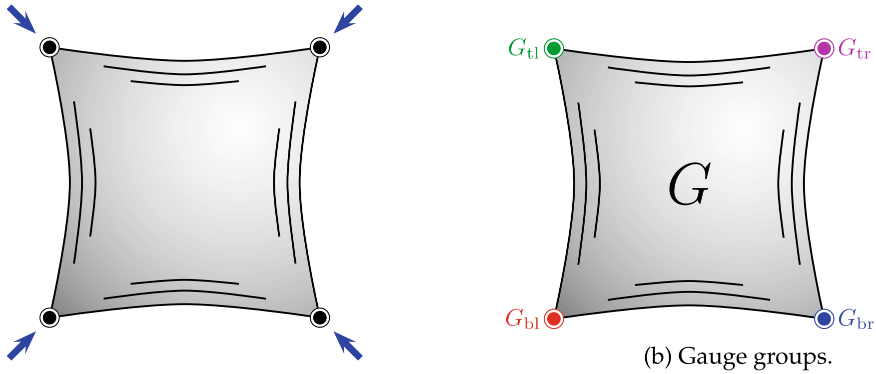
$$\mu \sim m_{3/2} \quad \text{and} \quad \kappa_{gfk\ell}^{(i)} \sim \frac{m_{3/2}}{M_{\mathrm{P}}^2} \ll \frac{10^{-8}}{M_{\mathrm{P}}}\,. \tag{18.5}$$

The statement on the μ term can be thought of as the Kim–Nilles [13] and Giudice–Masiero [14] mechanisms being at work, but the $\mathbb{Z}_4^R$ offers an explanation for why these are the *only* contributions.

Where can one get this $\mathbb{Z}_4^R$ from? As already mentioned, not from 4D GUTs. However, they do arise in orbifold compactifications of the heterotic string [15, 16], which we discuss in what follows.

18.3 Orbifold Compactifications of the Heterotic String

A toroidal orbifold emerges by dividing a torus by some of its non–freely acting symmetries. The resulting space is smooth everywhere except for the orbifold fixed points (cf. Fig. 18.1). In general, these fixed points are special points at which (a) the gauge symmetry gets broken (b) localized "matter" fields live. Just by looking at Fig. 18.1 it is tempting to suspect that orbifold compactifications have plenty of discrete symmetries. In fact, as we shall discuss in Sect. 18.4, additional discrete transformations arise, some of which have only been noted recently; 2. Section 18.5, family symmetries appear naturally; 3. Section 18.6, some compacifications have built–in $\mathcal{CP}$ violation. With regards to the discussion in Sect. 18.2, discrete R symmetries emerge as discrete remnants of the Lorentz group. The so–called H–momentum conservation rule [17, 18] can be interpreted as an R symmetry [19]. It turns out that it is



(a) Fixed points.

(b) Gauge groups.

Fig. 18.1 Cartoon of an orbifold

rather straightforward to construct an explicit string model with the exact spectrum of the MSSM and a residual $\mathbb{Z}_4^R$ symmetry [20, 21]. As discussed in Sect. 18.2, this symmetry is instrumental to understand why the Higgs pair is massless prior to supersymmetry breaking. The alert reader may wonder how a discrete remnant of the Lorentz symmetry may appear anomalous. This is because some the residual symmetries in orbifolds are diagonal subgroups of the symmetries of the upstairs theory. Of course, in string theory, these anomalous looking symmetries are never really anomalies, but cancelled by the Green–Schwarz mechanism [22].

18.4 Discrete Remnants of Orbifolding

Given the phenomenological success of the orbifold models, it is imperative to carefully analyze the residual symmetries these constructions have. The standard lore used to be that the surviving symmetry consists of the transformations that commute with the orbifold action. This turns out to be not entirely correct [23].

Consider a higher–dimensional gauge theory with gauge fields $V_a^\mu(x, y)\,\mathsf{T}_a^{(\mathrm{CW})}$, where $\mathsf{T}_a^{(\mathrm{CW})}$ denote the generators in the Cartan–Weyl basis. Now compactify on an orbifold with action on the extra coordinates y and generators

$$y \stackrel{P}{\mapsto} \vartheta\, y \quad \text{and} \quad \mathsf{T}_a^{(\mathrm{CW})} \stackrel{P}{\mapsto} P\,\mathsf{T}_a^{(\mathrm{CW})}\,P^{-1}\,. \tag{18.6}$$

We demand that performing first a gauge transformation and then the orbifold transformation, or reversing the order of the operations leads to the result,

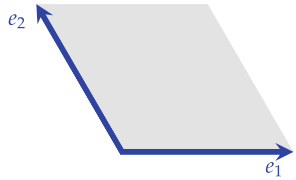


Fig. 18.2 Simple roots of SU(3)

$$
\begin{tikzcd}
V_a^\mu(x,y)\,\mathsf{T}_a^{(\mathrm{CW})} \arrow[r, mapsto, "P\in O"] \arrow[dd, mapsto, "U\in G"] & V_a^\mu(x,\vartheta^{-1}y)\,P\,\mathsf{T}_a^{(\mathrm{CW})}\,P^{-1} \arrow[d, mapsto, "U\in G"] \\
 & V_a^\mu(x,\vartheta^{-1}y)\,U\,P\,\mathsf{T}_a^{(\mathrm{CW})}\,P^{-1}\,U^{-1} \arrow[d, phantom, "\rotatebox{-90}{$\overset{!}{=}$}"] \\
V_a^\mu(x,y)\,U\,\mathsf{T}_a^{(\mathrm{CW})}\,U^{-1} \arrow[r, mapsto, "P\in O"] & V_a^\mu(x,\vartheta^{-1}y)\,P\,U\,\mathsf{T}_a^{(\mathrm{CW})}\,U^{-1}\,P^{-1}\,.
\end{tikzcd}
\tag{18.7}
$$

Using Schur's lemma, this leads to the condition that $P^{-1}U^{-1}P\,U$ is in the center of the group G [23], which is weaker than the traditional condition that P commutes with U.

Note that even if one demands that P commutes with U, some important symmetries have been missed in the past, with the perhaps most important example being the so–called left–right parity or D–parity of the Pati–Salam [24] or left–right symmetric model [25],

$$[\mathrm{SU}(4)\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}]\rtimes\mathbb{Z}_2\,. \tag{18.8}$$

It is known that this $\mathbb{Z}_2$ can be obtained in 4D SO(10) GUTs by giving a VEV to a **54**–plet [26, 27]. However, it has been only noted recently that this symmetry is automatically there if one breaks SO(10) by the action of a $\mathbb{Z}_2$ orbifold [23]. This symmetry illustrates a generic feature of these discrete remnants: they are typically outer automorphisms of the continuous residual gauge symmetry.

An example in which the fact that the "survival" condition is weaker than previously assumed is important in the $\mathbb{T}^2/\mathbb{Z}_3$ orbifold with an SU(3) gauge symmetry. Here the torus lattice coincides with the root lattice of SU(3) (Fig. 18.2). The associated gauge embedding is

$$P~=~\begin{pmatrix}\omega & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & 1\end{pmatrix}~\in~\mathrm{SU}(3)\;, \tag{18.9}$$

where $P^3 = \mathbb{1}$. The condition for unbroken gauge symmetries is

$$[P,U_{(k)}]~=~\exp\left(\frac{2\pi\,\mathrm{i}\,k}{3}\right)\mathbb{1}\quad\text{where}\quad k~\in~\{0,1,2\}\;. \tag{18.10}$$

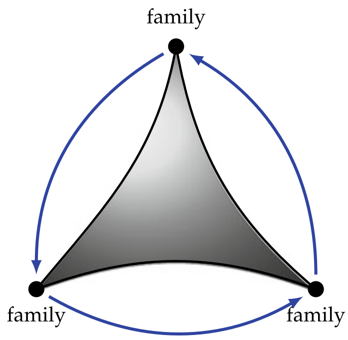


Fig. 18.3 Geometric origin of family symmetries

Therefore, the residual symmetries are

$$U_{(0)} = \begin{pmatrix} e^{i(\alpha+\beta)} & 0 & 0 \\ 0 & e^{i(\alpha-\beta)} & 0 \\ 0 & 0 & e^{-2i\alpha} \end{pmatrix} \quad \text{and} \quad U_{(1)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} . \tag{18.11}$$

So $\mathrm{SU}(3) \xrightarrow{\mathbb{Z}_3^{\mathrm{orb.}}} \Big[\mathrm{U}(1) \times \mathrm{U}(1)\Big] \rtimes \mathbb{Z}_3$. The explanation of the additional $\mathbb{Z}_3$ factor in terms of gauge symmetries completes the analysis by Beye et al. [28, 29], who explore the gauge origin of family symmetries in string theory. Away from the critical radius R_{crit} the U(1) symmetries get broken to $\mathbb{Z}_3$ subgroups such that

$$\mathrm{SU}(3) \xrightarrow{\mathbb{Z}_3^{\mathrm{orb.}}} \Big[\Big[\mathrm{U}(1) \times \mathrm{U}(1)\Big] \rtimes \mathbb{Z}_3\Big] \rtimes \mathbb{Z}_2 \xrightarrow{R \neq R_{\mathrm{crit}}} \Big[\Big[\mathbb{Z}_3 \times \mathbb{Z}_3\Big] \rtimes \mathbb{Z}_3\Big] \rtimes \mathbb{Z}_2 = \Delta(54) .$$

The $\mathbb{Z}_2$ factor is the outer automorphism of SU(3). This explicitly demonstrates the gauge origin of the full $\Delta(54)$ flavor symmetry. This will be discussed in more detail elsewhere.

18.5 Family Symmetries

As discussed in the previous section, family symmetries can arise from orbifolding. In fact, they arise very naturally in heterotic orbifolds [30, 31]. One way to understand how whey arise is to look at the geometry of compact space. As illustrated in Fig. 18.3 for the $\mathbb{Z}_3$ orbifold plane, the repetition of families may be related to the geometrical properties of the orbifold such as the existence of equivalent fixed points. It is then not too surprising that certain permutation symmetries arise. To obtain the full symmetry group, one has to work a bit harder. In general, they are obtained as the outer automorphism group of the space group [32, 33]. Even though the discussion at the end of Sect. 18.4 only concerns the $\Delta(54)$ symmetry, it strongly suggests that all the other flavor symmetries derived from string compactifications originate completely from gauge symmetries.

In what follows, we will discuss that the string–derived $\Delta(54)$ symmetry has another, rather surprising property.

18.6 $\mathcal{CP}$ Violation from Strings

It has been pointed out that there is a deep, group–theoretical connection between flavor symmetries and $\mathcal{CP}$ violation [34]. Certain discrete groups clash with $\mathcal{CP}$ conservation [34, 35] (see [36] for a recent review). There are simple group–theoretic indicators that allow one to tell $\mathcal{CP}$–violating groups from those which are consistent with $\mathcal{CP}$ apart [35]. All odd order non–Abelian finite groups clash with $\mathcal{CP}$, yet there are also even–order groups of that type, and intereresting the above–mentioned $\Delta(54)$ symmetry belongs to this class.

As discussed above, the $\Delta(54)$ flavor symmetry emerges from the $\mathbb{Z}_3$ orbifold plane. Already the very first string–derived 3–generation models [37] have this symmetry (although this has not been spelled out at the time when these models were found), so this is not at all an exotic property. One therefore expects that these models have a built–in means of $\mathcal{CP}$ violation, which has been confirmed in [38]. That is, these flavor symmetries, which have been explicitly shown to be gauged and can be understood as outer automorphisms of the so–called space group, "destroy" $\mathcal{CP}$, an outer automorphism of the Lorentz group.

18.7 Summary

Given all the strong arguments that all symmetries, including discrete ones, need to be gauged, in these proceedings we studied to which extent this is the case the discrete symmetries in string compactifications. While this is straightforward to see for most of the discrete symmetries, it is a bit harder to make this explicit for flavor symmetries. Only after a recent careful reanalysis of the residual symmetries of orbifolding the gauge origin of all symmetry factors could be established.

Altogether we have reviewed the conceivable roles of explicitly string–derived discrete symmetries in physics beyond the standard model. In particular:

1. Discrete R symmetries can be understood as discrete remnants of the Lorentz symmetry of compact space. They appear to be instrumental to solve the problems of supersymmetric extensions of the standard model.
2. There are symmetries after orbifolding that have been missed until recently. These symmetries comprise the left–right parity of left–right symmetric and Pati–Salam models, and other outer automorphism symmetries of the low–energy continuous gauge group.

3. Discrete flavor symmetries can be completely traced back to continuous gauge symmetries in higher dimensions. They also provide a possible origin of $\mathcal{CP}$ violation.

Acknowledgements Big thanks go to the organizers of FHEP 2019 for putting together such nice conference, and to Patrick Vaudrevange for comments. This work is supported by NSF Grant No. PHY-1719438.

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