# **Merchant Power Flow Controllers**

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Abstract: The transmission system in the U.S. is under stress, leading to high congestion costs. To address this issue, more efficient utilization of the existing network is a paramount alternative to building new transmission lines. Significant transfer capability enhancement can be readily achieved via a number of mature technologies that enable power flow control. Despite the promise of power flow controllers (PFC), their deployment has been very limited, due to a number of reasons, including heavy economic regulation. This has many drawbacks, including lengthy planning and approval time, lack of incentives for efficient planning and operation, and transfer of the investment risks to the ratepayers. This paper argues that PFCs pose characteristics that fit well within the framework of merchant transmission without its drawbacks, such as lumpy investments. This paper, thus, proposes to assign financial transmission rights (FTR) to merchant PFC owners based on the additional transfer capability that they offer to the system. The owners are expected to recover their investment costs through the revenues they collect from such FTRs. Unlike regulated rate of return payment, the proposed model provides the right incentive for efficient planning and operation of PFCs. The paper also proves FTR revenue adequacy in presence of the PFCs by developing a simultaneous feasibility test model. The performance of the method as well as its revenue adequacy are demonstrated, first, on a twobus system, and then, on a three-bus system in presence of loop flows. The paper concludes that opening the electricity markets to merchant PFC projects would reveal profitable investment opportunities to improve the efficiency of the system.

**Keywords:** Deregulation, electricity markets, financial transmission rights, merchant transmission, power flow controller, power transmission.

JEL Classification: D4, Q4.

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### 1. Introduction

The transmission system in the United States is under stress, which leads to costly congestion in the grid (Spencer, 2002; Snarr, 2009). Figure 1 shows the annual congestion rent in the U.S. for a select number of independent system operators (ISO) and regional transmission organizations (RTO) in 2015 (California ISO, 2016; Potomac Economics, 2016a; ISO New England Inc. Internal Market Monitor, 2016; Monitoring Analytics LLC, 2016; Potomac Economics, 2016b; Potomac Economics, 2016c; SPP Market Monitoring Unit, 2016). Congestion revenue is presented here as a proxy to congestion cost, on which public data is not available. The total congestion revenue for the presented areas in Figure 1, adds to five billion dollars. Assuming that congestion rent is a good proxy for congestion cost, this expense will be transferred to electricity ratepayers. The new congestion patterns created by increased penetration from intermittent renewable energy resources is only expected to aggravate this problem (Sang, et al., 2017).

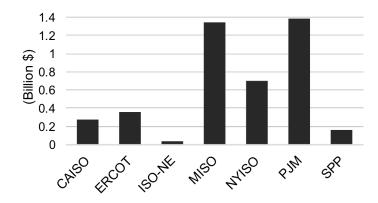


Figure 1 - Annual congestion cost in select U.S. independent system operators and regional transmission organizations in 2015.

While building new transmission lines can offer an effective solution to the congestion problem, new transmission projects are extremely costly, take a long time to complete, and face substantial permitting barriers. Alternatively, power flow controllers (PFC) can significantly enhance the transfer capability over the existing network, through utilizing its unused capacity (Hug, 2008). The increase in transfer capability can be as large as 50% according to the literature (Amin, 2004) and provide substantial savings in terms of avoided congestion costs and deferred transmission investment costs. Power flow control enables rerouting of the power to the paths that are not congested. This is shown schematically for PJM in Figure 2, where an actual map of real-time prices is presented on the left. Due to the transmission system limits, the prices are very high in the eastern parts of the system near the load centers in Philadelphia and Washington, DC, while the prices are very low in southwest Virginia. This large price difference signals clear inefficiencies, as the cheap energy produced in Kentucky, southwest Virginia, and West Virginia cannot be transferred to the locations with high demand. Thus, local expensive power plants near the load centers are required to produce energy at a much higher cost to meet the energy demand in the system. Additionally, the figure has implications regarding system reliability. As most of the generation capacity in the northeastern part of the system is utilized to produce energy, there is little reserve (extra capacity) left for contingency response. Figure 2-right shows how PFCs can improve the transfer capability and allow additional flow of power from the cheap resources to the electric load. Enhancement of the transfer capability will improve economic efficiency by replacing some of the expensive power plants near the load centers with cheaper resources in the southwestern part of the system. Consequently, the reserve capacity near the load centers will increase, which would translate in reliability improvements.

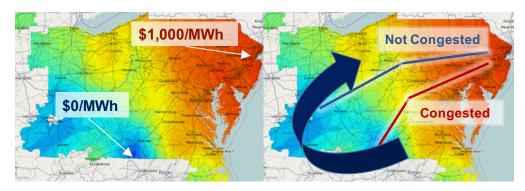


Figure 2 - Left: a map of real-time prices in PJM, with prices as high as \$1000/MWh in the east and as low as \$0/MWh in the soutwestern parts of the system; Right: with power flow control, the unused capacity over the existing transmission system can be utilized to improve both economic efficiency and system reliability.

Power flow control can be achieved via a number of different technologies, such as topology control, voltage phase shift, or impedance control (Sahraei-Ardakani, et al., 2016; Gotham & Heydt, 1998; Zhang & Sahraei-Ardakani, 2018). This paper focuses on the latter technology, as many of the existing PFC devices rely on impedance control. The grid is also expected to be equipped with more variable-impedance PFCs, as a distributed and relatively cheap version of such devices has been successfully introduced to the market (Smart Wires Inc.). PFCs are already a part of the North American grid. To mention a few, five EPRI-sponsored FACTS devices are currently operating in AEP's territory (Kentucky), BPA (Oregon), CSW (Texas), TVA (Tennessee), and NYPA (New York) (Basler, et al., 2012). Recently, Smart Wires has also completed the installation of a distributed series reactor device for Minnesota Power (Smart Wires, 2017).

Despite their potential in improvement of transfer capability, PFC installations have been relatively limited. Moreover, the set point of the existing PFCs are not optimized alongside generation dispatch within the energy management systems (Sahraei-Ardakani & Hedman, 2016; Sahraei-Ardakani & Hedman, 2017). There are two reasons for such underutilization: (i) PFC modeling involves computational complexities that are challenging to handle (Sahraei-Ardakani & Hedman, 2016; Sahraei-Ardakani & Hedman, 2017); and (ii) PFCs are regulated as a part of monopoly transmission system (Sahraei-Ardakani & Blumsack, 2016; Sahraei-Ardakani & Blumsack, 2012). Effective handling of the computational challenges involved in PFC operation has received significant attention recently (Ziaee & Choobineh, 2017a; Ziaee & Choobineh, 2017b; Sahraei-Ardakani & Hedman, 2016a; Sahraei-Ardakani & Hedman, 2017; Sahraei-Ardakani & Hedman, 2016b; Sang & Sahraei-Ardakani, 2017). However, addressing the inherent inefficiencies of regulation remains to be an unresolved barrier.

The existing PFCs, similar to any transmission asset, receive a fixed regulated rate of return (RoR) on their investment. The RoR compensation structure does not provide any incentive for efficient operation. On the contrary, frequent adjustment of the PFC set point would increase the maintenance costs, which are not desirable. Therefore, PFC owners under an RoR payment structure, would prefer to keep the set point of their devices unchanged for as long as they can. Moreover, a badly located PFC will receive the same compensation as a well-planned PFC, as long as they are both permitted. This paper aims to offer a solution to these problems through a merchant model, where the payments to the PFC owners are based on their performance. We, first, develop a convex model for PFCs and show that financial transmission right (FTR) market revenue adequacy is maintained in presence of PFCs, with such a convex model. Then, we calculate the additional FTRs that can be supported in a network that is equipped with PFCs. This paper argues that the additional FTRs should be assigned to PFC owners, through which they may recover their investment costs and make extra profit. The proposed structure would transfer the investment risks to the PFC owners and provide the right incentive for efficient operation.

The rest of this paper is organized as follows: section II develops a convex model for PFCs. Section III provides a proof for FTR revenue adequacy in presence of PFCs. A Case study on a two-bus system is provided in section IV, and finally section V concludes the paper.

# 2. PFC Modeling and Convexification

The power flow on a transmission line can be calculated through the shift factors and nodal injections, according to the linear dc power flow equation, as shown in (1).

$$f_l = \sum_{i=1}^{N} \varphi_{li} I_i \tag{1}$$

 $f_l$  is the flow on line l, N is the number of nodes in the network,  $\varphi_{li}$  is the sensitivity of  $f_l$  to injection at node i,  $I_i$ . For a network with fixed topology, shift factors ( $\varphi$ ) are constant, making (1) a linear equation. However, when a transmission line's impedance is controllable through a PFC, (1) is no longer valid, because the shift factors change. To keep the shift factors constant, PFC can be represented via a pair of injections, as shown in Figure 3 (Sahraei-Ardakani & Hedman, 2017). In the figure, a single line k is shown with its "from" and "to" nodes and its susceptance,  $b_k$ . The PFC is represented by the change in the susceptance of the line,  $\Delta b_k$ . This susceptance change is equivalent to an injection pair at the "from" and "to" nodes of line k, as shown in Figure 3.

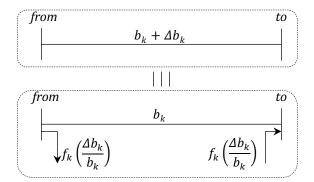


Figure 3 - Conversion of the susceptance control impacts of PFC on a line, to an injection pair, while keeping the line's susceptance and the system's shift factors unchanged.

The injection pair, representing the PFC, will affect the flow of all the other lines in a meshed network, which have nonzero shift factors associated to the "from" and "to" node of line k. For instance, the flow on an arbitrary line l, can be calculated as shown in (2).

$$f_{l} = \sum_{i=1}^{N} \varphi_{li} I_{i} + \sum_{k=1}^{K} \nu_{k} \left( \varphi_{lk_{to}} - \varphi_{lk_{fr}} \right)$$
 (2 - a)

$$v_k = \left(\frac{\Delta b_k}{b_\nu}\right) f_k \tag{2-b}$$

$$\Delta b_k^{min} \le b_k \le \Delta b_k^{max} \tag{2-c}$$

 $v_k$  represents the equivalent injection for the PFC, installed on line k;  $b_k$  is the initial susceptance of line k, and  $\Delta b_k$  is the change in line k's susceptance through the PFC.  $k_{to}$  and  $k_{fr}$  are the "to" and "from" buses of transmission line k; and K is the number of lines equipped with PFCs. Each PFC has a certain control range that is reflected in (2-c). Generally, PFCs can operate in both capacitive and inductive modes, to reduce or increase the susceptance.  $\Delta b_k^{min}$  identifies the control limit in the inductive mode, while  $\Delta b_k^{max}$  represents the limit in

the capacitive mode. It is apparent from (2-b) that the power flow equations, considering the impacts of PFC, are no longer linear, as both  $f_k$  and  $\Delta b_k$  are variables.

It is shown in (Sahraei-Ardakani & Hedman, 2017) that the nonlinear equation, (2-b), can be reformulated via linear inequality constraints, shown in (3).

$$if f_k \ge 0$$
  $\left(\frac{\Delta b_k^{max}}{b_k}\right) f_k \le \nu_k \le \left(\frac{\Delta b_k^{min}}{b_k}\right) f_k$  (3 - a)

$$if \ f_k < 0 \qquad \left(\frac{\Delta b_k^{min}}{b_k}\right) f_k \le \nu_k \le \left(\frac{\Delta b_k^{max}}{b_k}\right) f_k \tag{3-b}$$

Both (3-a) and (3-b) are linear and represent convex constraints; however, the combination of the two identifies a nonconvex feasible set. As the direction of the flow identifies the direction of the constraints for the PFC model in (3), the PFC can be presented via purely linear constraints, as long as that the direction of the flow is known.

PFCs are usually installed on major lines, where the power flow direction is rather predictable. For instance, it is trivial to predict the flow direction on key corridors such as the California-Oregon intertie (COI). The operator can also solve an initial optimal power flow model without considering PFCs and assign the same power flow directions to (3). It is not likely that the flow directions change after PFC adjustments, on the lines equipped with PFC. On the contrary, it is expected that the flows increase, in the same direction, on the paths parallel to the congested lines. Using this engineering insight, only (3-a) or (3-b) needs to be included within the model, keeping it linear.

Although the full representation of the constraints, identified in (3) is a non-convex set, elimination of one segment of (3), will leave us with a convex feasible set. This is important in the next section, where FTR revenue adequacy is proved.

# 3. FTR Revenue Adequacy in Presence of PFCs

Hogan showed that for a constant network topology with loss-less dc set of power flow equations, the congestion rent collected from market is larger than the FTR settlements, if the FTRs pass a simultaneous feasibility test (SFT) (Hogan, 1992). The proof does not hold for a general case with ac power flow constraints or even with representation of losses (Alsaç, et al., 2004). In the particular case of a lossy but linear network, revenue adequacy may still be achieved with some modeling tricks (Sarkar & Khaparde, 2008). However, it can be shown that non-convexity in the set of feasible power injections may lead to revenue inadequacy (Lesieutre & Hiskens, 2005). Non-convexity can occur in form of non-convex but continuous set of ac power flows or a discrete change in the network topology (Hedman, et al., 2011).

In the case of a convex feasible set, social welfare maximization (cost minimization for inelastic demand) would be equivalent to congestion rent maximization (Oren, 2013). In this section, we follow the proof presented in (Philpott & Pritchard, 2004), and demonstrate revenue adequacy in presence of PFCs in a dc optimal power flow (DCOPF)-based market. DCOPF seems to be an appropriate framework, because every single North American market chooses to employ one or another form of a linearized market solver (Stott, et al., 2009).

System operators employ DCOPF to minimize the total cost of producing energy, subject to physical constraints of the network. The problem, including the modeling of PFCs, can be stated as follows:

$$\min \sum_{g} c_g P_g \tag{4}$$

$$\{(1), (2-a), (3)\},$$

$$-f_l^{max} \le f_l \le f_l^{max}$$

$$\sum_{a \in G(n)} P_g + \sum_{l \in \delta^+(n)} f_l - \sum_{l \in \delta^-(n)} f_l = d_n$$
(7).

The marginal cost of generator g is represented via  $c_g$ , whereas its generation output is shown with  $P_g$ . G(n) is the set of generators connected to node n,  $\delta^+(n)$  is the set of transmission lines that flow to node n, and  $\delta^-(n)$  is the set of transmission lines that flow from node n.  $d_n$  is the demand at node n. The above DCOPF is a linear problem as long as only one segment of (3) is selected for the lines that are equipped with PFC. In such a case, all the constraints will remain linear, and the feasible set will be convex.

Proposition: FTR revenue adequacy holds in presence of PFCs, when power flow direction on PFCs is predetermined.

*Proof*: To simplify the problem, let us define a set for feasible power flows and PFC injections  $u = \{f, v\} \in U$ , and another set for feasible generation outputs  $p \in P$ . It follows the definition of a linear program that U and P are convex sets. Thus, the DCOPF can be represented as:

$$min(4)$$
 s.t.  $\{u \in U, p \in P, (5)\}.$  (8)

The Lagrangian for this problem can be represented as follows:

$$\mathcal{L} = \sum_{g} c_g P_g + \sum_{n} \lambda_n \left( d_n - \sum_{g \in G(n)} P_g - \sum_{l \in \delta^+(n)} f_l + \sum_{l \in \delta^-(n)} f_l \right)$$
(9)

After solving the market and finding the optimal values of generation output, PFC injections, and power flows, the system operator will collect the following congestion rent, which is the difference between the load payment and generators' revenue:

$$CR = \sum_{n} \lambda_n \left( d_n - \sum_{g \in G(n)} P_g^* \right)$$
 (10).

Using (7), (10) can be recalculated as follows:

$$CR = \sum_{n} \lambda_n \left( \sum_{l \in \delta^+(n)} f_l^* - \sum_{l \in \delta^-(n)} f_l^* \right)$$
 (11).

Simultaneous feasibility test ensures that the set of accepted FTRs would result in flows that would not violate the physical constraints of the system. Thus, the flows resulting from the allocated FTRs should belong to U. To prove revenue adequacy, we need to show that the congestion rent is larger than the payment to FTR owners under any such feasible set of FTR-based flows. This follows the Lagrangian calculated in (9). It should be noted that the Lagrangian is minimized at the optimal solution. Since the only term in Largangian that includes power flow is  $\sum_{n} \lambda_{n} \left( -\sum_{l \in \delta^{+}(n)} f_{l} + \sum_{l \in \delta^{-}(n)} f_{l} \right)$ , the following holds for the optimal solution:

$$\sum_{n} \lambda_{n} \left( -\sum_{l \in \delta^{+}(n)} f_{l}^{*} + \sum_{l \in \delta^{-}(n)} f_{l}^{*} \right) \leq \sum_{n} \lambda_{n} \left( -\sum_{l \in \delta^{+}(n)} f_{l} + \sum_{l \in \delta^{-}(n)} f_{l} \right) \tag{12}.$$

Multiplying both sides of (12) with -1, attains revenue adequacy.

$$\sum_{n} \lambda_{n} \left( \sum_{l \in \delta^{+}(n)} f_{l}^{*} - \sum_{l \in \delta^{-}(n)} f_{l}^{*} \right) \ge \sum_{n} \lambda_{n} \left( \sum_{l \in \delta^{+}(n)} f_{l} - \sum_{l \in \delta^{-}(n)} f_{l} \right)$$

$$(13)$$

The proof, presented above shows that FTR revenue adequacy holds even in presence of PFCs, as long as the direction of the power flow on the lines equipped with PFC is pre-determined. It should be noted that the PFC setpoints in the DCOPF solution,  $v^*$ , do not have to be the same as their value in the simultaneous feasibility test. Furthermore, as shown previously by Hogan, the simultaneous feasibility test does not require all the assigned FTRs to be feasible with the PFC setpoints,  $\nu^*$ , optimized in the DCOPF (Hogan, 2002). The PFC setpoints calculated in the SFT are optimized to maximize the FTR auction revenue, while the PFC setpoints in the DCOPF are optimized to minimize the operation cost. The two solutions can be different and this difference does not affect revenue adequacy.

#### **Impact of Contingency Constraints** 3.1.

For the sake of simplicity, the DCOPF and SFT, presented above, did not include contingency constraints. Here, the results obtained above are extended to the case where contingency constraints are present. These constraints ensure that the power flow on a line will remain within its contingency limit after the outage of another line. Contingency constraints are modeled using shift factors and line outage distribution factors (LODF), both of which are constant in a linear network. For instance, the flow on line l, after the outage of line k,  $f_l^k$ , can be calculated as shown in (14).

$$f_l^k = f_l^0 + \psi_l^k f_k^0 \tag{14 - a}$$

$$f_l^k = f_l^0 + \psi_l^k f_k^0$$
 (14 - a)  
$$f_l^0 = \sum_{i=1}^N \varphi_{li} I_i$$
 (14 - b)

$$f_k^0 = \sum_{i=1}^N \varphi_{ki} I_i$$
 (14 - c)

 $f^0$  indicates the pre-outage flow on the lines, and  $\psi_l^k$  is the LODF sensitivity, which determines the portion of pre-outage flow on line *k* that will be transferred to line *l*, should line *k* go out of service.

To ensure that the flow on line l remains within the limits, even after the outage of line k, both the normal and contingency limits should be enforced. The normal limits are represented in (6) and the contingency limits can be modeled as shown in (15).

$$-f_l^{max_{cont}} \le f_l^k \le f_l^{max_{cont}} \tag{15}$$

The contingency limits,  $f_l^{max_{cont}}$ , are often larger than the normal limits; the operator will have a limited amount of time to bring the system back to the normal operation after the contingency occurs. In case that either or both of the lines k and l are equipped with PFC, the same linearization technique, introduced in (3), can be used to convert (14) into linear equations (Sahraei-Ardakani & Hedman, 2017). As the post-contingency flows, shown in (14-a), are calculated from the pre-contingency flows, shown in (14-b)-(14-c), no additional assumption with respect to flow directions will be required. Once the normal flow direction for the lines equipped with PFC is identified, contingency constraints can be straightforwardly added to the DCOPF problem. The proof of revenue adequacy can, thus, be easily extended to the case with explicit modeling of contingency constraints. The difference in the proof would be that contingency flows  $(f_l^k)$  will need to be included in (9), (11)-(13), alongside normal flows.

It should be noted that contingency constraints are preventive measures, which affect the normal operation in order to protect the system against overload, in the case of a contingency. The proof, presented above, also relates to normal operation, before the occurrence of the contingency. Since the absolute majority of contingency constraints are neglected due to computational challenges, and only the most critical ones are modeled within energy and market management systems, revenue adequacy may or may not hold at the post-contingency stage (Oren, et al., 2010).

#### 3.2. FTR Allocation to PFC Owners

PFCs will affect the transmission network, and if properly located, will enhance the transfer capability of the grid. Thus, with PFCs, allocation of new FTRs, which were not feasible before, will become possible. Similar to the method, proposed by (Bushnell & Stoft, 1996a; Bushnell & Stoft, 1997; Bushnell & Stoft, 1996b), we allow the PFC owner to pick the FTRs that they would like to hold as long as they pass the SFT presented above, along with the existing FTRs. Since the PFC set point can always be picked at zero, the pre-existing FTRs are guaranteed to be feasible after inclusion of the PFCs. Due to the flexibility of the PFCs, additional FTRs will also become possible, from which the owner can choose a set of FTRs to hold. The FTRs requested by the PFC owners are node to node FTRs, without any specification of the PFC control. As long as some PFC setpoint makes the requested FTR feasible, the assignment will be allowed.

The assumption behind the framework developed and discussed in this paper is that the PFC setpoints are co-optimized alongside generation dispatch, as reflected in the DCOPF and SFT formulated above. In such case, the PFC setpoints are changed only if such change improves the social welfare (reduce the total cost with inelastic demand) in the DCOPF. With this framework, it is not possible to create harmful congestion patterns with the PFCs. In the SFT, the PFC setpoints are picked, so that the FTR auction revenue is maximized. Note that the setpoints calculated in the SFT are never physically implemented as FTRs are purely financial instruments and the main function of SFT is to ensure FTR revenue adequacy. Furthermore, the PFC setpoints in the SFT and DCOPF do not have to match.

It should be noted that revenue adequacy was proved only when the same segment of (3) is picked in simultaneous feasibility test and market operation, as the PFC injections are also a part of set U. Each segment of (3) relates to one direction of power flow on the PFC. If different power flow directions are chosen at different stages, revenue adequacy will not be guaranteed. The PFC owners, however, can choose different set of FTRs for each power flow direction, as long as those FTRs are simultaneously feasible with the pre-existing FTRs. In such case, revenue adequacy will hold for any direction of power flow on the lines equipped with PFC, and the PFC owners will receive payments based on the FTRs they hold under the direction of the flow, identified in the market.

#### 3.3. The Issue of Lumpy Investments

It is worth noting that the merchant transmission model has received criticism, as to why it may not necessarily be effective (Joskow & Tirole, 2005). One of the main reasons that leads to inefficiency in merchant transmission model is the lumpy nature of transmission projects, implied by economies of scale. New transmission investments result in large and discontinuous changes in the transfer capability. This is not the case for PFCs. As discussed earlier in the paper, there are multiple PFC technologies available to the investors, including conventional FACTS devices, and the recently developed and commercialized modular PFCs. The control range of conventional PFCs depends on the design; however, they are usually not built with a large control range, due to stability concerns among other reasons. For instance, the UPFC, which was built in AEP footprint in Kentucky, has a control range of about  $\pm 10\%$  of the line's capacity (Renz, et al., 1999). Unlike transmission lines that cannot be built with a marginal capacity, PFCs with small capacity can be realistically designed and built. In fact, the distributed and modular PFCs, which have been commercialized recently, have a rather small adjustment range. Their aggregated control range increases linearly with the number of modular PFCs installed on a line and so does their cost. Thus, both with the conventional and distributed technologies, a PFC can be built at any desirable size. Hence, we believe that PFCs fit very well within the framework of merchant transmission model, and do not suffer from some of the undesirable characteristics of transmission itself.

#### 3.4. FTR Revenue Adequacy in Practice

Although all the North American electricity markets use a linearized and convex model of the power flows (Stott, et al., 2009), for which FTR revenue adequacy can be proved, some markets have experienced consistent underfunding of their long-term FTRs (Hogan, 2013). The revenue inadequacy occurs due to a number of reasons such as prolonged transmission outages and incomplete modeling of the transmission network (Hogan, 2013). Long-term FTR revenue adequacy is crucial for the functionality of the merchant transmission model, and this insufficiency is another reason as to why the merchant transmission model has not been able to attract substantial levels of investment into the power transmission sector. We acknowledge the existence of this challenge, which will affect the theory developed in this paper as well, but do not discuss it in further details as it falls outside the scope of this paper. We suffice to mention that the issue has been brought up to Federal Energy Regulatory Commission's attention along with a number of potential solutions, such as the practice at New York ISO (Hogan, 2013).

# 4. Case Study

In this section, we show how the proposed methodology would work, first on a simple two-bus system, and then on a three-bus system with loop flows.

#### 4.1. Two-Bus System

The two-bus test case, shown in Figure 4, includes a cheap generator at bus one and a relatively expensive generator at bus 2. There are two transmission lines connecting the two nodes in the system, each of which has the same susceptance of -1 per unit. Line 1 has a thermal capacity of 200 MW, while the thermal capacity for line 2 is only 100 MW. The load is located at bus two, with a total demand of 250 MW.

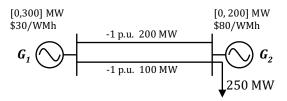


Figure 4 – Two-bus system with two transmission links.

As the susceptance of the two lines are equal, without any power flow controller, the system can only handle 200 MW of transfer from bus one to bus two without causing any network violation. The congestion pattern in this system is from bus one to bus two, and FTRs in this direction would be profitable. Thus, the total FTR that the system can handle without violating the limits would also be 200 MW from bus one to bus two. For the base case, the optimal dispatch would be:

$$P_1 = 200 \; MW, \quad P_2 = 50 \; MW, \quad f_1 = 100 \; MW, \quad f_2 = 100 \; MW, \quad \lambda_1 = \$30/\text{MWh}, \qquad \lambda_2 = \$80/\text{MWh}$$
 
$$Total \; Cost = \$10,000/h, \quad Congestion \; Rent = (\lambda_2 - \lambda_1)(f_2 + f_1) = \$10,000/h.$$

With 200 MW of transfer between the two buses, there will be 100 MW of unused capacity on line 1, which can be utilized through PFCs. To enhance the transfer capability, a PFC on line 1 would decrease the impedance to pull more power towards line 1. Alternatively, a PFC on line 2 would push power away from line1, by increasing its impedance.

Let us assume that there is a PFC on line 1, which can control the susceptance between -1 p.u. and -1.2 p.u. Following the model developed in Section 2, the PFC can be modeled with an injection pair as shown in Figure 5.

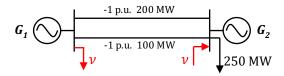


Figure 5 - Two-bus system with PFC injections.

Without the PFC, as the susceptances of the lines are equal, they each get half of the injection at node 1, which is  $P_1/2$ . Defining the flow directions from node 1 to node 2, (3) provides the following constraints for PFC injections,  $\nu$ :

if 
$$f_1 \ge 0$$
  $\left(\frac{0}{-1}\right) f_1 \le \nu \le \left(\frac{-0.2}{-1}\right) f_1$ 

if 
$$f_1 < 0$$
  $\left(\frac{-0.2}{-1}\right) f_1 \le \nu \le \left(\frac{0}{-1}\right) f_1$ 

The two equations, together, represent a non-convex set; however, as discussed before, the direction of the flow can often be guessed. In this particular example, the flow will be from node 1 to node 2, as the operator would prefer to use the cheaper generator located at node 1 as much as possible. Accordingly, the second part of the above set can be eliminated and the feasible set will become convex. With  $0 \le v \le 0.2 f_1$ , and  $f_1 = 100 \, MW$ , which is the power flow when the PFC impact is isolated via the injection pair model, the PFC can facilitate an additional transfer of 20 MW. The optimal dispatch, thus, will change to:

$$P_1 = 220 \; MW, \; P_2 = 30 \; MW, \; f_1 = 120 \; MW, \; f_2 = 100 \; MW, \; \lambda_1 = \$30/\text{MWh}, \; \lambda_2 = \$80/\text{MWh}$$
   
  $Total \; Cost = \$9,000/h, \; Congestion \; Rent = (\lambda_2 - \lambda_1)(f_2 + f_1) = \$11,000/h.$ 

The dispatch with PFC is 10% cheaper than the initial dispatch, which shows the benefit of power flow control. Since the PFC enables 20 MW of additional transfer, the PFC owner can choose to receive FTR for 20 MW from node 1 to node 2. The value of such FTR would be the difference between the price at the two nodes, \$50/MWh, multiplied by the FTR quantity, 20 MW, which is \$1000/h. It is worth noting that the results presented here are consistent with those of (Gribik, et al., 2005; Baldick, 2007), where the transmission upgrades are awarded with the difference in power flow before and after the upgrade (20 MW in this case) multiplied by the difference in nodal price at the two ends of the link (\$50/MWh).

An important implication of the method proposed in this paper is that a merchant PFC owner would receive market-based payments, if the device is operated in a way that enables the additional FTR allocation. Consequently, it is apparent that the FTR-based payment signals efficient operation.

#### 4.2. Three-Bus System with Loop Flows

In order to show the impacts of loop flow, a three-bus system, shown in Figure 6, is studied here. The generators are picked with the same cost and limits, as in the two-bus system. The transmission lines all have the same susceptance of -1 p.u. The only line that has a limit in the scale of this problem is the line connecting nodes 1 and 3, which has a capacity limit of 150 MW. Similar to the two-bus system, the demand for electricity is 250 MW, which is located on bus 3.

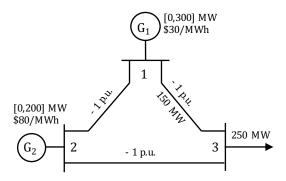


Figure 6 - Three-bus system with loop flow.

Using superposition over the network, the power flows can be calculated as follows:

$$f_{12} = \frac{1}{3}P_1 - \frac{1}{3}P_2, \qquad f_{13} = \frac{2}{3}P_1 + \frac{1}{3}P_2, \qquad f_{23} = \frac{1}{3}P_1 + \frac{2}{3}P_2.$$

Ideally, the entire 250 MW of load would be produced by generator 1, which is cheaper. However, 250 MW of production from generator 1 would lead  $F_{13}$  to take a value of 166.6 MW, which violates the line's capacity limit. By enforcing the capacity limit, the following dispatch solution will be obtained:

$$P_1 = 200 \; MW, \; P_2 = 50 \; MW, \; f_{12} = 50 \; MW, \; f_{13} = 150 \; MW, \; f_{23} = 100 \; MW$$
  $\lambda_1 = \$30/\text{MWh}, \; \lambda_2 = \$80/\text{MWh}, \; \lambda_3 = 2 \times 80 - 30 = \$130/\text{MWh}$   $\lambda_3 = 2 \times 80 - 30 = \$130/\text{MWh}$   $\lambda_4 = \$10.000/h, \; Congestion \; Rent = \$22.500/h, \; Conges$ 

The PFC in this case can be placed on any of the three lines. Here, we study a case where PFC is placed on line 1-2. We further assume that the PFC has the capability of controlling the susceptance of the line between -0.5 and -1.5 p.u. This range is equivalent to a 50% susceptance adjustment range both in capacitive and inductive directions. The injection-pair representation of the PFC is shown in Figure 7.

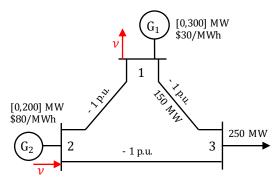


Figure 7 – Injection pair representation of the PFC on line 1-2.

As the flow on line 1-2 is positive, according to (3), the limits on  $\nu$  can be identified as follows:

$$\left(\frac{0.5}{-1}\right) f_{12} \le \nu \le \left(\frac{-0.5}{-1}\right) f_{12}.$$

In this case, the operator would prefer to reduce the impedance (increase the absolute value of susceptance) of the line in an attempt to pull more power towards line 1-2, and enable more production from generator 1. Thus, the PFC injection  $\nu$  will take the maximum value of  $0.5f_{12}$ . To calculate the dispatch under this scenario, we would need to enforce the limit on line 1-3.

$$f_{13} = \frac{2}{3}(P_1 - \nu) + \frac{1}{3}(P_2 + \nu) = \frac{2}{3}P_1 + \frac{1}{3}P_2 - \frac{1}{3}(0.5f_{12}) = \frac{2}{3}P_1 + \frac{1}{3}P_2 - \frac{1}{6}f_{12}$$

Now  $f_{12}$  needs to be recalculated based with the added injection pair.

$$\begin{split} f_{12} &= \frac{1}{3}(P_1 - \nu) - \frac{1}{3}(P_2 - \nu) = \frac{1}{3}P_1 - \frac{1}{3}P_2 - \frac{2}{3}\nu = \frac{1}{3}P_1 - \frac{1}{3}P_2 - \frac{2}{3} \times \frac{1}{2}f_{12} \\ f_{12} &= \frac{1}{3}P_1 - \frac{1}{3}P_2 - \frac{1}{3}f_{12} \Rightarrow \frac{4}{3}f_{12} = \frac{1}{3}P_1 - \frac{1}{3}P_2 \Rightarrow f_{12} = \frac{1}{4}P_1 - \frac{1}{4}P_2 \end{split}$$

This value of  $f_{12}$  can be replaced in  $f_{13}$  calculations to obtain the final value of  $f_{13}$  based solely on  $P_1$  and  $P_2$ :

$$f_{13} = \frac{2}{3}P_1 + \frac{1}{3}P_2 - \frac{1}{6}f_{12} = \frac{2}{3}P_1 + \frac{1}{3}P_2 - \frac{1}{6}\left(\frac{1}{4}P_1 - \frac{1}{4}P_2\right) = \frac{5}{8}P_1 + \frac{3}{8}P_2.$$

By enforcing this transmission limit and balancing the total supply and demand, the following dispatch can be achieved:

$$P_1 = 225 \ MW, \ P_2 = 25 \ MW, \ f_{12} = 75 \ MW, \ f_{13} = 150 \ MW, \ f_{23} = 100 \ MW$$
  $\lambda_1 = \$30/\text{MWh}, \ \lambda_2 = \$80/\text{MWh}, \ \lambda_3 = -1.5 \times 30 + 2.5 \times 80 = \$155/\text{MWh}$  
$$Total \ Cost = \$8,750/h, \ \ Congestion \ Rent = \$30,000/h.$$

With the PFC, the dispatch becomes 12.5% cheaper, while the congestion rent increases. The PFC on line 1-2 has enabled an additional 25 MW transfer from bus 1 to bus 2. The PFC owner, thus, can acquire an FTR with the quantity of 25 MW from bus 1 to bus 2. This FTR will have the value of  $25 \times (80 - 30) = \$1,250/h$ . Due to the existence of loop flows, the operation of the PFC will substantially affect the prices without changing the marginal generating units. Such impact on the prices will change the value of other existing FTRs. For the sake of analysis, let us assume that prior to the PFC installation, there were two FTR assignments of 200 MW from bus 1 to bus 3, and 50 MW from bus 2 to bus 3. Those FTRs had a value of  $200 \times (130 - 30) = \$20,000/h$  and  $50 \times (130 - 80) = \$2,500/h$  respectively. However, after PFC installation, the value of those FTRs will change to  $200 \times (155 - 30) = \$25,000/h$  and  $50 \times (155 - 80) = \$3,750/h$  respectively. The FTR values, both before and after the installation of the PFC, sum up to the total congestion rent, which confirms FTR revenue adequacy.

#### 5. Conclusions

A compensation mechanism for power flow controllers, based on the concept of merchant transmission, was proposed in this paper. The proposed method has the potential to enable active participation of PFCs within the electricity markets. The additional transfer capability, offered through operation of PFCs, would allow for allocation of additional FTRs in the SFT. Those FTRs can, then, be assigned to the owners of PFCs. Consequently, the PFC owners would collect revenue from the FTR market. Unlike the regulated rate of return payment structure, where the compensation is not related to efficient planning or operation, the proposed payment structure would only generate revenue when PFC is planned and operated efficiently, in a way that enhances transfer capability. Thus, the method developed in this paper would incentivize investment in appropriate locations and efficient real-time PFC operation. Revenue adequacy of the modified FTR market was proved with a convex PFC model even in presence of contingency constraints. Results on a two-bus and a three-bus test system demonstrated the performance and effectiveness of the proposed model.

#### References

Alsaç, O. O. et al., 2004. The rights to fight price volatility. IEEE Power and Energy Magazine, July, 2(4), pp. 47-57.

Amin, M., 2004. North American electricity infrastructure: system security, quality, reliability, availability, and efficiency challenges and their societal impacts," Continuing Crises in National Transmission Infrastructure: Impacts and Options for Modernization. s.l.:National Science Foundation.

Baldick, R., 2007. Border Flow Rights and Contracts for Differences of Differences: Models for Electric Transmission Property Rights. *IEEE Transactions on Power Systems*, November, 22(4), pp. 1495-1506.

Basler, S. et al., 2012. Effective grid utilization: A technical assessment and application guideline, Golden, CO: National Renewable Energy Laboratory.

Bushnell, J. & Stoft, S., 1996. Electric Grid Investment Under a Contract Network Regime. *Journal of Regulatory Economics*, 1(10), pp. 61-79.

Bushnell, J. & Stoft, S., 1996. Grid Investment: Can a Market Do the Job?. The Electricity Journal, January.pp. 74-79.

Bushnell, J. & Stoft, S., 1997. Improving Private Incentives for Electric Grid Investment. *Resources and Energy Economics*, Volume 19, pp. 85-108.

California ISO, 2016. 2015 Annual Report on Market Issues and Performance. [Online] Available at: <a href="https://caiso.com/Documents/2015AnnualReportonMarketIssuesandPerformance.pdf">https://caiso.com/Documents/2015AnnualReportonMarketIssuesandPerformance.pdf</a>

Gotham, D. J. & Heydt, G. T., 1998. Power flow control and power flow studies for systems with FACTS devices. February, 13(1), pp. 60-65.

Gribik, P., Shirmohammadi, D., Graves, J. & Kritikson, J., 2005. Transmission Rights and Transmission Expansions. *IEEE Transactions on Power Systems*, November, 20(4), pp. 1728-1737.

Hedman, K. W., Oren, S. S. & O'Neill, R. P., 2011. Optimal transmission switching: economic efficiency and market implications. *Journal of Regulatory Economics*, 40(2), pp. 111-140.

Hogan, W., 1992. Contract Networks for Electric Power Transmission. *Journal of Regulatory Economics*, 4(3), pp. 211-242.

Hogan, W., 2002. Financial transmission right formulations, Cambridge, MA.: Center for Business and Government.

Hogan, W. W., 2013. Financial Transmission Rights, Revenue Adequacy and Multi-Settlement Electricity Markets, s.l.: Harvard University.

Hug, G., 2008. *Coordinated power flow control to enhance steady state security in power systems,.* Zurick: Swiss Federal Institute of Technology.

ISO New England Inc. Internal Market Monitor, 2016. 2015 Annual Markets Report. [Online] Available at: <a href="https://www.iso-ne.com/static-assets/documents/2016/05/2015">https://www.iso-ne.com/static-assets/documents/2016/05/2015</a> imm amr final 5 25 2016.pdf

Joskow, P. & Tirole, J., 2005. Merchant transmission investment. *The Journal of industrial economics*, 53(2), pp. 233-264.

Lesieutre, B. C. & Hiskens, I. A., 2005. Convexity of the Set of Feasible Injections and Revenue Adequacy in FTR Markets. *IEEE Transactions on Power Systems*, November, 20(4), pp. 1790-1798.

Monitoring Analytics LLC, 2016. *State of the Market Report for PJM.* [Online] Available at: <a href="http://www.monitoringanalytics.com/reports/PJM State of the Market/2015/2015-som-pjm-volume2.pdf">http://www.monitoringanalytics.com/reports/PJM State of the Market/2015/2015-som-pjm-volume2.pdf</a>

Oren, S., 2013. Point to Point and Flow-Based Financial Transmission Rights: Revenue Adequacy and Performance Incentives. In: *Financial Transmission Rights*. London: Springer, pp. 77-94.

Oren, S., Hedman, K. & O'Neill, R., 2010. *Optimal Transmission Switching: When Economic Efficiency and FTR Markets Collide.* Toulouse, France, s.n.

Philpott, A. & Pritchard, G., 2004. Financial transmission rights in convex pool markets. *Operations Research Letters*, 32(2), pp. 109-113..

Potomac Economics, 2016. 2015 State of the Market Report for MISO Electricity Markets. [Online] Available at: <a href="https://www.misoenergy.org/Library/Repository/Report/IMM/2015%20State%20of%20the%20Market%20Report.pd">https://www.misoenergy.org/Library/Repository/Report/IMM/2015%20State%20of%20the%20Market%20Report.pd</a> f

Potomac Economics, 2016. 2015 State of the Market Report for the ERCOT Wholesale Electricity Markets. [Online] Available at: <a href="http://www.puc.texas.gov/industry/electric/reports/ERCOT">http://www.puc.texas.gov/industry/electric/reports/ERCOT</a> annual reports/2015annualreport.pdf

Potomac Economics, 2016. 2015 State of the Market Report for the New York ISO Markets. [Online]

Available at: <a href="http://www.nyiso.com/public/webdocs/markets-operations/documents/Studies-and-Reports/Reports/Market-Monito-ring-Unit Reports/2015/NYISO%202015%20SOM%20Report-5-23-2016-CORRECTED.pdf">http://www.nyiso.com/public/webdocs/markets-operations/documents/Studies-and-Reports/Reports/Market-Monito-ring-Unit Reports/2015/NYISO%202015%20SOM%20Report-5-23-2016-CORRECTED.pdf</a>

Renz, B. et al., 1999. AEP unified power flow controller performance. *IEEE Transactions on Power Delivery*, 14(4), pp. 1374-1381.

Sahraei-Ardakani, M. & Blumsack, S., 2012. Market Equilibrium for dispatchable transmission using FACT devices. San Diego, CA, s.n.

Sahraei-Ardakani, M. & Blumsack, S., 2016. Transfer Capability Improvement Through Market-Based Operation of Series FACTS Devices. *IEEE Transactions on Power Systems*, Sept., 31(5), pp. 3702 - 3714.

Sahraei-Ardakani, M. & Hedman, K. W., 2016. A Fast LP Approach for Enhanced Utilization of Variable Impedance Based FACTS Devices. *IEEE Transactions on Power Systems,* May, 31(3), pp. 2204 - 2213.

Sahraei-Ardakani, M. & Hedman, K. W., 2016. Day-Ahead Corrective Adjustment of FACTS Reactance: A Linear Programming Approach. *IEEE Transactions on Power Systems*, Jul., 31(4), pp. 2867 - 2875.

Sahraei-Ardakani, M. & Hedman, K. W., 2017. Computationally Efficient Adjustment of FACTS Set Points in DC Optimal Power Flow With Shift Factor Structure. *IEEE Transactions on Power Systems*, May, 32(3), pp. 1733 - 1740.

Sahraei-Ardakani, M. et al., 2016. Real-time contingency analysis with transmission switching on real power system data. *IEEE Transactions on Power Systems*, May, 31(3), pp. 2501-2502.

Sang, Y. & Sahraei-Ardakani, M., 2017. The Interdependence between Transmission Switching and Variable-Impedance Series FACTS Devices. *IEEE Transactions on Power Systems,* Volume early access.

Sang, Y., Sahraei-Ardakani, M. & Parvania, M., 2017. Stochastic Transmission Impedance Control for Enhanced Wind Energy Integration. *IEEE Transactions on Sustainable Energy*, Volume Early Access.

Sarkar, V. & Khaparde, S. A., 2008. A comprehensive assessment of the evolution of financial transmission rights. *IEEE Transactions on Power Systems*, November, 23(4), pp. 1783-1795.

Smart Wires Inc., n.d. *Smart Wires -- Technology.* [Online] Available at: <a href="https://www.smartwires.com/technology/">https://www.smartwires.com/technology/</a>

Smart Wires, 2017. *Minnesota Power Deploys Smart Wires to Optimize Grid and Save Customers Money.* [Online] Available at: <a href="https://www.smartwires.com/2017/03/03/minnesota-power/">https://www.smartwires.com/2017/03/03/minnesota-power/</a>

Snarr, S. W., 2009. The commerce clause and transmission infrastructure development: an answer to jurisdictional issues clouded by protections. *Electricity Journal*, June, 22(5), pp. 8-18.

Spencer, A., 2002. *National transmission grid study.* [Online] Available at: <a href="http://www.ferc.gov/industries/electric/gen-info/transmission-grid.pdf">http://www.ferc.gov/industries/electric/gen-info/transmission-grid.pdf</a>.

SPP Market Monitoring Unit, 2016. *2015 State of the Market.* [Online] Available at: <a href="https://www.spp.org/documents/41597/spp">https://www.spp.org/documents/41597/spp</a> mmu state of the market report 2015.pdf

Stott, B., Jardim, J. & Alsaç, O., 2009. DC power flow revisited. *IEEE Transactions on Power Systems*, 24(3), pp. 1290-1300..

- Zhang, Q. & Sahraei-Ardakani, M., 2018. Distributed DCOPF with Flexible Transmission. *Electric Power System Research,* January, Volume 154, pp. 37-47.
- Ziaee, O. & Choobineh, F., 2017. Optimal Location-Allocation of TCSC Devices on a Transmission Network. *IEEE Transactions on Power Systems,* January, 32(1), pp. 94-102.
- Ziaee, O. & Choobineh, F., 2017. Optimal Location-Allocation of TCSCs and Transmission Switch Placement Under High Penetration of Wind Power. *IEEE Transactions on Power Systems*, July, 32(4), pp. 3006 3014.