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Reliable location of first responder stations for cooperative response to disasters

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ABSTRACT

Strategic positioning and allocation of emergency responders and/or resources to potential emergency incidents are very important decisions for disaster management programs. In this paper, a reliable multi-type joint-service facility location model is proposed, which takes into consideration the need for cooperative service from multiple types of responder stations, as well as the probabilistic risk of station disruptions. The problem is formulated as a mixed-integer non-linear program and solved via a set of customized linear program and Lagrangian relaxation based algorithms. Numerical experiments on hypothetical and full-scale cases are conducted to demonstrate the applicability of the model and to draw managerial insights.

1. Introduction

Natural catastrophes and human-made disasters pose significant threats to society. In 2018 alone, they caused a total economic loss of approximately $ 165 billion worldwide (McCarthy, 2019). Strategically prepositioning response and rescue resources is one of the most important parts of a disaster management cycle. Many emergency incidents, especially those of large-scale impacts, require cooperative dispatches of resources from multiple responder stations or jurisdictions. One reason is that individual responder stations can be limited by resources, equipment, organizational structure, and staffing strategies. For example, it was reported that 65% of all U.S. firefighters are part-time, on-call volunteers (Ben and Gary, 2017) and as a result, in case of major incidents, multiple stations (or even jurisdictions) might need to cooperate with each other to form an effective response. Another reason is that large-scale incidents usually require services from different types of resources. For example, medical staff, reconstruction technicians, and security personnel had to be dispatched together in order to effectively respond to the 2010 Haiti earthquake (Margesson and Taft-Morales, 2010). During the devastating 2019 Australian bush-fires, more than 3000 reservists, 3700 firefighters, and 440 emergency personnel are dispatched simultaneously from multiple jurisdictions as the disaster developed rapidly (BBC, 2020).

The situation with large-scale disasters is challenging also because built emergency-response systems themselves can be subject to random service disruptions, possibly due to physical damage, resource shortage, or temporary absences of employees as a result of the disasters. Such disruptions of first responder stations during/after disasters will lead to even more damage and loss. It was reported that during Hurricane Maria in Puerto Rico, a third of deaths were caused by healthcare service disruptions (Bryant, 2018). When stations are disrupted, they fail to respond to emergency incidents as originally planned, and service must be delivered through backup stations that are less convenient or responsive. As such, to enhance

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disaster preparedness, emergency management agencies need to determine where to position the first responder stations of each type, which combinations of responder stations to cover a certain potential incident, and in what sequence (and with what probability) to use backups as needed.

There is a large body of literature on facility location problem, which, depending on the objective and constraints, can be formulated as covering-, center-, or median-type models and solved by various discrete and continuous optimization techniques. The covering-type model aims at providing guaranteed service coverage to demand points with the least number of facilities. Toregas et al. (1971) proposed a set covering model for emergency facility location problem which was solved by linear programming type techniques. Several generalized covering models were later developed; for example, Church and ReVelle (1974) developed a maximal covering model that does not enforce coverage of all demand points. The center-type model, as described in Kariv and Hakimi (1979), tries to position a given number of facilities to minimize the worst service time or cost experienced by any potential incident. This type of model is widely used in designing emergency facilities such as medical service facilities (Jia et al., 2007) and fire stations (Serra and Marianov, 1998). The median-type model, in which the objective is to minimize the average or total service time or cost of all potential incidents, was introduced by Hakimi (1964) and later studied extensively in the context of business facilities (Rolland et al., 1997; Charikar and Guha, 1999; Arya et al., 2004).

Most of the emergency facility location problems were formulated as center or covering models where each service demand point is typically served by a single facility; interested readers are referred to Li et al. (2011) and Caunhye et al. (2012) for reviews. Also, most of the aforementioned models assumed that the facilities will be functioning once installed; hence, the system is not designed to be resilient to disruptions – the service quality and system performance might be significantly degraded if the installed stations are disrupted. In light of this, reliable facility location problems have been studied during the past few years to account for probabilistic disruptions of facilities. Daskin (1983) introduced the maximum expected set covering location problem which considered the probabilities that facilities are unable to provide service. Repede and Bernardo (1994) extended the model by incorporating time-varying demand. Snyder and Daskin (2005) proposed reliability versions of the median problem and the uncapacitated fixed charge location problem, where the expected service costs under a large set of facility disruption scenarios are considered. A Lagrangian relaxation algorithm was developed to solve the problem. Cui et al. (2010) studied the reliable uncapacitated fixed charge location problem, in which each facility is subject to independent but location-specific probabilistic disruptions. A mixed integer program formulation and a continuum approximation approach are both developed to solve the problem. Li and Ouyang (2010) started to address correlations among facility disruptions due to interdependencies or shared hazards. This line of work on correlated disruptions was followed by a series of efforts (Li et al., 2013; Xie et al., 2015; 2019). An et al. (2015) further considered the queuing and congestion effect in reliable facility location problems, and a scenario-based stochastic mixed-integer non-linear program and a customized approximation solution algorithm were developed. Xie et al. (2016) then proposed an integer programming formulation of a reliable location-routing problem, in which outbound delivery routing, facility setup, and backup plans are jointly optimized. More recently, Xie and Ouyang (2019) augmented the service network structure by critical network access points, such that the disruption of service facilities could be due to blockage of network access points. All the above reliable facility location literature deal with a single type of facility. Due to complexity, there is very limited literature on the reliable facility location problem with cooperative services from multiple types of stations/jurisdictions. The most relevant problem, to the best of our knowledge, is a sensor location problem as described in An et al. (2018), in which multiple spatially distributed sensors are required to work together to trace a target object via trilateration. However, in that paper, all sensors are identical, and each target object will only use an equal number of sensors. Such assumptions might not be totally suitable for emergency response planning which often requires cooperation across varying numbers and different types of responders.

In all the previous studies on reliable facility locations, there is only a single type of facility which are functionally identical (i.e., in terms of providing the same service), while in this paper, we generalize previous models by considering, in addition to (i) disruption risks, location of (ii) multiple types of facilities (e.g., hospital, fire station, police station) that can provide complementing services, while (iii) each demand point (e.g., fire incident) require cooperative responses from a certain combination of facility types. It is very challenging to incorporate joint services from multiple stations of different types into the reliable facility location problem, as the solution space now must cover discrete location and allocation decisions, combination of facilities as service units, and backup plans under probabilistic facility disruptions. Solving the problem for even small scale instances is very difficult. In spite of these challenges, however, this paper aims at developing a framework of model formulations and solution algorithms for the reliable multi-type joint-service facility location problem. Specifically, this paper develops a systematic approach to track joint service of multi-type facilities when some of the facilities may be subject to disruptions. The problem is formulated as a compact mixed-integer non-linear program to minimize expected systemwide cost across all disruption scenarios. A set of customized algorithms, including linear program and Lagrangian relaxation based algorithms, as well as approximation subroutines for lower bounds, is proposed to solve the problem effectively. A series of numerical experiments are conducted to draw insights on the optimal station deployment and assignment plans under various parameter settings.

The remainder of this paper is organized as follows. Section 2 introduces the mathematical formulation of the proposed reliable multi-type joint-service facility location model. Section 3 discusses the set of customized solution approaches. Section 4 presents the numerical experiments, based on which managerial insights are drawn. Section 5 summarizes the paper and discusses future research directions.

2. Mathematical modeling

We consider a set of discrete demand points $I = \{1, 2, \dots, |I|\}$ in a region where emergency incidents may occur. Each point $i \in I$ has a service demand v_i (e.g., expected risk, probability times consequence). Let $J = \{1, 2, \dots, |J|\}$ be a set of candidate locations for building emergency response stations. We denote the set of emergency station types in the system as $\mathcal{M} = \{1, 2, \dots, |\mathcal{M}|\}$. At location $j \in J$, a type $m \in \mathcal{M}$ station can be built with a fixed set-up cost f_{jm}. We further assume that an incident at i needs cooperative dispatch of resources from a combination of stations, which includes n_{im} type-m stations, where nonnegative integer $n_{im} \in \mathbb{Z}_+ \cup \{0\}$. The set of station types required by demand i is hence denoted as $\mathcal{M}_i = \{m | n_{im} \in \mathbb{Z}_+\} \subseteq \mathcal{M}$. We denote the set of all station combinations by K, such that a combination $k \in K$ includes a set of location-type tuples. For example, if a combination k includes a type-m_1 station located at j_1 and a type-m_2 station located at j_2, then $k = \{(j_1, m_1), (j_2, m_2)\}$. A combination k can be used to serve demand i if it contains n_{im} type m stations, $\forall m \in \mathcal{M}_i$.[1] We introduce a set of binary parameters $\{a_{ik}\}$ to describe the relationship between demand i and combination k, where $a_{ik} = 1$ if demand i can be served by station combination k, or 0 otherwise. A set of parameters $\{b_{kjm}\}$ is also defined to represent the combination-location-type structure, where $b_{kjm} = 1$ if combination k contains a type m station at location j, or 0 otherwise.

A station located at j might fail to respond to an incident with probability p_j. To enhance reliability, we must assign more than n_{im} stations $\forall m \in \mathcal{M}_i$ to provide backups for each demand i. We assume that failure to serve demand point i results in a monetary loss of $e_i^{\max}$. The monetary saving for serving demand i from station combination k is denoted as e_{ik}. We assume this saving is decreasing with the increase of waiting time before service is delivered. These quantities shall be related. If the service can be delivered to instantaneously using station combination k (e.g., in an idealized world, where facilities are perfectly reliable and co-located with the demand point), then e_{ik} is equal to $e_i^{\max}$; i.e., demand point i incurs no economic loss. In contrast, if the required time for service delivery from combination k is too large, then e_{ik} approaches zero, indicating that the demand point i shall incurs the entire economic loss of $e_i^{\max}$. For each demand i, we generally assume that the available stations are preferred based on a generally defined "distance" – when choices are available, stations that are closer to the demand will provide a better service quality and hence be used at a higher priority, while those farther away shall be used only if the high-priority ones are disrupted. For modeling convenience, we also assume there are $N_m = \max_{i \in I}\{n_{im}\}$ type-m virtual dummy stations in the system $\forall m \in \mathcal{M}$ such that there are always at least n_{im} type m stations available for each demand point i even when all the regular stations are disrupted. The dummy stations have no contribution to the system service quality, i.e., when any demand is served by any dummy stations in any scenario, service is effectively discontinued and the service effectiveness is equal to 0. The dummy stations incur zero installation costs and are immune from failure. Let $\tilde{J}_m$ be the set of type m dummy stations and $\tilde{J}$ be the set of all dummy stations, where $|\tilde{J}| = \sum_m N_m$, and $\mathcal{J} = J \cup \tilde{J}$ be the set of all stations.

The main decision variables include those on station installation location, $\mathbf{X} := \{X_{jm}\}$, where $X_{jm} = 1$ if a type m service station is installed at location j, or $X_{jm} = 0$ otherwise. We use binary variables $\mathbf{Z} := \{Z_{ijmr}\}$ to denote service assignments, where $Z_{ijmr} = 1$ if a type m station installed at j is assigned to demand i with backup level r, or 0 otherwise. Higher priority of a station will be represented by a lower value of backup level r (starting from 0 for the highest priority). The assignment of combination k to demand is also traced by a level vector $\vec{r} = (r_1, r_2, \dots, r_m, \dots, r_{|M|}) \in \mathcal{R} = (\mathbb{Z}_+ \cup \{0\})^{|M|}$, where the value of r_m is the largest level index among all type-m station in k. For modeling convenience, we also use $\mathbf{Y} := \{Y_{ik\vec{r}}\}$ to directly describe the assignment of station combinations to the demand, where $Y_{ik\vec{r}} = 1$ if demand i uses combination k whose stations' largest level indices by type are given by $\vec{r}$, or 0 otherwise. Note that $\mathbf{Y}$ is redundant because it can be uniquely determined from the backup levels of stations that are assigned to the demand $\mathbf{Z}$. Finally, we use continuous variables $\mathbf{P} := \{P_{ik\vec{r}}\}$ to denote the pseudo-probability variables, where $P_{ik\vec{r}}$ is the probability that combination k is used by demand i if $Y_{ik\vec{r}} = 1$.

This reliable multi-type joint-service facility location problem can be formulated as the following mixed-integer non-linear program:

$$(\text{RFLP}) \min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{P}} \quad \sum_{m\in\mathcal{M}} \sum_{j\in J} f_{jm} X_{jm} + \sum_{i\in I} \left(e_i^{max} - \sum_{k\in K} \sum_{\vec{r}\in\mathcal{R}} v_i e_{ik} P_{ik\vec{r}} Y_{ik\vec{r}} \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{m=1}^{|\mathcal{M}|} X_{jm} \le 1, \ \forall j \in J, \tag{2}$$

$$\sum_{r=1}^{|\mathcal{J}|} Z_{ijmr} \le X_{jm}, \ \forall i \in I, j \in J, m \in \mathcal{M}_i, \tag{3}$$

[1] Traditional non-cooperative response in the literature (i.e., those provided by one station) can be considered as a special case of our model, in which we can simply define each combination to contain only a single station.

$$\sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|} Z_{ijmr} \le 1, \ \ \forall i \in I, j \in J, \tag{4}$$

$$\sum_{r=1}^{|\mathcal{J}|} Z_{ijmr} = 1, \ \ \forall i \in I, j \in \tilde{J}_m, m \in \mathcal{M}_i, \tag{5}$$

$$Z_{ijmr} = Z_{i,j+1,m,r+1}, \ \ \forall i \in I, j \in \tilde{J}_m \backslash \left\{\max\left\{\tilde{J}_m\right\}\right\}, r = 1, 2, \cdots, |\mathcal{J}| - 1, m \in \mathcal{M}_i, \tag{6}$$

$$\sum_{j\in J} Z_{ijmr} + \sum_{s=1}^{r} Z_{i,j',m,s} = 1, \ \ \forall i \in I, r = 1, \cdots, |\mathcal{J}|, m \in \mathcal{M}_i, j' = \min\left\{\tilde{J}_m\right\} \tag{7}$$

$$Y_{ik\vec{r}} \le \frac{1}{\sum_m n_{im}} \sum_{j\in\mathcal{J}} \sum_{m\in\mathcal{M}_i} \sum_{s=1}^{r_m} a_{ik} b_{kjm} Z_{ijms}, \ \ \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, \tag{8}$$

$$Y_{ik\vec{r}} \le \sum_{j\in\mathcal{J}} a_{ik} b_{kjm} Z_{ijmr_m}, \ \ \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, m \in \mathcal{M}_i, \tag{9}$$

$$P_{ik\vec{r}'} = \sum_{j\in\mathcal{J}} \left(p_j\right)^{1_{[j\in J]}} Z_{i,j,m,(r_m+1)} P_{ik,\vec{r}}, \quad \forall i \in I, k \in K, \vec{r} = (r_1, \ldots, r_m, \ldots, r_{|\mathcal{M}|}), \vec{r}\,' = (r_1, \ldots, r_m + 1, \ldots, r_{|\mathcal{M}|}) \in \mathcal{R}, m \in \mathcal{M}_i, \tag{10}$$

$$P_{ik\vec{0}} = \prod_{m\in\mathcal{M}_i} \prod_{j\in J} \left(1 - p_j\right)^{a_{ik}b_{kjm}} \left(p_j\right)^{-a_{ik}b_{kjm}}, \ \ \forall i \in I, k \in K, \tag{11}$$

$$X_{jm}, Z_{ijmr}, Y_{ik\vec{r}} \in \{0, 1\}, \ \ \forall k \in K, j \in \mathcal{J}, \vec{r} \in \mathcal{R}, r = 1, \cdots, |\mathcal{J}|, i \in \mathcal{I}, m \in \mathcal{M}. \tag{12}$$

The objective function (1) contains the station setup costs and the expected economic loss due to disasters. Constraints (2) require that at most one station can be installed at each candidate location.[2] Constraints (3) ensure that demand can be served by only installed stations. Note that Constraints (4) are included here for modeling convenience in the remaining part of this paper though they can be derived from (2) and (3). Constraints (5) state that each dummy station must be assigned to a demand point at a certain backup level, while the same dummy station could be shared by multiple demand points at various levels. Constraints (6) enforce that if a dummy station $j \in \tilde{J}_m$ is assigned to demand i at level r, then dummy station $j+1$ must be assigned to i at level $r+1$ as well. As such, the dummy stations will be used only as the last resort. Constraints (7) require that demand i cannot use a regular station at level r if it uses the first dummy station at one of the higher-backup levels, $s \le r$. Constraints (8) enforce that combination k is available to demand i only if all stations in k are installed. These constraints are enforced by (i) to serve demand i using combination k at backup level $\vec{r}$, all relevant assignments variables Z_{ijmr} must be set to 1; and (ii) by enforcing constraints (3), Z_{ijmr} can be set to 1 only if a type m station is installed at location j, i.e., $X_{jm} = 1$. Constraints (9) relate the level vector of a station combination to the largest backup level index of any station type in that combination. Constraints (10) and (11) recursively define the assignment probability P_{ikr} for $Y_{ikr} = 1$, in which $1_{[\cdot]} = 1$ if condition $[\cdot]$ is true, or 0 otherwise. The correctness of these assignment probability formulas are proven in the following proposition.

Proposition 1. *The recursive equations (10)-(11) define the assignment probabilities.*

Proof. When $Y_{ik\vec{r}} = 0$, the value of $P_{ik\vec{r}}$ has no effect on the objective function, hence we will only prove the correctness of $P_{ik\vec{r}}$ when $Y_{ik\vec{r}} = 1$. Expanding the recursive formula (10)-(11), we see that $P_{ik\vec{r}}$ can be written as

$$P_{ik\vec{r}} = \prod_{m\in\mathcal{M}_i} \prod_{j\in J} \left(1 - p_j\right)^{a_{ik}b_{kjm}} \frac{\prod_{m\in\mathcal{M}_i} \prod_{s\le r_m} \left[\sum_{j\in\mathcal{J}} Z_{ijms} \left(p_j\right)^{1_{[j\in J]}}\right]}{\prod_{m\in\mathcal{M}_i} \prod_{j\in J} p_j^{a_{ik}b_{kjm}}}. \tag{13}$$

It is essentially the product of two probabilities. The term $\prod_{m\in\mathcal{M}_i} \prod_{j\in J} \left(1 - p_j\right)^{a_{ik}b_{kjm}}$ is equal to the probability that all stations in combination k are functioning. The remainder term, $\frac{\prod_{m\in\mathcal{M}_i} \prod_{s\le r_m} \left[\sum_{j\in\mathcal{J}} Z_{ijms} (p_j)^{1_{[j\in J]}}\right]}{\prod_{m\in\mathcal{M}_i} \prod_{j\in J} p_j^{a_{ik}b_{kjm}}}$, represents the probability that all the installed stations, by type, with priorities higher than those in k are disrupted. This completes the proof. □

[2] This is not restrictive. Any location that allow multiple facility installation can be duplicated accordingly.

3. Solution approach

3.1. Linearization approach

Problem (RFLP) can be linearized using the technique introduced in Sherali and Alameddine (1992) (also discussed in Li and Ouyang (2012); An et al. (2018)). Observing that $P_{ik\vec{r}}Y_{ik\vec{r}}$ and $Z_{i,j,m,(r_m+1)}P_{ik,\vec{r}}$ are both bounded between 0 and 1, we will replace them by new variables $\mathbf{W} = \{W_{ik\vec{r}}\}$ and $\mathbf{L} = \left\{L_{i,j,m,k,\vec{r}}\right\}$, respectively, and then add the following set of constraints,

$$W_{ik\vec{r}} \le P_{ik\vec{r}} + 1 - Y_{ik\vec{r}}, \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, \tag{14}$$

$$W_{ik\vec{r}} \ge P_{ik\vec{r}} + Y_{ik\vec{r}} - 1, \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, \tag{15}$$

$$W_{ik\vec{r}} \le Y_{ik\vec{r}}, \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, \tag{16}$$

$$W_{ik\vec{r}} \ge -Y_{ik\vec{r}}, \forall i \in I, k \in K, \vec{r} \in \mathcal{R}, \tag{17}$$

$$L_{i,j,m,k,\vec{r}} \le P_{ik\vec{r}} + 1 - Z_{i,j,m,r_m+1}, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}, k \in K, \vec{r} \in \mathcal{R}, \tag{18}$$

$$L_{i,j,m,k,\vec{r}} \ge P_{ik\vec{r}} + Z_{i,j,m,r_m+1} - 1, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}, k \in K, \vec{r} \in \mathcal{R}, \tag{19}$$

$$L_{i,j,m,k,\vec{r}} \le Z_{i,j,m,(r_m+1)}, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}, k \in K, \vec{r} \in \mathcal{R}, \tag{20}$$

$$L_{i,j,m,k,\vec{r}} \ge -Z_{i,j,m,(r_m+1)}, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}, k \in K, \vec{r} \in \mathcal{R}, \tag{21}$$

$$L_{i,j,m,k,\vec{r}}, W_{ik\vec{r}} \ge 0, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}, k \in K, \vec{r} \in \mathcal{R}. \tag{22}$$

Then (RFLP) is transformed into the following linearized reliable facility location problem (LRFLP).

$$\text{(LRFLP)} \min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{P},\mathbf{W},\mathbf{L}} \quad \sum_{m\in\mathcal{M}} \sum_{j\in J} f_{jm}X_{jm} + \sum_{i\in I} \left(e_i^{max} - \sum_{k\in K} \sum_{\vec{r}\in\mathcal{R}} v_i e_{ik} W_{ik\vec{r}} \right) \tag{23}$$

$$\begin{aligned} \text{s.t.} \quad & (2)-(9), (11)-(12), (14)-(22) \\ & P_{ik\vec{r}'} = \sum_{j\in\mathcal{J}} \left(p_j\right)^{1_{[j\in J]}} L_{i,j,m,k,\vec{r}}, \\ & \qquad \forall i \in I, k \in K, \vec{r} = (r_1, \dots, r_m, \dots, r_{|\mathcal{M}|}), \vec{r}\,' = (r_1, \dots, r_m + 1, \dots, r_{|\mathcal{M}|}) \in \mathcal{R}, m \in \mathcal{M}_i. \end{aligned} \tag{24}$$

Problem (LRFLP) is a mixed-integer linear program and can be solved using existing commercial solvers (for example, Gurobi). However, as we will observe in Section 4, the problem becomes very challenging for commercial solvers to handle even for relatively small problem instances due to the large number of interrelated variables. Hence a customized algorithm is developed in Section 3.2 to solve the problem more efficiently.

3.2. Lagrangian relaxation based approach

3.2.1. Lagrangian relaxation framework

In (RFLP), the location decision variables **X** and sensor assignment variables **Z** are related through constraints (3). This is one of the primary sources of model complexity. A Lagrangian relaxation (LR) approach can be adopted to decompose the problem into subproblems. In so doing, we relax constraints (3) and then add them to the objective function (1) using a set of non-negative Lagrangian multipliers $\boldsymbol{\mu} = \left\{\mu_{ij}, \forall i \in I, j \in J\right\}$, to obtain the following relaxed problem (RRFLP).

$$\text{(RRFLP)} \min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{P}} \quad \sum_{m\in\mathcal{M}} \sum_{j\in J} \left(f_{jm} - \sum_{i\in\{i:m\in\mathcal{M}_i\}} \mu_{ijm} \right) X_{jm} + \sum_{i\in I} \left(e_i^{max} - \sum_{k\in K} \sum_{\vec{r}\in\mathcal{R}} v_i e_{ik} P_{ik\vec{r}} Y_{ik\vec{r}} \right) + \sum_{i\in I} \sum_{j\in J} \sum_{m\in\mathcal{M}_i} \sum_{r=1}^{|\mathcal{J}|} \mu_{ijm} Z_{ijmr} \tag{25}$$

$$\text{s.t.} \quad (2), (4)-(12).$$

Problem (RRFLP) has two separable parts that can be solved separately for any given set of multipliers $\boldsymbol{\mu}$. The first part involves only location variables $\mathbf{X}$, i.e.,

$$\min_{\mathbf{X}} \quad \sum_{m\in\mathcal{M}}\sum_{j\in J}\left(f_{jm} - \sum_{i\in\{i:m\in\mathcal{M}_i\}}\mu_{ijm}\right)X_{jm} + \sum_{i\in I} e_i^{max},$$
$$\text{s.t. } (2),$$
$$X_{jm}\in\{0,1\}, \forall j\in J, m\in\mathcal{M}.$$

It can be solved easily by inspecting the cost coefficient, i.e., $f_{jm} - \sum_{i\in\{i:m\in\mathcal{M}_i\}}\mu_{ij}$, for each X_{jm}. The second part involves variables $\mathbf{Y}$ and $\mathbf{Z}$ but can be solved separately for each demand i. This sub-problem, formulated for each demand i, is as follows,

$$(\text{RRFLP}_i)\ \min_{\mathbf{Y},\mathbf{Z},\mathbf{P}} \quad \sum_{j\in J}\sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|}\mu_{ijm}Z_{ijmr} - \sum_{k\in K}\sum_{\vec{r}\in\mathcal{R}} v_i e_{ik} P_{ik\vec{r}} Y_{ik\vec{r}} \tag{26}$$

$$\text{s.t.} \quad \sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|} Z_{ijmr} \le 1, \ \ \forall j\in J, \tag{27}$$

$$\sum_{r=1}^{|\mathcal{J}|} Z_{ijmr} = 1, \ \ \forall j\in\tilde{J}_m, m\in\mathcal{M}_i, \tag{28}$$

$$Z_{ijmr} = Z_{i,j+1,m,r+1}, \ \ \forall j\in\tilde{J}_m\backslash\left\{\max\left\{\tilde{J}_m\right\}\right\}, r=1,2,\cdots,|\mathcal{J}|-1, m\in\mathcal{M}_i, \tag{29}$$

$$\sum_{j\in J} Z_{ijmr} + \sum_{s=1}^{r} Z_{i,j',m,s} = 1, \ \ \forall r=1,\cdots,|\mathcal{J}|, m\in\mathcal{M}_i, j'=\min\left\{\tilde{J}_m\right\}, \tag{30}$$

$$Y_{ik\vec{r}} \le \frac{1}{\sum_m n_{im}} \sum_{j\in\mathcal{J}}\sum_{m\in\mathcal{M}_i}\sum_{s=1}^{r_m} a_{ik} b_{kjm} Z_{ijms}, \ \ \forall k\in K, \vec{r}\in\mathcal{R}, \tag{31}$$

$$Y_{ik\vec{r}} \le \sum_{j\in\mathcal{J}} a_{ik} b_{kjm} Z_{ijmr_m}, \ \ \forall k\in K, \vec{r}\in\mathcal{R}, m\in\mathcal{M}_i, \tag{32}$$

$$P_{ik\vec{r}'} = \sum_{j\in\mathcal{J}} \left(p_j\right)^{1_{[j\in J]}} Z_{i,j,m,(r_m+1)} P_{ik,\vec{r}},$$
$$\forall k\in K, \vec{r} = (r_1,\ldots,r_m,\ldots,r_{|\mathcal{M}|}), \vec{r}\,' = (r_1,\ldots,r_m+1,\ldots,r_{|\mathcal{M}|})\in\mathcal{R}, m\in\mathcal{M}_i, \tag{33}$$

$$P_{ik\vec{0}} = \prod_{m\in\mathcal{M}_i}\prod_{j\in J}\left(1-p_j\right)^{a_{ik}b_{kjm}}\left(p_j\right)^{-a_{ik}b_{kjm}}, \ \ \forall k\in K, \tag{34}$$

$$Z_{ijmr}, Y_{ik\vec{r}} \in \{0,1\}, \ \ \forall k\in K, j\in\mathcal{J}, \vec{r}\in\mathcal{R}, r=1,\cdots,|\mathcal{J}|, i\in\mathcal{I}, m\in\mathcal{M}. \tag{35}$$

We notice that, even with relaxation, (RRFLP$_i$) remains challenging due to the recursive computation of $\mathbf{P}$ in (33) and the nonlinearity introduced by $P_{ik\vec{r}}Y_{ik\vec{r}}$ in (26). To further reduce the computation complexity, we replace $P_{ik\vec{r}}$ with a set of constants following the ideas proposed by An et al. (2018). For each combination k, we identify a set of candidate locations that are not used in k, denoted as $T_k = \left\{j_1,\ldots,j_{|T_k|}\right\}$. Then we sort the candidate locations in T_k by the site-dependent disruption probability in descending order, i.e., $p_{j_1} \ge p_{j_2} \ge \ldots \ge p_{j_{|T_k|}}$, and replace $P_{ik\vec{r}}$ with $\beta_{ik\vec{r}} = \prod_{m\in\mathcal{M}_i}\prod_{j\in J}\left(1-p_j\right)^{a_{ik}b_{kjm}}\prod_{m\in\mathcal{M}_i}\prod_{t=1}^{r_m-n_{im}} p_{j_t}$, in which j_t is the t^{th} element in T_k. By replacing $P_{ik\vec{r}}$ with $\beta_{ik\vec{r}}$, we will have the following program.

$$(\text{RRFLP}_i\text{-1})\ \min_{\mathbf{Y},\mathbf{Z}} \quad \sum_{j\in J}\sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|}\mu_{ijm}Z_{ijmr} - \sum_{k\in K}\sum_{\vec{r}\in\mathcal{R}} v_i e_{ik}\beta_{ik\vec{r}} Y_{ik\vec{r}} \tag{36}$$
$$\text{s.t.} \quad (27)-(32), (35).$$

Proposition 2. *The optimal objective value of* (RRFLP$_i$-1) *provides a lower bound to* (RRFLP$_i$).

Proof. Since RRFLP$_i$ is a minimization problem, the optimal objective will only decrease if we remove constraints (33)-(34). Also, if we consider the objective function of RRFLP$_i$, the value of $P_{ik\vec{r}}$ has no impact if $Y_{ik\vec{r}} = 0$. When $Y_{ik\vec{r}} = 1$, we notice that by the definition of $\beta_{ik\vec{r}}$, the probabilities of the stations in k being functioning are the same as those of $P_{ik\vec{r}}$. While $P_{ik\vec{r}}$ includes the probability of the stations that are not in k being disrupted, the upper bound of this probability is used in $\beta_{ik\vec{r}}$. Hence, $P_{ik\vec{r}}$ is bounded from the above by $\beta_{ik\vec{r}}$, and the optimal objective value of RRFLP$_i$-1 provides a lower bound to RRFLP$_i$. □

If (RRFLP$_i$-1) can be effectively solved at each Lagrangian iteration, then together with the solution to the first part of (RRFLP), we have already obtained a lower bound to the original problem (RFLP), as well as the station installation decisions **X** (from the first part of RRFLP). Following the convention, we can build a feasible solution based on **X**, by assigning demand to the nearest stations sequentially following the ideas proposed in An et al. (2018). For each demand i and a station type m, sort the set of installed type-m stations based on increasing distance from customer i, as $J_{im} = \left\{j_{1,m}, j_{2,m}, j_{|J_{im}|,m}\right\}$, and set $Z_{ij_{km}mk} = 1$ for all $k = 1, \dots, |J_{im}|$, or 0 otherwise. And then determine **Y** from **Z** accordingly. This feasible solution gives us an upper bound to the original problem RFLP. Then we can use these bounds to update the Lagrangian multipliers using the sub-gradient techniques proposed in Fisher (1981). To further close the gap, we embed the LR algorithm into a branch-and-bound (B&B) framework. The basic idea is to branch on variables **X** and build a search tree. We perform a breadth-first-search in the search tree, and at each tree node, we call the LR algorithm to obtain a lower bound. The final upper bound would be the best upper bound obtained at each node of the search tree. For the final lower bound of the root problem, we first compute the smallest lower bound at each fully explored level of the search tree, and then use the maximum of these lower bounds as the final lower bound.

3.2.2. Lower bounds approximation

Unfortunately, solving (RRFLP$_i$-1) is still very time-consuming owing to the similarity to a generalized assignment problem and the large number of assignment variables **Y**. As such, two approximation approaches, i.e., (RRFLP$_i$-2) and (RRFLP$_i$-3), are developed in this subsection to reduce the computation time needed for obtaining lower bounds.To this end, we consider ways to replace **Y** and approximate the service quality. First, we define a set of variables, $\mathbf{U} := \left\{U_{ijm\vec{r}}\right\}$, and a set of parameters, $C_{ijm\vec{r}} = \max_{k\in K}\left\{a_{ik}e_{ik}\beta_{ik\vec{r}}\rho_{ikjm}\right\}$, where $\rho_{ikjm} = 1$ if j is the remotest type-m station for demand i in combination k or 0 otherwise. Also, we let $N_{i\vec{r}} = \prod_{m\in\mathcal{M}_i}\binom{r_m-1}{n_{im}-1}$, and $\gamma_{ijm\vec{r}} = \frac{C_{ijm\vec{r}}N_{i\vec{r}}}{|\mathcal{M}_i|}$. Then we can formulate (RRFLP$_i$-2) as follows to obtain another lower bound.

$$(\text{RRFLP}_i\text{-2}) \min_{\mathbf{Z},\mathbf{U}} \quad \sum_{j\in J}\sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|}\mu_{ijm}Z_{ijmr} - \sum_{\vec{r}\in\mathcal{R}}\sum_{j\in J}\sum_{m\in\mathcal{M}_i}\nu_i\gamma_{ijm\vec{r}}U_{ijm\vec{r}} \tag{37}$$

$$\text{s.t.} \quad (27)-(30),$$

$$U_{ijm\vec{r}} \le Z_{i,j,m,r_m}, \forall m\in\mathcal{M}_i,\ j\in J, \vec{r} = (r_1,\dots,r_m,\dots,r_{|\mathcal{M}|})\in\mathcal{R}, \tag{38}$$

$$U_{ijm\vec{r}} \le \sum_{j'\in J\setminus\{j\}} Z_{ij'm'r_{m'}},\quad \forall\vec{r} = (r_1,.r_{m'},.r_{|\mathcal{M}|})\in\mathcal{R},\ j\in J, m\in\mathcal{M}_i, m'\in\mathcal{M}_i\setminus\{m\}, \tag{39}$$

$$Z_{ijmr}, U_{i\bar{j}\vec{r}} \in \{0,1\},\ \ \forall j\in\mathcal{J}, \vec{j}\in\bar{J}, \vec{r}\in\mathcal{R}, r = 1,\cdots,|\mathcal{J}|, i\in\mathcal{I}, m\in\mathcal{M}_i. \tag{40}$$

Proposition 3. *The optimal objective value of problem (RRFLP$_i$-2) provides a lower bound to (RRFLP$_i$-1)*

Proof. Suppose the optimal solution to (RRFLP$_i$-1) is $(\mathbf{Z}^*, \mathbf{Y}^*)$ and the corresponding optimal objective value is denoted $F(\mathbf{Z}^*, \mathbf{Y}^*)$. We will show that there always exists a feasible solution $(\mathbf{Z}', \mathbf{U})$ to (RRFLP$_i$-2), such that the associated objective value is no larger than $F(\mathbf{Z}^*, \mathbf{Y}^*)$. First, we set $Z'_{ijmr} = Z^*_{ijmr}, \forall j\in\mathcal{J}, m\in\mathcal{M}_i, r = 1,\cdots,|\mathcal{J}|$, and $U_{ijm\vec{r}} = \min\left\{Z^*_{i,j,m,r_m}, \min_{m'\in\mathcal{M}_i\setminus\{m\}}\left\{\sum_{j'\in\mathcal{J}\setminus\{j\}} Z^*_{ij'm'r_{m'}}\right\}\right\}$. Obviously, Z' and U satisfy constraints (38)-(40). Then we will show that the objective function (37) is no larger than $F(\mathbf{Z}^*, \mathbf{Y}^*)$. For each $\vec{r}$, we define a set $K_{i\vec{r}} = \{k : Y^*_{i,k,\vec{r}} = 1\} \subseteq K$. We first notice that if $K_{i\vec{r}}$ is non-empty, then in the optimal solution to (RRFLP$_i$-1), for any $m\in\mathcal{M}_i$, all combinations $k\in K_{i\vec{r}}$ share the same station $\tilde{j}_m(\vec{r})$ in level r_m, i.e., the remotest type-m station for demand i in any combination $k\in K_{i\vec{r}}$ is $\tilde{j}_m(\vec{r})$, and we know $Z^*_{i,\tilde{j}_m(\vec{r}),m,r_m} = 1$ (as enforced by constraints (30) and (32)). Also we know that $U_{i,\tilde{j}_m(\vec{r}),m,\vec{r}} = 1$ for all $m\in\mathcal{M}_i$ from (32) and our construction of $U_{i,\tilde{j}_m(\vec{r}),m,\vec{r}}$. Finally, we can see that $|K_{i\vec{r}}| = N_{i\vec{r}}$. Hence we have the following for any $\vec{r}\in\mathcal{R}$,

$$\sum_{k\in K} e_{ik}\beta_{ik\vec{r}}Y^*_{ik\vec{r}} = \sum_{k\in K_{i\vec{r}}} e_{ik}\beta_{ik\vec{r}}Y^*_{ik\vec{r}} \le N_{i\vec{r}}\frac{\sum_{m\in\mathcal{M}_i}\max_{k\in K_{i\vec{r}}}\left\{e_{ik}\beta_{ik\vec{r}}b_{ik\tilde{j}_m(\vec{r})m}\right\}}{|\mathcal{M}_i|}$$
$$\le \sum_{m\in\mathcal{M}_i}\gamma_{i,\tilde{j}_m(\vec{r}),m,\vec{r}}U_{i,\tilde{j}_m(\vec{r}),m,\vec{r}} \le \sum_{m\in\mathcal{M}_i}\sum_{j\in\mathcal{J}}\gamma_{ijm\vec{r}}U_{ijm\vec{r}}.$$

As such, we can conclude that the solution to problem (RRFLP$_i$-2) provides a lower bound to (RRFLP$_i$-1). □

An even tighter lower bound can be obtained, if we are tolerant to a larger relaxed problem size and a slightly higher computational cost. To this end, we define a set of vectors $\vec{j} = \left(j_1, j_2, \dots, j_m, \dots, j_{|M|}\right) \in \bar{J}$, in which $j_m \in J \cup \left\{\hat{j}\right\}$ and $\hat{j}$ is a placeholder. To approximate the quality of service and avoid computation burden introduced by $\mathbf{Y}$, we define a set of new variables, $\mathbf{V} := \left\{V_{i\vec{j}\vec{r}}\right\}$, and a set of parameters, $C'_{i\vec{j}\vec{r}} = \max_{k\in K}\left\{a_{ik}e_{ik}\beta_{ik\vec{r}}\eta_{k\vec{j}}\right\}$, in which $\eta_{ik\vec{j}} = 1$ if j_m (the m^{th} entry of $\vec{j}$) is the remotest type-m station for demand i in combination k for all $m \in \mathcal{M}_i$ or 0 otherwise. Also, similarly we let $\gamma_{i\vec{j}\vec{r}} = C'_{i\vec{j}\vec{r}}N_{i\vec{r}}$. Then we can define another approximate problem, (RRFLP$_i$-3), as follows. It gives another lower bound.

$$(\text{RRFLP}_i\text{-3}) \min_{\mathbf{Z},\mathbf{V}} \quad \sum_{j\in J}\sum_{m\in\mathcal{M}_i}\sum_{r=1}^{|\mathcal{J}|}\mu_{ijm}Z_{ijmr} - \sum_{\vec{r}\in\mathcal{R}}\sum_{\vec{j}\in\bar{J}}\nu_i\gamma_{i\vec{j}\vec{r}}V_{i\vec{j}\vec{r}} \tag{41}$$

$$\text{s.t.} \quad (27)-(30),$$

$$V_{i\vec{j}\vec{r}} \le Z_{i,j_m,m,r_m}, \forall m \in \mathcal{M}_i, \vec{j} = (j_1, \dots, j_m, \dots, j_M) \in \bar{J}, \vec{r} = (r_1, \dots, r_m, \dots, r_{|\mathcal{M}|}) \in \mathcal{R} \tag{42}$$

$$Z_{ijmr}, V_{i\vec{j}\vec{r}} \in \{0,1\}, \quad \forall j \in \mathcal{J}, \vec{j} \in \bar{J}, \vec{r} \in \mathcal{R}, r = 1, \cdots, |\mathcal{J}|, i \in \mathcal{I}, m \in \mathcal{M}1. \tag{43}$$

Proposition 4. *The optimal objective value of problem (RRFLP$_i$-3) provides a lower bound to (RRFLP$_i$-1)*

Proof. The proof uses a similar idea as that for Proposition 3. We will show that there is a feasible solution $(\mathbf{Z}', \mathbf{V})$ to (RRFLP$_i$-3) whose objective value is no larger than the optimal objective value of (RRFLP$_i$-1), denoted by $F(\mathbf{Z}^*, \mathbf{Y}^*)$. Similarly, we set $Z'_{ijmr} = Z^*_{ijmr}, \forall j \in \mathcal{J}, m \in \mathcal{M}_i, r = 1, \cdots, |\mathcal{J}|$, $V_{i\vec{j}\vec{r}} = \min_{m\in\mathcal{M}_i}\left\{Z^*_{i,j_m,m,r_m}\right\}$, and $K_{i\vec{r}} = \{k : Y^*_{i,k,\vec{r}} = 1\} \subseteq K$ for each $\vec{r}$. Obviously, Z' and W satisfy the constraints in (RRFLP$_i$-3). To see the objective function in (41) is no larger than $F(\mathbf{Z}^*, \mathbf{Y}^*)$, we first notice that in the optimal solution to (RRFLP$_i$-1), for any $m \in \mathcal{M}_i$, all combinations $k \in K_{i\vec{r}}$ share the same station j_m in level r_m, i.e., the remotest type-m station for demand i in any combination $k \in K_{i\vec{r}}$ is j_m, and we know $Z^*_{i,j_m,m,r_m} = 1$, which is enforced by constraints (30) and (32). We collect all such j_m in a vector $\vec{j}'(\vec{r})$ (using placeholder $\hat{j}$ if $m \notin M_i$), and we know that $V_{i,\vec{j}'(\vec{r}),\vec{r}} = 1$ by construction. Hence, we have the following, for any $\vec{r} \in \mathcal{R}$:

$$\sum_{k\in K} e_{ik}\beta_{ik\vec{r}}Y^*_{ik\vec{r}} = \sum_{k\in K_{i\vec{r}}} e_{ik}\beta_{ik\vec{r}}Y^*_{ik\vec{r}} \le N_{i\vec{r}}\max_{k\in K_{i\vec{r}}}\left\{e_{ik}\beta_{ik\vec{r}}b_{k\vec{j}'(\vec{r})}\right\} \le \gamma_{i,\vec{j}'(\vec{r}),\vec{r}} = \gamma_{i,\vec{j}'(\vec{r}),\vec{r}}V_{i,\vec{j}'(\vec{r}),\vec{r}} \le \sum_{\vec{j}\in\bar{J}}\gamma_{i\vec{j}\vec{r}}V_{i\vec{j}\vec{r}}.$$

As such, we conclude that the solution to problem (RRFLP$_i$-3) provides a lower bound to (RRFLP$_i$-1). □

It shall be noted that the number of variables in (RRFLP$_i$-3) is much larger than that in (RRFLP$_i$-2), mainly due to the existence of $\vec{j}$ However, as shown in the following proposition, the solution to (RRFLP$_i$-3) provides a tighter lower bound to (RRFLP$_i$-1).

Proposition 5. *The optimal objective value of (RRFLP$_i$-2) provides a lower bound to (RRFLP$_i$-3). In other words, the optimal objective value of (RRFLP$_i$-3) provides a tighter lower bound to (RRFLP$_i$-1) than the optimal objective value of (RRFLP$_i$-2).*

Proof. The logic behind the proof is again through construction. Given any optimal solution to $(\mathbf{Z}^*, \mathbf{V}^*)$ to (RRFLP$_i$-3) with optimal objective function $F'(\mathbf{Z}^*, \mathbf{V}^*)$, we will show that there is always a feasible solution $(\mathbf{Z}, \mathbf{U})$ to (RRFLP$_i$-2), and the objective function is no larger than $F'(\mathbf{Z}^*, \mathbf{V}^*)$. Then, the optimal objective value of (RRFLP$_i$-2) must be no larger than $F'(\mathbf{Z}^*, \mathbf{V}^*)$ as well. To do this, we set $Z_{ijmr} = Z^*_{ijmr}, \forall i \in I, j \in \mathcal{J}, m \in \mathcal{M}_i, r = 1, \cdots, |\mathcal{J}|$. Also, for any $\vec{j}$ and $\vec{r}$ such that $V^*_{i\vec{j}\vec{r}} = 1$, we can set $U_{ijm\vec{r}} = 1$ if j is the m^{th} entry in $\vec{j}, \forall m \in \mathcal{M}_i$ and $j \in J$. And we will set other Z_{ijmr} and $U_{ijm\vec{r}}$ to 0. The feasibility of $(\mathbf{Z}, \mathbf{U})$ is enforced by (42). For any $\vec{r}$, if we have $\gamma_{i,\vec{j}'(\vec{r}),\vec{r}}V^*_{i,\vec{j}(\vec{r})',\vec{r}} \ne 0$ for some $\vec{j}(\vec{r})' \in \bar{J}$, then we define $\tilde{j}_m(\vec{r})$ as the m^{th} entry in $\vec{j}(\vec{r})'$. Note that by the definition of $\gamma_{i,\vec{j}'(\vec{r}),\vec{r}}$, the value of $\vec{j}'(\vec{r})$ is unique. Also, we have $U_{i\tilde{j}_m(\vec{r})m\vec{r}} = 1$ from the construction of $\mathbf{U}$. As such, we have the following for each $\vec{r}$,

$$\sum_{\vec{j}\in\bar{J}}\gamma_{i\vec{j}\vec{r}}V^*_{i\vec{j}\vec{r}} = \gamma_{i,\vec{j}'(\vec{r}),\vec{r}}V^*_{i,\vec{j}'(\vec{r}),\vec{r}} = \sum_{m\in\mathcal{M}_i}\frac{\gamma_{i,\vec{j}'(\vec{r}),\vec{r}}}{|\mathcal{M}_i|}U_{i\tilde{j}_m(\vec{r})m\vec{r}} \le \sum_{m\in\mathcal{M}_i}\sum_{j\in J}\gamma_{ijm\vec{r}}U_{ijm\vec{r}}.$$

Note the second inequality holds since $C'_{i\vec{j}\vec{r}}$ is defined by taking the maximum over a smaller set as compared to what we used for $C_{ijm\vec{r}}$. Hence, we see that the solution to (RRFLP$_i$-2) is a lower bound to (RRFLP$_i$-3). □

Approximate problems (RRFLP$_i$-2) or (RRFLP$_i$-3) can be solved by commercial solvers such as Gurobi to obtain better lower bounds to (RFLP). The rest of the LR algorithm as well as the use of B&B framework remains the same.It should be noted, however, that Proposition 5 only holds under the same multipliers; it does not necessarily indicate that the LR gaps obtained using (RRFLP$_i$-3) is smaller than those obtained using (RRFLP$_i$-2). The reason is that, in the LR algorithm, the multipliers might converge to different values, and in turn, possibly yield looser lower bounds. Also, the computation time per Lagrangian iteration can be significantly different, and hence use of (RRFLP$_i$-2) may still have a better overall convergence rate and performance within a limited computation time.

4. Numerical results

In this section, we demonstrate the applicability of our modeling approach by running a series of numerical experiments over a set of hypothetical examples and a Chicago railroad network example. All numerical experiments are coded in Python and tested on a 3.4 GHz Intel i7 laptop with 16 GB RAM.

4.1. Hypothetical case

We consider 3 hypothetical grid networks as shown in Figs. 1a-1c. The cases include $|J| = 2 \times 2, 3 \times 3,$ and 4×4 candidate station locations, where 2 types of stations can be built to serve $|I| = 2 - 8$ demand points. We assume service demand $v_i = 1, \forall i,$ and consider the following service station-type requirements: for network (i), $n_{11} = 1, n_{12} = 1, n_{21} = 0, n_{22} = 2$; for network (ii), $n_{i1} = 1, n_{i2} = 1, \forall i \in \{1, 2\}, n_{i2} = 2, \forall i \in \{3, 4\}$ and $n_{im} = 0$ otherwise; for network (iii), $n_{i1} = 1, n_{i2} = 1, \forall i \in \{1, \ldots, 4\}, n_{i2} = 2, \forall i \in \{5, \ldots, 8\}$ and $n_{im} = 0$ otherwise. We assume $e_{ik} = \sum_{j \in J} \sum_{m \in \mathcal{M}} \frac{a_{ik} b_{kjm} e_i^{\max}}{2+20 * d_{ij}}$, $\forall i \in I, k \in K$, in which d_{ij} is the Euclidean distance from demand point i to station location j. Parameter $e_i^{\max}$ is assumed to be 300 \$. To evaluate the performance of our model, we try different station failure probabilities, i.e., $p_j = p \in \{0.05, 0.1\}, \forall j \in J,$ as well as different values of station installation cost $f_{jm} = f \in \{5, 10\}, \forall j \in J, m \in \mathcal{M}$. The performance of (LRFLP) formulation and the two LR approaches, i.e., (RRFLP$_i$-2) and (RRFLP$_i$-3), which will be referred to as LR-1 and LR-2, respectively.

The results are summarized in Table 1. All the cases are solved for 7200 seconds and the optimality gaps are less than 3%. It should be noted that Gurobi failed to find feasible solutions if we directly use the (LRFLP) formulation for cases 5–12. For cases 1–4, our algorithms were able to find the same best feasible solution as Gurobi. For cases 5–12, the (approximate) optimality gap is computed as (UB-LB)/UB, where upper bound UB is used as the benchmark because we want to evaluate the difference between UB and LB against the actual system cost of a feasible solution, i.e., UB. We can see that as the station setup cost and failure probability increase, the total cost will increase for most cases. Also, we can observe different optimal deployment decisions from Figs. 2a and 2c. In all three cases, the optimal decision is to install both type 1 and type 2 stations at the bottom part of the region where demand requires service from both types of stations. From cases 1–8, we can see that when there is a small number of candidate locations and demand, (B&B+LR-2) can obtain better lower bounds and smaller gaps as compared with (B&B+LR-1). However, when the problem size becomes large, the computation

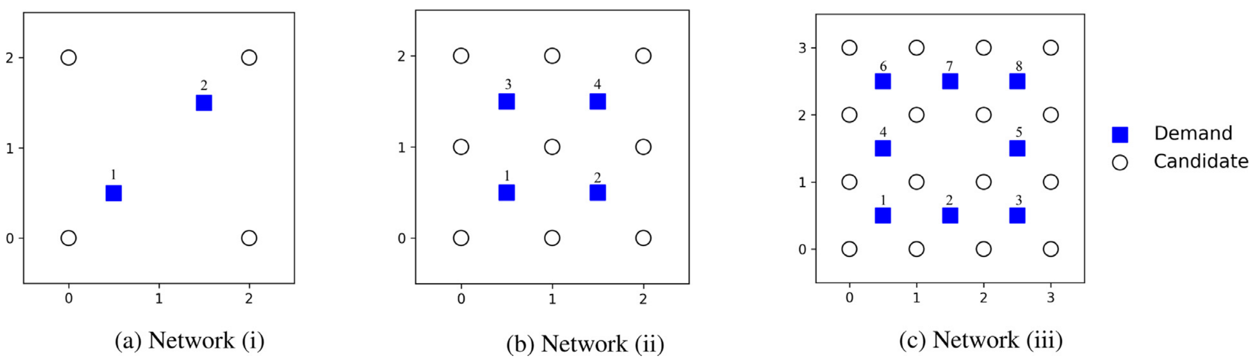
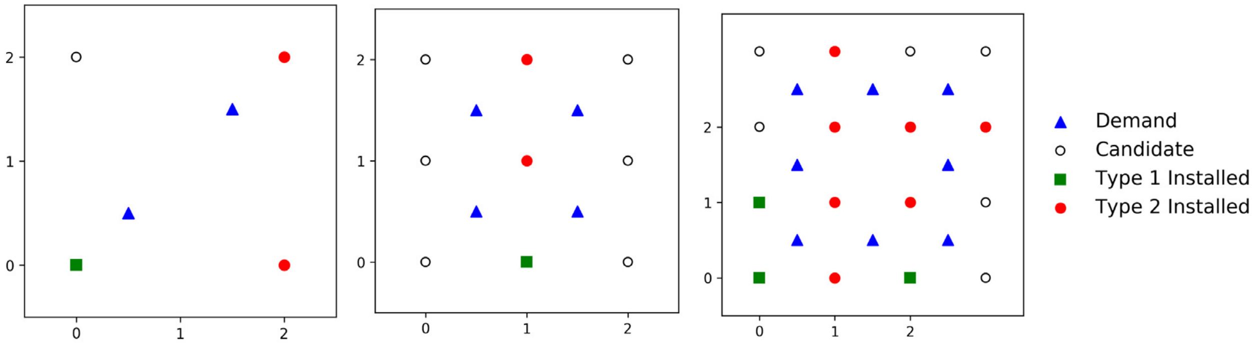


Fig. 1. Hypothetical network.

Table 1
Numerical results for hypothetical network case.

Case	Network	p	f	LRFLP	B&B+LR-1				B&B+LR-2			
					LB	UB	Gap	# of installed stations	LB	UB	Gap	# of installed stations
1	i	0.05	10	579	578	579	0.17%	3	578	579	0.17%	3
2	i	0.05	5	564	562	564	0.36%	3	564	564	0.0%	3
3	i	0.10	10	583	582	583	0.17%	3	582	583	0.17%	3
4	i	0.10	5	568	567	568	0.18%	3	567	568	0.18%	3
5	ii	0.05	10	-	1086	1093	0.64%	3	1091	1093	0.18%	3
6	ii	0.05	5	-	1073	1078	0.47%	4	1073	1077	0.37%	4
7	ii	0.10	10	-	1097	1103	0.55%	4	1097	1103	0.55%	4
8	ii	0.10	5	-	1078	1083	0.46%	5	1078	1083	0.46%	5
9	iii	0.05	10	-	2155	2198	2.0%	6	2148	2198	2.33%	6
10	iii	0.05	5	-	2131	2157	1.22%	10	2133	2167	1.59%	10
11	iii	0.10	10	-	2154	2198	2.04%	7	2151	2214	2.93%	9
12	iii	0.10	5	-	2138	2163	1.17%	10	2132	2161	1.36%	7



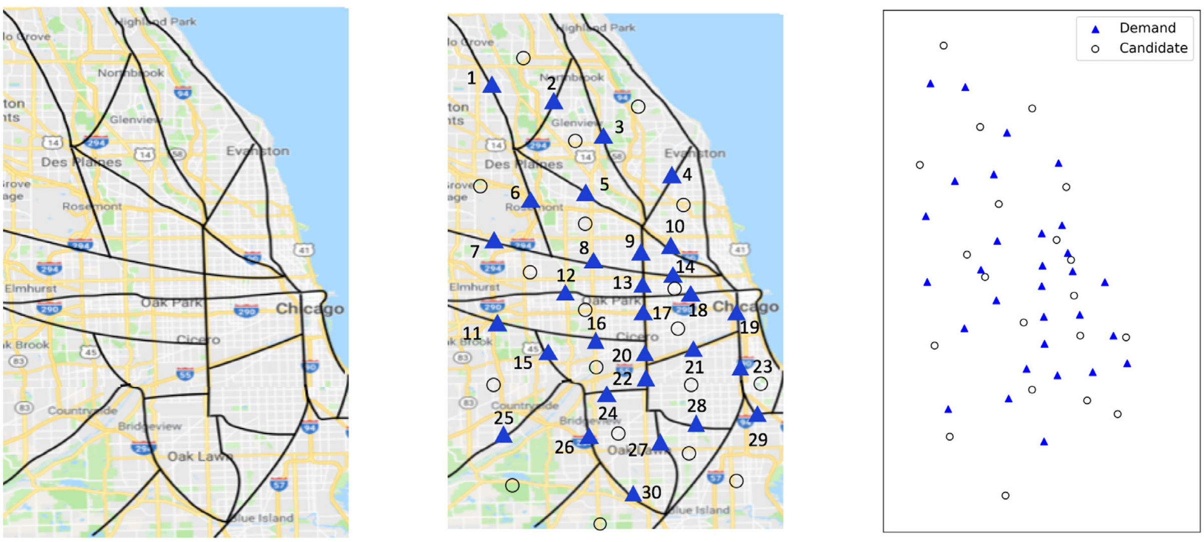
(a) Case 1 (B&B+LR-1) (b) Case 5 (B&B+LR-1) (c) Case 10 (B&B+LR-1)

Fig. 2. Optimal station location.

time per iteration for LR-2 becomes relatively large and thus the overall performance will degrade. Hence, we can see that even though the subproblem in LR-2 is more complex and harder to solve, it is possible that the tighter lower bound from it can improve LR convergence and reduce optimality gaps; some evidence is shown in these numerical experiments, in which LR-2 seems to work better for smaller size network.

4.2. Chicago example

The proposed model is also tested with a Chicago railroad network example as shown in Fig. 3a. The data was originally used in Xie and Ouyang (2019). The 30 railroad segments face heavy hazardous material traffic and are subject to incidents such as fires and chemical spills, and thus can be considered as service demand points. We assume these railroad segments might require 2 types of service facilities, e.g., fire stations and hospitals, and there are 20 candidate locations where these 2 types of stations can be installed, as shown in Fig. 3b-3c. We design two service need patterns: (i) $n_{i1} = 1, n_{i2} = 1, \forall i \in \{1, \dots, 15\}, n_{i2} = 2, \forall i \in \{16, \dots, 30\}$ and $n_{im} = 0$ otherwise; (ii) $n_{i1} = 1, n_{i2} = 1, \forall i \in I$. The service quality function and the remaining parameters are the same as those for the hypothetical network, except that we are using city street network instead of Euclidean distance. To evaluate the performance of our model, we try different station failure probabilities, i.e., $p_j = p \in \{0.05, 0.1\}, \forall j \in J$, different values of station installation cost $f_{jm} = f \in \{5, 10\}, \forall j \in J, m \in \mathcal{M}$, as well as different monetary value of loss, $e_i^{\max} = e^{\max} \in \{200, 500, 800\}, \forall i \in I$.



(a) Chicago railroad (b) Demand + candidate (c) Abstract representation

Fig. 3. Chicago railroad network.

Table 2
Numerical results for Chicago network case.

Case	Demand pattern	p	f	e_i^{max}	B&B+LR-1				SA Result
					# of installed stations	LB	UB	Gap	
1	i	0.05	5	200	7	5922	5953	0.5%	5968
2	i	0.05	10	200	4	5934	5982	0.8%	6012
3	i	0.1	5	200	6	5926	5953	0.5%	5970
4	i	0.1	10	200	3	5942	5978	0.6%	6014
5	i	0.05	5	500	14	14760	14817	0.4%	14865
6	i	0.05	10	500	9	14783	14874	0.6%	14906
7	i	0.1	5	500	10	14771	14827	0.4%	14870
8	i	0.1	10	500	10	14797	14877	0.5%	14912
9	i	0.05	5	800	18	23606	23696	0.4%	23754
10	i	0.05	10	800	8	23638	23731	0.4%	23799
11	i	0.1	5	800	8	23609	23705	0.4%	23763
12	i	0.1	10	800	12	23645	23745	0.4%	23808
13	ii	0.05	5	200	5	5909	5949	0.7%	5963
14	ii	0.05	10	200	3	5934	5974	0.7%	6004
15	ii	0.1	5	200	10	5914	5960	0.8%	5965
16	ii	0.1	10	200	3	5933	5975	0.7%	6006
17	ii	0.05	5	500	14	14746	14818	0.5%	14846
18	ii	0.05	10	500	6	14759	14868	0.7%	14888
19	ii	0.1	5	500	14	14748	14825	0.5%	14852
20	ii	0.1	10	500	6	14766	14880	0.8%	14893
21	ii	0.05	5	800	15	23582	23688	0.4%	23706
22	ii	0.05	10	800	12	23621	23747	0.5%	23772
23	ii	0.1	5	800	11	23581	23688	0.5%	23724
24	ii	0.1	10	800	12	23609	23747	0.6%	23781

For the Chicago example, direct application of commercial solvers such as Gurobi will certain not work. Hence, we compare our algorithms with a simulated annealing (SA) algorithm. In the SA algorithm, we will randomly perturb 3 entries in binary location decisions $\mathbf{X}$ at each perturbation, i.e., 3 facilities will be installed/uninstalled at certain locations. Given the location decisions $\mathbf{X}$, we obtain the optimal assignment decisions, i.e., $\mathbf{Z}$ and $\mathbf{Y}$, conditional on $\mathbf{X}$ using the same approach as described in Section 3.2, and we evaluate the objective function value. Here we run B&B+LR-1 algorithm and the heuristic algorithm for 7200 seconds and the results are summarized in Table 2. Note here only B&B+LR-1 algorithm is used since the problem size of B&B+LR-2 becomes very large and thus Gurobi fails to solve it within the time limit.

We can see that the optimality gaps are less than 1% for all cases. Also, the total cost will increase as f increases for all cases. If we compare cases 1 and 2, we can observe that fewer stations will be installed as f increases. When we compare cases 6 and 8, we can see that the number of installed stations changes from 9 to 10 when the failure probability changes from 0.05 to 0.1, which indicates that it is more profitable to install more stations to hedge against the service quality reduction due to station disruptions. However, if we compare case 9 with case 11, we can see although the failure probability increases to 0.1, the number of installed stations actually decreases from 18 to 8, which indicates that the marginal installation cost exceeds the marginal increase of service quality. Hence, how the number of installed stations will change with the failure probability depends on the trade-off between the increase of the expected marginal service quality and the marginal installation cost. For example, it is possible that the monetary value of effectiveness improvement from building additional stations is larger than the cost of setting up the facilities, in which we should set up more stations. If the monetary value of the expected service quality improvement of building more stations is smaller than the facility setup cost, for example, the probabilities of using newly installed stations is very small while the facility setup cost is relatively large, then it is not cost-effective to set up these additional stations. A similar observation was also made in An et al. (2018).

We can also see that in Fig. 4a and Fig. 4b, notably more type-2 stations are installed in order to serve demand pattern (i), and most type-2 stations are installed at the southern part of the city, where more type-2 stations are needed. While for demand pattern (ii), we can see a more homogenous installation distribution; for example, in Fig. 4c, the numbers of type-1 and type-2 stations installed are 5 and 7, respectively. This is probably because in pattern (ii), all demand points require service from a type 1 station and a type 2 station. Also, we can compare the best feasible objective functions obtained from our proposed algorithm (i.e., UB) and the SA algorithm, which are summarized in column 8 and column 10 of Table 2, respectively. We can see that the proposed algorithm can (i) outperform the SA algorithm (and proven to be effective) for large size networks, by obtaining solutions with smaller total costs for all 24 cases; and (ii) yield a tight lower bound which can be utilized to evaluate the quality of solutions, i.e., the optimality gaps, as shown in column 9 of Table 2. Finally, it should be noted that our observation is based on the feasible solutions we get from our algorithms (which might not reflect the true optimal results), but we believe the insights are correct and useful.

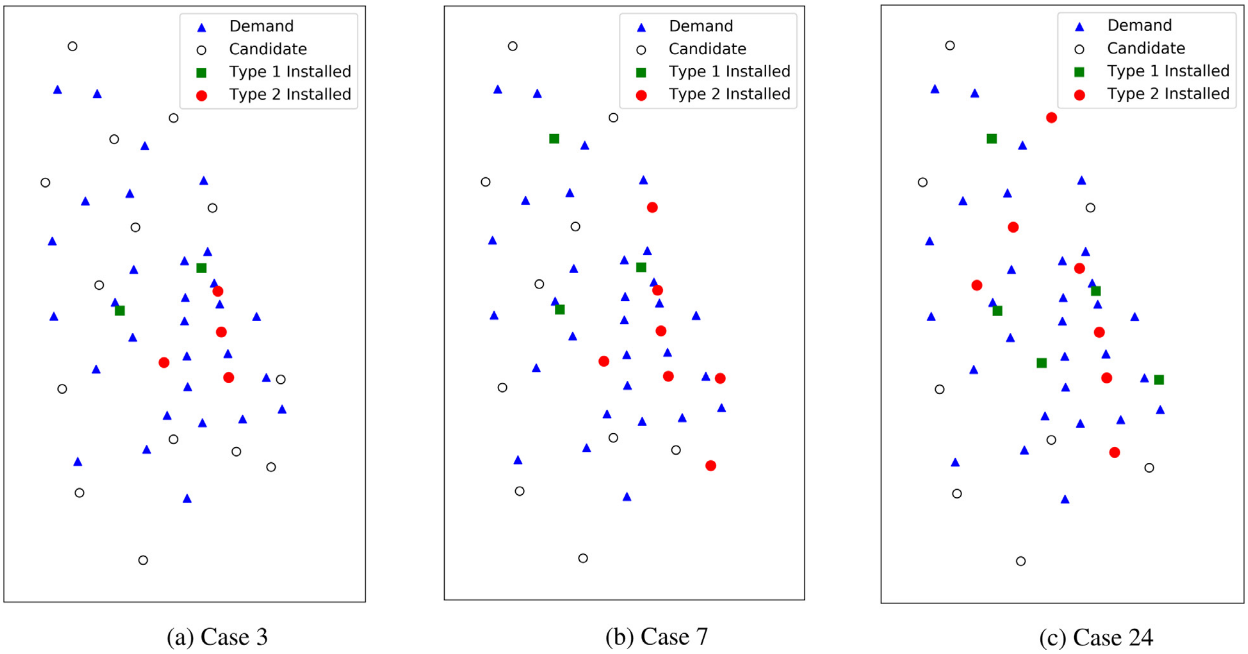


Fig. 4. Optimal station installation.

5. Summary and future research

In many emergency response systems, cooperation across different types of responders is required. However, it is very challenging to incorporate joint service decisions into the reliable facility location problem due to (i) most existing literature on reliable facility location problem assumes there is only a single type of facility which are functionally identical (i.e., in terms of providing the same service), while the literature on systems with multi-type facilities and evaluation of the performance of such systems under probabilistic disruption and reliability considerations is quite lacking; (ii) it is difficult to model the interrelationship among multi-type facility location decisions, assignment of facility combinations of proper types to serve demand points with different service need patterns, and backup plans (also in terms of facility combinations) under different disruption scenarios; and (iii) when multi-type stations and the cooperation dispatching strategies are considered, the number of variables and constraints becomes significantly larger than that of the models for single-type facilities, which makes it very challenging to solve the model. In this paper, we developed a reliable multi-type joint-service facility location model to optimally position and allocate emergency response resources such that the expected quality of service across multiple stations is maximized under the risk of station disruptions. The model is formulated as a mixed-integer non-linear program, and then solved using a set of customized algorithms based on linear program relaxation, Lagrangian relaxation, as well as approximate formulation methods for enhanced lower bounds. A series of numerical experiments and sensitivity analyses are conducted to demonstrate how the proposed model could be applied to both hypothetical and full-scale cases, and how the proposed algorithm far outperforms commercial solvers in terms of solution quality and computation time.

Future studies can be extended in several directions. First, this paper assumes that each station is disrupted independently. It would be interesting to look at the possibility of correlated disruptions (e.g., due to shared hazards or spatial proximity); see Xie et al. (2015, 2019). Some large-scale disasters often cause degradation to the supporting transportation networks as well, and this effectively contributes to disruption of the services (beyond the failures of the stations themselves), as discussed in Bell et al. (2014); Xie and Ouyang (2019). We would also like to investigate how to deploy resources for joint services from a combination of unreliable stations through unreliable roadway networks. Furthermore, when the size of network is extremely large, the discrete mixed-integer models in this paper can suffer from prohibitive computation costs. Hence, it may be very important to derive more efficient solution algorithms, such as those based on continuum approximation and following the ideas in Cui et al. (2010); Li and Ouyang (2010), for large-scale networks. Also, no capacity constraints have been considered for the stations. In reality, a station might only be able to respond to a limited number of incidents at the same time (especially under disruptions). It will be of interest to explore how capacity constraints can be incorporated into the proposed model. Finally, this paper assumes that the demand is deterministic while in the real world, the demand might be stochastic which should affect the optimal installation strategy and service plan.

Declaration of Competing Interest

No.

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