

A Decomposition and Coordination Approach for Large Sub-hourly Unit Commitment

Jianghua Wu, *Student Member, IEEE*, Peter B. Luh, *Life Fellow, IEEE*, Yonghong Chen, *Senior Member, IEEE*, Bing Yan, *Member, IEEE*, Mikhail A. Bragin, *Member, IEEE*

Abstract -- Sub-hourly Unit Commitment (UC) problems have been suggested as a way to improve power system efficiency. Such problems, however, are much more difficult than hourly UC problems. This is not just because of the increased number of period to consider, but also because of much reduced unit ramping capabilities leading to more complicated convex hulls. As a result, state-of-the-art and practice methods such as branch-and-cut suffer from poor performance. In this paper, our recent Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) method, which overcame major difficulties of standard Lagrangian Relaxation, is enhanced by synergistically incorporating the concept of Ordinal Optimization (OO). By using OO, solving subproblems becomes much faster. Testing of Midcontinent ISO (MISO)'s problem with 15 minutes as the time interval over 36 hours involving about 1,100 units and 15000 virtuals demonstrates that the new method obtains near-optimal solutions efficiently and significantly outperforms branch-and-cut.

Index Terms -- Sub-hourly Unit Commitment; Mixed Integer Linear Programming; Surrogate Absolute-Value Lagrangian Relaxation; Ordinal Optimization

I. INTRODUCTION

Unit Commitment (UC) is an important problem formulated as a Mixed Integer Linear Programming (MILP) problem. While traditionally, UC problems are solved with hourly time resolution, the Federal Energy Regulatory Commission (FERC) reported in 2012 that traditional hourly schedules are insufficient for operators to manage power systems efficiently [1]. To provide a more reliable power system, sub-hourly schedules are suggested by FERC. The comparison of results based on 5, 15, 30, and 60-minute intervals [2] shows that sub-hourly schedules lead to more realistic estimations of total generation costs over traditional hourly schedules. In [3, 4], 15-minute schedules under high levels of renewable penetration produce substantial savings. Such sub-hourly problems, however, are much more difficult than hourly problems. This is not just because of the increased

number of periods to consider, but also because of much reduced unit ramping capabilities per period leading to more complicated convex hulls as shown in Figure 9 in [5]. As a result, state-of-the-art and practice methods such as branch-and-cut suffer from poor performance [6].

To overcome the complexity difficulty, Lagrangian Relaxation (LR) has been traditionally used [7]. A problem's complexity is significantly reduced after relaxing coupling constraints and decomposing the relaxed problem into subproblems. However, LR suffers from zigzagging of multipliers thereby resulting in slow convergence. Moreover, the method requires the knowledge of the optimal dual value for convergence proof and for practical implementations [8]. Surrogate Lagrangian Relaxation (SLR) overcame major difficulties of traditional LR [9]. Its convergence has recently been accelerated by introducing linearizable absolute-value penalty terms in the Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) method [10]. However, if solving subproblems takes a long time, the performance of SAVLR may not be good enough.

In this paper, large and complicated 15-minute interval UC problems within Midcontinent ISO (MISO)'s context [6] is considered. The problem formulation, including virtuals and dispatchable demand, is presented in section III. Transmission capacity constraints are modeled as soft constraints, which allow violations with penalties.

In view of the complexity of sub-hourly UC problems as explained above, SAVLR is enhanced by incorporating the Ordinal Optimization (OO) concept in Section IV. The idea is to obtain "good enough" solutions (subjecting to the surrogate optimality condition to be explained later) for subproblems based on simplified models to reduce computation efforts [11]. Specific methods to reduce CPU time and enhance performance are studied.

In section V, a MISO problem with 15 minutes as the time interval over 36 hours involving about 1,100 units and 15000 virtuals is tested. Testing results demonstrate that the new method obtains near-optimal solutions within a specified amount of time, and significantly outperforms branch-and-cut.

II. LITERATURE REVIEW

In subsection II.A, the main ideas and major difficulties of branch-and-cut (B&C) are reviewed, together with those of traditional Lagrangian Relaxation (LR), Augmented Lagrangian Relaxation (ALR), and Alternate Direction Method of Multipliers (ADMM). Our recent Surrogate Lagrangian

This work is supported in part by the Midcontinent ISO and by the National Science Foundation under grants ECCS-1810108, and CNS-1647209. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the MISO and NSF.

J. Wu, P. B. Luh, and M. A. Bragin are with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT, 06269 (e-mails: jianghua.wu; peter.luh; mikhael.bragin@uconn.edu).

Y. Chen and is with MISO Energy, Carmel, IN, 46032 (e-mails: ychen@misoenergy.org).

B. Yan is with the Department of Electrical and Microelectronic Engineering, Rochester Institute of Technology, Rochester, NY, 14623 (e-mail: bxyee@rit.edu).

relaxation (SLR) and Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) are reviewed in subsection II.B. The Ordinal Optimization (OO) concept is then presented in subsection II.C.

A. Branch-and-Cut and Lagrangian Relaxation

Currently, commercial Mixed Integer Linear Programming (MILP) solvers are widely based on the B&C method. The method applies valid cuts trying to obtain the smallest convex set that encloses the set of feasible solutions (the convex hull). If the convex hull is obtained, then the problem is reduced to an LP problem, whose optimal feasible solution is located at one of the convex hull's vertices and can be easily obtained. However, the convex hull may not always be obtained in a computationally efficient way, since valid cuts are problem dependent and facet-defining cuts are generally difficult to find. The time-consuming branch-and-bound method and heuristics may then be needed to obtain feasible solutions. For a large UC problem, we have vividly witnessed the poor performance or breakdown of branch-and-cut [13].

Lagrangian Relaxation (LR) is a traditional method to solve MILP problems based on the concept of decomposition and coordination. A problem's complexity is significantly reduced after coupling constraints are relaxed and the relaxed problem is decomposed into subproblems. However, performance of standard LR is limited because of its major difficulties: 1) substantial effort to obtain a subgradient – requiring all subproblems to be solved; 2) zigzagging of Lagrangian multipliers because of non-differentiability of dual functions caused by the presence of discrete decision variables in the original formulation; 3) reliance on the knowledge of the optimal dual value for convergence proof and for practical implementation [7].

Augmented Lagrangian Relaxation (ALR) improves the convergence of standard LR by introducing quadratic penalties for constraint violations [14]. Its variation, the Alternate Direction Method of Multipliers (ADMM), alternatively solves two subproblems. When problems are large or complicated, subproblems may still be difficult to solve, and the convergence of ADMM cannot be ensured with the presence of discrete decision variables [15]. In addition, MILP solvers cannot be directly used to solve ALR or ADMM subproblems in view of the presence of quadratic penalty terms.

B. Our Recent Developments: SLR and SAVLR

Surrogate Lagrangian relaxation (SLR) overcame the major difficulties of traditional LR. It obtains surrogate subgradient directions by solving one or a few subproblems subject to the “surrogate optimality conditions.” Then it uses surrogate subgradient directions to update Lagrangian multipliers, significantly reducing computational efforts and multiplier zigzagging. Moreover, its convergence proof is based on the contraction mapping concept and does not require the knowledge of the optimal dual value [9].

The convergence of SLR has recently been accelerated by introducing linearizable absolute-value penalty terms – the Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) method [10]. With exact linearization through extra variables and constraints, it is demonstrated that SAVLR has

significantly better performance as compared to methods such as LR, ADMM, SLR and B&C [10].

C. Ordinal Optimization

The idea of OO has two aspects: 1) an “order” is much easier to determine than a “value”; 2) the problem becomes easier after softening the optimization goal. Compared to an optimal solution, “good enough” solutions are easier to obtain [11]. Many examples of applying OO to accelerate the computation of the simulation processes are shown in [12].

III. PROBLEM FORMULATION

In this section, a UC problem with I conventional units, V virtuals, Y dispatchable demand and L transmission lines over T 15-min time intervals (or looking-ahead $T/4$ hours) is considered. The formulation is based on those in [6], [8] and [16-18]. Constraints considered include (1) coupling system demand, reserve and transmission capacity constraints; (2) conventional unit-level generation capacity, minimum up- and down-time and ramp-rate constraints; and (3) virtual capacity and dispatchable demand constraints. Transmission capacity constraints are modeled as soft constraints, which allow violations with penalties.

Constraints

System Demand Constraints (Energy Balance). System demand should equal total generation at each time period, i.e.,

$$G_t = D_t, \forall t \in T, \quad (1)$$

$$G_t = \sum_{i \in I} p_{i,t} + \sum_{i \in V} x_{i,t} - \sum_{i \in Y} y_{i,t}, \forall t \in T. \quad (2)$$

Here, the total generation at time t is denoted by G_t , and the net system demand at time t is denoted by D_t . At time t , generation level of unit i ($i \in I$) is denoted by $p_{i,t}$ (MW), energy produced by virtual i ($i \in V$) $x_{i,t}$ and energy dispatched by demand ($i \in Y$) $y_{i,t}$.

Transmission Capacity Constraints. The net flow in a transmission line is limited by the capacities of the line for each period:

$$f_{t,l} - \bar{s}_{t,l} \leq \bar{F}_l, \forall t \in T, \forall l \in L, \quad (3)$$

$$f_{t,l} + \underline{s}_{t,l} \geq \underline{F}_l, \forall t \in T, \forall l \in L, \quad (4)$$

$$f_{t,l} = \sum_{i \in I} \alpha_{i,l} p_{i,t} + \sum_{i \in V} \alpha_{i,l} x_{i,t} - \sum_{i \in Y} \alpha_{i,l} y_{i,t} - \sum_{i \in D} \alpha_{i,l} d_{i,t} \quad \forall t \in T, \forall l \in L. \quad (5)$$

For each transmission line l and time t , $\underline{F}_l$ and $\bar{F}_l$ are capacity limits of line l ; $f_{t,l}$ is the power flow, and it is obtained by calculating the net injection from all nodes weighted by generation shift factor $\alpha_{i,l}$; and $d_{i,t}$ is the fixed demand. These constraints are soft and have non-negative slack variables $\bar{s}_{t,l}$ and $\underline{s}_{t,l}$ with the penalty coefficient C^P as will be seen in (11).

System Reserve Requirements. There are minimum amount requirements for the total regulating reserve, the total spinning reserve and the sum of all reserves [16]:

$$\sum_{i \in I} rr_{i,t} \geq R_t^r, \forall t \in T, \quad (6)$$

$$\sum_{i \in I} rs_{i,t} \geq R_t^s, \forall t \in T, \quad (7)$$

$$\sum_{i \in I} (rr_{i,t} + rs_{i,t} + rsn_{i,t} + rsf_{i,t}) \geq R_t^{all}, \forall t \in T. \quad (8)$$

The amounts of the regulating reserve and spinning reserve are denoted by $rr_{i,t}$ and $rs_{i,t}$, respectively; and $rsn_{i,t}$ and $rsf_{i,t}$ are the amounts of online and offline supplemental reserves. The required amounts of these reserves are denoted by R_t^r , R_t^s , and R_t^{all} .

Conventional Unit Constraints. Conventional unit constraints including max-up constraints, maximum start-up constraints, maximum energy constraints and reserve capacity constraints follow the modeling of (2) in [5]. Different types of start-up (hot start-up, intermediate start-up and cold start-up) and shut-down requirements follow the modeling of (2)-(3) in [17]; and the state transition logic, generator capacity constraints and ramp-up/down constraints follow the modeling of (8)-(13) in [17]. Min-up/down constraints follow the modeling of (1)-(2) in [18]. *Individual Virtual and Dispatchable Demand Constraints.* Virtuals are subject to energy capacity constraints:

$$\underline{X}_{i,t} \leq x_{i,t} \leq \bar{X}_{i,t}, \forall i \in V, \forall t \in T, \quad (9)$$

where $x_{i,t}$ is the energy provided by virtual i ($i \in V$) at time t . Similarly, dispatchable demand i ($i \in Y$) has limits on its level $y_{i,t}$ for time t :

$$0 \leq y_{i,t} \leq \bar{Y}_{i,t}, \forall i \in Y, \forall t \in T. \quad (10)$$

Objective Function

For a conventional unit i at time t , there are no-load cost $C_{i,t}^{NL}$, start-up cost $C_{i,t}^{Start}$, energy cost $C_{i,t}^E$ (depending on the generation level $p_{i,t}$), and reserve costs $C_{i,t}^{RR}$, $C_{i,t}^{RS}$, $C_{i,t}^{RSN}$ and $C_{i,t}^{RSF}$. For a virtual (or dispatchable demand), it has energy cost $C_{i,t}^V$ (or $C_{i,t}^Y$). The objective is to minimize the total cost, which consists of costs from all resources plus the penalties for the violations of transmission capacity constraints, i.e.,

$$\min_{\delta, u, p, x, y, r} \left\{ \begin{aligned} & \sum_{i \in I} \sum_{t \in T} (\sum_{s \in S} C_{i,s,t}^{Start} \delta_{i,s,t} + C_{i,t}^{NL} u_{i,t} + C_{i,t}^E \\ & + C_{i,t}^{RR} rr_{i,t} + C_{i,t}^{RS} rs_{i,t} + C_{i,t}^{RSN} rsn_{i,t} + C_{i,t}^{RSF} rsf_{i,t}) \\ & + \sum_{i \in V} \sum_{t \in T} (C_{i,t}^V x_{i,t}) - \sum_{i \in Y} \sum_{t \in T} (C_{i,t}^Y y_{i,t}) \\ & + C^P \sum_{t \in T} \sum_{l \in L} (\bar{s}_{t,l} + \underline{s}_{t,l}) \end{aligned} \right\}. \quad (11)$$

IV. SOLUTION METHODOLOGY

The decomposition and coordination Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) method is presented in subsection IV.A. The synergistic combination of SAVLR, Ordinal Optimization (OO) and Branch-and-cut (B&C) is developed in subsection IV.B. In subsection IV.C, specific methods to reduce CPU time and enhance performance are discussed.

A. The Decomposition and Coordination SAVLR Method

Based on [10], system demand constraints (1) are relaxed by using Lagrangian multiplier λ_t^k , and reserve constraints (6), (7) and (8) are relaxed by $\mu_t^{rr,k}$, $\mu_t^{rs,k}$ and $\mu_t^{r,k}$, respectively. The violations of these constraints are penalized by absolute-value

penalty terms with the penalty coefficient $\frac{c^k}{2}$. Then the objective function of the relaxed problem becomes:

$$\min_{\delta, u, p, x, y, r} \left\{ \begin{aligned} & \sum_{i \in I} \sum_{t \in T} (\sum_{s \in S} C_{i,s,t}^{Start} \delta_{i,s,t} + C_{i,t}^{NL} u_{i,t} + C_{i,t}^E \\ & + C_{i,t}^{RR} rr_{i,t} + C_{i,t}^{RS} rs_{i,t} + C_{i,t}^{RSN} rsn_{i,t} + C_{i,t}^{RSF} rsf_{i,t}) \\ & + \sum_{i \in V} \sum_{t \in T} (C_{i,t}^V x_{i,t}) - \sum_{i \in Y} \sum_{t \in T} (C_{i,t}^Y y_{i,t}) \\ & + \sum_{t \in T} \lambda_t^k (D_t - G_t) + \frac{c^k}{2} \sum_{t \in T} |D_t - G_t| \\ & + \sum_{t \in T} \mu_t^{rr,k} (R_t^r - \sum_{i \in I} rr_{i,t}) + \frac{c^k}{2} \sum_{t \in T} |R_t^r - \sum_{i \in I} rr_{i,t}| \\ & + \sum_{t \in T} \mu_t^{rs,k} (R_t^s - \sum_{i \in I} rs_{i,t}) + \frac{c^k}{2} \sum_{t \in T} |R_t^s - \sum_{i \in I} rs_{i,t}| \\ & + \sum_{t \in T} \mu_t^{r,k} [R_t^{all} - \sum_{i \in I} (rr_{i,t} + rs_{i,t} + rsn_{i,t} + rsf_{i,t})] \\ & + \frac{c^k}{2} \sum_{t \in T} |R_t^{all} - \sum_{i \in I} (rr_{i,t} + rs_{i,t} + rsn_{i,t} + rsf_{i,t})| \\ & + C^P \sum_{t \in T} \sum_{l \in L} (\bar{s}_{t,l} + \underline{s}_{t,l}) \end{aligned} \right\}. \quad (12)$$

The number of subproblems and the number of conventional units per subproblem are chosen by balancing subproblems' complexity and model loading efforts. Rather than grouping virtuals into a single subproblem, all virtuals are included in each subproblem to avoid the dramatic change of results across iterations.

Subproblem j is formed by taking out all variables belonging to units and virtuals of the subproblem while keeping variables associated with other units and virtuals at their latest available values:

$$\min_{\delta, u, p, x, y, r \in j} \left\{ \begin{aligned} & \sum_{i \in I_j} \sum_{t \in T} (\sum_{s \in S} C_{i,s,t}^{Start} \delta_{i,s,t} + C_{i,t}^{NL} u_{i,t} + C_{i,t}^E \\ & + C_{i,t}^{RR} rr_{i,t} + C_{i,t}^{RS} rs_{i,t} + C_{i,t}^{RSN} rsn_{i,t} + C_{i,t}^{RSF} rsf_{i,t}) \\ & + \sum_{i \in V} \sum_{t \in T} (C_{i,t}^V x_{i,t}) - \sum_{i \in Y} \sum_{t \in T} (C_{i,t}^Y y_{i,t}) \\ & + \sum_{t \in T} \lambda_t^k (\tilde{D}_t) + \frac{c^k}{2} \sum_{t \in T} |\tilde{D}_t| \\ & + \sum_{t \in T} \mu_t^{rr,k} (\tilde{R}_t^r) + \frac{c^k}{2} \sum_{t \in T} |\tilde{R}_t^r| \\ & + \sum_{t \in T} \mu_t^{rs,k} (\tilde{R}_t^s) + \frac{c^k}{2} \sum_{t \in T} |\tilde{R}_t^s| \\ & + \sum_{t \in T} \mu_t^{r,k} (\tilde{R}_t^{all}) + \frac{c^k}{2} \sum_{t \in T} |\tilde{R}_t^{all}| \\ & + C^P \sum_{l \in L} \sum_{t \in T} (\bar{s}_{t,l} + \underline{s}_{t,l}) \end{aligned} \right\}. \quad (13)$$

s.t. all individual unit-level constraints. In the above,

$$\tilde{G}_t = D_t - (\sum_{i \in I_j} p_{i,t} + \sum_{i \notin I_j} p_{i,t}^{k-1} + \sum_{i \in V} x_{i,t} - \sum_{i \in Y} y_{i,t}), \forall t \in T, \quad (14)$$

$$\tilde{R}_t^r = R_t^r - \sum_{i \in I_j} rr_{i,t} - \sum_{i \notin I_j} rr_{i,t}^{k-1}, \forall t \in T, \quad (15)$$

$$\tilde{R}_t^s = R_t^s - \sum_{i \in I_j} rs_{i,t} - \sum_{i \notin I_j} rs_{i,t}^{k-1}, \forall t \in T, \quad (16)$$

$$\tilde{R}_t^{all} = R_t^{all} - \sum_{i \in I_j} (rr_{i,t} + rs_{i,t} + rsn_{i,t} + rsf_{i,t}) - \sum_{i \notin I_j} (rr_{i,t}^{k-1} + rs_{i,t}^{k-1} + rsn_{i,t}^{k-1} + rsf_{i,t}^{k-1}), \forall t \in T, \quad (17)$$

$$\tilde{f}_{t,l} - \bar{s}_{t,l} \leq \bar{F}_l, \forall t \in T, \forall l \in L, \quad (18)$$

$$\tilde{f}_{t,l} + \underline{s}_{t,l} \geq \underline{F}_l, \forall t \in T, \forall l \in L, \quad (19)$$

$$\tilde{f}_{t,l} = \sum_{i \in I_j} p_{i,t} \alpha_{i,l} + \sum_{i \in I_j} p_{i,t}^{k-1} \alpha_{i,l} + \sum_{i \in V} x_{i,t} \alpha_{i,l} - \sum_{i \in Y} y_{i,t} \alpha_{i,l} - F_D, \forall t \in T, \forall l \in L. \quad (20)$$

Subproblem j can be linearized by introducing continuous decision variables q_t^D , q_t^{rr} , q_t^{rs} and q_t^R and the following constraints:

$$-q_t^D \leq \tilde{D}_t \leq q_t^D, \forall t \in T, \quad (21)$$

$$-q_t^{rr} \leq \tilde{R}_t^r \leq q_t^{rr}, \forall t \in T, \quad (22)$$

$$-q_t^{rs} \leq \tilde{R}_t^s \leq q_t^{rs}, \forall t \in T, \quad (23)$$

$$-q_t^R \leq \tilde{R}_t^{all} \leq q_t^R, \forall t \in T. \quad (24)$$

The problem formulation can then be put into the following MILP form:

$$\min_{\substack{\delta, u, p \\ x, y, r \\ \in j}} \left\{ \begin{aligned} & \sum_{i \in I_j} \sum_{t \in T} (\sum_{s \in S} C_{i,s,t}^{start} \delta_{i,s,t} + C_{i,t}^{NL} u_{i,t} + C_{i,t}^E \\ & + C_{i,t}^{RR} r r_{i,t} + C_{i,t}^{RS} r s_{i,t} + C_{i,t}^{RSN} r s n_{i,t} + C_{i,t}^{RSF} r s f_{i,t}) \\ & + \sum_{i \in V} \sum_{t \in T} (C_{i,t}^V x_{i,t}) - \sum_{i \in Y} \sum_{t \in T} (C_{i,t}^Y y_{i,t}) \\ & + \sum_{t \in T} \lambda_t^k (\tilde{D}_t) + \sum_{t \in T} \mu_t^{rr,k} (\tilde{R}_t^r) \\ & + \sum_{t \in T} \mu_t^{rs,k} (\tilde{R}_t^s) + \sum_{t \in T} \mu_t^{r,k} (\tilde{R}_t^{all}) \\ & + \frac{c_k}{2} \sum_{t \in T} (q_t^D + q_t^{rr} + q_t^{rs} + q_t^{all}) \\ & + C^P \sum_{l \in L} \sum_{t \in T} (\tilde{s}_{t,l} + \underline{s}_{t,l}) \end{aligned} \right\}, \quad (25)$$

s.t. (14) - (24) and all individual unit/virtual constraints.

The above subproblems are iteratively solved subject to the surrogate optimality condition ((14) and (15) of [10]). After each subproblem is solved, multipliers are updated based on (17)-(19) and penalty coefficient is updated based on (20) and (21) of [10].

B. Synergistic Combination of SAVLR+OO+B&C

As an iterative method, SAVLR requires solving subproblems multiple times. Subproblems are generally solved by using B&C subject to the surrogate optimality condition. Solving a subproblem by using B&C requires solving the corresponding LP relaxation subproblem and adding valid cuts before searching for a feasible solution. This process can be very time-consuming for complicated sub-hourly subproblems. Moreover, although each subproblem is smaller in size and complexity as compared to the original problem, the overall computational effort may be significant. Therefore, it is important to solve subproblem in a very efficient way.

To speed up the subproblem solving process, the OO concept is synergistically incorporated within the SAVLR framework. The solution of a subproblem only requires to be “good enough”. A “good enough” solution here is simply a solution satisfying the surrogate optimality condition. Before applying B&C, “good enough” solutions are searched by adjusting or repairing possible solution candidates. For the first iteration, candidates are LP solutions with appropriate rounding as needed; and for subsequent iterations, candidates are solutions from previous iterations. Such a process is generally much faster than solving a subproblem by B&C. If a “good enough” solution can be obtained, then B&C is skipped, and we move on to solve the next subproblem. Otherwise, B&C is used to solve the subproblem.

C. Specific Methods to Enhance Performance

Initialize multipliers. The quality of initial multipliers affects the performance of SAVLR. In hourly UC, multipliers of the corresponding LP problem are good initial multipliers. However, likely caused by the large number of generation resources, the 15-minute interval LP problem itself requires a long time to solve, and it is not worthy to initialize multipliers by solving 15-minute interval LP problem directly. Inspired by OO, precise multipliers are not strictly required. Considering the similarity of four 15-min intervals within the same hour, multipliers of the hourly LP problem are extended to all four 15-min intervals as initial multipliers.

Build generic models. Within SAVLR, a subproblem is solved multiple times in an iterative manner. A subproblem model, whose parameters are associated with other subproblems' latest solutions, needs to be loaded every time before solving the subproblem. Building a new model for a subproblem in each iteration, however, takes a substantial amount of time. To address this issue, a generic model is constructed for each subproblem just once; and for each iteration, each subproblem's model parameters are updated according to the latest available subproblem solutions.

Use aggregated values. As shown in (14)-(17) and (20), system-wide constraints require variables from all generation resources. Because of the large number of generation resources, processing all variables takes a substantial amount of time. To save time, the values from other subproblems are aggregated, and these aggregated values are iteratively updated. For example, the aggregated generation level is initialized and updated as below:

$$\sum_{i \in I_j} p_{i,t} = P_{j,t}, \forall t \in T, j = 1, \quad (26)$$

$$P_{j+1,t} = P_{j,t} + \sum_{i \in I_j} p_{i,t} - \sum_{i \in I_{j+1}} p_{i,t}^{k-1}, \forall t \in T, k \in N^+. \quad (27)$$

V. NUMERICAL TESTING

The above SAVLR+OO+B&C approach has been implemented by using Python 2.7 and Gurobi 7.5.0, and tested on MISO's server with Intel® Core (TM) i7-7600U CPU @ 2.80 GHz RAM 16 GB. A MISO's UC problem, which contains 1,105 conventional units, 15,843 virtuals, 75 dispatchable demand and 227 transmission lines with 36 hours looking ahead (144 time intervals), is tested to demonstrate the performance and computational efficiency of the new method.

As shown in the last row of Table I, a near-optimal solution with a duality gap of 0.77% is obtained after 3237s, where the best known lower bound (\$9,337,931) obtained in advance is used to calculate the gap. The total solving time is 1484s, and the rest are model loading and miscellaneous times. Also for this particular testing, “good enough” solutions for subproblems are always obtained before applying B&C.

Results obtained by using B&C and SAVLR+B&C are shown in the first two rows of Table I as well as in Figure 1 for comparison purposes. Standard B&C cannot obtain a reasonable solution before 3,300s, and it obtains a feasible solution with a MIP gap of 0.9% after 5000s. For SAVLR+B&C, a solution with a duality gap of 0.88% is obtained after roughly 3500s. Our new approach thus significantly outperforms B&C and SAVLR+B&C.

TABLE I
PERFORMANCE OF SAVLR+OO+B&C, PURE B&C, AND SAVLR+B&C.

Method	Solving Time (s)	Total Time (s)	Feasible Cost (\$)	Lower Bound (\$)	Gap (%)
B&C	5211	5443	9,422,880	9,337,931	0.90
SAVLR+B&C	3537	5623	9,421,401		0.88
SAVLR+OO+B&C	1484	3237	9,410,518		0.77

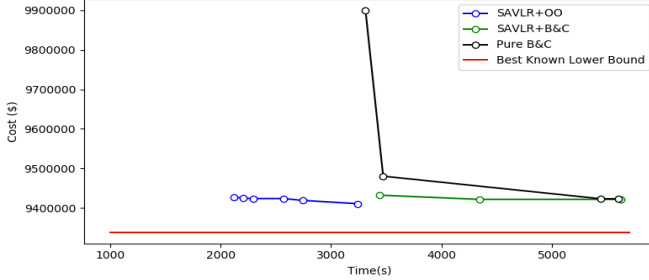


Fig. 1. Performance of SAVLR+OO+B&C, Pure B&C and SAVLR+B&C.

The average solving time for a subproblem in our method is 53s, which is much less than that in SAVLR+B&C of 162s. The difference is attributed to the fact that within our method, subproblem solutions satisfying the surrogate optimality condition are obtained in a much faster way as compared with that for SAVLR+B&C.

VI. CONCLUSION

This paper illustrates how sub-hourly UC problems are solved through the novel combination of SAVLR, OO and B&C. Testing results show that the new method has high computational efficiency. More importantly, the approach opens a new way to address other MILP problems in power systems and beyond.

VII. ACKNOWLEDGMENT

The authors would like to express their sincere appreciation to Professor Yu-Chi Ho of Harvard University for his inspiration on combining mathematical optimization with Ordinal Optimization; and to Mr. Anbang Liu of Tsinghua University in Beijing for his pioneering work on combining mathematical optimization with Ordinal Optimization to solve generalized assignment problems.

REFERENCES

- [1] "FERC Order 764," Tech. Rep. [Online]. Available: <https://www.ferc.gov/whats-new/comm-meet/2012/062112/E-3.pdf>.
- [2] J.P.Deane, G.Drayton, B.P.Ó Gallachóir, "The impact of sub-hourly modelling in power systems with significant levels of renewable generation," *Applied Energy*, Vol. 113, pp. 152–158, 2014.
- [3] S. Pineda, R. Fernández-Blanco, and J. Miguel Morales, "Time-adaptive unit commitment," *IEEE Transactions on Power Systems*, Vol. 34, No. 5, pp. 354–359, 2019.
- [4] M. Kazemi, P. Siano, D. Sarno, and A. Goudarzi, "Evaluating the impact of sub-hourly unit commitment method on spinning reserve in presence of intermittent generators," *Energy*, Vol. 113, pp. 338–354, 2016.
- [5] B. Yan, P. B. Luh, T. Zheng, D. Schiro, M. A. Bragin, F. Zhao, J. Zhao, and I. Lelic, "A systematical formulation tightening approach for unit commitment problems," *IEEE Transactions on Power Systems*, Vol. 35, pp. 782–794, 2020.
- [6] Y. Chen, A. Casto, F. Wang, Q. Wang, X. Wang, and J. Wan, "Improving large scale day-ahead security constrained unit commitment performance," *IEEE Transactions on Power Systems*, Vol. 31, No. 6, pp. 4732–4743, 2016.
- [7] S. Virmani, E. C. Adrian, K. Imhof, and S. Mukherjee, "Implementation of a Lagrangian relaxation based unit commitment problem," *IEEE Transactions on Power Systems*, Vol. 4, No. 4, pp. 1373–1380, 1989.
- [8] X. Guan, P. B. Luh, H. Yan, and J. A. Amalfi, "An optimization-based method for unit commitment," *Int. J. Elec. Power Energy System*, Vol. 14, No. 1, pp. 9–17, 1992.
- [9] M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the surrogate Lagrangian relaxation method," *J. Optimization Theory and Applications*, Vol. 164, No. 1, pp. 173–201, 2015.
- [10] M. A. Bragin, P. B. Luh, B. Yan, and X. Sun, "A scalable solution methodology for mixed-integer linear programming problems arising in automation," *IEEE Transactions on Automation Science and Engineering*, Vol. 16, No. 2, pp. 531–541, 2018.
- [11] M. Deng and Y. C. Ho, "An ordinal optimization approach to optimal control problems," *Automatica*, Vol. 35, pp. 331–338, 1999.
- [12] Y. Ho, Q. Zhao, Q. Jia, *Ordinal Optimization: Soft optimization for hard problems*, Springer US, 2007.
- [13] X. Sun, P. B. Luh, M. A. Bragin, Y. Chen, J. Wan, F. Wang, "A novel decomposition and coordination approach for large day-ahead unit commitment with combined cycle units," *IEEE Transactions on Power Systems*, Vol. 33, pp. 5297–5308, 2018.
- [14] M. J. D. Powell, "A method for nonlinear constraints in minimization problems," in *Optimization*, (R. Fletcher, ed.), Academic Press, 1969.
- [15] D. Gabay, and B. Mercier, "A dual algorithm for the solution of nonlinear variational problems via finite element approximations," *Computers and Mathematics with Applications*, Vol. 2, pp. 17–40, 1976.
- [16] NERC BAL-001-2. [Online]. Available: <https://www.nerc.com/files/BAL-001-2.pdf>.
- [17] G. Morales-España, J. M. Latorre and A. Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Transactions on Power Systems*, Vol. 28, pp. 4897–4908, 2013.
- [18] D. Rajan and S. Takriti, "Minimum up/down polytopes of the unit commitment problem with start-up costs," IBM, 2005. [Online]. Available: <http://domino.research.ibm.com/library/cyberdig.nsf/1e4115aea78b6e7c85256b360066f0d4/cdcb02a7c809d89e8525702300502ac0?OpenDocument>.