Cooperative Adaptive Containment Control with Parameter Convergence via Cooperative Finite-Time Excitation

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***Abstract*— This paper addresses the problem of cooperative adaptive containment control for multi-agent systems, which specifies the objective of jointly achieving containment control and accurate adaptive learning/identification of unknown system parameters. We consider a class of linear uncertain multi-agent systems with multiple leaders subject to bounded unmeasurable inputs and multiple followers subject to unknown system dynamics. A novel cooperative adaptive containment control architecture is proposed, which consists of a discontinuous nonlinear state-feedback control law and a filter-based cooperative adaptation law. This new control architecture is compelling in the sense that exponential convergence of both containment tracking errors to zero and adaptation parameters to their true values can be achieved simultaneously under a mild cooperative finite-time excitation condition. This condition significantly relaxes existing ones (e.g., persistent excitation and finite-time excitation) for parameter identification in adaptive control systems. Effectiveness of the proposed approach has been demonstrated through both rigorous analysis and a case study.**

***Index Terms*— Cooperative adaptive learning control, cooperative finite-time excitation, containment control**

I. Introduction

In the controls community, multi-agent systems (MASs) have been well recognized as a useful paradigm to formalize complex distributed control problems involving multiple/many dynamical agents working in a cooperative fashion. Fruitful results can be found in the literature concerning various MAS distributed control tasks, such as consensus control [1], formation control [2], bipartite consensus [3], and containment control [4], etc. In particular, containment control is fundamental for enabling coordinations among multiple leaders and followers. It aims to drive all the followers into a convex-hull space spanned by the leaders [5]. Such a behavior is highly relevant to many important applications. One typical example is multi-robot coordination (e.g., [30]), where movements of a team of slave/follower robots are constrained within a region spanned by some master/leader robots in order to avoid any of the slave robots venturing into hazardous/prohibited areas.

A large majority of existing containment control techniques (e.g., [4]–[6]) consider MASs with precisely known dynamics, which is deemed too restrictive in practice. This is because it is generally difficult or even impossible to acquire accurate models of all the agents especially when the number of agents is large. Robust control techniques [7] and adaptive control techniques [8] provide promising solutions for addressing the associated uncertainty issues. Compared to robust control, adaptive control is more appropriate in handling time-invariant or slowly time-varying uncertainties [8]. This has been motivating considerable research efforts to develop new adaptive control techniques for uncertain MASs, e.g., [2], [9]–[11].

Despite rich literature, we observe that existing cooperative adaptive containment control techniques are largely focused on "control", while the "learning" capability of adaptive controllers has been poorly explored. Specifically, existing adaptive containment controllers are capable of rendering stability and containment tracking control performance, but very few of them can guarantee convergence of the associated adaptation/estimate parameters (e.g., estimate unknown plant parameters or adaptive controller gains) to their true/optimal values. One technical challenge lies in satisfaction of the so-called persistent excitation (PE) condition [8], which requires some system signals (e.g., inputs/states) to be sufficiently rich (e.g., containing sufficiently large number of distinct frequencies for linear systems). This condition is in general difficult to satisfy and even verify *a priori*, especially when both control and learning objectives are jointly concerned. Attempting efforts have been made to overcome this challenge. In particular, a new concept of cooperative PE (cPE) was proposed in [12]–[14] for accurate identification of unknown MASs, which was extended by [2], [11] to composite adaptive formation control and cooperative learning of nonlinear MASs. Such a cPE condition relaxes the traditional PE condition in the sense that it can be satisfied by incorporating multiple system signals with each of them not necessarily PE. Moreover, a concurrent learning control scheme was proposed in [15] by combining the use of instantaneous and past/historical data for online parameter adaptation. With this scheme, exponential convergence for both control tracking errors and adaptation parameters can be jointly achieved by satisfying a finite-time excitation (FTE) condition. This condition further relaxes the traditional PE condition, as it only requires the system signals to be exciting over a finite time but not necessarily persistently. This concurrent learning control scheme was subsequently refined in [16], [17] by employing a series of filtering techniques to avoid using state-derivative feedback. However, these results pertain to single adaptive systems, while extending them to MASs is non-trivial due to the complexity of network structures and limited information accessibility in MASs.

In this paper, our objective is to address the cooperative adaptive control and learning problem for MASs by jointly accounting for the specifications of containment control and adaptive identification of unknown system parameters without resorting to the PE/cPE/FTE conditions. Specifically, the problem will be discussed for a class of linear MASs with multiple leaders and multiple followers; all the leaders are subject to bounded reference inputs which are assumed to be unmeasurable for any follower, and all the followers' dynamics are also unknown for local controller design. More detailed technical definitions of the leaders and followers will be provided in the next section. To this end, we propose a novel cooperative adaptive containment control protocol, which, compared to existing ones (e.g., [12], [16], [17]), is more advanced and compelling in the following three aspects: (i) It jointly renders containment control performance and cooperative adaptive learning performance for MASs. (ii) It utilizes

This work was supported in part by the National Science Foundation under Grant CMMI-1952862.

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both local agent's information (e.g., local plant's instantaneous and integrated states, and local estimate parameters) and neighboring agent's information (e.g., relative plant states, and relative estimate parameters) for feedback control and adaptive learning. (iii) Exponential convergence of both containment tracking errors (to zero) and adaptation parameters (to their true values) can be guaranteed under a new cooperative finite-time excitation (cFTE) condition, which significantly relaxes existing ones including PE/cPE/FTE. Moreover, this paper advances our previous work [18] by addressing a more challenging containment control problem, providing more rigorous analysis on system stability and parameter convergence, and considering more realistic mobile robot systems in simulation study.

II. Preliminaries and Problem Statement

A. Notation and Graph Theory

$\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}_+$ denotes the set of positive real numbers. $\mathbb{R}^{m\times n}$ is the set of real $m\times n$ matrices, and $\mathbb{R}^n$ is the set of real $n\times 1$ vectors. I_n and $\mathbf{1}_n$ denote the identity matrix of dimension n and an n-dimensional column vector with all elements being 1, respectively. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ denote the sets of real symmetric $n\times n$ matrices and positive definite matrices, respectively. A block diagonal matrix with matrices $X_1, X_2, \cdots, X_p$ on its main diagonal is denoted by $diag\{X_1, X_2, \cdots, X_p\}$. $\otimes$ denotes the Kronecker product. $col\{x_1, \cdots, x_n\}$ denotes a column vector by stacking column vectors $x_1, \cdots, x_n$ together. For two integers $k_1 < k_2$, $\mathbf{I}[k_1, k_2] := \{k_1, k_1+1, \cdots, k_2\}$. For $x \in \mathbb{R}^n$, $\|x\| := (x^T x)^{1/2}$. The distance from $x \in \mathbb{R}^n$ to a set $\mathcal{C} \subset \mathbb{R}^n$ is denoted by $dist(x, \mathcal{C}) := \inf_{y\in\mathcal{C}} \|x-y\|$. The convex hull of a finite set of points $X = \{x_1, x_2, \cdots, x_q\}$ is defined by $Co(X) := \{\sum_{i=1}^q \alpha_i x_i | x_i \in X, \alpha_i \in \mathbb{R}, \alpha_i \geq 0, \sum_{i=1}^q \alpha_i = 1\}$.

A graph is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the elements of $\mathcal{V} = \{1, 2, \cdots, N\}$ are called vertices, the elements of $\mathcal{E}$ are pairs (i, j) with $i, j \in \mathcal{V}, i \neq j$, called edges, and the matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ is called the adjacency matrix. If $(i, j) \in \mathcal{E}$, it means agent i can receive information from agent j where these two agents are called adjacent. The adjacency matrix is thus defined as $\mathcal{A} = [a_{ij}]_{N\times N}$, with $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The graph $\mathcal{G}$ is called undirected if for every $(i, j) \in \mathcal{E}$ also $(j, i) \in \mathcal{E}$. The Laplacian matrix of a given graph is defined as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = \sum_{j\neq i} a_{ij}, l_{ij} = -a_{ij}, i \neq j$. If the graph is undirected, then $\mathcal{L}$ is a positive semi-definite real symmetric matrix, so all eigenvalues of $\mathcal{L}$ are non-negative real. Zero is always an eigenvalue of $\mathcal{L}$, so it has rank at most $N-1$. Furthermore, an undirected graph is called connected if for every pair of distinct vertices i and j there exists a path from i to j, i.e., a finite set of edges (i_k, i_{k+1}) with $k = 1, 2, \cdots, r-1$ such that $i_1 = i$ and $i_r = j$. An undirected graph is connected if and only if its Laplacian has rank $N-1$. In that case the zero eigenvalue of $\mathcal{L}$ has multiplicity one.

B. Problem Statement

Consider a linear MAS consisting of N followers and M leaders. The dynamics of the ith follower is described as:

$$\dot{x}_i = Ax_i + Bu_i, \quad \forall i \in \mathcal{F}, \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^{n_u}$ denote the state and control input, respectively. $\mathcal{F} = \{1, \cdots, N\}$ denotes the set of indices for the followers. The dynamics of the kth leader is given by:

$$\dot{w}_k = A_0 w_k + B_0 r_k, \quad \forall k \in \mathcal{R}, \tag{2}$$

where $w_k \in \mathbb{R}^n$ is the state, $r_k \in \mathbb{R}^{n_r}$ is a bounded input, $\mathcal{R} = \{N+1, \cdots, N+M\}$ denotes the set of indices for the leaders. Let $\mathcal{G}$ and $\mathcal{G}_s$ denote the graphs of the $N+M$ agents and the N followers defined above, respectively. The adjacency matrix and Laplacian matrix associated with $\mathcal{G}_s$ are denoted as $\mathcal{A}$ and $\mathcal{L}$, respectively. The weight associated with the connection edge directed from a leader $k \in \mathcal{R}$ to the ith follower ($i \in \mathcal{F}$) is denoted by δ_i^k. Specifically, $\delta_i^k = 1$ if the kth leader is connected to the ith follower, otherwise $\delta_i^k = 0$. We define diagonal matrices $\Delta_k = diag\{\delta_1^k, \cdots, \delta_N^k\}$ for all $k \in \mathcal{R}$.

Given the MAS (1)–(2), we consider the following setting. First, considering that it is usually difficult to precisely obtain the model of many physical systems and motivated from existing literature (e.g., [21], [28]), for each follower considered in this paper, we assume that B is available but A is unknown. Note that this can be relaxed to a more general case with both A and B being unknown by employing the pre-filtering approach from [19], [20], interested readers are referred to these references for more technical details. Second, the leader's system matrices A_0 and B_0 are assumed available for followers, while their states w_k are measurable only for their neighboring followers and their inputs r_k are not measurable for any follower (but the upper bounds of r_k are available), as motivated from [5], [26], [28]. The following assumptions are further made.

Assumption 1: There exist constant matrices $K_1 \in \mathbb{R}^{n\times n_u}$ and $K_2 \in \mathbb{R}^{n_r\times n_u}$, such that $A_0 = A + BK_1^T$ and $B_0 = BK_2^T$.

Assumption 2: (A_0, B_0) is stabilizable, and r_k are bounded, i.e., $\|r_k\| \leq r_k^*$ for all $k \in \mathcal{R}$, where r_k^* are positive constants.

Assumption 3: $\mathcal{G}_s$ is undirected and connected, and there is at least one leader that has a directed path to each follower.

Assumption 1 includes the so-called model matching condition that frequently arises in the literature of model reference adaptive control [16], [17], [21]–[23]. It essentially requires that: there must exist two matrices K_1, K_2 such that the follower model (1) can be transformed to match the leader model (2). More discussions on such a condition including its motivations from practical applications can be found in [8] and the references therein. Under this assumption, since A is unknown, only existence of K_1 is guaranteed. In contrast, since B_0 and B are given, K_2 can be calculated off-line for each follower. Assumption 2 is made to ensure a meaningful bounded convex-hull containment envelop generated by the leaders. With Assumption 3, according to [24], we have $H_k := \frac{1}{M}\mathcal{L} + \Delta_k$ for all $k \in \mathcal{R}$, and $\Phi := \sum_{k=N+1}^{N+M} H_k$ are all positive definite.

Our objective is to design a cooperative adaptive containment control protocol for the N followers in (1), such that the following joint objectives can be achieved.

1) *Containment Control:* All the followers' states x_i in (1) will converge to a dynamic convex hull spanned by the leaders' states w_k in (2) as $t \to \infty$, i.e., $\lim_{t\to\infty} dist\,(x_i(t), Co(w_k(t), k \in \mathcal{R})) = 0, \forall i \in \mathcal{F}$.
2) *Cooperative Learning:* The unknown constant matrix K_1 can be accurately identified by every follower without requiring satisfaction of the PE/cPE/FTE conditions[1].

To fulfill the above objectives, in the following sections, a novel filter-based adaptive learning control protocol will be proposed, followed by rigorous analysis on system stability and parameter convergence, and a case study on a group of realistic mobile robots.

III. Main Results

[1]We refer the readers to [8], [12], and [16] for detailed definitions of the PE, cPE, and FTE conditions, respectively.

A. Controller Structure

We propose the following control protocol for the N followers in (1):

$$u_i = \hat{K}_{1,i}^T x_i + K_2^T K_3 e_i + \beta K_2^T f_1(K_4 e_i), \quad \forall i \in \mathcal{F}, \tag{3}$$

where e_i is an integrated error signal defined by $e_i = \sum_{j=1}^{N} a_{ij}(x_j - x_i) + \sum_{k=N+1}^{N+M} \delta_i^k (w_k - x_i)$, $\hat{K}_{1,i} \in \mathbb{R}^{n \times n_u}$ is a time-varying controller gain used to estimate the true value K_1. $K_2 \in \mathbb{R}^{n_r \times n_u}$ can be obtained from Assumption 1. $K_3, K_4 \in \mathbb{R}^{n_u \times n}$ and $\beta \in \mathbb{R}_+$ are constant design parameters. $f_1(K_4 e_i)$ is a nonlinear function defined by: $f_1(K_4 e_i) = \frac{K_4 e_i}{\|K_4 e_i\|}$, if $\|K_4 e_i\| \neq 0$; otherwise $f_1(K_4 e_i) = 0$.

For accurate identification of K_1 via $\hat{K}_{1,i}$, we seek to develop a new online adaptation law for $\hat{K}_{1,i}$ such that convergence of $\hat{K}_{1,i}$ to K_1 for all $i \in \mathcal{F}$ can be achieved without resorting to PE/cPE/FTE. To this end, motivated by [16], [17], we first construct a series of filters. Specifically, from Assumption 1 and system (1), we have $\dot{x}_i - Bu_i + BK_1^T x_i = A_0 x_i$ for all $i \in \mathcal{F}$, which can be rewritten as $\hat{X}_i := u_i + (B^T B)^{-1} B^T A_0 x_i - (B^T B)^{-1} B^T \dot{x}_i = K_1^T x_i$, where the matrix B is assumed to be full rank, ensuring invertibility of $B^T B$. Then, consider the following filter equations for all $i \in \mathcal{F}$:

$$\dot{N}_i = -kN_i + x_i, \quad N_i(0) = 0 \tag{4}$$

$$\dot{g}_i = -kg_i + \hat{X}_i, \quad g_i(0) = 0 \tag{5}$$

where $N_i \in \mathbb{R}^n$ and $g_i \in \mathbb{R}^{n_u}$ denote the respective filter states, $k > 0$ is a scalar. Solving the above two equations yields

$$N_i(t) = \mathrm{e}^{-kt} \int_0^t \mathrm{e}^{k\tau} x_i(\tau) d\tau, \quad \forall i \in \mathcal{F}, \tag{6}$$

$$g_i(t) = \mathrm{e}^{-kt} \int_0^t \mathrm{e}^{k\tau} \hat{X}_i(\tau) d\tau, \quad \forall i \in \mathcal{F}, \tag{7}$$

which implies that $g_i(t) = K_1^T N_i(t)$. Note that $N_i(t)$ can be computed in real-time for each follower via (4) as x_i is measurable, but $g_i(t)$ is not measurable from (5) as $\hat{X}_i$ containing the agent's state derivative information $\dot{x}_i$ is not available. To overcome this, we note that (7) can be reformulated using the by-parts rule of integration, such that $g_i(t)$ can be computed online via

$$\begin{aligned} g_i(t) = &-(B^T B)^{-1} B^T \left(x_i(t) - \mathrm{e}^{-kt} x_i(0) - kN_i(t) \right) \\ &+ (B^T B)^{-1} B^T A_0 N_i(t) + h_i(t), \quad \forall i \in \mathcal{F}, \end{aligned} \tag{8}$$

where $h_i \in \mathbb{R}^{n_u}$ is the state of

$$\dot{h}_i = -kh_i + u_i, \quad h_i(0) = 0. \tag{9}$$

We further introduce two additional filters for all $i \in \mathcal{F}$:

$$\dot{M}_i = N_i N_i^T, \quad \dot{G}_i = g_i N_i^T, \quad (M_i(0) = 0, \; G_i(0) = 0) \tag{10}$$

where $M_i \in \mathbb{R}^{n \times n}$ and $G_i \in \mathbb{R}^{n_u \times n}$ are two respective filter states. Solving the above two equations gives

$$M_i(t) = \int_0^t N_i(\tau) N_i^T(\tau) d\tau, \quad G_i(t) = K_1^T M_i(t), \tag{11}$$

for all $t \geq 0$ and $i \in \mathcal{F}$. It is easy to verify that $M_i(t) \geq 0$ for all $t \geq 0$ and all $i \in \mathcal{F}$, and $M_i(t)$ is a nondecreasing function of time, i.e., $M_i(t_2) \geq M_i(t_1)$ for any $t_2 \geq t_1 \geq 0$ and all $i \in \mathcal{F}$.

Finally, we propose a new adaptation law for $\hat{K}_{1,i}$ as follows:

$$\dot{\hat{K}}_{1,i} = \gamma x_i e_i^T PB + f_{2,i} + f_{3,i}, \quad \forall i \in \mathcal{F}, \tag{12}$$

where $\gamma \in \mathbb{R}_+$ and $P \in \mathbb{S}_+^n$ are two constant design parameters, $f_{2,i} := \gamma \left(G_i^T - M_i \hat{K}_{1,i} \right)$, and $f_{3,i} := \gamma \sum_{j=1}^{N} a_{ij} \left(\hat{K}_{1,j} - \hat{K}_{1,i} \right)$.

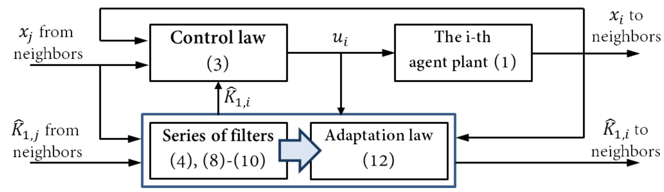

Fig. 1: The proposed controller structure.

In summary, the proposed cooperative adaptive containment control protocol consists of the adaptive control law (3) and the cooperative adaptation law (12) with filters (4) and (8)–(10). To understand this new control architecture, a block diagram is given in Fig. 1. Specifically, the first two linear terms of (3) is used to stabilize the overall MAS and attain containment control performance, while the nonlinear term $f_1(\cdot)$ of (3) is inspired from the sliding-mode control theory [5], [25], [26] for eliminating the effects of unmeasurable leaders' input signals r_k for all $k \in \mathcal{R}$. For the adaptation law (12), the first term is similar to traditional Lyapunov/gradient-based forms for ensuring Lyapunov stability, while its novelty is reflected from the last two terms $f_{2,i}$ and $f_{3,i}$. In particular, $f_{2,i}$ is motivated from the concurrent learning strategy [15], aiming to online adjust the adaptation parameters $\hat{K}_{1,i}$ by integrating local state information. The last term $f_{3,i}$ is inspired from the MAS consensus control theory [1], [12] for enabling cooperative learning among neighboring agents through sharing knowledge (i.e., $\hat{K}_{1,i}$). Neighboring agents will need to share both plant and controller state information (x_i and $\hat{K}_{1,i}$).

B. Stability and Parameter Convergence Analysis

Interconnecting the controller (3) and (12) to (1), the closed-loop dynamics of each follower can be obtained as

$$\begin{aligned} \dot{x}_i &= \left(A + B\hat{K}_{1,i}^T \right) x_i + BK_2^T K_3 e_i + \beta BK_2^T f_1(K_4 e_i), \\ \dot{\hat{K}}_{1,i} &= \gamma x_i e_i^T PB + \gamma M_i \left(K_1 - \hat{K}_{1,i} \right) + f_{3,i}, \quad \forall i \in \mathcal{F}. \end{aligned} \tag{13}$$

Then, we define $\bar{r}_k = \mathbf{1}_N \otimes r_k$, $\bar{w}_k = \mathbf{1}_N \otimes w_k$ for all $k \in \mathcal{R}$, $e = col\{e_1, \cdots, e_N\}$, $x = col\{x_1, \cdots, x_N\}$, and $\tilde{x} = x - \left(\Phi^{-1} \otimes I_n \right) \sum_{k=N+1}^{N+M} (H_k \otimes I_n) \bar{w}_k$. To facilitate the subsequent derivations, we list the following useful facts: $(\Delta_k \otimes I_n)(\mathbf{1}_N \otimes w_k) = (H_k \otimes I_n)(\mathbf{1}_N \otimes w_k)$, $(\forall k \in \mathcal{R})$; $e = \sum_{k=N+1}^{N+M} (H_k \otimes I_n)\bar{w}_k - (\Phi \otimes I_n)x = -\sum_{k=N+1}^{N+M} (H_k \otimes I_n)\tilde{x} = -(\Phi \otimes I_n)\tilde{x}$, and $e_i = -\sum_{j=1}^{N} h_{ij}\tilde{x}_j$, $(\forall i \in \mathcal{F})$, where h_{ij} denotes the (i,j)-th entry of the matrix $\Phi = \sum_{k=N+1}^{N+M} H_k$, and $\tilde{x}$ is partitioned as $\tilde{x} = col\{\tilde{x}_1, \cdots, \tilde{x}_N\}$ with $\tilde{x}_i \in \mathbb{R}^n$ for all $i \in \mathcal{F}$.

Lemma 1: Let ℓ_{ik} denote the i-th entry of the vector $\Phi^{-1} H_k \mathbf{1}_N \in \mathbb{R}^N$ for all $k \in \mathcal{R}$. Then, under Assumption 3, we have $\sum_{k=N+1}^{N+M} \ell_{ik} = 1$ for all $i \in \mathcal{F}$.

Proof: Since $\Phi = \sum_{k=N+1}^{N+M} H_k$, we have $\sum_{k=N+1}^{N+M} \Phi^{-1} H_k \mathbf{1}_N = \Phi^{-1} \left(\sum_{k=N+1}^{N+M} H_k \right) \mathbf{1}_N = \mathbf{1}_N$. As a result, $\sum_{k=N+1}^{N+M} \ell_{ik} = 1$ can be concluded for all $i \in \mathcal{F}$. ■

The overall closed-loop dynamics can be obtained from (13):

$$\begin{aligned} \dot{\tilde{x}} = &\left[I_N \otimes A_0 - \Phi \otimes B_0 K_3 \right] \tilde{x} + (I_N \otimes B)\tilde{K}_{1d}^T x + \beta (I_N \otimes B_0) \\ &\times F_1(\tilde{x}) + (I_N \otimes B_0) \left(\Phi^{-1} \otimes I_{n_r} \right) \sum_{k=N+1}^{N+M} (H_k \otimes I_{n_r}) \bar{r}_k, \\ \dot{\tilde{K}}_1 = &\gamma diag\{x_1 e_1^T, \cdots, x_N e_N^T\}(I_N \otimes PB) - \gamma (M + \mathcal{L} \otimes I_n)\tilde{K}_1, \end{aligned} \tag{14}$$

where

$$\hat{K}_1 = diag\{\hat{K}_{1,1}, \cdots, \hat{K}_{1,N}\},$$
$$\tilde{K}_{1d} = diag\{\tilde{K}_{1,1}, \cdots, \tilde{K}_{1,N}\}, \quad \tilde{K}_{1,i} = \hat{K}_{1,i} - K_1, \ (\forall i \in \mathcal{F}),$$
$$\tilde{K}_1^T = \begin{bmatrix} \tilde{K}_{1,1}^T & \cdots & \tilde{K}_{1,N}^T \end{bmatrix}^T, \quad M = diag\{M_1, \cdots, M_N\},$$
$$F_1(\tilde{x}) = \begin{bmatrix} f_1(K_4 e_1) \\ \vdots \\ f_1(K_4 e_N) \end{bmatrix} = \begin{bmatrix} f_1(-K_4 \sum_{j=1}^N h_{1j}\tilde{x}_j) \\ \vdots \\ f_1(-K_4 \sum_{j=1}^N h_{Nj}\tilde{x}_j) \end{bmatrix}.$$

Before presenting the main theorem, a cooperative finite-time exciting (cFTE) condition on the filtered signals $N_i(t)$ for all $i \in \mathcal{F}$ is introduced. Specifically, $N_i(t)$'s are said to satisfy the cFTE condition with degree of excitation $\alpha > 0$, if there exists $t_1 \geq 0$, $T > 0$ such that during the finite time interval $[t_1, t_1 + T]$,

$$\int_{t_1}^{t_1+T} \sum_{i=1}^{N} N_i(\tau) N_i^T(\tau) d\tau \geq \alpha I_n. \tag{15}$$

Theorem 1: Consider the MAS consisting of the followers (1), the leaders (2), and the controller (3) and (12). Under Assumptions 1–3,

(i) if there exist a $\hat{P} \in \mathbb{S}_+^n$ and a $\hat{K}_3 \in \mathbb{R}^{n_r \times n}$ such that

$$A_0\hat{P} + \hat{P}A_0^T - \lambda_i(\Phi)(B_0\hat{K}_3 + \hat{K}_3^T B_0^T) < -\rho\hat{P}, \tag{16}$$

holds for all $i \in \mathcal{F}$ and some constant $\rho \in \mathbb{R}_+$, where $\lambda_i(\Phi)$ denotes the ith eigenvalue of Φ; and the controller coefficients are chosen as $\gamma \in \mathbb{R}_+$, $\beta \geq \max_{k\in\mathcal{R}} r_k^*$, and $P = \hat{P}^{-1}$, $K_3 = \hat{K}_3\hat{P}^{-1}$, $K_4 = B_0^T\hat{P}^{-1}$;

(ii) and if the filtered signals $N_i(t)$ for all $i \in \mathcal{F}$ satisfy the cFTE condition (15) with degree of excitation $\alpha > 0$,

then, the states of (14) are uniformly ultimately bounded for $t \geq 0$, x_i will exponentially converge to $Co(w_k, k \in \mathcal{R})$, and $\hat{K}_{1,i}$ will exponentially converge to K_1 for all $i \in \mathcal{F}$ and $t \geq t_1 + T$.

Before proving this theorem, we establish the following lemma.

Lemma 2: Consider a group of time-varying square matrices $M_i(t)$ defined in (11) for all $i \in \mathcal{F}$. If N_i for all $i \in \mathcal{F}$ satisfy the cFTE condition (15), and if Assumption 3 holds, then there exists a positive constant $\alpha' \in \mathbb{R}_+$ such that $M(t) + \mathcal{L} \otimes I_n \geq \alpha' I_{Nn}$, $\forall t \geq t_1 + T$, where $M(t) = diag\{M_1(t), \cdots, M_N(t)\}$.

Proof: With Assumption 3, $\mathcal{L}$ has only one zero eigenvalue whose unit eigenvector is $\frac{1}{\sqrt{N}}\mathbf{1}_N$, and accordingly $\mathcal{L} \otimes I_n$ has n zero eigenvalues whose orthogonal unit eigenvectors are $\nu_1 = \frac{1}{\sqrt{N}}\mathbf{1}_N \otimes \varepsilon_1, \cdots, \nu_n = \frac{1}{\sqrt{N}}\mathbf{1} \otimes \varepsilon_m$, where $\varepsilon_i \in \mathbb{R}^n$ represents a unit vector whose ith element is one. The other eigenvalues of $\mathcal{L} \otimes I_n$ are positive and denoted as $0 < \lambda_{n+1} \leq \cdots \leq \lambda_{Nn}$, whose orthogonal unit eigenvectors are denoted correspondingly as $\nu_{n+1}, \cdots, \nu_{Nn}$. For an arbitrary nonzero vector $\xi \in \mathbb{R}^{Nn}$, it can always be expressed as $\xi = \sum_{i=1}^n c_i\nu_i + \sum_{i=n+1}^{Nn} c_i\nu_i$. We consider the following two cases.

- When $\sum_{i=n+1}^{Nn} c_i^2 \neq 0$, we have $\xi^T(M(t) + \mathcal{L} \otimes I_n)\xi = \xi^T M(t)\xi + \sum_{i=n+1}^{Nn} \lambda_i c_i^2 \geq \sum_{i=n+1}^{Nn} \lambda_i c_i^2 > 0$.
- When $\sum_{i=n+1}^{Nn} c_i^2 = 0$, which means that $\sum_{i=1}^n c_i^2 \neq 0$ and $\xi = \sum_{i=1}^n c_i\nu_i$ since ξ is a nonzero vector, we have $\xi^T(M(t) + \mathcal{L} \otimes I_n)\xi = \left(\sum_{i=1}^n c_i\nu_i\right)^T M(t) \left(\sum_{i=1}^n c_i\nu_i\right) = \mathcal{C}^T\mathcal{V}^T M(t)\mathcal{V}\mathcal{C}$, where $\mathcal{C} = col\{c_1, \cdots, c_n\}$ and $\mathcal{V} = [\nu_1, \cdots, \nu_n]$. Since N_i for all $i \in \mathcal{F}$ satisfy the cFTE condition (15), and based on the definition of M_i in (11), it can be verified that for all $t \geq t_1 + T$, $\mathcal{V}^T M(t)\mathcal{V} = \frac{1}{N}\sum_{i=1}^N M_i(t) \geq \frac{\alpha}{N} I_n$, which leads to $\xi^T(M(t) + \mathcal{L} \otimes I_n)\xi \geq \frac{\alpha}{N}\sum_{i=1}^n c_i^2 > 0$.

As such, we have shown that $M(t) + \mathcal{L} \otimes I_n > 0$ for all $t \geq t_1 + T$. We need to further show that there exists a positive constant α' such that $M(t) + \mathcal{L} \otimes I_n \geq \alpha' I_{Nn}$ for all $t \geq t_1 + T$. This amounts to show that all of the eigenvalues of the time-varying positive definite matrix $M(t) + \mathcal{L} \otimes I_n$ must have a lower bound $\alpha' > 0$. We will prove this by contradiction. Suppose that for all $t \geq t_1 + T$, there exists an eigenvalue $\underline{\lambda}(t)$ and a time sequence $\{t^k\}_{k=1}^\infty$ with $t^k \geq t_1 + T$ such that $\lim_{k\to\infty} \underline{\lambda}(t^k) = 0$. Denote the unit eigenvector of $\underline{\lambda}(t^k)$ as $\eta(t^k)$, that is, $\|\eta(t^k)\| = 1$. Then, we have

$$\begin{aligned} &\lim_{k\to\infty} \eta^T(t^k)(M(t^k) + \mathcal{L} \otimes I_n)\eta(t^k) \\ &= \lim_{k\to\infty} \eta^T(t^k)\underline{\lambda}(t^k)\eta(t^k) = 0. \end{aligned} \tag{17}$$

However, since $\eta(t^k)$ can also be written as $\eta(t^k) = \sum_{i=1}^{Nn} c_i(t^k)\nu_i$ with $\sum_{i=1}^{Nn} c_i^2(t^k) = 1$, following the above two-case discussions, we have the following observations.

- If $\sum_{i=n+1}^{Nn} c_i^2(t^k)$ has a positive lower bound $\underline{c}$, then we have $\lim_{k\to\infty} \eta^T(t^k)(M(t^k) + \mathcal{L} \otimes I_n)\eta(t^k) \geq \lim_{k\to\infty} \sum_{i=n+1}^{Nn} \lambda_i c_i^2(t^k) \geq \sum_{i=n+1}^{Nn} \lambda_i \underline{c} > 0$, which contradicts (17).
- If $\sum_{i=n+1}^{Nn} c_i^2(t^k)$ does not have a positive lower bound, then there must exist a time subsequence $\{t^{k_\ell}\}_{\ell=1}^\infty$ corresponding to the time sequence $\{t^k\}_{k=1}^\infty$ such that $\lim_{\ell\to\infty} \sum_{i=n+1}^{Nn} c_i^2(t^{k_\ell}) = 0$, which means that $\lim_{\ell\to\infty} \sum_{i=1}^{n} c_i^2(t^{k_\ell}) = 1$. Denote $\eta(t^{k_\ell}) = \eta_1(t^{k_\ell}) + \eta_2(t^{k_\ell})$ where $\eta_1(t^{k_\ell}) = \sum_{i=1}^n c_i(t^{k_\ell})\nu_i$ and $\eta_2(t^{k_\ell}) = \sum_{i=n+1}^{Nn} c_i(t^{k_\ell})\nu_i$. Obviously, $\lim_{\ell\to\infty} \eta_2(t^{k_\ell}) = 0$, and then $\lim_{\ell\to\infty} \eta^T(t^{k_\ell})(M(t^{k_\ell}) + \mathcal{L} \otimes I_n)\eta(t^{k_\ell}) = \lim_{\ell\to\infty} \eta_1^T(t^{k_\ell})M(t^{k_\ell})\eta_1(t^{k_\ell}) \geq \lim_{\ell\to\infty} \frac{\alpha}{N}\sum_{i=1}^n c_i^2(t^{k_\ell}) = \frac{\alpha}{N} > 0$, which also contradicts (17).

Thus, we can conclude that there exists a positive constant α' such that $M(t) + \mathcal{L} \otimes I_n \geq \alpha' I_{Nn}$ holds for all $t \geq t_1 + T$. ■

Proof: [Proof of Theorem 1] Consider (14), we chose a Lyapunov function $V = \tilde{x}^T(\Phi \otimes P)\tilde{x} + \frac{1}{\gamma}\text{tr}\left(\tilde{K}_1^T\tilde{K}_1\right)$ to yield

$$\begin{aligned} \dot{V} =& \tilde{x}^T\left(\Phi \otimes (PA_0 + A_0^T P) - \Phi^2 \otimes (PB_0K_3 + K_3^T B_0^T P)\right)\tilde{x} \\ &+ 2\tilde{x}^T(\Phi \otimes PB_0)\left(\beta F_1(\tilde{x}) - \left(\Phi^{-1} \otimes I_{n_r}\right) \right. \\ &\left. \times \sum_{k=N+1}^{N+M} (H_k \otimes I_{n_r})\bar{r}_k\right) + 2\tilde{x}^T(\Phi \otimes PB)\tilde{K}_{1d}^T x \\ &- 2\sum_{i=1}^N \text{tr}\left(\tilde{K}_{1,i}^T x_i \left(\sum_{j=1}^N h_{ij}\tilde{x}_j\right)^T PB\right) \\ &- 2\text{tr}\left(\tilde{K}_1^T(M + \mathcal{L} \otimes I_n)\tilde{K}_1\right). \end{aligned}$$

We first examine the term: $\tilde{x}^T(\Phi \otimes PB)\tilde{K}_{1d}^T x = \tilde{x}^T(\Phi \otimes I_n)(I_N \otimes PB)\tilde{K}_{1d}^T x = \sum_{i=1}^N\sum_{j=1}^N h_{ij}\tilde{x}_j^T PB\tilde{K}_{1,i}^T x_i$. Since $\sum_{j=1}^N h_{ij}\tilde{x}_j^T PB\tilde{K}_{1,i}^T x_i = \text{tr}\left(\tilde{K}_{1,i}^T x_i \left(\sum_{j=1}^N h_{ij}\tilde{x}_j\right)^T PB\right)$, we have $2\tilde{x}^T(\Phi \otimes PB)\tilde{K}_{1d}^T x - 2\sum_{i=1}^N \text{tr}\left(\tilde{K}_{1,i}^T x_i \left(\sum_{j=1}^N h_{ij}\tilde{x}_j\right)^T PB\right) = 0$. Therefore, we get

$$\begin{aligned} \dot{V} =& \tilde{x}^T\left(\Phi \otimes (PA_0 + A_0^T P) - \Phi^2 \otimes (PB_0K_3 + K_3^T B_0^T P)\right)\tilde{x} \\ &+ 2\tilde{x}^T(\Phi \otimes PB_0)\left(\beta F_1(\tilde{x}) - \left(\Phi^{-1} \otimes I_{n_r}\right) \right. \\ &\left. \times \sum_{k=N+1}^{N+M} (H_k \otimes I_{n_r})\bar{r}_k\right) - 2\text{tr}\left(\tilde{K}_1^T(M + \mathcal{L} \otimes I_n)\tilde{K}_1\right). \end{aligned} \tag{18}$$

We consider the following three cases for (18).

(i) When for all $i \in \mathcal{F}$, $\|K_4 \sum_{j=1}^{N}(h_{ij}\tilde{x}_j)\| \neq 0$, in light of Assumption 2 and Lemma 1, we have

$$\begin{aligned} & -2\tilde{x}^T(\Phi \otimes PB_0)(\Phi^{-1} \otimes I_{nr}) \sum_{k=N+1}^{N+M} (H_k \otimes I_{nr})\bar{r}_k \\ =& -2\left[\sum_{j=1}^{N} h_{1j}\tilde{x}_j^T PB_0 \quad \cdots \quad \sum_{j=1}^{N} h_{Nj}\tilde{x}_j^T PB_0\right] \\ & \times \sum_{k=N+1}^{N+M} (\Phi^{-1}H_k\mathbf{1}_N \otimes r_k) \\ =& -2\sum_{i=1}^{N}\sum_{j=1}^{N} h_{ij}\tilde{x}_j^T PB_0 \sum_{k=N+1}^{N+M} \ell_{ik} r_k \\ \leq& 2\sum_{i=1}^{N} \|B_0^T P \sum_{j=1}^{N} h_{ij}\tilde{x}_j\| \max_{k\in\mathcal{R}} r_k^*. \end{aligned} \tag{19}$$

On the other hand, with $K_4 = B_0^T P$, we have

$$\begin{aligned} & 2\beta\tilde{x}^T(\Phi \otimes PB_0)F_1(\tilde{x}) = 2\beta\tilde{x}^T(\Phi \otimes I_n) \\ & \times (I_N \otimes PB_0)F_1(\tilde{x}) = -2\beta\sum_{i=1}^{N}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\|. \end{aligned} \tag{20}$$

Consequently, combining (18)–(20), we obtain

$$\begin{aligned} \dot{V} \leq& \tilde{x}^T\Big(\Phi \otimes (PA_0 + A_0^T P) - \Phi^2 \otimes (PB_0K_3 \\ & + K_3^T B_0^T P)\Big)\tilde{x} - 2\mathrm{tr}\left(\tilde{K}_1^T(M + \mathcal{L} \otimes I_n)\tilde{K}_1\right) \\ & + 2\sum_{i=1}^{N}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\| \max_{k\in\mathcal{R}} r_k^* \\ & - 2\beta\sum_{i=1}^{N}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\|. \end{aligned}$$

Obviously, if $\beta \geq \max_{k\in\mathcal{R}} r_k^*$, it yields

$$\begin{aligned} \dot{V} \leq& \tilde{x}^T\Big(\Phi \otimes (PA_0 + A_0^T P) - \Phi^2 \otimes (PB_0K_3 \\ & + K_3^T B_0^T P)\Big)\tilde{x} - 2\mathrm{tr}\left(\tilde{K}_1^T(M + \mathcal{L} \otimes I_n)\tilde{K}_1\right). \end{aligned} \tag{21}$$

(ii) When for all $i \in \mathcal{F}$, $\|K_4 \sum_{j=1}^{N} h_{ij}\tilde{x}_j\| = 0$, with $K_4 = B_0^T P$, follow the similar line of deriving (19) and (20), we have $-2\tilde{x}^T(\Phi \otimes PB_0)(\Phi^{-1} \otimes I_{nr})\sum_{k=N+1}^{N+M}(H_k \otimes I_{nr})\bar{r}_k \leq 2\sum_{i=1}^{N}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\| \max_{k\in\mathcal{R}} r_k^* = 0$, and $2\beta\tilde{x}^T(\Phi \otimes PB_0)F_1(\tilde{x}) = -2\beta\sum_{i=1}^{N}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\|^2 = 0$. It immediately yields the same result of (21).

(iii) When for some $i \in \mathcal{F}$, $\|K_4 \sum_{j=1}^{N} h_{ij}\tilde{x}_j\| \neq 0$, and all other $v \in \mathcal{F}$, $\|K_4 \sum_{j=1}^{N} h_{vj}\tilde{x}_j\| = 0$. In this case, without loss of generality, we assume that $i \in \mathbf{I}[1, N_1]$ and $v \in \mathbf{I}[N_1 + 1, N]$ with $2 \leq N_1 \leq N - 1$. Then, combining the discussions for the previous two cases, it is easy to deduce that $-2\tilde{x}^T(\Phi \otimes PB_0)(\Phi^{-1} \otimes I_{nr})\sum_{k=N+1}^{N+M}(H_k \otimes I_{nr})\bar{r}_k \leq 2\max_{k\in\mathcal{R}} r_k^* \sum_{i=1}^{N_1}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\|$, and $2\beta\tilde{x}^T(\Phi \otimes PB_0)F_1(\tilde{x}) = -2\beta\sum_{i=1}^{N_1}\|B_0^T P\sum_{j=1}^{N} h_{ij}\tilde{x}_j\|$. Then, with $\beta \geq \max_{k\in\mathcal{R}} r_k^*$, we arrive at (21).

Summarizing all the above discussions, with condition (16), we obtain $\dot{V} \leq -\rho\tilde{x}^T(\Phi \otimes P)\tilde{x} - 2\mathrm{tr}\left(\tilde{K}_1^T(M + \mathcal{L} \otimes I_n)\tilde{K}_1\right)$. Apparently, since for all $t \geq 0$, $M + \mathcal{L} \otimes I_n > 0$, we have $\dot{V} \leq 0, \forall t \geq 0$, which implies that all states of (14) are bounded. Furthermore, since Lemma 2 states that $M + \mathcal{L} \otimes I_n > \alpha' I_N n$ for all $t \geq t_1 + T$, we can obtain $\dot{V} \leq -\bar{\rho}V$, where $\bar{\rho} = \min\{\rho, 2\gamma\alpha'\}$. Solving the inequality yields $0 \leq V(t) \leq V(0)e^{-\bar{\rho}t}, \forall t \geq t_1 + T$, which implies that $\tilde{x}$ and $\tilde{K}_1$ exponentially converge to zero with a convergence rate no less than $\bar{\rho}/2$ for all $t \geq t_1 + T$. Based on the fact that convergence of $\tilde{x}$ to zero implies that e is also converging to zero exponentially, according to Lemma 8 of [22], the containment control objective is achieved. Exponential convergence of $\tilde{K}_1 \to 0$ indicates cooperative adaptive learning, i.e., $\hat{K}_{1,i} \to K_1$ for all $i \in \mathcal{F}$. ■

Remark 1: Regarding solvability of condition (16), note that if we specify a particular solution with $\hat{K}_3 = \kappa B_0^T$ for some κ such that $\kappa \geq \frac{1}{\min_{i\in\mathcal{F}}\{\lambda_i(\Phi)\}}$, then a sufficient condition to ensure solvability of (16) is $A_0\hat{P} + \hat{P}A_0^T - 2B_0B_0^T < -\rho\hat{P}$. According to [5], [26], [27], given a sufficiently small ρ, a necessary and sufficient condition for existence of a $\hat{P} \in \mathbb{S}_+^n$ to satisfy the above condition is that (A_0, B_0) is stabilizable, which is guaranteed under Assumption 2. In summary, solvability of condition (16) is ensured with a sufficiently small $\rho > 0$ under Assumption 2. It should be pointed out that solving condition (16) requires the information $\lambda_i(\Phi)$, which might not be always feasible under the distributed control context. Possible solutions to overcome this issue are to utilize self-tuning adaptation techniques as proposed in [5], which however is out of the scope of this paper and will be pursued in our future research.

Remark 2: It is seen that the proposed cFTE condition (15) can be satisfied whenever the associated group of signals N_i for all $i \in \mathbf{I}[1, N]$ are exciting in a cooperative (cumulative) fashion over a finite-time window $[t_1, t_1 + T]$. Specifically, it does not necessarily require each individual signal N_i to be persistently exciting or even finite-time exciting, which significantly relaxes existing excitation conditions of PE [8] and FTE [16]. In addition, it does not necessarily require the cumulative signal (i.e., $\sum_{i=1}^{N} N_iN_i^T$) to be exciting persistently, which also relaxes the cPE condition of [12].

Remark 3: The proposed cooperative adaptive containment control scheme distinguishes itself from existing methods (e.g., [5], [12], [16], [17]) in the following aspects:

- Advanced over the method of [5] where only the containment control objective is concerned, it succeeds to fulfill the joint objectives of containment control and accurate parameter identification without requiring satisfaction of PE.
- Advanced over the method of [12], instead of dealing with the parameter identification problem only, it renders simultaneously containment control performance and accurate identification of unknown system parameters for MASs, and accurate parameter identification is ensured without resorting to cPE.
- Advanced over the methods of [16], [17] which are applicable to single dynamical systems only, it addresses the direct adaptive control problem for MASs, and parameter convergence can be guaranteed without imposing FTE on each follower.

IV. A Case Study

Consider a group of N unicycle mobile robots as followers, each robot's linearized motion equations are borrowed from [29], i.e., $\dot{p}_{x,i} = w_{x,i}$, $\dot{p}_{y,i} = w_{y,i}$, $m\dot{w}_{x,i} = u_{x,i}$, $m\dot{w}_{y,i} = u_{y,i}$, $(\forall i \in \mathcal{F})$, where $(p_{x,i}, p_{y,i})$, $(w_{x,i}, w_{y,i})$, and $(u_{x,i}, u_{y,i})$ denote the robot positions, angular velocities, and control inputs along X-Y axes, respectively. m is the mass with unknown value equal to 1 kg. The goal is to apply the proposed control scheme to realize containment control and parameter identification for these robots. It is clear that the X-axis and Y-axis dynamics are decoupled, we thus only consider the X-axis dynamics for simplicity. Moreover, in order to fit into the proposed design framework with all the unknown parameters contained in A of (1), we need to introduce the X-axis dynamics a pre-filter using method of [20], i.e., $\dot{x}_{u,i} = A_u x_{u,i} + B_u u_i$, $u_{x,i} = C_u x_{u,i}$, $(\forall i \in \mathcal{F})$. This leads to the state-space MAS model in the form of (1) with $x_i = col\{p_{x,i}, w_{x,i}, x_{u,i}\}$ and $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, $B = [0, 0, 1]^T$ by choosing $(A_u, B_u, C_u) = (-1, 1, 1)$. The M (virtual) leaders' dynamics (2) are constructed with $A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{bmatrix}$ and $B_0 = [0, 0, 1]^T$. For control design, we specify $N = 100$ and $M = 3$ with an undirected connected topology given in Fig. 2. Assumption 1 is satisfied with

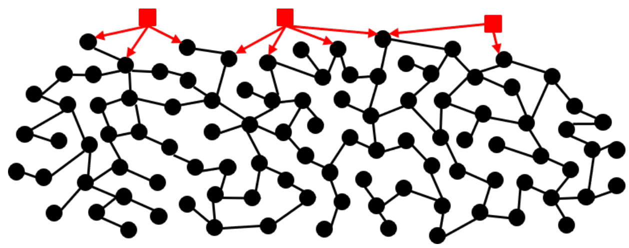

Fig. 2: Network graph. Black circles: followers ($\mathcal{F} = \{1, \cdots, 100\}$). Red squares: leaders ($\mathcal{R} = \{101, 102, 103\}$).

$K_1 = [-1, -2, -3]^T$ and $K_2 = 1$. For simulation study, we further specify the leaders' inputs as $r_{101}(t) = 5\sin(t)$, $r_{102}(t) = 3$, $r_{103}(t) = -5\sin(t)$, which gives an upper bound $\max_{r \in \mathcal{R}} r_k^* = 5$. Controller gains are then synthesized by solving condition (16) with $\gamma = 2, \rho = 0.2$, $\beta = 7.5$, and $k = 1$. With random initial conditions for both leader and follower states and $\hat{K}_{1,i} = 0$ $(\forall i \in \mathcal{F})$, simulation results are plotted in Figs. 3–4. Fig. 3 shows the containment error signals e_i all converging to zero rapidly, thus confirming fulfillment of containment control. The converging behaviors of the estimate parameters $\hat{K}_{1,i}$ $(\forall i \in \mathcal{F})$ to their true values are witnessed in Fig. 4, demonstrating accurate learning capability of the proposed controller.

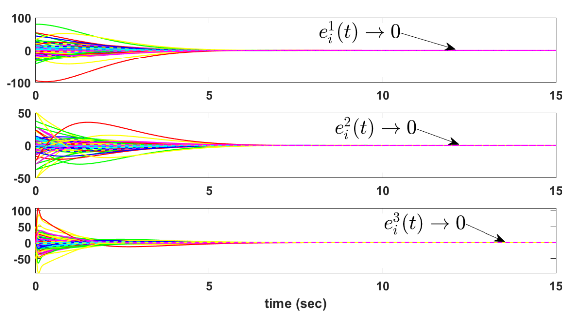

Fig. 3: Containment tracking control errors ($i = 1, \cdots, 100$).

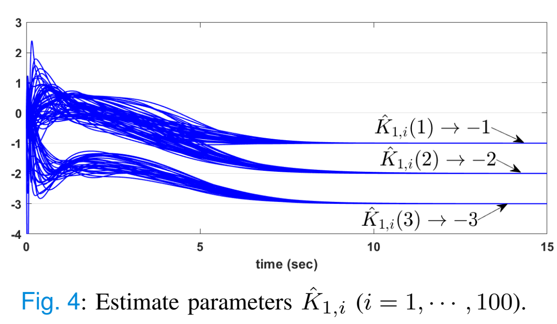

Fig. 4: Estimate parameters $\hat{K}_{1,i}$ ($i = 1, \cdots, 100$).

V. Conclusions

A new cooperative adaptive control scheme has been proposed to jointly achieve containment control and accurate cooperative learning/identification of unknown system parameters for a class of MASs subject to unmeasurable leader inputs and uncertain follower dynamics. An important novelty of this new control scheme lies in its capability of rendering jointly the containment control and accurate learning performance via a mild cFTE condition, which significantly relaxes existing ones (e.g., PE/cPE/FTE) for accurate parameter identification in adaptive control systems.

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