

Galaxy spin direction distribution in HST and SDSS

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ABSTRACT

Several recent observations of datasets of spiral galaxies show non-random distribution of the spin directions of spiral galaxies, even when the galaxies are too far from each other to have gravitational interaction. Here, a dataset of $\sim 8.7 \cdot 10^3$ spiral galaxies imaged by Hubble Space Telescope is used to test and profile a possible asymmetry between galaxy spin directions. The asymmetry between galaxies with opposite spin directions is compared to the asymmetry of galaxies from the Sloan Digital Sky Survey. The two datasets contain different galaxies at different redshift ranges, and each dataset was annotated in a different annotation method. The results show that both datasets show a similar asymmetry in the COSMOS field, which is covered by both telescopes. Fitting the asymmetry of the galaxies to cosine dependence shows a dipole axis with probabilities of $\sim 2.8\sigma$ and $\sim 7.38\sigma$ in HST and SDSS, respectively. The most likely dipole axis identified in the HST galaxies is at $(\alpha = 78^\circ, \delta = 47^\circ)$, and is well within the 1σ error range compared to the location of the most likely dipole axis in the SDSS galaxies with $z > 0.15$ at $(\alpha = 71^\circ, \delta = 61^\circ)$.

Keywords: Galaxy: general – galaxies: spiral

1. INTRODUCTION

Recently, several experiments using large datasets of galaxies imaged by several different instruments have shown asymmetry between galaxies with opposite spin directions (Longo 2011; Shamir 2012, 2013; Hoehn & Shamir 2014; Shamir 2016a, 2017a,b,c; Lee et al. 2019a,b; Shamir 2019, 2020a,b). The asymmetry is re-

flected by difference in the number of galaxies with opposite spin directions (Shamir 2012, 2019; Lee et al. 2019b; Shamir 2020b), and it changes with the directions of observation (Shamir 2012) and the redshift (Shamir 2016b, 2019, 2020b). Other experiments showed differences in the brightness of the galaxies (Shamir 2016a, 2017c).

Early experiments used manually annotated galaxies, and showed no statistically significant difference between the number of galaxies with

opposite spin directions. However, these experiments were limited by the number of galaxies that could be classified manually, and therefore the number of annotations was relatively small (Iye & Sugai 1991). The deployment of digital sky surveys powered by robotic telescopes introduced far stronger data collection capabilities, allowing to image a large number of astronomical objects. In the absence of reliable algorithms at the time, a first attempt to use the power of digital sky surveys to test for asymmetry between the number of galaxies with opposite spin directions was based on crowdsourcing (Land et al. 2008). The results showed a very strong preference of $\sim 20\%$ for galaxies with clockwise spin compared to galaxies with counterclockwise spin direction, and correction for a possible bias provided no statistically significant difference. However, these results could have been biased by the human perception of the non-professional volunteers who annotated the data, and analysis that corrected for these biases showed an asymmetry that was not statistically significant (Land et al. 2008). It was also found that volunteers annotating the same galaxies tended to classify elliptical galaxies with no apparent spin direction as spiral galaxies that spin clockwise, and therefore leading to a difference in the number of galaxies (Hayes et al. 2017). Another experiment that used manual analysis of the data was based on five undergraduate students annotating $\sim 1.5 \cdot 10^4$ galaxies. In that experiment the galaxies were also mirrored in attempt to correct for a possible human bias, and the results showed a difference of $\sim 7\%$ between the number of clockwise and counterclockwise galaxies (Longo 2011).

With the availability of very large astronomical databases, the algorithmic foundations gradually closed the gaps between the magnitudes of the image data and the supply of existing computational methods that can analyze them.

The ability to automate the annotation of the spin direction of spiral galaxies allowed to annotate far larger datasets that can provide strong statistical signal and profile a possible asymmetry between galaxies with opposite spin directions. It should be noted that the advantage of eliminating the human perception bias is compromised when using machine learning for the annotation, since machine learning algorithms are based on “ground truth” training data that is annotated manually, and the trained model can therefore still be biased by the data it was trained with.

By using model-driven automatic annotation algorithms (Shamir 2011a), very large datasets of galaxies showed asymmetry between the number of galaxies with opposite spin directions, and the asymmetry direction and magnitude change based on the direction of observation (Shamir 2012, 2019, 2020b) and the redshift (Shamir 2016b, 2020b). The asymmetry was identified in data collected by the Sloan Digital Sky Survey (Shamir 2012, 2016a), and showed good agreement with the asymmetry identified in data collected by the Panoramic Survey Telescope and Rapid Response System (Shamir 2017c, 2019, 2020b).

Experiments with smaller datasets annotated manually also showed patterns of spin directions of galaxies (Slosar et al. 2009), and alignment of spin directions was identified with quasars (Hutsemékers et al. 2014). More recently, consistency in spin directions was also observed with galaxies that are too distant from each other to have any kind of gravitational interactions (Lee et al. 2019b). These links are defined as “mysterious”, leading to the assumption of a link between galaxy rotation and the motion of the large-scale structure (Lee et al. 2019b).

This paper shows an analysis of the asymmetry between galaxies with opposite spin directions observed when using spiral galaxies from different parts of the sky. The main dataset used in this study is taken from Hubble Space Telescope, and the asymmetry in that dataset is compared to the asymmetry in a galaxy dataset from SDSS used in previous experiments (Shamir 2019, 2020b).

2. DATA

The dataset of spiral galaxies was taken from the Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (Grogin et al. 2011; Koekemoer et al. 2011). The initial dataset contained 114,529 galaxies taken from the Great Observatories Origins Deep Survey North (GOODS-N), the Great Observatories Origins Deep Survey South (GOODS-S), the Ultra Deep Survey (UDS), the Extended Groth Strip (EGS), and the Cosmic Evolution Survey (COSMOS) fields. The galaxy images were separated from the F814W band FITS images using the *mSubimage* tool included in the *Montage* package (Berriman et al. 2004), and were converted into 122×122 TIF (Tagged Image File) images.

The separation of the galaxies into galaxies with clockwise and counterclockwise spin directions was done manually. In previous experiments automatic analysis was used (Shamir 2013, 2017b, 2019, 2020b). However, while automatic analysis that is not based on machine learning is unbiased and capable of analyzing very large databases, it is limited by its ability to classify all galaxies. Therefore, the spin direction of many galaxies cannot be determined, and therefore these galaxies are excluded from the analysis. In sky surveys such as SDSS the number of galaxies is high, and therefore sacrificing some of the galaxies still leaves a sufficient number of accurately annotated galaxies, and does not

affect the analysis as long as the algorithm is fully symmetric. However, the HST fields are far smaller than sky surveys such as SDSS, and sacrificing some of the galaxies can reduce the number of galaxies in the dataset. Another reason for using manual analysis is to use a highly accurate method that is different from the methods used in previous experiments.

The analysis was done by first randomly mirroring half of the images, and then identifying all galaxies with clockwise spin direction and separating them from the rest of the galaxies. Then, all galaxy images were mirrored, and the clockwise galaxies were again separated from the rest of the galaxies. Each of these two datasets was then inspected to ensure that all galaxies are classified correctly. In the end of the process, 200 galaxies with clockwise spin direction, 200 galaxies with counterclockwise spin direction, and 200 galaxies that their spin direction could not be determined were inspected carefully. All 600 galaxies were annotated correctly. That provided a very clean dataset that is also symmetric in the annotations of the galaxies due to the random mirroring, and the identification of just clockwise galaxies. But unlike previous datasets, it is also complete in the sense that all galaxies that their spin direction could be determined are indeed annotated. The process was labor-intensive, and required ~ 250 hours of work to complete. It provided a clean dataset of 8,690 galaxies with identifiable spin direction. The distribution of the galaxies in the different fields are summarized in Table 1. The Subaru *g* magnitude and the photometric redshift distribution of these galaxies are shown in Figure 1.

The distribution of spin directions in the HST galaxies was compared to datasets of SDSS and Pan-STARRS galaxies that were used in previous experiments (Shamir 2017a,b, 2019, 2020b). These datasets were annotated automatically

Field	Field center (degrees)	# All galaxies	# Clockwise galaxies	# Counterclockwise galaxies
GOODS-N	189.23,62.24	5,931	396	373
GOODS-S	53.12,-27.81	5,024	276	264
COSMOS	150.12,2.2	84,424	3,116	2,965
UDS	214.82,52.82	14,245	323	293
EGS	34.41,-5.2	4,905	355	329

Table 1. The number of galaxies in each of the five fields.

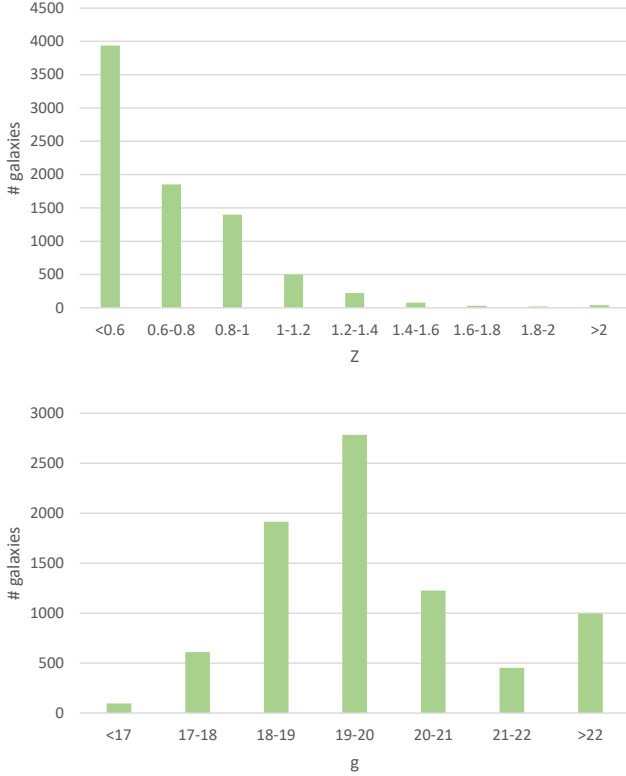


Figure 1. The redshift and g magnitude distribution of the HST galaxies.

by the Ganalyzer (Shamir 2011a,b) algorithm. Ganalyzer is a model-driven algorithm that is based on clear and defined rules. It is not based on machine learning or deep neural networks, and therefore cannot be biased by the training set or by complex non-intuitive rules typical to machine learning systems. Most importantly, Ganalyzer is based on symmetric rules. In addition to the theoretical evidence, it also showed

empirical evidence obtained by mirroring a large number of galaxy images. Full details about the galaxy annotation method and the dataset can be found in (Shamir 2017a,b,c, 2019, 2020b).

3. RESULTS

The distribution of galaxies in HST shows that the number of clockwise galaxies is higher, but the number of galaxies in the different fields is too low to allow statistical analysis. The only exception is the COSMOS field, where the number of galaxies is far higher than in any of the other HST fields used in this study. To compare the asymmetry in that field to galaxies imaged by SDSS and Pan-STARRS, the SDSS and Pan-STARRS galaxies in the 10×10 degrees around the center of COSMOS were examined. The reason for using a larger field is because COSMOS is far deeper than SDSS and Pan-STARRS, and therefore SDSS and Pan-STARRS have a much smaller number of galaxies in a field of the same size. The difference between the size of the fields naturally makes the comparison indirect, as the fields being compared are different. But although the fields are not identical, such comparison can provide certain information regarding the agreement between the populations of galaxies in the different fields.

Datasets that were used in previous studies were examined, all of them were annotated automatically. These included a dataset of SDSS

(Shamir 2017b), and a dataset of Pan-STARRS objects (Shamir 2020b). Because the dataset used in (Shamir 2017b) contained photometric objects of extended sources, some of the photometric measurements were made from photometric objectss inside the same extended source. To avoid the presence of duplicate objects, all objects that had another object within 0.01° or less were removed. Detailed information about these datasets and the distribution of redshift and magnitude of the galaxies they contain are described in the relevant papers (Shamir 2017b,c, 2020b). Table 2 shows the number of galaxies by their spin directions in each of the instruments. As the table shows, all datasets show a higher number of clockwise galaxies in that field. The statistical significance is not strong in the Pan-STARRS field, as expected due to the lower number of galaxies compared to COSMOS, but these fields do not conflict with the distribution of galaxy population in COSMOS. Assuming equal probability of having clockwise and counterclockwise galaxies, the probability of having that asymmetry in all of these fields is $2 \times 0.027 \times 0.017 \times 0.06 \simeq 5 \cdot 10^{-5}$.

Previous experiments showed evidence of non-random patterns of the asymmetry between the number of galaxies with opposite spin directions in different parts of the sky (Shamir 2012, 2019, 2020b). That was done by identifying the (α, δ) at which the asymmetry of the galaxy spin directions had best fit to cosine dependence. The HST galaxies data used in this experiment include several different fields in different parts of the sky. That allows to fit the distribution of the spin directions of these galaxies to cosine dependence. Fitting the galaxy spin directions to cosine dependence can indicate whether the galaxy spin directions are aligned in a form of a possible dipole axis, and can also provide the statistical significance of such axis.

To test the probability that the spin direction asymmetry exhibits a dipole axis, the same method used in (Shamir 2012, 2019, 2020b) was applied. Each galaxy was assigned with a value within the set $\{-1, 1\}$. Galaxies with clockwise spin direction were assigned with 1, and galaxies with counterclockwise spin direction were assigned with -1. Then, χ^2 statistics was used such that for each possible integer (α, δ) combination, the angular distance ϕ between (α, δ) and the celestial coordinates of each galaxy in the dataset was computed. The $\cos(\phi)$ of the galaxies were then fitted into $d \cdot |\cos(\phi)|$, such that d is the spin direction of the galaxy (a value within the set $\{-1, 1\}$). The χ^2 was computed 1000 times such that in each time the galaxies were assigned with random spin directions, and the mean and standard deviation were computed for each possible (α, δ) . The χ^2 mean computed with the random spin directions was then compared to the χ^2 when d was assigned to the real spin directions. The σ difference between the χ^2 of the real spin directions and the mean χ^2 when using the random spin directions shows the likelihood of an axis at (α, δ) . When the likelihood of all (α, δ) was computed, the (α, δ) where a dipole axis is the most likely could be identified. Figure 2 shows the probability of a dipole axis in all integer (α, δ) combinations. The most likely axis was identified at $(\alpha = 78^\circ, \delta = 47^\circ)$, with probability of $\sim 2.83\sigma$. The 1σ error for that axis is $(58^\circ, 184^\circ)$ for the right ascension, and $(6^\circ, 73^\circ)$ for the declination.

The SDSS data are distributed more uniformly in the sky compared to the HST data, and therefore allow comparing the asymmetry in different hemispheres. Figure 3 shows the asymmetry between the number of clockwise galaxies and the number of counterclockwise galaxies in the 180° hemisphere centered at each RA, as well as the same measurement made in the opposite hemisphere. The figure shows that the strongest

Surveys	# Clockwise galaxies	# Counterclockwise galaxies	P value
COSMOS	3,116	2,965	0.027
SDSS ($10^\circ \times 10^\circ$)	350	295	0.017
Pan-STARRS ($10^\circ \times 10^\circ$)	222	190	0.06

Table 2. Number of clockwise and counterclockwise galaxies in the COSMOS field and in the $10^\circ \times 10^\circ$ field of SDSS and Pan-STARRS centered around COSMOS. The P value reflects the binomial probability of having asymmetry equal or greater than the observed asymmetry when assuming that a galaxy has 0.5 probability of having clockwise or counterclockwise spin direction. All of these datasets were annotated in an automatic process.

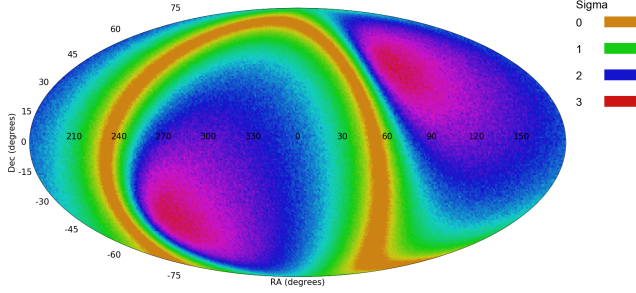


Figure 2. Probability of cosine dependence of the spin directions of HST galaxies from every possible integer (α, δ) combination.

asymmetry is observed in the hemisphere centered at $(\alpha = 90^\circ)$. In that hemisphere there are 14,403 galaxies with clockwise spin and 15,101 galaxies with counterclockwise spin. The one-tailed probability of having such a difference or greater by chance is $(P \simeq 0.000024)$, and the two-tailed probability is $(P \simeq 0.000048)$. The opposite hemisphere has 17,263 galaxies with clockwise spin direction, and 16,980 galaxies with counterclockwise spin direction. The probability of that difference is ~ 0.06 . Although that asymmetry is not significant, it also does not conflict with the asymmetry in the other hemisphere for the assumption that these two hemisphere exhibit a possible dipole. Clearly, SDSS galaxies are not evenly distributed in the sky, and the population varies significantly in different RA and declination ranges.

The dipole axis identified in the HST galaxies was compared to the dipole axis identified in

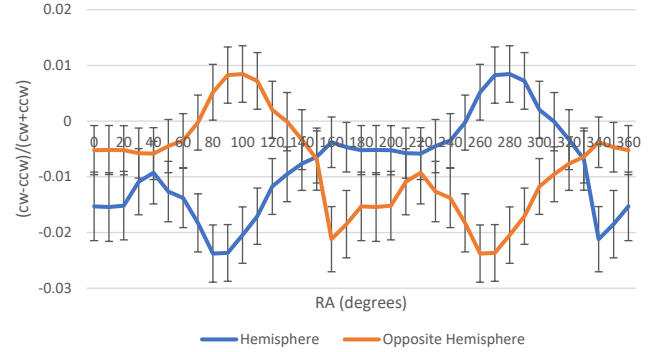


Figure 3. The asymmetry between the number of galaxies that spin clockwise and the number of galaxies that spin counterclockwise in hemispheres centered at different RAs. The blue line shows the asymmetry in the hemisphere centered at the RA of the x-axis, and the orange line shows the same measurement in the opposite hemisphere. The error bars are the normal distribution standard error of $\frac{1}{\sqrt{n}}$, where n is the total number of galaxies in the hemisphere.

SDSS galaxies that were annotated automatically (Shamir 2020b). Figure 4 shows the probability of a dipole axis identified in each possible pair of integer (α, δ) in the SDSS galaxies, when using the galaxies with $z > 0.15$ used in (Shamir 2020b). That dataset included 15,863 galaxies annotated automatically by their spin direction. The most likely axis is identified at $(\alpha = 71^\circ, \delta = 61^\circ)$, with $\sigma \simeq 7.38$. That most likely axis is close to the most likely dipole axis identified in the HST galaxies, and well within the 1σ error.

Unlike the HST galaxies that were classified manually, the dataset of SDSS galaxies was classified automatically by using a software. That leads to the assumption that the asymmetry is the result of a software error. An error in the software could lead to a higher number of one type of galaxies over the other. That can happen when using machine learning for galaxy classification, where complex rules are determined by the training set, and a small bias in these rules is very difficult to identify. The method used in this study is based on a model-driven algorithm, that works by defined rules, and is not based on machine learning (Shamir 2011a). The assumption that a software error led to the asymmetry is also challenged by the observation that galaxies in different parts of the sky exhibit different directions of asymmetry (Shamir 2020b). A software error is expected to be consistent in all directions of observation, showing the same asymmetry in all parts of the sky. Additionally, like in previous experiments (Shamir 2012, 2013), the experiment was repeated by mirroring the galaxy images (Shamir 2020b), leading to the expected inverse results. Figure 5 shows the most likely dipole axis when the galaxies are assigned with random spin directions.

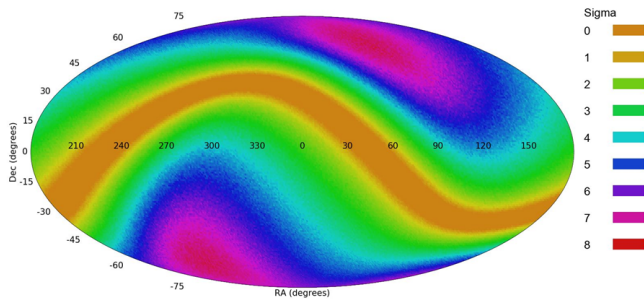


Figure 4. Cosine dependence probability of the spin directions of SDSS galaxies from every possible integer (α, δ) combination.

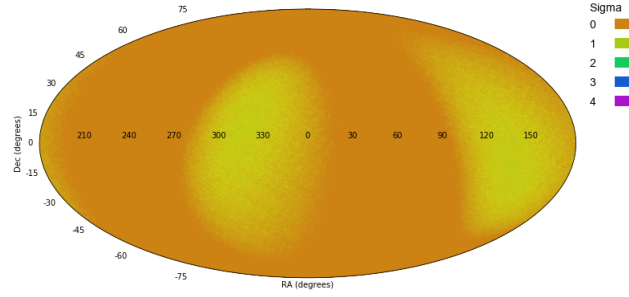


Figure 5. Probability of cosine dependence of the spin directions of SDSS galaxies from every possible integer (α, δ) combination when the galaxies are assigned with random spin directions.

Results from different datasets of galaxies imaged by two different instruments show similar asymmetry between galaxies with opposite spin directions. Each dataset contains different galaxies, and the galaxies in each dataset were annotated using a different method. Both datasets show a statistically significant dipole axis, and the location of the most likely axis is consistent in both datasets. Despite the difference in redshift, the two datasets show fairly similar location of the most likely dipole axis, and well within 1σ error. The statistical significance of the HST data of $\sim 2.8\sigma$ is not entirely impossible, but it is comparable to other puzzling phenomena such as the CMB Cold Spot (Cruz et al. 2007). It is also aligned with data collected from other instruments, reinforcing to investigate the nature of the observation.

While the observations are clearly provocative, it is difficult to identify an error that could lead to such results. The experiments are based on different instruments, and two different galaxy annotation methods. One of the instruments is space-based, reducing the possibility that the results are driven by an atmospheric effect. The distribution of the galaxies around the most populated field (COSMOS) show similar asymmetry, and the asymmetry is statistically significant. These results are consistent with previous similar experiments (Shamir

4. CONCLUSION

2013, 2016a, 2017a,b,c, 2019, 2020b). The automatic annotation method is model-driven, does not rely on machine learning, and consistent when the galaxy images are mirrored (Shamir 2017b). Previous experiments also showed that the asymmetry changes in different parts of the sky, which is not expected if the annotation method is biased (Shamir 2017c, 2019, 2020b). The mirroring of a random half of the galaxies and the annotation of just clockwise galaxies in each round should correct for any perceptual bias in the annotations. None of the datasets included duplicate objects. An error in a certain instrument or a photometric pipeline is also unlikely, as several different instruments provide consistent results (Shamir 2017c, 2020b). The galaxy spin direction is a crude measurement, and there is no known atmospheric or other effect that can make a galaxy that spin clockwise seem to spin the opposite way. In any case, one of the datasets used in this experiment is images by a space-based instrument, and therefore atmospheric effect cannot explain the asymmetry. Even if such effect existed, it is expected to have the same effect on clockwise and counterclockwise spiral galaxies, and differences in different parts of the sky are not expected.

It is naturally difficult to identify an immediate explanation for the observations. Lee et al. (2019b) identified consistency of spin directions of galaxies even if the galaxies are too far to interact gravitationally, and defined the observation as “mysterious” (Lee et al. 2019b). Explanations of the asymmetry can be related to parity-breaking gravitational waves, which can affect galaxy shape during inflation (Biagetti & Orlando 2020), and can provide an explanation to the asymmetry without violating the basic cosmological assumptions. Cosmological-scale anisotropy has been observed in the past with cosmic microwave background (Cline et al. 2003; Gordon & Hu 2004; Zhe et al. 2015).

These observations also challenge the basic cosmological assumptions and led to theories that differ from the standard cosmological models (Feng & Zhang 2003; Piao et al. 2004; Rodrigues 2008; Piao 2005; Jiménez & Maroto 2007; Bohmer & Mota 2008). These observations also led to the model of ellipsoidal universe (Campanelli et al. 2006, 2007; Gruppuso 2007), as well as a rotating universe (Gödel 1949; Ozsváth & Schücking 1962; Ozsvath & Schücking 2001; Sivaram & Arun 2012; Chechin 2016).

Cosmological isotropy and homogeneity are basic assumptions used in most standard cosmological theories, although spatial homogeneity is an assumption that cannot be verified directly (Ellis 1979). Some evidence of cosmological isotropy violation have been observed through other messengers such as radio sources (Bengaly et al. 2018), luminosity-temperature ratio (Migkas et al. 2020), short gamma ray bursts (Mészáros 2019), Ia supernova (Javanmardi et al. 2015), distribution of galaxy morphology (Javanmardi & Kroupa 2017), and cosmic microwave background (Aghanim et al. 2014; Hu & White 1997; Cooray et al. 2003; Ben-David et al. 2012; Eriksen et al. 2004). Future instruments such as the Earth-based Rubin observatory and the space-based Euclid can be used to validate whether the asymmetry is observed also in other instruments, and provide better profiling of the asymmetry.

Given the multiple reports on anomaly in the distribution of galaxies with opposite spin patterns (Longo 2011; Shamir 2012, 2019; Lee et al. 2019b; Shamir 2020b), it is important to continue the examination of such observations, verifying and profiling the possible non-random distribution, and identifying whether the reported observations can have non-astronomical explanations.

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REFERENCES

- Aghanim, N., Armitage-Caplan, C., Arnaud, M., et al. 2014, *A&A*, 571, A27
- Ben-David, A., Kovetz, E. D., & Itzhaki, N. 2012, *ApJ*, 748, 39
- Bengaly, C. A., Maartens, R., & Santos, M. G. 2018, *Journal of Cosmology and Astroparticle Physics*, 2018, 031
- Berriman, G. B., Good, J. C., Laity, A. C., et al. 2004, in *ADASS XIII*, Vol. 314, 593
- Biagetti, M., & Orlando, G. 2020, *Journal of Cosmology and Astroparticle Physics*, 2020
- Bohmer, C. G., & Mota, D. F. 2008, *PLB*, 663, 168
- Campanelli, L., Cea, P., & Tedesco, L. 2006, *PRL*, 97, 131302
- . 2007, *PRD*, 76, 063007
- Chechin, L. 2016, *Astronomy Reports*, 60, 535
- Cline, J. M., Crotty, P., & Lesgourgues, J. 2003, *Journal of Cosmology and Astroparticle Physics*, 2003, 010
- Cooray, A., Melchiorri, A., & Silk, J. 2003, *PLB*, 554, 1
- Cruz, M., Cayon, L., Martinez-Gonzalez, E., Vielva, P., & Jin, J. 2007, *ApJ*, 655
- Ellis, G. 1979, *General Relativity and Gravitation*, 11, 281
- Eriksen, H. K., Hansen, F. K., Banday, A. J., Gorski, K. M., & Lilje, P. B. 2004, *ApJ*, 605, 14
- Feng, B., & Zhang, X. 2003, *PLB*, 570, 145
- Gödel, K. 1949, *Reviews of Modern Physics*, 21, 447
- Gordon, C., & Hu, W. 2004, *PRD*, 70, 083003
- Grogan, N. A., Kocevski, D. D., Faber, S., et al. 2011, *ApJS*, 197, 35
- Gruppuso, A. 2007, *PRD*, 76, 083010

- Hayes, W. B., Davis, D., & Silva, P. 2017, *MNRAS*, 466, 3928
- Hoehn, C., & Shamir, L. 2014, *AN*, 335, 189
- Hu, W., & White, M. 1997, arXiv preprint astro-ph/9706147
- Hutsemékers, D., Braibant, L., Pelgrims, V., & Sluse, D. 2014, *A&A*, 572, A18
- Iye, M., & Sugai, H. 1991, *ApJ*, 374, 112
- Javanmardi, B., & Kroupa, P. 2017, *A&A*, 597, A120
- Javanmardi, B., Porciani, C., Kroupa, P., & Pflam-Altenburg, J. 2015, *ApJ*, 810, 47
- Jiménez, J. B., & Maroto, A. L. 2007, *PRD*, 76, 023003
- Koekemoer, A. M., Faber, S., Ferguson, H. C., et al. 2011, *ApJS*, 197, 36
- Land, K., Slosar, A., Lintott, C., et al. 2008, *MNRAS*, 388, 1686
- Lee, J. H., Pak, M., Lee, H.-R., & Song, H. 2019a, *ApJ*, 872, 78
- Lee, J. H., Pak, M., Song, H., et al. 2019b, *ApJ*, 884, 104
- Longo, M. J. 2011, *PLB*, 699, 224
- Mészáros, A. 2019, *AN*, 340, 564
- Migkas, K., Schellenberger, G., Reiprich, T., et al. 2020, *A&A*, 636, A15
- Ozsváth, I., & Schücking, E. 1962, *Nature*, 193, 1168
- Ozsvath, I., & Schücking, E. 2001, *Classical and Quantum Gravity*, 18, 2243
- Piao, Y.-S. 2005, *PRD*, 71, 087301
- Piao, Y.-S., Feng, B., & Zhang, X. 2004, *PRD*, 69, 103520
- Rodrigues, D. C. 2008, *Physical Review D*, 77, 023534
- Shamir, L. 2011a, *ApJ*, 736, 141
- . 2011b, *The Astrophysics Source Code Library*, ascl:1105.011
- . 2012, *PLB*, 715, 25
- . 2013, *Galaxies*, 1, 210
- . 2016a, *ApJ*, 823, 32
- . 2016b, arXiv:1601.04424v1
- . 2017a, *ApSS*, 362, 33
- . 2017b, *PASA*, 34, e011
- . 2017c, *PASA*, 34, e44
- . 2019, arXiv, 1912.05429
- . 2020a, *Open Astronomy*, 29, 15
- . 2020b, *ApSS*, 365, 136
- Sivaram, C., & Arun, K. 2012, *Open Astronomy*, 5, 7
- Slosar, A., Land, K., Bamford, S., et al. 2009, *MNRAS*, 392, 1225
- Zhe, C., Xin, L., & Sai, W. 2015, *Chinese Physics C*, 39, 055101