An Anti-Anti-Turbulence Conjecture

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ABSTRACT

Sandström's theorem suggested that buoyancy fluxes can not drive the overturning circulation, an idea that physical oceanographers continue to debate. This paper attempts to provide the most fundamental examination of Sandström's theorem that is possible. We conclude that the fundamental role of buoyant forcing in the modern ocean is to damp the circulation, that buoyance fluxes drive significant levels of turbulence, that buoyant turbulent generation is not solely proportional to molecular diffusivity and that some of the main pathways to turbulence are non-Boussineq. We conjecture that the ocean might not be constrained by the 'Anti-Turbulence' theorem.

1. Introduction

Sandström's theorem (Sandström (1908)), stating that horizontal convection does not drive large scale circulation, has a history that dates nearly to the inception of physical oceanography. The conclusion was based on laboratory experiments showing that the maintenance of a sustained circulation by buoyancy forcing required that the heat source be positioned at a level beneath the cooling source. Equivalently, oceanic buoyancy fluxes, perhaps most properly termed as driving 'horizontal convection', where the two sources are almost at the same geopotential level, could not be responsible for the observed large scale overturning circulation. That the theorem has been subject to several reinterpretations and restatements since it was first proposed cannot be denied. In spite of its continued debate in the community and investigation with advanced theoretical, numerical and laboratory techniques, the field remains divided in how to view Sandström's theorem. It is the objective of this paper to examine its validity from as fundamental a perspective as possible.

Several classical publications followed the original, notably Sandström (1916), Sandström (1922), Jeffreys (1925) and Defant (1961), where the conclusion and its interpretation were vigorously debated. In the last decade, interest in the ocean energy budget rekindled the discussion, with important contributions too numerous to effectively, or even usefully, cite. A recent overview is given in Coman et al. (2006), where attempts at reproduction of Sandström's original results failed, and it was concluded that the original experiments were flawed.

A powerful and elegant reexamination of the fundamental question raised by Sandström was provided by Paparella and Young (2002), who supported the basic Sandström result by arguing the flows driven by horizontal convection were non-turbulent, failing the 'zeroth law' of turbulence, which defines turbulent flow by its ability to sustain finite dissipation in the limit of vanishing viscosity. Thus, Paparella and Young (2002) imply diapycnal mixing in an ocean forced soley by horizontally placed buoyancy sources must be governed by molecular processes, in which case all stratification will be confined to a surface trapped boundary layer whose width is set by a diffusive length scale. This in no way resembles the observed abyssal stratification, leading to the conclusion that wind forcing is the primary source of interior ocean turbulence. Other notable publications endorsing or adopting this perspective include Huang (1998), Huang (1999), Wunsch and Ferrari (2004) and Ferrari and Wunsch (2009).

A vigorous counterpoint to this theme has been mounted by several, with a thorough discussion to be found in Hughes et al. (2009). More recently, Gayen et al. (2013) and Vreugdenhil et al. (2016) have argued that buoyancy effectively drives mixing through pathways that emphasize the generation of available potential energy. Clear evidence of large scale flows arising from purely horizontally driven convection have been found numerically and in the laboratory, although often confined to regions of weak stratification. Vreugdenhil et al. (2016) in a very dense paper discussing turbulence and convection resolving DNS simulations, find buoyancy forcing can maintain deep stratification by means of an extraordinarily high mixing efficiency.

The debate engages a very fundamental question in physical oceanography, namely the maintenance of the abyssal stratification. While it is generally accepted that ocean diapycnal mixing requires turbulence (i.e. the levels of required mixing are beyond those that can be achieved by molecular processes), the energy sources behind that turbulence remain unclear. With the supposition that ocean mixing proceeds with a gross 'efficiency' of about 20%, something like $2TW$ ($1TW = 10^{12}W$) must be provided to the ocean interior to maintain the observed stratification (Munk and Wunsch (1998), Wunsch and Ferrari (2004)). Existing estimates have assumed buoyancy forcing plays a negligible role in supplying this energy, and this is the position most hotly debated in the literature.

In our view, at least part of the disparity in viewpoints can be assigned to the use of the Boussinesq equations in examining this problem. The problem here is that Boussinesq thermodynamics remain somewhat foggy, to say the least, particularly with regards to the role of internal energy. This general topic received a significant boost by Young (2010) who put Boussinesq thermodynamics on a much firmer foundation by identifying dynamic enthalpy as the effective potential energy of a Boussinesq fluid with a realistic equation of state. With this identification, the resulting equations could be shown to have an energy conservation principle, up to turbulent mixing processes. While applauding this effort, we note that full inclusion of non-conservative processes was not considered. Divorcing dynamics from thermodynamics, i.e. the primary advantage of the Boussinesq equations, is very useful, but the danger is the ignored energy reservoir, internal energy, is generally several orders of magnitude greater than either kinetic or potential energy. Although the Boussinesq equations are a relatively accurate approximation of the full compressible equations, even tiny errors in estimating the internal energy budget can cause overwhelming errors in the kinetic and potential energy budgets.

We address this point in the present paper by examining the full compressible equations in order to extract exact statements about fluid energetics subject to external inputs of mechanical and thermal energy. As such, these statements are not subject to uncertainties in their interpretations due to approximations. They also allow us to identify pathways between dynamics, thermodynamics and dissipation, and to classify them as to their Boussinesq or non-Boussinesq foundations.

Accordingly, we find that the role of buoyant forcing in the modern ocean is to damp the modern general circulation. We also argue that buoyant forcing generates dissipation that cannot be simply related to molecular diffusion and so does not necessarily disappear in the limit of a vanishing molecular diffusivity. In this sense, we suggest a fully compressible fluid might not be subject to the ‘Anti-Turbulence’ theorem of Paparella and Young (2002). We find the buoyant forcing pathways leading to dissipation are non-Boussinesq, and an estimate based on modern observations is surprisingly large. Ultimately, we suggest that horizontal convection can drive turbulent flow and contributes importantly to global dissipation levels.

The next section reviews the derivation of the energy equations for a fully compressible fluid. Integral constraints then demonstrate the ultimately damping role of buoyancy fluxes on the modern circulation. When restricted to horizontal convection, net buoyancy fluxes must vanish. The resulting balances are then interpeted in terms of the roles played by convection in generating dissipation, and their basis as Boussinesq or non-Boussinesq. We conclude with a discussion.

2. Background

Paparella and Young (2002) argued that horizontal convection was non-turbulent, and thus that buoyancy fluxes did not significantly energize the circulation. The proof essentially reduced to examining the steady kinetic energy equation of a Boussinesq fluid, and arguing the level of dissipation for horizontal convection could be bounded by an integral proportional to the molecular diffusivity. In the limit of vanishing viscosity at a fixed Prandtl number, dissipation then vanished. This is a violation of the 'zeroth law' of turbulence, and leads eventually to the conclusion that horizontal convection cannot sustain the observed deep stratification. Although the Boussinesq equations are universal in ocean modeling, we ask how this prediction fares if one examines the fully compressible equations.

The exact stratified Navier-Stokes momentum and mass equations for a compressible binary fluid are

$$\boldsymbol{u}_t + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} = -\frac{\nabla P}{\rho} - g\boldsymbol{k} + \frac{\nabla \cdot \rho \underline{\boldsymbol{\tau}}}{\rho} \tag{1}$$

$$\rho_t + \nabla \cdot \boldsymbol{u}\rho = 0 \tag{2}$$

where $\boldsymbol{u}$ is velocity, $\boldsymbol{f}$ is the Coriolis parameter, P is total pressure, $\rho = 1/\alpha$ is fluid density and g is gravity. The quantity $\underline{\boldsymbol{\tau}}$ is the viscous stress tensor, whose i, j entry is

$$\tau_{i,j} = \nu \frac{\partial}{\partial x_j} u_i \tag{3}$$

with molecular viscosity ν and where Einstein summation notation is implied.

A kinetic energy equation is obtained from (1) by vector multiplying by $\boldsymbol{u}$

$$K_t + \boldsymbol{u} \cdot \nabla K = -\frac{\boldsymbol{u} \cdot \nabla P}{\rho} - wg - \varepsilon + \frac{\nabla \cdot \boldsymbol{u}\rho \cdot \underline{\boldsymbol{\tau}}}{\rho} \tag{4}$$

where $K = \boldsymbol{u} \cdot \boldsymbol{u}/2$ is kinetic energy and ε is positive definite molecular dissipation

$$\varepsilon = \nu \left(\frac{\partial}{\partial x_j} u_i\right)\left(\frac{\partial}{\partial x_j} u_i\right) \tag{5}$$

Using mass conservation, (4) can be written as

$$(\rho K)_t + \nabla \cdot (\rho \boldsymbol{u} K) = -\boldsymbol{u} \cdot \nabla P - \rho w g - \rho \varepsilon + \nabla \cdot \rho \boldsymbol{u} \cdot \underline{\boldsymbol{\tau}} \tag{6}$$

The full internal energy equation of a binary compressible fluid is

$$\frac{d}{dt} e = -P \frac{d}{dt} \alpha + T \frac{d}{dt} \eta + \mu \frac{d}{dt} S \tag{7}$$

where e is internal energy, T is in-situ temperature, η is entropy, μ is the relative chemical potential of seawater and S is salinity. In view of the fundamental thermodynamic relation (IOC, SCOR and IAPSO (2010))

$$T \frac{d}{dt} \eta + \mu \frac{d}{dt} S = -\frac{1}{\rho} \nabla \cdot \boldsymbol{F}_Q + \varepsilon \tag{8}$$

where $\boldsymbol{F}_Q$ is the generalized heat flux of a two component fluid, we have

$$(\rho e)_t + \nabla \cdot (\boldsymbol{u} \rho e) = -P \nabla \cdot \boldsymbol{u} - \nabla \cdot \boldsymbol{F}_Q + \rho \varepsilon \tag{9}$$

where mass conservation has been used. A curious fact is that (6) and (9), comprising a full statement of fluid energetics, make no explicit mention of salinity, although its presence is implicit in $\boldsymbol{F}_Q$.

We now integrate (6) and (9) over a closed domain. The nature of the boundaries will depend on the application. When discussing the ocean, we classify the bottom as rigid, no slip and non-conductive, while the free surface will be permeable to both momentum and heat. We will also consider Rayleigh-Bernard convection, where the upper and lower boundaries will both be heat conductive, but stress-free. In all cases, we neglect boundary mass flux (e.g., evaporation and precipitation). The results are

$$\frac{\partial}{\partial t}\int_V \rho K = -\int_V \boldsymbol{u}\cdot\nabla P dV - \int_V \rho\varepsilon dV - \int_V \rho w g dV + \int_S \rho \boldsymbol{u}\cdot\underline{\boldsymbol{\tau}_o}\cdot\hat{\boldsymbol{n}} dS \tag{10}$$

where $\hat{\boldsymbol{n}}$ is the unit outward pointing normal at the free surface, $\boldsymbol{\tau}_o$ is the surface wind stress (if present) and

$$\frac{\partial}{\partial t}\int_V \rho e dV = -\int_V P\nabla\cdot\boldsymbol{u} dV + \int_V \rho\varepsilon dV - \int_S \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS \tag{11}$$

Multiplying mass conservation by z and integrating yields

$$\frac{\partial}{\partial t}\int_V \rho z dV = \int_V w\rho dV \tag{12}$$

Note that combining (10), (11) and (12) leads to

$$\frac{\partial}{\partial t}\int_V \rho(K+e+gz) dV = -\int_S \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS + \int_S \rho\boldsymbol{u}\cdot\underline{\boldsymbol{\tau}_o}\cdot\hat{\boldsymbol{n}} dS \tag{13}$$

Equivalently, the total energy budget in the basin is controlled by momentum and generalized heat exchanges with the external environment.

For application to the ocean, the surface is the only location where heat exchange and mechanical work are allowed. Assuming further that the ocean is in steady state, following Paparella and Young (2002), the result is

$$\int_S \boldsymbol{F}_Q\cdot\hat{\boldsymbol{n}} dS = \int_S \rho\boldsymbol{u}\cdot\underline{\boldsymbol{\tau}_o}\cdot\hat{\boldsymbol{n}} dS \tag{14}$$

Eq. (14), equating net mechanical forcing and buoyancy fluxes, at first perhaps surprising, emerges upon reflection as obvious. The assumption that the circulation is in steady state requires total energy fluxes into the system must be exactly compensated by total energy fluxes out. In a general statement of energy, the only available fluxes are those of kinetic energy and heat, hence they must balance. There is no explicit statement here about potential energy because of the steady form of (12).

Modern ocean circulation is characterized by net wind work on the ocean (see Wunsch (1998), Zhai et al. (2012)), thus the mechanical energy input to the ocean is exactly balanced by the equivalent extraction of energy in the form of heat. This leads to the first conclusion of this paper, namely that the fundamental role of buoyancy fluxes in the modern ocean is to energetically damp the flow. We are unaware of any proof that the winds must provide net work to the ocean. Indeed, Hazewinkel et al. (2012) describe circulations in which wind stress acts counter to the surface flow, in which case it is the winds that energetically damp the circulation and the buoyancy fluxes that provide the primary driving. However, for observed conditions, the oceans are characterized by thermal damping.

It is key that the pressure appearing in all the equations so far is total pressure. Normally the Boussinesq equations work with the dynamic pressure, p, which is the pressure associated with fluid movements and forcing. These various pressure forms are related via

$$P = P_s + p = P_o - \rho_o g z + p \tag{15}$$

where P_o is a constant reference surface pressure and 'static' pressure is given by $P_s = P_o - \rho_o g z$. Note that static pressure is aware of gravity; in fact in the exact integral balances considered previously, this is the only place where gravity makes an explicit appearance.

Introducing this pressure decomposition into (10)

$$0 = \rho_o g \int_V w dV - \int \boldsymbol{u} \cdot \nabla p dV - \int_V \rho \varepsilon + \int_S \rho \boldsymbol{u} \cdot \underline{\boldsymbol{\tau}_o} \cdot \hat{\boldsymbol{n}} dS \tag{16}$$

Performing some algebra on the steady form of (12)

$$\rho_o g \int_V w dV = \int_V \rho w b dV \tag{17}$$

where

$$b = \frac{-g(\rho - \rho_o)}{\rho} \tag{18}$$

is the buoyancy variable introduced in Young (2010).

To this point, all results are exact. To probe the consequences of the Boussinesq approximation, ρ is replaced by ρ_o on the right hand side of (17), and Boussinesq incompressibility is introduced into (16)

$$0 = \rho_o \int_V w b dV - \int_V \rho \varepsilon dV + \int_S \rho \boldsymbol{u} \cdot \underline{\boldsymbol{\tau}_o} \cdot \hat{\boldsymbol{n}} dS \tag{19}$$

as

$$\int_V \nabla \cdot p \boldsymbol{u} dV = \int_S p \boldsymbol{u} \cdot \hat{\boldsymbol{n}} dS = 0 \tag{20}$$

The Boussinesq buoyancy equation is

$$b_t + \nabla \cdot b \boldsymbol{u} = \nabla \cdot K \nabla b \tag{21}$$

where K is molecular diffusivity. Integrating (21) from the ocean floor to a depth z and over the horizontal yields

$$\int_{A(z)} w b dA = \int_{A(z)} K b_z dA \tag{22}$$

where steady conditions have been assumed and $A(z)$ is the basin area at level z. Integrating (22) over the entire fluid column

$$\int_V w b dV = K \int_S (b_s - b_b) dA \tag{23}$$

where $b_{(s,b)}$ is the (surface, bottom) fluid buoyancy. Substitution of (23) into (19) leads to

$$\rho_o K \int_S (b_s - b_b) dA + \int_S \rho \boldsymbol{u} \cdot \underline{\boldsymbol{\tau}_o} \cdot \hat{\boldsymbol{n}} dS = \int_V \rho \varepsilon dV \tag{24}$$

For a Boussinesq ocean without wind forcing ($\underline{\tau_o} = 0$), total dissipation on the right hand side is bounded by an integral proportional to molecular diffusivity. As viscosity vanishes, for fixed Prandtl number, net dissipation must vanish, which is the statement of the 'Anti-Turbulence' theorem of Paparella and Young (2002). The equality in (24) also holds in the presence of wind forcing, in which case, it is possible to estimate the Boussinesq statement of buoyantly generated turbulence as compared to wind generation. With a thermal diffusivity of $K = 10^{-7} m^2/s$, an average surface to bottom buoyancy difference of

$$b_s - b_b \approx g\frac{\Delta\rho}{\rho_o} = 0.1 m/s^2 \tag{25}$$

and an ocean surface area of $A \approx 10^{14} m^2$, (23) is roughly

$$\rho_o \int_A K(b_s - b_b) dA = 10^3 \frac{Kg}{m^3}\ 10^{-7}\frac{m^2}{s}\ .01\frac{m}{s^2}\ 10^{14} m^2 = 10^9 W = 1 GW \tag{26}$$

which is quite small compared to typical wind work energy fluxes of $O(1TW)$. This can be interpreted to mean buoyant effects generate negligible, if non-zero, turbulence in the real ocean.

It is useful at this point to think through the application of these constraints to the well known problem of Rayleigh-Bernard convection. In this case, mechanical stresses vanish, and so (14) insures that net heat fluxes vanish

$$\int_A \boldsymbol{F}_Q \cdot \hat{\boldsymbol{n}} dA = 0 \tag{27}$$

The steady internal energy equation (11) becomes

$$\int_V P \nabla \cdot \boldsymbol{u} dV = \int_V \rho \varepsilon dV \tag{28}$$

Rayleigh-Bernard flow is generally highly turbulent, and thus highly dissipative. The right hand side of (28) is positive definite, implying that strong dissipation requires the ability of the fluid to correlate low pressures with fluid contractions and high pressures with fluid expansions. Recalling that pressure in a gravitational potential involves a gravitationally aware static pressure, turbulence as a result of gravitational convection can now be understood. Rayleigh-Bernard convection, either heated from below or cooled from above, meets this condition by forcibly contracting fluid at low pressures high in the fluid and expanding it at high pressures deep in the fluid. The net work needed to achieve this comes from the fluid reserves of internal energy, effectively ‘cooling’ the internal energy. However, in steady state, as assumed here, this ‘cooling’ of internal energy is exactly balanced by the heat production driven by dissipation. Net heat fluxes into the system from the environment are unaware of, and need not account for, this dissipatively generated heat. The oppositely forced case of cooling from below and heating from above eventually arrives at a steady state, and meets (27), but cannot tap into internal energy to drive flows that dissipate. Instead, molecular conduction will result in a temperature distribution that conducts heat from source to sink in the absence of fluid flow.

The discussion of Rayleigh-Bernard convection emphasizes that the surface heating and cooling enable different processes for accessing internal energy. In particular, regions of surface cooling can locate zones of fluid contraction near the surface that can subsequently reach deep into the fluid. Surface heating zones will drive a response that remains very much trapped to the surface. With this view of convection, we return to the exact statement (11). To estimate the net diabatic dissipation production in (11), we note that

$$\nabla \cdot \boldsymbol{u} = -\frac{1}{\rho}\frac{d}{dt}\rho \approx -\frac{1}{\rho}\rho_\theta \frac{d}{dt}\theta \tag{29}$$

In regions like the Gulf Stream and other western boundary currents, typical annual average heat losses to the atmosphere are $F_o = 200W/m^2$. If we assume this heat flux is confined to a mixed layer of $h_m = O(100m)$, fluid convergence is approximately

$$\nabla \cdot \boldsymbol{u} \approx \frac{-F_o}{h_m}\,\frac{1}{\rho C_P}\,\frac{\rho_\theta}{\rho} = \frac{-200W}{m^2 100m}\,\frac{m^3}{10^3 Kg}\,\frac{Kg^oK}{4\times 10^3 J}\,\frac{2\times 10^{-4}}{^oK} = -10^{-10}s^{-1} \tag{30}$$

Thus

$$-\int_z P\nabla\cdot\boldsymbol{u}dz \approx \rho_o g\ 10^{-10}s^{-1}\int_{-100m}^{o} zdz = 10^3\frac{Kg}{m^3}\ 10\frac{m}{s^2}\ 5000m^2\ 10^{-10}s^{-1} = .01W/m^2 \quad (31)$$

which leads to a large, if surface trapped, dissipation. Scaling this number up to a global value requires multiplying by the total ocean surface area times the fraction, f, covered by regions of strong heat loss. Maps of surface energy exchange suggest this is roughly 10% of the total surface area

$$\int_V \rho\varepsilon dV = \int_V P\nabla\cdot\boldsymbol{u}dV = .01W/m^2 10^{14}m^2\ f = 0.1TW \quad (32)$$

There are at least two surprises here. First, although smaller than modern global wind input of $1TW$, it is non-negligible, suggesting buoyant forcing contributes importantly to the global energetics budget. Second, the value is two orders of magnitude larger than that obtained from the Boussinesq estimate, which argues that the primary buoyant energy pathways of the fluid are non-Boussinesq.

3. Conclusions and Discussion

We argue several somewhat counterintuitive ideas in this paper. Perhaps our most firm conclusion is that buoyant forcing in the modern ocean adds about $0.1TW$ to the overall ocean energy budget. This estimate is drawn from modern observations made in a wind-driven ocean while, ultimately, Sandström's theorem and the 'Anti-Turbulence' theorem are meant to describe a purely buoyantly forced ocean. Pure horizontal convection is normally defined by the absence of wind work, in which case the total dissipation is entirely set by the previously described convective process

$$\int_V \rho\varepsilon dV = \int_V P\nabla\cdot\boldsymbol{u}dV \quad (33)$$

It is unclear that such an ocean can find its way to a configuration where waters made dense by buoyancy flux are situated over lighter waters, but we remark that purely buoyantly forced oceans develop horizontal circulations and boundary currents that separate from the coasts and invade the interior (Vreugdenhil et al. (2016)). As they do, they bring with them waters that are often made buoyantly unstable. Of course, this is not a proof that convection will occur; those solutions are from Boussinesq models that employ diffusivities larger than molecular. We admit that a stagnant abyss full of dense water and surface confined buoyancy layer remains a possible solution for a fully compressible ocean model subject only to buoyancy forcing. However, the current demonstration that this must be true is based on the Boussinesq equations, and we feel the above arguments give us reason to question the result. In the absence of a formal proof to the contrary, we prefer to think of the present effort as an ‘Anti-Anti Turbulence Conjecture.’

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